Unit - V Elementary Number theory and Cryptography

Fundamentals of logic

Proposition or Statement:

A proposition or a statement is a sentence which can be either true or false but not both.

For Ex :—

1) Bangalore is a IT hub of India (true)
2) 1 is an odd numbers (true)
3) $2 + 3 = 4$ (False)

Logical Connections :-

Compound propositions / statement obtained by the use of words or phrases likes 'not', 'and', 'or', 'if', .. 'then' and "if and only if" are called logical connectives

1) Negation ('not' or ~)

If P is a proposition, then negation of P is denoted by $\sim P$ and defined by 'not P'.

For example :
p : mumbai is a capital of maharastra
$\sim p$: Mumbai is not a capital of maharastra

Truth Table:—

p	$\sim p$
T	F
F	T

2) Conjunction ('and' OR $\wedge$)
A compound proposition obtained by combaining 2 given propositions p and q by inserting a word 'and' in b/w is called conjunction and it is denoted by 'p and q' or '$p \wedge q$'

Remark:— The Conjunction $p \wedge q$ is true only when both p and q are true ;
in all other cases it is false

T.T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3) Disjunction ('or' OR $\vee$)

A compound proposition or statement obtained by combining two given proposition p and q by inserting a word 'or' in b/w is called Disjunction and it is denoted by 'p or q' or '$p \vee q$'

Remark :- The disjunction $p \vee q$ is true only when one of both or both are true

Otherwise false (i.e when both are false)

T. T

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4) Conditional ($p \rightarrow q$)

A compound proposition obtained by combining two given propositions p and q by using the words 'if' and 'then' at appropriate places is called Conditional and it is denoted by '$p \rightarrow q$'

Remark :- The conditional $p \rightarrow q$ is false only when p is true and q is false in all other cases its true.

p : 2 is a prime number $\sim p$: 2 is not a prime number

q : 2 is divided by 1 $\sim q$: 2 is not divided by 1

① $p \rightarrow q$: If 2 is a prime no, then 2 divided by 1 (T)

② $p \rightarrow \sim q$: If 2 is a prime num, then 2 does not divide by 1 (F)

③ $\sim p \rightarrow q$: If 2 is not a prime no, then 2 divided by 1 (T)

④ $\sim p \rightarrow \sim q$: If 2 is not a prime number, then 2 does not divided by 1 (T)

T.T

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

5) Biconditional $(p \leftrightarrow q)$

Let p & q are 2 proposition. Then the conjunction of the conditionals $p \rightarrow q$ and $q \rightarrow p$ is called ~~the~~ Biconditional of p and q. And it is denoted by $p \leftrightarrow q$

i.e $(p \rightarrow q) \wedge [q \rightarrow q) = p \leftrightarrow q$

T. T

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

6) Tautology

A compound proposition which is true for all the possible truth values of its components is called tautology

For Ex: $p \vee \sim p$ is a tautology

T. T

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Contradiction

A compound proposition which is false for all the possible truth values of its components is called contradication

For Ex:—

$p \wedge \sim p$ is a contradiction

T.T :

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

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Laws of Logic :—

(1) Law of double negation

If P is any proposition, then $\sim(\sim P) \equiv P$

Proof :-

P	$\sim P$	$\sim(\sim P)$
T	F	T
F	T	F

$\therefore \sim(\sim P) \equiv P$

(2) Idempotent Laws

If P is any proposition, then

(i) $(P \vee P) \equiv P$

(ii) $(P \wedge P) \equiv P$

(i)

P	P	$P \wedge P$
T	T	T
F	F	F

$(P \vee P) \equiv P$

(ii)

P	P	$P \vee P$
T	T	T
F	F	F

$(P \wedge P) \equiv P$

3) Identity Laws.

If P is any proposition and T_0 and F_0 are Tautology and contradication respectively, then

(i) $(P \wedge T_0) \equiv P$ (ii) $(P \vee F_0) \equiv P$

Proof:- (i)

P	T_0	$P \wedge T_0$
T	T	T
F	T	F

(ii)

P	F_0	$P \vee F_0$
T	F	T
F	F	F

$\therefore (P \wedge T_0) \equiv P$

4) Inverse Laws

If P is any proposition, then

(i) $(P \vee \sim P) \equiv T_0$, (ii) $(P \wedge \sim P) \equiv F_0$

when T_0 & F_0 are tautology and Contradication respectively

Proof: (i)

