HW#2

(1) $Poi(x|\lambda) = e^{-\lambda} \cdot \frac{\lambda^x}{x!}$ for $x \in \{0,1$
$\lambda \geq 0$

$$f(\lambda) = \prod_{i=1}^{n} e^{-\lambda} \cdot \frac{\lambda^{x_i}}{x_i!} = \frac{(e^{-\lambda} \cdot \lambda^{\bar{x}})^n}{X_1! \dots X_n!}$$

$$\bar{X} = \frac{(X_1 + \dots + X_n)}{n}$$

$$g(\lambda) = e^{-\lambda} \cdot \lambda^{\bar{x}} = e^{(-\lambda + \bar{x} \cdot \ln(\lambda))}$$

Apply ln

$$g'(\lambda) = \left(-1 + \frac{\bar{x}}{\lambda}\right) \cdot e^{-\lambda} \cdot \lambda^{\bar{x}}$$

$\bar{X} = \lambda$ function is maximized

(2) $p(x/_\theta) = \theta e^{-\theta x}$ for $x \geq 0, \theta > 0$

(a)

$$f(\theta) = \prod_{i=1}^{n} \theta e^{-\theta x_i} = (\theta e^{-\theta \bar{x}})^n$$

$$LL(f(\theta)) = (\theta e^{-\theta x})'$$

$$= \cancel{\theta e^{-\theta}}$$

$$= e^{-\theta x} - e^{-\theta x} \cdot \theta x$$

$$\cancel{e^{-\theta x}} = \cancel{e^{-\theta x}} \theta x$$

$$1 = \theta x$$

$$\frac{1}{\theta} \cancel{= x}$$

$$\boxed{\theta = \frac{1}{x}}$$

(b) $x_1 = 5 \rightarrow \theta = \cancel{1/5}$

$$\frac{1}{5+6+4} = \frac{1}{\cancel{15}/5} \cdot \cancel{1/3}\ 3\ 1 = \boxed{1/5}$$

(c)

Prove expectation of exp fn is $\frac{1}{\lambda}$

(c)

$P(\theta) = \text{Exponential}(\theta | \lambda)$

$p(\theta) = \lambda e^{-\lambda\theta}$

$E(\theta) = \int \lambda e^{-\lambda\theta} \cdot \theta \, d\theta$

$Ga(\theta |_{a,b}) \propto \theta^{a-1} \cdot e^{-\theta b}$

when $b = 3$

$E(3) = 1/3$

$\frac{1}{b}$

$\theta^{1-1} e^{-\theta \cdot 3}$

$1 \cdot e^{-3\theta} = e^{-3\theta}$

(d)

$p(\theta | x = \hat{x})$

Write formula for posterior

$P(\theta | x = \hat{x}) = \frac{P(\theta \cap \hat{x})}{P(\hat{x})} = \frac{P(\theta) \cdot P(\hat{x} | \theta)}{P(\hat{x})}$

(c) yes it is a conjugate because they both have a similar form

(F) $\boxed{\text{Posterior mean} = \int \lambda e^{-\lambda \theta} \cdot \hat{\lambda}\, d\lambda}$

③ $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ $\mu = ax_i + b$

ⓐ

$$\boxed{f(y_i)} = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{(y_i - \mu_i)^2}{2\sigma_i^2}}$$

$$L = \prod f(y_i) = \left[\frac{1}{(2\pi\sigma_i^2)^{1/2}}\right]^N \cdot e^{-1/2 \cdot \sum \frac{(y_i - \mu_i)^2}{\sigma_i^2}}$$

$$LL = \underbrace{-\frac{N}{2} \cdot \log(2\pi\sigma_i^2)}_{\text{Totally irrelevant to a}} - \frac{1}{2} \sum \frac{(y_i - \mu_i)^2}{\sigma_i^2}$$

$$\frac{\partial LL}{\partial a} \rightarrow = \frac{\partial (y_i - \mu_i)^2}{\partial a} = 0$$

$$\frac{\partial \Delta^2}{\partial a} = 0 \rightarrow 2\Delta \cdot \frac{\partial \Delta}{\partial a} = 0 \quad \frac{1}{\sigma^2} \sum 2(y_i - \mu_i) \frac{\partial (y_i}{\partial a}$$

$$= 2(y_i - \mu_i) \cdot (-x_i$$

$$\sum 2(y_i - ax_i - b_i) \cdot (-x_i) = 0$$

$$\frac{\sum (y_i - b) x_i}{\sum x_i^2} = a_{MLE}$$

$\underline{b}$

$\mu = ax + b$

$\sum (y_i - \mu_i)^2 \qquad \frac{\partial \mu}{\partial b} = 1$

$2(y_i - \mu_i) \cdot \frac{\partial (y_i - \mu_i)}{\partial b}$

$\sum (y_i - \mu_i) \cdot (1)$

$\cancel{\sum y_i - \mu_i}$

$\sum (y_i - ax_i - b) \cdot 1$

$\boxed{b_{MLE} = \sum y_i - ax_i}$

$a_{MLE} = \frac{\sum (y_i - b_i)}{\sum x_i^2}$

$= \frac{\sum (\cancel{y_i} - (\cancel{y_i} - ax_i))}{\sum x_i^2} = \boxed{\frac{\sum a x_i}{\sum x_i^2}} = a_{MLE}$

$\frac{\partial LL}{\partial \sigma^2} = \frac{-N}{2} \cdot \log(2\pi\sigma^2) - \frac{\sum (y_i - \mu)}{\sigma^2}$

$\boxed{\sigma^2_{MLE} = \frac{-n}{2} + \frac{(y_i - \mu_i)^2}{x^3}}$

3b. Check matlab code for graph

3c. Normal prior