Volvox Parameters

1 Nondimensionalizing the Equation Being Solved

The equation being solved is

$$6\pi\mu a \begin{bmatrix} 1+\frac{9a}{16z} & 0 & 0 \\ 0 & 1+\frac{9a}{16z} & 0 \\ 0 & 0 & 1+\frac{9a}{8z} \end{bmatrix} \mathbf{v}^* = \lambda(r^* - r_0^*)\hat{\mathbf{e}}_r - f^{\text{drive}}\hat{\mathbf{e}}_\phi. \tag{1}$$

However, the paper sets up everything in terms of a nondimensional spring parameter $\Lambda = \frac{\lambda d}{f^{\text{drive}}}$ where d is the height of the center of the orbit of the rotor above the wall and λ is the stiffness of the radial spring. Λ can be thought of as controlling the stiffness of the preferred orbit. To get Eq 1 in terms of this parameter we first nondimensionalize the distances. We'll let * after a variable represent the dimensional version of that variable. Then the variable without the * will represent the nondimensional version of that variable.

We'll first focus on distances. If we take the characteristic length to be 10 μm then it is like taking our ruler to be 10 μm and we will think of all distances in terms of this ruler. Thus nondimensional distances will be obtained by dividing dimensional distances by 10 μm. So for example if $r^* = 8$ then this would represent 8 micrometers. The nondimensional distance would be thinking about this in units of our ruler of 10 micrometers. Thus the nondimensional distance would be $r = r^*/10 = 8/10 = 0.8$. A way to think about this in general is letting $d = 10$ be the 'characteristic length'. Then the nondimensional distances can be obtained by dividing by d. So we have

$$r^* = dr$$

Using this in Eq 1 and then factoring out the d in the first term on the right we have:

$$6\pi\mu a \begin{bmatrix} 1+\frac{9a}{16z} & 0 & 0 \\ 0 & 1+\frac{9a}{16z} & 0 \\ 0 & 0 & 1+\frac{9a}{8z} \end{bmatrix} \mathbf{v}^* = \lambda d(r - r_0)\hat{\mathbf{e}}_r - f^{\text{drive}}\hat{\mathbf{e}}_\phi.$$

So now r and r_0 are nondimensional. Now we divide by f^{drive} which yields

$$\frac{6\pi\mu a}{f^{\text{drive}}} \begin{bmatrix} 1+\frac{9a}{16z} & 0 & 0 \\ 0 & 1+\frac{9a}{16z} & 0 \\ 0 & 0 & 1+\frac{9a}{8z} \end{bmatrix} \mathbf{v}^* = \frac{\lambda d}{f^{\text{drive}}}(r - r_0)\hat{\mathbf{e}}_r - \hat{\mathbf{e}}_\phi.$$

The paper we are following sets f^{drive} as $\frac{6\pi\mu a(2\pi r_0^*)}{T}$ where $T = 1/33$ seconds will be the characteristic time scale (this is the time it takes for the rotor to complete one orbit). Plugging this in for f^{drive}, simplifying, and replacing $\frac{\lambda d}{f^{\text{drive}}}$ with Λ yields

$$\frac{T}{2\pi r_0^*} \begin{bmatrix} 1+\frac{9a}{16z} & 0 & 0 \\ 0 & 1+\frac{9a}{16z} & 0 \\ 0 & 0 & 1+\frac{9a}{8z} \end{bmatrix} \mathbf{v}^* = \Lambda(r - r_0)\hat{\mathbf{e}}_r - \hat{\mathbf{e}}_\phi.$$

We almost have everything nondimensionalized. The only thing left is $\mathbf{v}^*$ is the dimensional velocity and r_0^* is the dimensional radius of the preferred orbit. The nondimensional radius of the preferred orbit is obtained as $r_0^* = dr_0$. And the nondimensional velocity is obtained as $\mathbf{v}^* = \mathbf{v}\frac{d}{T} \rightarrow \frac{T}{d}\mathbf{v}^* = \mathbf{v}$.

Thus rewritting $r_0^* = dr_0$ then we have a factor of $\frac{T}{d}$, which if we think of this multiplying $\mathbf{v}^*$ then we get our nondimensional equation

$$\frac{1}{2\pi r_0}\begin{bmatrix} 1+\frac{9a}{16z} & 0 & 0 \\ 0 & 1+\frac{9a}{16z} & 0 \\ 0 & 0 & 1+\frac{9a}{8z} \end{bmatrix}\mathbf{v} = \Lambda(r-r_0)\hat{\mathbf{e}}_r - \hat{\mathbf{e}}_\phi. \tag{2}$$

2 Setting Up The Parameters

In application, the code should solve Eq 2 at every time step for the velocity, and then use this velocity to update the location of the rotor. At the beginning of the program you should define the needed characteristic dimensions

1. The time scale $T = 1/33$
2. The length scale $d = 10$

Then define the needed dimensional distance parameters:

1. $r_0 = 5$ (the preferred radius of the sphere's orbit)
2. $a = 1$ (the radius of the sphere)

Then nondimensionalize these by dividing by the characteristic length $d = 10$. A few other things you will need:

1. The center of orbit of the rotor should be located a distance of 10 micrometers above the wall. For simplicity, assume the wall is at $z = 0$ assume the center of orbit is at $(0, 0, 10)$. Note the 10 is dimensional, so it should also be nondimensionalized by dividing by $d = 10$ so the nondimensional center of orbit is at $(0, 0, 1)$.
2. Set the time step dt as $1/30$. Assuming this parameter is nondimensionalized then it means that are measure of time is in terms of the characteristic time scale $T = 1/33$ seconds the length of time it takes to complete a single orbit. Thus a time step of $dt = 1/30$ means that if everything is working properly, a complete orbit should be completed in 30 time steps.
3. Set $\Lambda = 0.1$. This is already nondimensionalized, so it does not need to be scaled.

This should be all the parameters you need. I find nondimensionalization to be one of the most confusing topics. Hopefuly this makes sense. In the end, nondimensionalization shouldn't really change any of the code, it just influences the numbers getting plugged into the code.