P	$\sim P$	$P \vee \sim P$
T	F	T
F	T	T

T_0

$(P \vee \sim P) \equiv T_0$

(ii)

p	~p	p ∧ ~p
T	F	F
F	T	F

$(p \wedge \sim p) = F_0$

5) Domination Laws

If p is any proposition, then

(i) $(p \vee T_0) \equiv T_0$ (ii) $(p \wedge F_0) \equiv F_0$

where T_0 and F_0 are tautology and contradication respectively

Proof: (i)

p	T_0	$p \vee T_0$
T	T	T
F	T	T

(ii)

p	F_0	$p \wedge F_0$
T	F	F
F	F	F

$(p \vee T_0) \equiv T_0$ $(p \wedge F_0) \equiv F_0$

6) Commutative Law

If p and q are any proposition, then

(i) $(p \wedge q) \equiv (q \wedge p)$ (ii) $(p \vee q) \equiv (q \vee p)$

Proof:-

p

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$(p \wedge q) \equiv (q \wedge p)$

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

$(p \vee q) \equiv (p \vee q)$

7) Absorption Law

If p and q are any 2 propositions then

(i) $[p \vee (p \wedge q)] \equiv p$ (ii) $[p \wedge (p \vee q)] \equiv p$

Proof: [i]

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	F	T	T
T	F	F	T
F	T	F	F
F	F	F	F

$\therefore \ p \vee (p \wedge q) \equiv p$

(ii)

p	q	p∨q	[p∧(p∨q)]
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

$\therefore p \wedge (p \vee q) \equiv p$

18) De-morgan's Law.

If p and q are any two propositions then ~~the first one is negation of p~~.

(i) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

(ii) $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

(i)

p	q	~p	~q	p∨q	~(p∨q) ✓	(~p)∧(~q) ✓
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

$$\therefore \sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$$

(ii)

p	q	~p	~q	p∧q	~(p∧q)	(~p)∨(~q)
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$$\sim(p \wedge q) = (\sim p) \vee (\sim q)$$

(29) Associative Laws

If p, q, r are any 3 propositions, then

(i) $p \vee (q \vee r) \equiv (p \vee q) \vee r$, (ii) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

(i)

(i) p	(ii) q	(iii) r	(iv) q∨r	(vi) p∨q	(v) p∨(q∨r)	vii (p∨q)∨r
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T
F	F	F	F	F	F	F

(ii)

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

L 10 Distributive Law

If p, q, r, are any three propositions, then

(i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

(ii)

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q)$	$(p \wedge r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

Law of the Negation of the conditional

If p and q are any 2 propositions and given conditional is $p \rightarrow q$, then negation of $p \rightarrow q$ is obtained by the following

$$\sim(p \rightarrow q) \equiv p \wedge (\sim q)$$

p	q	$\sim q$	$(p \rightarrow q)$	$\sim(p \rightarrow q)$ *	$p \wedge (\sim q)$ *
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

$$\therefore \sim(p \rightarrow q) \equiv p \wedge (\sim q)$$

Date
Page 9

Remarks:—

1) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

2) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

3) $\sim(p \rightarrow q) \equiv p \wedge \sim q$

4) $p \rightarrow q \equiv \sim[\sim(p \rightarrow q)]$ ($\because$ By the law of double negat^n)

$\equiv \sim[(p \wedge \sim q)]$ $\because$ By 3

$\equiv \sim p \vee \sim(\sim q)$ ($\because$ By 2)

$p \rightarrow q \equiv \sim p \vee q$

Transistive Rule :—

If p, q, r are any propositions, such that $p \equiv q$ and $q \equiv r$, then $p \equiv r$ is called transitive rule.

Substitution Rule :—

1) Suppose a compound statement u is a tautology, and p is a component of u, If we replace each occurence p in u by the proposition q, then resulting compound proposition v is also a tautology. This rule is called substitution rule.

$u : (p \wedge \sim r) \wedge p \longrightarrow$ Tautology

$v : (q \wedge \sim r) \wedge q \longrightarrow$ "

2) Suppose u is a compound proposition which contains a compound p. Let q be a proposition such that $p \equiv q$ if we replace one or more occurrences of p by q and obtained a compound proposition v, then $u \equiv v$. This is also called a substitution rule.

Examples

① Let x be a specified number. Write down the negation of the following conditional
"If x is an integer, then x is a rational number"

Sol: Given conditional is
"If x is an integer, then x is a rational number"
p: x is an integer
q : x is a rational number
∴ Given conditional can also be written as
$$p \rightarrow q$$
∴ Negation of the above conditional is
$$\sim(p \rightarrow q) \equiv p \wedge \sim q$$

$\therefore$ "x is an integer and it is not a rational number"

2. Let x be a specified number. Write down the negation of the following proposition
"If x is not a ~~really~~ no. then it is not a rational number and not an irrational number"

$\rightarrow$ Sol^n Given Compound proposition is
"If x is not a real no. then it is not a rational no and not an irrational number

p: x is a real num $\quad$ $\sim p$: not real no
q: x is a rational num $\quad$ $\sim q$: not a rational no
r: x is a irrational num $\quad$ $\sim r$: not a irrational no

Above compound statement

$$\sim p \rightarrow (\sim q \wedge \sim r)$$

$\therefore$ Negation of the above statement is

$$\sim[\sim p \rightarrow (\sim q \wedge \sim r)] \equiv$$

$$\sim p \wedge \sim(\sim q \wedge \sim r) \quad (\because p \wedge \sim q)$$

$$\equiv \sim p \wedge [\sim(\sim q) \vee \sim(\sim r)] \quad (\because \sim(p \wedge q) \equiv \sim p \vee \sim q)$$

$$\equiv \sim p \wedge q \vee r$$

∴ " x is not a ~~a~~ real number and it is rational number or ~~it is~~ an irrational number".

