

UniDyn--Study-04.nb

John A. Marohn
jam99@cornell.edu
Cornell University

Abstract: This demonstration notebook loads the **UniDyn** package runs an **Evolver** rotations involving the harmonic oscillator.

Set the path to the package

Check the Mathematica version number.

```
In[60]:= $VersionNumber
Out[60]= 12.3
```

Tell *Mathematica* the path to the directory containing the packages and the /studies directory.

EDIT THE FOLLOWING PATH STRINGS:

```
In[61]:= $UniDynPath =
"/Users/jam99/Dropbox/MarohnGroup__Software_Library/UniDyn/
unidyn";
```

YOU SHOULD NOT NEED TO EDIT ANYTHING FROM HERE ONWARDS.

Load and test the packages

Append the package path to the system path. Before trying to load the package, ask *Mathematica* to find it. This is a test that we directed *Mathematica* to the correct directory. The output of this command should be the full system path to the UniDyn.m file.

```
In[62]:= $Path = AppendTo[$Path, $UniDynPath];
FindFile["UniDyn`"]
Out[63]= /Users/jam99/Dropbox/MarohnGroup__Software_Library/UniDyn/unidyn/UniDyn.m
```

Now that we are confident that the path is set correctly, load the **Unidyn** package. Setting the global \$VerboseLoad variable to True will print out the help strings for key commands in the package.

```
In[64]:= $VerboseLoad = True;
Needs["UniDyn`"]
```

Carry out the unit tests for the **Unidyn** functions.

```
In[66]:= SetDirectory[$UniDynPath];
In[67]:= fn = FileNames["*-tests.m"];
test$report = TestReport /@ fn;
TableForm[Table[test$report [[k]], {k, 1, Length[test$report]}]]
```

Out[69]//TableForm=



Harmonic oscillator

```
In[95]:= Clear[aL, aR, ω, Q$sym, P$sym, Q, P, ω, δq, δp, A, φ, Δ, H$0, H$1, H$1$φ, H$2]
OscSingle$CreateOperators[aL, aR];
CreateScalar[ω, δq, δp, A, φ, Δ];
... OscSingle$CreateOperators: Creating oscillator operators.
... OscSingle$CreateOperators: Adding oscillator commutations relations.
```

Define the number operator.

```
In[98]:= Nop = Mult[aR, aL];
```

Define position and momentum operators.

```
In[99]:= Q$sym = (aR + aL) / Sqrt[2];
P$sym = I (aR - aL) / Sqrt[2];
```

Define a rule for rewriting raising and lowering operators in terms of position and momentum

operators.

```
In[101]:= CreateOperator[{{Q, P}}]
```

```
QP$rules = {aR → (Q - I P) / Sqrt[2], aL → (Q + I P) / Sqrt[2]};
```

Define the harmonic oscillator Hamiltonian.

```
In[103]:= H$0 = ω (Nop + 1/2);
```

Evolve the position and momentum operators under the harmonic oscillator Hamiltonian.

```
In[104]:= {Evolver[H$0, t, Q$sym], Evolver[H$0, t, P$sym]} /. QP$rules // Simplify
```

$$\text{Out}[104]= \left\{ \frac{1}{2} e^{-i t \omega} \left(i (-1 + e^{2 i t \omega}) P + (1 + e^{2 i t \omega}) Q \right), P \cos[t \omega] + Q \sin[t \omega] \right\}$$

A position kick. (Change the sign here from Study-03.)

```
In[105]:= {Evolver[δq P$sym, 1, Q$sym], Evolver[δq P$sym, 1, P$sym]} /. QP$rules // Simplify
```

$$\text{Out}[105]= \{Q - \delta q, P\}$$

Examine this evolution more closely.

```
In[120]:= Evolver[δq P$sym, τ, Q$sym, quiet → False]
```

$$\rho \text{ matrix} = \begin{pmatrix} \frac{aL+aR}{\sqrt{2}} \\ -\delta q \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Omega = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{system of equations} = \left\{ \begin{array}{l} \{\text{Private}`x4$67145`[\text{Private}`time$67145] == 0, \text{Private}`x3$67145`[\text{Private}`time$67145] == 0, \dots \} \end{array} \right.$$

