- We can assume a normal prior on $\underline{a}$ and $\underline{b}$
- We assume $\underline{a}$ and $\underline{b}$ priors are defined independently.

$$p(a,b \mid a_0, b_0, \sigma_a^2, \sigma_b^2) \sim p(a \mid a_0, \sigma_a^2)\, p(b \mid b_0, \sigma_b^2)$$

$$\simeq N(a; a_0, \sigma_a^2)\, N(b; b_0, \sigma_b^2)$$

- We define our likelihood function over N samples.

$$L = \prod_{i=1}^{N} p(y_i \mid x_i, a, b) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(y_i - a x_i - b)^2}{2\sigma^2}}$$

$$= \frac{1}{(\sqrt{2\pi})^N \sigma^N} e^{-\frac{\left(\sum_{i=1}^{N} y_i^2 + a^2 \sum_{i=1}^{N} x_i^2 + b^2 N - 2\sum_{i=1}^{N} x_i y_i a - 2\sum_{i=1}^{N} y_i b + 2\sum x_i ab\right)}{2\sigma^2}}$$

$\Rightarrow$ Let's define: $S_1 = \sum_{i=1}^{N} x_i^2$

$S_2 = N$

$S_3 = \sum_{i=1}^{N} x_i y_i$

$S_4 = \sum y_i$

$S_5 = \sum x_i$

$$L = C\, e^{-\frac{(a^2 S_1 + b^2 S_2 - 2S_3 a - 2S_4 b + 2S_5 ab)}{2\sigma^2}}$$

C: all constant terms which are not function of a or b

$\Rightarrow$ to build the posterior on a, b; we combine likelihood and prior:

$$p(a,b \mid (y_i, x_i)_{i=1}^{N}, a_0, b_0, \sigma_a^2, \sigma_b^2) = C_1 e^{-\frac{(a^2 S_1 + b^2 S_2 - 2S_3 a - 2S_4 b + 2S_5 ab)}{2\sigma^2}} \times e^{-\frac{(a^2 - 2aa_0)}{2\sigma_a^2}}\, e^{-\frac{(b^2 - 2bb_0)}{\sigma_b^2}}$$

We combined all constant terms in C_1.

$$= C_1 e^{-\frac{1}{2}\left(a^2\left(\frac{S_1}{\sigma^2}+\frac{1}{\sigma_a^2}\right)+b^2\left(\frac{S_2}{\sigma^2}+\frac{1}{\sigma_b^2}\right)-2\left(\frac{S_3}{\sigma^2}+\frac{a_0}{\sigma_a^2}\right)a-2\left(\frac{S_4}{\sigma^2}+\frac{b_0}{\sigma_b^2}\right)b+\frac{2S_5}{\sigma^2}ab\right)}$$

note this one has the same shape of the prior; thus, the posterior will ~~can~~ be a multivariate normal.

$$= C_1 e^{-\frac{1}{2}\left(a^2 A_1 + b^2 A_2 - 2A_3 - 2A_4 + 2A_5 ab\right)}$$

$$\therefore \quad A_1 = \frac{S_1}{\sigma^2} + \frac{1}{\sigma_a^2}$$

$$A_2 = \frac{S_2}{\sigma^2} + \frac{1}{\sigma_b^2}$$

$$A_3 = \frac{S_3}{\sigma^2} + \frac{a_0}{\sigma_a^2}$$

$$A_4 = \frac{S_4}{\sigma^2} + \frac{b_0}{\sigma_b^2}$$

$$A_5 = \frac{S_5}{\sigma^2}$$

now; we can find the mean and variance of posterior

look this: $\frac{1}{\sigma_p^2} = \frac{S_1}{\sigma^2} + \frac{1}{\sigma_a^2}$

$\Downarrow$

this means as the number of samples grows; S_1 grows and the posterior variance (σ_p^2) becomes small and smaller

$$\Sigma^{-1} = \begin{bmatrix} A_1 & A_5 \\ A_5 & A_2 \end{bmatrix}$$

$$\mu = \begin{bmatrix} \quad \end{bmatrix}$$

we can find this too