3) Simplify the following compound propositions using the laws of logic

(i) $(p \vee q) \wedge [\sim(\sim p \wedge q)]$ (ii) $(p \vee q) \wedge [\sim(\sim p \vee q)]$

(i) $(p \vee q) \wedge [\sim(\sim p) \vee \sim q]$ ($\because$ Demorgan's Law)

$\equiv (p \vee q) \wedge [p \vee \sim q]$ ($\because p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$)

$\equiv p \vee (q \wedge \sim q)$ ($p \wedge \sim p \equiv F_0$)

$\equiv p \vee (F_0)$

$\equiv p$

(ii) $(p \vee q) \wedge [\sim(\sim p \vee q)]$

$\equiv (p \vee q) \wedge [\sim(\sim p) \wedge \sim q]$ (D.M)

$\equiv (p \vee q) \wedge (p \wedge \sim q)$

$\equiv [(p \vee q) \wedge p] \wedge \sim q$ (Associative Law)

$\equiv [p \wedge (p \vee q)] \wedge \sim q$ ($\because$ Commutative Law)

$\equiv p \wedge \sim q$ $\therefore$ (Absorption Law)

$\therefore (p \vee q) \wedge [\sim(\sim p \vee q)] \equiv p \wedge \sim q.$

classmate
Date
Page

4) Prove the following logical equivalence without using T.T.

(i) $[p \vee q \vee (\sim p \wedge \sim q \wedge r)] \equiv (p \vee q \vee r)$

(ii) $[(\sim p \vee \sim q) \longrightarrow (p \wedge q \wedge r)] \equiv (p \wedge q)$

Sol^n (i) Given that

$[p \vee q \vee (\sim p \wedge \sim q \wedge r)]$

$\equiv p \vee q \vee \{(\sim p \wedge \sim q) \wedge r\}$ (Associative Law)

$\equiv [p \vee q \vee \{\sim(p \vee q) \wedge r\}]$ ($\because$ D.M Law)

$\equiv (p \vee q) \vee \{\sim(p \vee q) \wedge r\}]$ (Distributive Law)

$\equiv [(p \vee q) \vee \sim(p \vee q)] \wedge [(p \vee q) \vee r]$

$\equiv \underbrace{(p \vee q)}_{T_0} \wedge [(p \vee q) \vee r]$

$\equiv T_0 \wedge [(p \vee q) \vee r]$ ($\because p \vee \sim p \equiv T_0$)

$\equiv (p \vee q \vee r) \wedge T_0$ ($\because$ Commutative Law)

$\equiv (p \vee q \vee r)$ ($\because$ Identity Law

~~$p \vee q$~~

(ii) $[(\sim p \vee \sim q) \longrightarrow (p \wedge q \wedge r)]$

$(\because p \to q \equiv \sim p \vee q)$

$\equiv \sim(\sim p \vee \sim q) \vee (p \wedge q \wedge r)$

$\equiv \sim(\sim p) \wedge (\sim(\sim q)) \vee (p \wedge q \wedge r)$

$(\because$ DM Law $)$

$\equiv (p \wedge q) \vee (p \wedge q \wedge r)$ (Law of Double Negatⁿ)

$\equiv (p \wedge q) \vee [(p \wedge q) \wedge r]$ (Absorptⁿ Law)

$\equiv (p \wedge q)$

$\therefore [(\sim p \vee \sim q) \longrightarrow (p \wedge q \wedge r)] \equiv (p \wedge q)$

5 Prove the following logical equivalence that is

$(p \longrightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] \equiv \sim(p \vee q)$

Solⁿ $(p \longrightarrow q) \wedge [\sim q \wedge (r \vee \sim q)]$ (commutative Law

$\equiv (p \to q) \wedge [\sim q \wedge (\sim q \vee r)]$

$\equiv (p \to q) \wedge (\sim q)$ (Absorption Law)

$\equiv \sim[(p \to q) \to q]$ $(\because \sim(p \to q) \equiv p \wedge \sim q)$

~~$\equiv \sim[p(\sim p \vee q) \to q$~~

$\equiv \sim[\sim(p \to q) \vee q]$ $(\because p \to q \equiv \sim p \vee q)$

$\equiv \sim[(p \wedge \sim q) \vee q]$ $(\because \sim p \to q \equiv p \wedge \sim q)$

$\equiv \sim [q \vee (p \wedge \sim q)]$ ($\because$ Commutative Law)

$\equiv \sim [(q \vee p) \wedge (q \vee \sim q)]$ [Distributive Law]

$\equiv \sim [(q \vee p) \wedge T_0]$ ($\because$ Inverse Law)

$\equiv \sim (q \vee p)$ (Identity Law)

$\equiv \sim (p \vee q)$ ($\because$ Commutative Law)

$\therefore (p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] \equiv$

$\sim (p \vee q)$ //

Duality

Suppose u is a compound statement / proposition that contains the connectives $\wedge$ and $\vee$, also ~~cond~~ contains the components T_0 and F_0.

