(1)

(a) $p_1 = P(y=1|x)$ $\quad P(y=0|x) \cdot \lambda_{10} < P(y=1|x)\lambda_{01}$

$$(1-p_1)\cdot\lambda_{10} < p_1 \cdot \lambda_{01}$$

$$\lambda_{10} - \lambda_{10} p_1 < p_1 \lambda_{01}$$

$$\lambda_{10} - \lambda_{10} p_1 - p_1 \lambda_{01} < 0$$

$$\cancel{\lambda_{10} + p(\lambda_{10} + \lambda_{01}}$$

$$\lambda_{10} - p_1(\lambda_{10} - \lambda_{01}) < 0$$

$$\lambda_{10} < p_1(\lambda_{10} - \lambda_{01})$$

$$\boxed{p_1 > \frac{\lambda_{10}}{\lambda_{10} - \lambda_{01}}}$$

(b)

$$\begin{bmatrix} 0 & .9 \\ .1 & 0 \end{bmatrix}$$

(2)

a)

$$\lambda(a_i|Y=j) = \begin{cases} 0 & \text{if } i=j \text{ and } i,j \in \{1,\dots,c\} \\ \lambda_r & \text{if } i=c+1 \\ \lambda_s & \text{otherwise} \end{cases}$$

Minimum risk: $Y=j$ if $p(y=j|x) \geq p(y=k|x)$ and $p(y=j|x) \geq 1-\lambda_r/\lambda_s$

Risk of choosing a class

$$R(w_i|x) = \sum_{j=1}^{c} \lambda(w_i|w_j)P(w_j|x)$$

$\lambda(w_i|w_j)$ = mean of cost function when predicting $w_i$ but real class is $w_j$

$$= 0 \cdot P(w_i|x) + \sum_{j=1, j!=i}^{c} \lambda_s P(w_j|x)$$

$$R(w_i|x) = \lambda_s(1-P(w_i|x))$$

$$\boxed{\begin{gathered}\lambda_s(1-P(w_i|X)) \leq \lambda_r \\ P(w_i|x) \geq 1-\frac{\lambda_r}{\lambda_s}\end{gathered}}$$

(b)

As $\lambda_r/\lambda_s$ increases the chance of rejecting grows, ~~as the~~ as Towards 0 it becomes more likely to ~~cla~~ classify instead of reject

c) $P(w_i|X) \geq 1 - \frac{\lambda_r}{\lambda_s}$

$\hookrightarrow 1 - 0 = 1$ Reject always

$P(w_i|X) \geq 1$

$\frac{\lambda_r}{\lambda_s} = 1/2$

$P(w_i|X) \geq 1 - 1/2$

$P(w_i|X) \geq 1/2$

$\frac{\lambda_r}{\lambda_s} = 1$ $\begin{bmatrix} 0 & \lambda_s \\ \lambda_s & 0 \end{bmatrix}$

$\hookrightarrow P(w_i|X) \geq 1 - 1$

$P(w_i|X) \geq 0$ - Never reject

$1/2 = N(-1,1) \leftarrow$ plug in pdf

③

$$B = \frac{p(D|H_1)}{p(D|H_0)}$$

$$= \frac{p(H_1|D) \cdot p(H_0)}{p(H_0|D) \cdot p(H_1)}$$

$$B \overset{\propto}{=} \frac{p}{1-p}$$

Thus the curves will be/look the same

④ $X_N \sim N(\mu, \sigma^2)$

$$\frac{\partial LL}{\partial \sigma^2} = \left(\frac{-N}{2} \cdot \log(2\pi\sigma_i^2) - \frac{1}{2} \sum \frac{(y_i - \mu)^2}{\sigma_i^2}\right)$$

$$\sigma_{MLE} = \frac{-n}{\sigma} + \frac{(y-\mu)^2}{x^3}$$

The rest of the problems are within the code