A *ring* a set R with two binary operations $+$ and $\cdot$, called addition and multiplication, which maps every pair of elements of R to a unique element of R. In order to qualify as a ring, these operations must satisfy the *ring axioms*, which must be true for all $a, b, c \in R$. The ring axioms are:

1. $(a + b) + c = a + (b + c)$ ($+$ is associative)
2. There is an element $0 \in R$ such that $0 + a = a$ (0 is the *zero element*
3. $a + b = b + a$ ($+$ is commutative)
4. For each $a \in R$ there exists $-a \in R$ such that $a + (-a) = (-a) + a = 0$ ($-a$ is the inverse element of a)
5. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ ($\cdot$ is associative)
6. $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ (left distributivity)
7. $(b + c) \cdot a = (b \cdot a) + (c \cdot a)$ (right distributivity)
8. There is an element $1 \in R$ such that $a \cdot 1 = 1 \cdot a = a$ (multiplicative identity)

A *group* is a set, G, with an operation, $\cdot$, that combines any two elements a and b to form another element, written as $a \cdot b$ or ab. In order to qualify as a group, the set and the operation, (G, cot), must satisfy the *group axioms* which are as follows.

Closure: For all $a, b \in G$, the result of the operation, $a \cdot b$, is also in G.

Associativity: For all a, b and $c \in G, (a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Identity element: There exists an element $e \in G$, such that for every element $a \in G, e \cdot a = a \cdot e = a$.

Inverse element: For each $a \in G$, there exists an element $b \in G$ such that $a \cdot b = b \cdot a = e$, where e is the identity element.

A *field* is a set F with two operations, addition and multiplication, such that the following axioms hold:

Closure: For all $a, b \in F$ both $a + b$ and $a \cdot b$ are in F

Associativity: For all $a, b,$ and $c \in F$ the following equalities hold: $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

Commutativity: For all a and $b \in F$ the following equalities hold: $a + b = b + a$ and $a \cdot b = b \cdot a$

Existence of Identity Elements: There exists an element of F, called the *additive identity* and denoted by 0, such that for all $a \in F, a + 0 = a$. Likewise, there is an element, called the *multiplicative identity* and denoted by 1, such that for all $a \in F, a \cdot b = a$. The additive and multiplicative identities are required to be distinct.

Existence of Inverse Elements: For every a in F, there exists an element $-a$ in F, such that $a + (-a) = 0$. Similarly, for any a in F, other than 0, there exists an element a^{-1} in F such that $a \cdot a^{-1} = 1$

Distributivity: For all $a, b,$ and $c \in F$ the following equality holds: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$