Suppose we replace each occurrences of $\wedge$, $\vee$, T_0 and F_0 by $\vee$, $\wedge$, F_0 & T_0 respectively.

Then resulting Compound proposition is good dual of u And it is denoted by u^d

For Ex:- Given compound proposition is

$u : P \vee (q \wedge \sim r) \vee (r \wedge T_0)$

Then ~~du~~ dual of u is given by

$$u^d = P \wedge (q \vee \sim r) \wedge (r \vee F_0)$$

Remark:-

1) If u is compound proposition and u^d is its dual, then $(u^d)^d \equiv u$

2) For any 2 compound proposition u & v, if $u \equiv v$, then $u^d \equiv v^d$. This is known as principle of duality.

Ex:-

1) Write down the duals of following proposition

(i) $u: p \rightarrow q \equiv \sim p \vee q$

$u \equiv \sim p \vee d$

$\therefore u^d$

dual of u is $u^d: \sim p \wedge d$

(ii) $(p \rightarrow q) \rightarrow r \equiv (\sim p \vee q) \rightarrow r$

$(\because p \rightarrow q \equiv \sim p \vee q)$

$\equiv \sim(\sim p \vee q) \vee r$

$= \sim(\sim p) \wedge \sim q \vee r$

$\equiv p \wedge \sim q \vee r$

$\therefore u : (p \rightarrow q) \rightarrow r \equiv (p \wedge \sim q) \vee r$

$\therefore \boxed{u^d : (p \vee \sim q) \wedge r}$

(iii) Given that $(\because p \rightarrow q \equiv \sim p \vee q)$

$u: p \rightarrow (q \rightarrow r) \equiv \sim p \vee (q \rightarrow r)$

$\equiv \sim p \vee (\sim q \vee r)$

$\therefore \quad \equiv \sim p \vee (\sim q \vee r)$

$u : (p \rightarrow q) \rightarrow r \equiv \sim p \vee (\sim q \vee r)$

$u^d: \sim p \wedge (\sim q \wedge r)$

Ex : 2

(2) Verify the principal of duality for the following logical equivalence

$$\sim(p \wedge q) \longrightarrow \sim p \vee (\sim p \vee q) \equiv (\sim p \vee q)$$

Soln:

~~NASS~~

$\therefore u : \sim(p \wedge q) \longrightarrow \sim p \vee (\sim p \vee q)$
$v : (\sim p \vee q)$.

(i) $u : \sim(p \wedge q) \longrightarrow \sim p \vee (\sim p \vee q)$

$\equiv \sim(p \wedge q) \longrightarrow (\sim p \vee \sim p) \vee q$ ($\because$ Associative Law)

$\equiv \sim(p \wedge q) \longrightarrow \sim p \vee q$ ($\because$ Idempotent Law)

$\equiv \sim[\sim(p \wedge q)] \vee (\sim p \vee q)$
$(p \rightarrow q \equiv \sim p \vee q)$

$\equiv (p \wedge q) \vee (\sim p \vee q)$

$\equiv (\sim p \vee q) \vee (p \wedge q)$
$\equiv [(\sim p \vee q) \vee p] \wedge [(\sim p \vee q) \wedge q]$

$\equiv [p \vee (\sim p \vee q)] \wedge [\sim p \vee (q \vee q)]$

$\equiv [(p \vee \sim p) \vee q] \wedge [\sim p \vee q]$

(∵ Associative Law & Idemp Law)

$\equiv [T_0 \vee q] \wedge [\sim p \vee q]$ (Domination Law)

$\equiv T_0 \wedge [\sim p \vee q]$

$\equiv \sim p \vee q$

$\therefore$ $u : \sim p \vee q$

$u^d : \sim p \wedge q$ dual of u

$v : \sim p \vee q$

$v^d : \sim p \wedge q$ dual of v

$\therefore$ $\boxed{u^d \equiv v^d}$

(3) Verify the principle of duality for the following logical equivalence

(i) $(p \vee q) \wedge [\sim p \wedge (\sim p \wedge q)] \equiv (\sim p \wedge q)$

Soln Given that:

$(p \vee q) \wedge [\sim p \wedge (\sim p \wedge q)] \equiv (\sim p \wedge q)$

$\therefore$ $u : (p \vee q) \wedge [\sim p \wedge (\sim p \wedge q)]$;

