Proof that, in the square multiplication table, the diagonal of squares describes a "ridge line" (imagine a roof), and as you "move off" the ridge, you go "downhill" (toward smaller numbers). So, for example, $900^2 = 810000$, but moving off the ridge (perpendicularly), you get smaller numbers, so $901 \times 899 = \ldots$

```
(%i20) 901*899;
```

(%o20) 809999

...a smaller number. Generalized proof follows. Pick any (x, x) pair of integers (i.e., a square on the "ridge line" and move any distance e along the southwest-northeast axis (where $(1, 1)$ is the top-left corner of the table).

```
(%i21) x*x-(x-e)*(x+e);
```

(%o21) $x^2 - (x - e) \cdot (x + e)$

```
(%i22) expand(x*x-(x-e)*(x+e));
```

(%o22) e^2

i.e., the difference between the starting point (the ridge line) and where you move to is always positive (the ridge line is always higher). The rest of this writeup assumes we're below the main diagonal. (The logic is the same for above the diagonal, obviously, although signs change.) And now, the d3 function I mentioned in the code comments:

```
(%i23) d3(n,m):=(836+n)*(836+m)-836^2;
```

(%o23) $\mathrm{d3}(n, m) := (836 + n) \cdot (836 + m) - 836^2$

```
(%i24) ratsimp(d3(n,m));
```

(%o24) $(m + 836) \cdot n + 836 \cdot m$

Find an expression that gives us m for a given n such that the delta will still be zero:

```
(%i25) solve(d3(n,m)=0,m);
```

(%o25) $[m = -\frac{836 \cdot n}{n + 836}]$

Make the solution a function, "m for n".

```
(%i26) m4(n):= 836*n/(n+836);
```

(%o26) $\mathrm{m4}(n) := \dfrac{836 \cdot n}{n + 836}$

Test. Back up one position on the x axis. (-1)

```
(%i27) m4(-1);
```

(%o27) $-\dfrac{836}{835}$

Need to shift y more than 1 $(+1)$. Verify:

```
(%i28) d3(-1,1);
```

(%o28) -1

```
(%i29) d3(-1,2);
```

(%o29) 834

What's the biggest y-offset from 836 we can tolerate? $999 - 836$.

```
(%i30) 999-836;
```

(%o30) 163

What x-offset will require that large a y-offset?

```
(%i31) solve(m4(n)=-163,n),numer;
```

rat: replaced 136.4044044044044 by 136268/999 = 136.4044044044044

(%o31) $[n = -136.4044044044044]$

Verify. Back x off by -136 and increase y the maximum (so... is $(836 - 136) \times (836 + 163) > 836^2$?):

```
(%i32) d3(-136,163);
```

(%o32) 404

```
(%i33) d3(-137,163);
```

(%o33) -595

Here's another way to do it, that doesn't require an alternative function like m4:

```
(%i34) solve(d3(n,163)=0,n),numer;
```

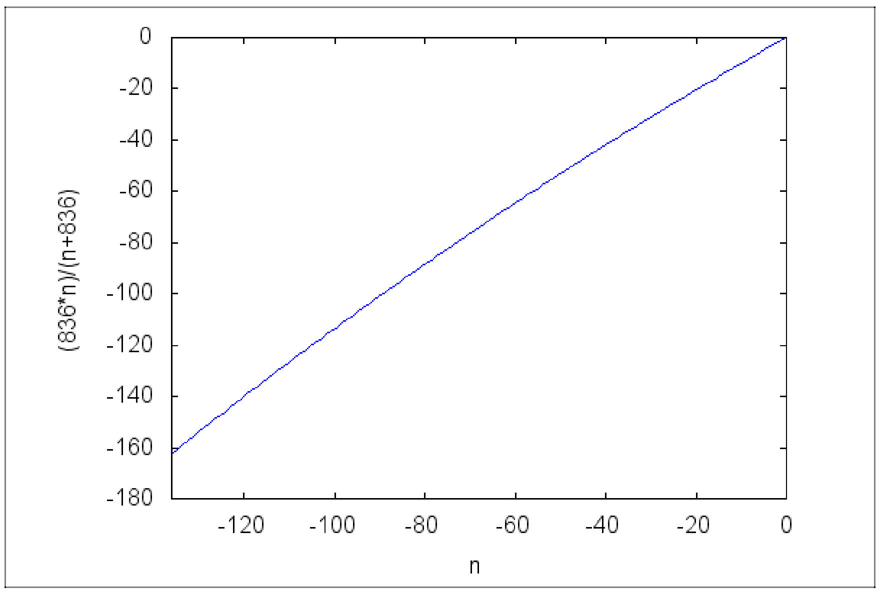
rat: replaced 136.4044044044044 by 136268/999 = 136.4044044044044

(%o34) $[n = -136.4044044044044]$

So, how did I know the relationship between n and m isn't linear? For starters, the function doesn't have a linear form, but also, if you plot it, it doesn't look like a straight line.

```
(%i35) wxplot2d(m4(n),[n,-136,0]);
```

(%t35)



(%o35)

The real proof is in the derivative. A straight line implies a constant derivative.

```
(%i36) diff(m4(n),n);
```

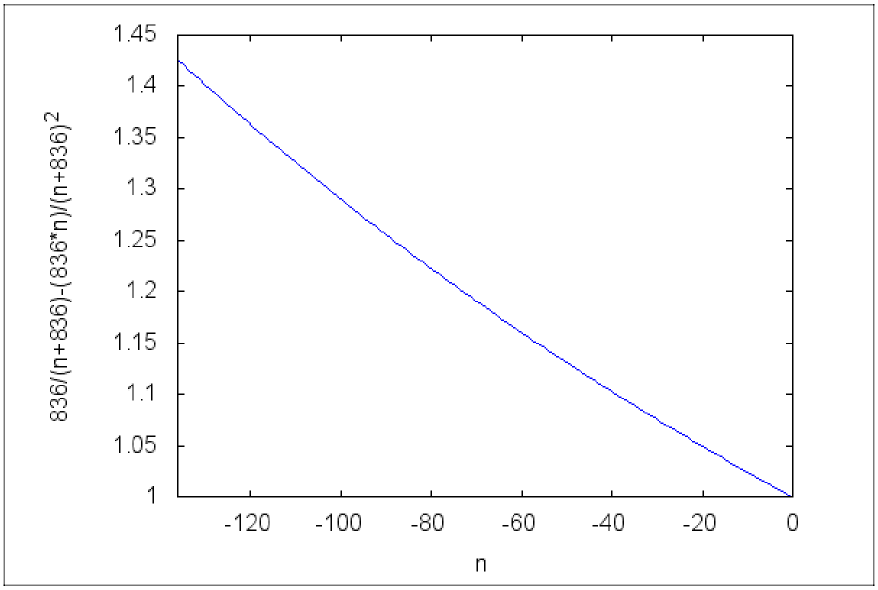
(%o36) $$\frac{836}{n+836} - \frac{836 \cdot n}{(n+836)^2}$$

```
(%i37) ratsimp(diff(m4(n),n),n);
```

(%o37) $$\frac{698896}{n^2 + 1672 \cdot n + 698896}$$

That ain't a constant, so the m4 function isn't linear.

```
(%i38) wxplot2d(diff(m4(n),n),[n,-136,0]);
```

(%t38) 

(%o38)

So, never mind trying to be too fancy. Just take all pairs in the triangle starting at $(700, 700)$ (which is $836 - 136$) and find the biggest palindrome. Not super-elegant, but not TOO wasteful.