1st solution = Private`x1\$67145 →

$$\text{Function}\left[\{\text{Private}`time$67145\}, \frac{1}{2} (\sqrt{2} aL + \sqrt{2} aR - 2 \text{Private}`time$67145 \delta q)\right]$$

$$\text{1st solution w/ substitution} = \frac{1}{2} (\sqrt{2} aL + \sqrt{2} aR - 2 \delta q \tau)$$

$$\text{Out}[120]= \frac{aL}{\sqrt{2}} + \frac{aR}{\sqrt{2}} - \delta q \tau$$

A momentum kick.

```
In[107]:= {Evolver[\delta p Q$sym, 1, Q$sym], Evolver[\delta p Q$sym, 1, P$sym]} /. QP$rules // Simplify
Out[107]= {Q, P + \delta p}
```

Evolution under a force .

```
In[108]:= H$1 = - A Q$sym;
In[109]:= {Evolver[H$1, t, Q$sym], Evolver[H$1, t, P$sym]} /. QP$rules // Simplify
Out[109]= {Q, P - A t}
```

Evolution under a force with a phase factor .

```
In[110]:= H$1$\phi = A \frac{1}{\sqrt{2}} (Exp[-I \phi] aR + Exp[I \phi] aL);
{Evolver[H$1$\phi, t, Q$sym], Evolver[H$1$\phi, t, P$sym]} /. QP$rules // Simplify
Out[111]= {Q + A t Sin[\phi], P + A t Cos[\phi]}
```

Evolution under a squeezing Hamiltonian.

```
In[112]:= H$2 = - \frac{\Delta}{2} I (Exp[-I \phi] Mult[aR, aR] - Exp[I \phi] Mult[aL, aL]);
In[113]:= ans = {Evolver[H$2, t, Q$sym], Evolver[H$2, t, P$sym]} /. QP$rules // Simplify //
ExpToTrig // Simplify
Out[113]= {Q Cosh[t \Delta] + (Q Cos[\phi] - P Sin[\phi]) Sinh[t \Delta], P Cosh[t \Delta] - (P Cos[\phi] + Q Sin[\phi]) Sinh[t \Delta]}
```

We can see that the squeezing is phase-dependent .

```
In[114]:= ans /. \phi \rightarrow 0 // TrigToExp
Out[114]= {e^{t \Delta} Q, e^{-t \Delta} P}
```

```
In[115]:= ans /. \phi \rightarrow \pi // TrigToExp
Out[115]= {e^{-t \Delta} Q, e^{t \Delta} P}
```

Evolution under the harmonic oscillator and squeezing Hamiltonians.

```
In[116]:= ans =
{Evolver[H$0 + H$2, t, Q$sym], Evolver[H$2, t, P$sym]} /. QP$rules // Simplify //
ExpToTrig // Simplify // Collect[#, {Q, P}] &
Out[116]= \left\{ -\frac{P (\omega + \Delta \sin[\phi]) \sinh[t \sqrt{\Delta^2 - \omega^2}]}{\sqrt{\Delta^2 - \omega^2}} + Q \left( \cosh[t \sqrt{\Delta^2 - \omega^2}] + \frac{\Delta \cos[\phi] \sinh[t \sqrt{\Delta^2 - \omega^2}]}{\sqrt{\Delta^2 - \omega^2}} \right), -Q \sin[\phi] \sinh[t \Delta] + P (\cosh[t \Delta] - \cos[\phi] \sinh[t \Delta]) \right\}
```

The position and momentum are somewhat simpler when $\phi=0$ and $\phi=\pi/2$.

```
In[117]:= ans /. ϕ → 0 // Simplify // Collect[#, {Q, P}] &
Out[117]= -P ω Sinh[t √Δ² - ω²] / √Δ² - ω² + Q (Cosh[t √Δ² - ω²] + Δ Sinh[t √Δ² - ω²] / √Δ² - ω²),
P (Cosh[t Δ] - Sinh[t Δ])}

In[118]:= ans /. ϕ → π/2 // Simplify // Collect[#, {Q, P}] &
Out[118]= {Q Cosh[t √Δ² - ω²] + P (-Δ - ω) Sinh[t √Δ² - ω²] / √Δ² - ω², P Cosh[t Δ] - Q Sinh[t Δ]}
```

Clean up

```
In[93]:= Clear[aL, aR, ω, Q$sym, P$sym, Q, P, ω, δq, δp, A, φ, Δ, H$0, H$1, H$1$φ, H$2]
```