$v : \sim p \wedge q$

Consider

$u : (p \wedge q) \wedge [\sim p \wedge (\sim p \wedge q)] \equiv (p \vee q) \wedge [(\sim p \wedge \sim p) \wedge q]$

($\because$ Associative law)

$\equiv (p \vee q) \wedge [\sim p \wedge q]$ ($\because$ Idempotent Law $p \wedge p \equiv p$)

$\equiv (\sim p \wedge q) \wedge (p \vee q)$

($\because$ Commutative law)

$\equiv [(\sim p \wedge q) \wedge p] \vee [(\sim p \wedge q) \wedge q]$

($\because$ Distributive law)

$\equiv [p \wedge (\sim p \wedge q)] \vee [(\sim p \wedge q) \wedge q]$

($\because$ Asso Commut Law)

$\equiv [(p \wedge \sim p) \wedge q] \vee [\sim p \wedge (q \wedge q)]$

$\equiv [F_0 \wedge q] \vee [\sim p \wedge q]$

$\equiv F_0 \vee [\sim p \wedge q) \equiv \sim p \wedge q$

(Dominat^ Law

$\therefore$ $u : (p \vee q) \wedge [\sim p \wedge (\sim p \wedge q)] \equiv \sim p \wedge q$

Dual of u is

$u^d : \sim p \vee q$

Dual of v is

$v^d : \sim p \vee q$

$\therefore u^d \equiv v^d$

$\therefore$ This proves the principle of duality

The Connective NAND and NOR

NAND

The Compound proposition $\sim(p \wedge q)$ is read as " not p and q " and it is denoted by $p \uparrow q$ (or $p \bar{\wedge} q$). This is called the connective NAND.

i.e $\boxed{p \uparrow q \equiv p \bar{\wedge} q \equiv \sim(p \wedge q) \equiv \sim p \vee \sim q}$

NOR

The compound proposition $\sim(p \vee q)$ is read as " not p or q " and it is denoted by $p \downarrow q$ (or $p \underline{\vee} q$). This is called the connective NOR i.e $\boxed{p \downarrow q \equiv p \underline{\vee} q \equiv \sim(p \vee q) \equiv \sim p \wedge \sim q}$

Remarks

(i) The connective NAND is combinat^n of the connective 'not' and 'AND'

(ii) The connective NOR is combinat^n of the connective 'not' and 'NOR'

(iii) Evidently p NAND($\uparrow$) q and p NOR $\downarrow$ q are duals of each other

Examples:

① Express the following proposition in terms of only NAND and only NOR connection

(i) $\sim p$ (ii) $p \wedge q$ (iii) $p \vee q$ (iv) $p \to q$

(i) By Idempotent law, we have

$p \wedge q \equiv p$ and $p \vee p \equiv p$

$\sim(p \wedge p) \equiv \sim p$ and $\sim(p \vee p) \equiv \sim p$

$\therefore$ $p \uparrow p \equiv \sim p$ and $p \downarrow p \equiv \sim p$

(ii) $p \wedge q \equiv \sim[\sim(p \wedge q)]$ ($\because$ Double negations)

$= \sim[\sim p \vee \sim q]$ ($\because$ De morgan's law)

$\equiv [\sim p \vee \sim q] \uparrow [\sim p \vee \sim q]$ (definat^n of NAND)

$p \wedge q \equiv (p \uparrow q) \uparrow (p \uparrow q)$

Also, $p \wedge q \equiv \sim[\sim(p \wedge q)]$

$\equiv \sim[\sim p \vee \sim q]$ (DeMorgan law)

$\equiv \sim(\sim p) \wedge \sim(\sim p)$

$\equiv \sim p \downarrow \sim q$

$\equiv (p \downarrow p) \downarrow (q \downarrow q)$

(iii) $p \vee q \equiv \sim[\sim(p \vee q)]$ ($\because$ Double negation)

$\equiv \sim[\sim p \wedge \sim q]$

$\equiv \sim((\sim p \wedge \sim q) \downarrow (\sim p \wedge \sim q))$

$\equiv (p \downarrow q) \downarrow (p \downarrow q)$

Also $p \vee q \equiv \sim[\sim(p \vee q)]$

$\equiv \sim[\sim p \wedge \sim q]$

$\equiv \sim(\sim p) \vee \sim(\sim q)$

$\equiv \sim p \uparrow \sim q$

$\equiv (p \uparrow p) \uparrow (q \uparrow q)$

(iv) $p \rightarrow q \equiv \sim p \vee q$

$\equiv \sim(p \wedge \sim q)$ ($\because$ De-Morgan's law)

$\equiv p \uparrow \sim q$ ($\because$ def of NAND)

$\equiv p \uparrow (q \uparrow q)$

$p \to q \equiv \sim p \vee q$

$\equiv \sim(p$

Also: $p \to q \equiv \sim[\sim(p \to q)]$ ($\because$ double negat^n)

