



[DOCUMENT TITLE]

CS234 Winter 2018



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Assignment 2

Q2

A:

Function quicksort(list):

If len(list) <= 1: return list	# O(1)
Pivot <- average(list)	# O(1)
Lesser <- all elements in list that are < pivot	# O(n)
Greater <- all elements in list that are > pivot	# O(n)
Equal <- all elements in list that are = pivot	# O(n)
Return quicksort(Lesser) + Equal + quicksort(Greater)	# 2 T(n/2) + O(1)

In this algorithm, pivot is the average of the list, it might exist or not exist in the list.

If the pivot is not exist in the list, I create three empty list for Lesser, Greater and Equal = [] and use the character k to represent the number in the for loop, like for k in rang(num).

Using a example list to explain the question: list_a: [9,1,6,3,6] Average(list_a): 5

At the beginning: k = 6, it is larger than 5, therefore, Greater.append(6), Greater: [9].

And then the k = 1, it is less than 5, Lesser.append(1), Lesser = [1]

And then go ahead, k = 9, it is bigger than 5, Greater.append(6), Greater: [9,6]

And then the k = 3, it is less than 5, Lesser.append(3), Lesser:[1,3]

And then the k = 6, it is bigger than 5, Greater.append(6), Greater.append(6), Greater: [9,6,6]

Return quicksort([1,3]) + [] + quicksort([9,6,6])

The next processes are like the processes as above

If the Pivot is exist in the list, it is similar to the original quicksort(), store the elements in list of Equal: [], and continue for the quicksort for lesser or greater.

Therefore, this algorithm always terminates.

B:

Basis on the Helpful recurrence relations

$$T(n) = 3 \cdot O(n) + 3 \cdot O(1) + 2 \cdot T(n/2) \rightarrow O(n) + 2 \cdot T(n/2) \rightarrow O(n \log n)$$

Therefore, the worst case is $O(n \log n)$, which is similar to the merge sort. In the worst case, the number of comparison of this algorithm makes is equal to or slightly smaller than $(n \log(n) - 2^{(\log n)} + 1)$, which is between $(n \log n - n + 1)$ and $(n \log(n) + n + O(\log n))$. Therefore, the worst case is $O(n \log n)$

An example worst case could be given, the elements in the list based on the rule form largest to smallest, like `my_list = [13,12,11,10,9,8,7,6,5,4,3,2,1,7]` `average(my_list): 7`

In the first run of `quicksort(my_list)`, Greater: `[13,12,11,10,9,8]`, Equal: `[7,7]`, Lesser: `[6,5,4,3,2,1]`, it will return `quicksort([13,12,11,10,9,8]) + [7,7] + quicksort([6,5,4,3,2,1])`

And then the `quicksort(Greater)` `average(Greater): 10.5`

In the list of Greater of `quicksort(Greater)`, it will return `quicksort([13,12,11]) + quicksort([10,9,8])`

And then `quicksort([13,12,11])` -> will return `quicksort([11]) + [12] + quicksort([13])`

....

Finally, For the `quicksort(Greater)`, `quicksort([13,12,11,10,9,8])` will produce `[8,9,10,11,12,13]`

Similar process to the `quicksort(Lesser)`

....

`Quicksort(my_list)` will produce `[1,2,3,4,5,6,7,7,8,9,10,11,12,13]`

Q3

A:

I suppose $\text{len}(\text{head1}) \geq \text{len}(\text{head2})$ $\text{len}(\text{head1}) = n$, $\text{len}(\text{head2}) = m$

$\text{head}.\text{element}$ will return the value of the Node, example, $a = \text{Node}(2, \text{Node}(3, \text{None}))$, $a.\text{element} == 2$

$\text{head}.\text{next}$ will return the next Node of the head, example, $a.\text{next} == \text{Node}(3, \text{None})$

function `find_common_node1`:

while head1 is not None:	# O(n) times
for head2 is not None:	# O(m) times
if head1 == head2: return head1.element or head2.element	# O(1)
head2 = head2.next	# O(1)
head1 = head1.next	# O(1)

$T(n) = O(n) * (O(m) * O(1)) = O(nm)$

Therefore, this algorithm that has $O(nm)$ time complexity

B:

I suppose $\text{len}(\text{head1}) \geq \text{len}(\text{head2})$ $\text{len}(\text{head1}) = n$, $\text{len}(\text{head2}) = m$

`head.remove_front(num)` will mutate the head, by getting rid of the num of item from head, example, `a = Node(2, Node(3, Node(4, None)))` after running `a.remove_front(2)`, a will become `a: Node(4, None)`

`head.element` will return the value of the Node, example, `a = Node(2, Node(3, None))`, `a.element == 2`

function `find_common_node2`:

```
n = len(head1)                                # O(n)
m = len(head2)                                # O(m)
new_head1 = head1.remove_front(n-m)           # O(1)
l = 0:                                         # O(1)
while l < m:                                   # O(m)
    if new_head1 == head2:                     # O(1)
        return head2.element                  # O(1)
    else:
        new_head1.remove_front(1)             # O(1)
        head2.remove_front(1)                 # O(1)
        l += 1                                # O(1)
return None
```

$T(n) = O(n) + O(m) = O(n + m)$

Therefore, this algorithm that has $O(n+m)$ time complexity