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DAA OBA-1

1.)

(i) $x(n) = x(n-1) + 5$, for $n > 1$, $x(1) = 0$

$$\begin{aligned} \rightarrow x(n) &= x(n-1) + 5 \\ &= [x(n-2) + 5] + 5 = x(n-2) + 5 \cdot 2 \\ &= [x(n-3) + 5] + 5 \cdot 2 = x(n-3) + 5 \cdot 3 \\ &= \dots \\ &= x(n-i) + 5 \cdot i \\ &= \dots \\ &= x(1) + 5 \cdot (n-1) = 5(n-1) \end{aligned}$$

$$x(n) = O(n)$$

(ii) $x(n) = x(n/2) + n$ for $n > 1$, $x(n) = 1$

\rightarrow

$$\begin{aligned} x(2^k) &= x(2^{k-1}) + 2^k \\ &= [x(2^{k-2}) + 2^{k-1}] + 2^k = x(2^{k-2}) + 2^{k-1} + 2^k \\ &= [x(2^{k-3}) + 2^{k-2}] + 2^{k-1} + 2^k = x(2^{k-3}) + 2^{k-2} + 2^{k-1} + 2^k \\ &= \dots \\ &= x(2^{k-i}) + 2^{k-i+1} + 2^{k-i+2} + \dots + 2^k \\ &= \dots \\ &= x(2^{k-k}) + 2^1 + 2^2 + \dots + 2^k = 1 + 2 + 2^2 + 2^3 + \dots + 2^k \\ &= 2^{k+1} - 1 = 2 \cdot 2^k - 1 \\ &= 2n - 1 \end{aligned}$$

$$x(n) = O(n)$$

2.) "Q U I C K S O R T"

Applying selection sort algorithm.

Writing the ASCII values we get

Q U I C K S O R T
81 85 73 67 75 83 79 82 84

The algorithm for selection sort is

for $i \leftarrow 0$ to $n-2$ do

$\text{min} \leftarrow i$

 for $j \leftarrow i+1$ to $n-1$ do

~~if $j \leftarrow i+1$ to $n-1$ do~~

 if $A[j] < A[\text{min}]$

$\text{min} \leftarrow j$

 swap $A[i]$ & $A[\text{min}]$

By tracing selection sort algorithm, we get

Q	U	I	C	K	S	O	R	T
81	85	73	67	75	83	79	82	84
67	85	73	81	75	83	79	82	84
67	73	85	81	75	83	79	82	84
67	73	75	81	85	83	79	82	84
67	73	75	79	85	83	81	82	84
67	73	75	79	81	83	85	82	84
67	73	75	79	81	82	85	83	84
67	73	75	79	81	82	83	85	84
67	73	75	79	81	82	83	84	85
67	73	75	79	81	82	83	84	85

After sorting we get: C I K O Q R S T U

2.) Performance :

* Worst case performance : $O(n^2)$

If we want to sort in ascending order & array is in descending order, worst case occurs

* Best case performance : $O(n^2)$

It occurs when array is already sorted

* Average case performance : $O(n^2)$

It occurs when elements of array are in jumbled order.

3.)

*> Maximum Element in array

MaxElement ($A[0 \dots n-1]$)

Input: An array $A[0 \dots n-1]$ of real numbers

Output: The value of largest element is A

Algorithm:

max $\leftarrow A[0]$

for $i \leftarrow 1$ to $n-1$ do

if $A[i] > \text{max}$

max $\leftarrow A[i]$

return max

*> Minimum element in array

MinElement ($A[0 \dots n-1]$)

Input: An array $A[0 \dots n-1]$ of real numbers

Output: The value of smallest element is A

Algorithm:

min $\leftarrow A[0]$

3.)

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for i ← 1 to n-1 do
  if A[i] < min
    min ← A[i]

return min

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* General plan for non-recursive algorithm to find order of growth of an element

→ Max Element

Input size: n

Basic operation: Comparison in for loop

if $A[i] > \text{max}$

$$C(n) = 1 + 1 + \dots + n - 1$$

$$= \sum_{i=1}^n 1 = (n-1) - 1 + 1$$

$$= n - 1$$

$$= \Theta(n)$$

→ Min Element

Input size: n

Basic operation: comparison in for loop

if $A[i] < \text{min}$

$$C(n) = 1 + 1 + \dots + n - 1$$

$$= \sum_{i=1}^{n-1} 1 = (n-1) - 1 + 1$$

$$= n - 1$$

$$\in \Theta(n)$$