$\cancel{\equiv \sim[\sim(\sim p}$

$\equiv \sim[p \wedge \sim q]$

$\cancel{\equiv p \uparrow \sim q}$

$\equiv (p \wedge \sim q) \downarrow (p \wedge \sim q)$ ($\because$ DM Law)

$\equiv \sim(\sim p \vee q) \downarrow (\sim(\sim p \vee q))$

$\equiv (\sim p \downarrow q) \downarrow (\sim p \downarrow q)$

$$\boxed{p \to q \equiv ((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)}$$

Converse, Inverse and Contrapositive of a Conditional

"If x is an even number, then it is divisible by 2"

p: x is an even number

q: x is divisible by 2

$\sim p$: x is not an even no

$\sim q$: x is not ~~as~~ divisible by 2

$\therefore p \longrightarrow q$

Converse of $p \longrightarrow q$

$q \longrightarrow p$: If x is divisible by 2, then it is an even number

Inverse of $p \longrightarrow q$

$\sim p \longrightarrow \sim q$: If x is not an even no, then x is not divisible by 2

Contrapositive of $p \longrightarrow q$

$\sim q \longrightarrow \sim p$: If x is not divisible by 2, then it is not an even number

Let p and q are 2 proposition and $p \to q$ is a conditional then

(i) $q \to p$ is called Converse of $p \to q$

(ii) $\sim p \to \sim q$ is called Inverse of $p \to q$

(iii) $\sim q \to \sim p$ is called Contrapositive of $p \to q$

Truth Table

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Remark : From above we have following results

(i) Conditional and its contrapositive are logically equivalent.

i.e $p \to q \equiv \sim q \to \sim p$

(ii) The Converse and Inverse of a conditional are logically equivalent i.e $q \to p \equiv \sim p \to \sim q$

Example

State the converse, inverse and contrapositive of the following conditionals

1) "If x is an even no, then it is divisible by 2"

2) "If a quadrilateral is a parallelogram, then its diagonal bisect each other".

3) "If a real number x^2 is greater than zero, then x is not equal to zero".

Sol^n (i) Given statement is

"If x is an even number, then it is divisible by 2"

p: x is an even number.

q: x is divisible by 2.

$\sim p$: x is not an even no

$\sim q$: x is not ~~on~~ divisible

Given statement can be written as

$p \longrightarrow q$

Converse of $p \longrightarrow q$

$q \longrightarrow p$: If x is divisible by 2, then it is an even number

Inverse of $p \rightarrow q$

$\sim p \rightarrow \sim q$: If x is not an even number, then it is not divisible by 2.

Contrapositive of $p \rightarrow q$

$\sim q \rightarrow \sim p$: If x is not divisible by 2, then it is not an even number

2) Given statement is
" If a quadrilateral is a parallelogram, then its diagonals bisect each other "

p: A quadrilateral is a parallelogram

q: Diagonals of a quadrilateral bisect each other

$\sim p$: A quadrilateral is not a parallelogram

$\sim q$: Diagonals of a quadrilateral do not bisect each other

← next (prev) page

Continue of Ex 2)

Given statement can also be written as
$p \rightarrow q$

Converse of $p \rightarrow q$

$q \rightarrow p$: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Inverse of $p \rightarrow q$

$\sim p \rightarrow \sim q$: If a quadrilateral is not a parallelogram, then its diagonal do not bisect each other

Contrapositive of $p \rightarrow q$

$\sim q \rightarrow \sim p$: If the diagonal of a quadrilateral do not bisect each other, then it is not a parallelogram.

3) ~~Given statement~~.

5) Given statement is
"If a real number x^2 is greater than zero, then x is not equal to zero"

p : A real number x^2 is greater than zero
q : x is not equal to zero
~~$\sim p$: x is~~
$\sim p$: A real number x^2 is not greater than zero
$\sim q$: x is equal to zero

Given statement can also be written as

$p \longrightarrow q$

Converse of $p \longrightarrow q$
$q \longrightarrow p$: If a real number x is not equal to zero, then x^2 is greater than zero

Inverse of $p \longrightarrow q$
$\sim p \longrightarrow \sim q$: If a real number x^2 is not greater than zero, then x is equal to zero.

Contrapositive of $p \longrightarrow q$
$\sim q \longrightarrow \sim p$: If a real number x is equal to zero, than x^2 is not equal to zero.

Implicative Statement or Hypothetical statement

The conditional $p \to q$, where p and q are any two propositions such that both are related in some way. So that the true value of q depends on the truth value of p and vice-versa. such conditional $p \to q$ are called implicative statement or hypothetical statement.

Logical implication

In an implicative statement $p \to q$, q is true whenever p is true, then we say the "p logically implies to q" or "p implies q". This is symbolically written as

$$p \Rightarrow q$$

In an implicative statement $p \to q$, q is not necessarily true when even p is true, then we say that "p does not implies q". This is symbolically written as

$$p \not\Rightarrow q$$

Date
Page

Example

1) Prove the following

(i) $[p \wedge (p \to q)] \Rightarrow q$, (ii) $[(p \to q) \wedge \sim q] \Rightarrow \sim p$

(iii) $[(p \vee q) \wedge \sim p] \Rightarrow q$

Solⁿ: The truth table is as following

p	q	$\sim p$	$\sim q$	$p \vee q$	$p \to q$
T	T	F	F	T	T
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	F	T

(i) ~~From the above truth table~~

$p \wedge (p \to q)$	$(p \to q) \wedge \sim p$	$(p \vee q) \wedge \sim p$
T	F	F
F	F	F
F	T	T
F	T	F

(i) From the above truth table $p \wedge (p \to q)$ true, then q is also true

$\therefore p \wedge (p \to q) \Rightarrow q$

(ii) From the above truth table, $(p \to q) \wedge \sim p$ is true then $\sim p$ is also true

$\therefore (p \longrightarrow q) \wedge \sim p \Rightarrow \sim p$

(iii) From the above truth table $(p \vee q) \wedge \sim p$ is true, then q is also true

$\therefore (p \vee q) \wedge \sim q \Rightarrow q$

2) Prove the following

(i) $p \wedge q \Rightarrow p \vee q$

(ii) $\sim p \Rightarrow (p \longrightarrow q)$

(iii) $q \Rightarrow (p \longrightarrow q)$

Sol^n Truth table

p	q	$\sim p$	$p \wedge q$	$p \to q$	$p \vee q$
T	T***	F	T*	T	T*
T	F	F	F	F	T
F	T***	T**	F	T**	T
F	F	T**	F	T**	F

(i) From the truth table, we found that, if

(i) $(p \wedge q)$ is true, then $(p \vee q)$ is also true
$\therefore (p \wedge q) \Longrightarrow (p \vee q)$

(ii) From the T.T, we found that, if $\sim p$ is true, then $(p \rightarrow q)$ is true
$\therefore \sim p \Longrightarrow (p \rightarrow q)$

(iii) From the T.T, we found that, if q is true
$\therefore q \Rightarrow (p \rightarrow q)$

3) Prove the following
(i) $p \wedge (q \vee r) \Longrightarrow (p \wedge q) \vee r$
(ii) $(p \rightarrow q) \Longrightarrow p \rightarrow (p \wedge q)$

Sol^n Truth Table

$\sim p$	p	q	r	$q \vee r$	$p \wedge q$	$p \rightarrow q$ (**)	$p \wedge (q \vee r)$ (*)	$(p \wedge q) \vee r$
F	T	T	T	T	T	T **	T *	T *
F	T	T	F	T	T	T **	T *	T *
F	T	F	T	T	F	F	T *	T *
F	T	F	F	F	F	F	F	F
T	F	T	T	T	F	T **	F	T
T	F	T	F	T	F	T **	F	F
T	F	F	T	T	F	T **	F	T
T	F	F	F	F	F	T **	F	F

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$p \rightarrow (p \wedge q)$
T **
T **
F
F
T **
T **
T **
T **

(i) ~~It~~ From T.T, we found that, if $p \wedge (q \vee r)$ is true, then $(p \wedge q) \vee r$ is true

$$p \wedge (q \vee r) \Longrightarrow (p \wedge q) \vee r$$

(ii) $(p \rightarrow q) \Rightarrow p \rightarrow (p \wedge q)$
From T.T we found that if $(p \rightarrow q)$ ~~$\Rightarrow p \rightarrow (p \wedge q)$~~ is true, then $p \rightarrow (p \wedge q)$

Rule of Inferences

Argument : Collection of proposition
$\{P_1, P_2, P_3 \dots P_n\}, q$

premises of an argument

conclusion of an argument

$(P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_n) \longrightarrow q$ then this is called an argument

p if $(P_1, P_2, P_3, \dots P_n) \longrightarrow$ True
then q is true

$(P_1 \wedge P_2 \wedge P_3 \wedge \dots P_n) \longrightarrow q$ is argument said to be valid if $(P_1 \wedge P_2 \wedge P_3 \dots P_2) \longrightarrow q$

Rule of Inferences

Argument

Let us consider a set of proposition $\{P_1, P_2, \dots P_n\}$ and another proposition q. Then a compound proposition of the form
$$(P_1 \wedge P_2 \wedge P_3, \dots \wedge P_n) \longrightarrow q$$
is called an argument

Here $P_1, P_2, P_3, \dots P_n$ are called premises of an argument and q is called conclusion of an argument.

The

$$\begin{array}{c} P_1 \\ P_2 \\ P_3 \\ \vdots \\ \underline{P_n} \\ \underline{q} \end{array}$$

The argument is said to be valid if each proposition of a premise $p_1, p_2, p_3 \ldots p_n$ are true, then conclusion q is also true. On Other words, the argument $(p_1 \wedge p_2 \wedge p_3 \wedge p_4 \ldots p_n) \longrightarrow q$ is valid, when $p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) \Rightarrow q$

We have the following rule of inference

1) Rule of conjective simplification

For any 2 proposition p and q, if $(p \wedge q)$ is true, then p is true.

i.e $(p \wedge q) \Rightarrow p$ (or $(p \wedge q) \Rightarrow q$)

Tabular form is

$$\frac{p \wedge q}{\therefore p} \quad \left(\text{or } \frac{p \wedge q}{q} \right)$$

2) Rule of Disjunctive Amplification

For any 2 proposition p and q, if p is true, then $(p \vee q)$ is true

i.

i.e $p \Rightarrow (p \vee q)$

∴ The tabular form is

$$\frac{p}{\therefore (p \vee q)}$$

3) <u>Rule of Syllogism</u>

For any three proposition p, q and r, if $(p \rightarrow q)$ is true and $(q \rightarrow r)$ is true, then $(p \rightarrow r)$ is true

i.e $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$

∴ R. Tabular form is

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$(p \rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

4) **Modus Pones (Rule of Detachment**

For any two propositions p and q, if p is true and $(p \rightarrow q)$ is true then q is true

i.e. $p \wedge (p \rightarrow q) \Rightarrow q$

∴ Tabular form is

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array} \quad \text{OR} \quad \begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$
T	T*	T	T*
T	F	F	F
F	T	T	F
F	F	T	F

5) **Modus Tollen's**

For any two propositions p and q, if $(p \rightarrow q)$ is true, and q is false, then p is false

i.e. $(p \rightarrow q) \wedge \sim q \Rightarrow \sim p$

$\therefore$ Tabular form is

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \\ \hline \end{array}$$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim q$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	F	T	F
F	F	T*	T	T	T*

6> <u>Rule of Disjunctive Syllogism</u>

For any two proposition p and q, if ~~p or q~~ $(p \vee q)$ is true and p is false, then q is true.

i.e. $(p \vee q) \wedge \sim p \Longrightarrow q$

$\therefore$ Tabular form is

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore \; q \\ \hline \end{array}$$

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$
T	T	F	T	F
T	F	F	T	F
F	T*	T	T	T*
F	F	T	F	F

7) Rule of Contradiction

For any proposition p and contradiction F_0, if $(\sim p \rightarrow F_0)$ is true, then p is true

$$(\sim p \longrightarrow F_0) \longrightarrow p$$

Tabular form

$$\begin{array}{l} \sim p \longrightarrow F_0 \\ \hline \therefore \; p \\ \hline \end{array}$$

p	F_0	$\sim p$	$\sim p \rightarrow F_0$
T*	F	F	T*
F	F	T	F

Examples

Test whether the following are valid argument

1) If sachin hits a century, then he gets free car
Sachin hits a century

∴ Sachin gets a free car

Given argument can be written as

p: Sachin hits century
q: Sachin gets free car

$$\begin{array}{c} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

By the rule of Modus Ponens the given argument is valid

2) If I drive to work, then I will arrive ~~tr~~ tired
I'm not tired

∴ I don't drive to work

Sol^n: Let p: I drive to work
~~p~~q: I will be tired

~p: I do not drive to work
~q: I am tired

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

∴ By the rule of Modus ~~Ponens~~ Tollens the given argument is valid

3) I will become famous or I will not become a musician
I will become musician

I will become famous

Sol^n: Let p: I will become famous
q: I will become a musician
~q: I will not become a musician

$q \rightarrow p \equiv \sim q \vee p$

∴ Given argument can be also written as

$$\begin{array}{l} p \vee \sim q \\ q \\ \hline p \end{array} \qquad \begin{array}{l} q \rightarrow p \\ q \\ \hline p \end{array}$$

OR

$$p \vee \sim q \equiv \sim q \vee p$$

$$\frac{\quad q \quad}{\sim p} \qquad \frac{\quad q \quad}{p}$$

$$\equiv q \rightarrow p$$

$$\frac{\quad q \quad}{\therefore p} \quad (\because q \rightarrow p \equiv \sim q \vee p)$$

$\therefore$ By the rules Modus Pones the given argument is valid

OR

" disjunctive syllogism

4) If there is a strike by students, then examination will be postponed
There is no strike by students

$\therefore$ The examination will not be postponed

There is
p: A strike by students
q: Examinations will be postpond
$\sim p$: There is no ~~no~~ strike by students
$\sim q$: Examination will not be postponed

$\therefore$ Given argument can be written as

$$\begin{array}{l} p \rightarrow q \\ \sim p \\ \hline \therefore \sim q \end{array}$$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim p$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	F ✗	T	T ✗
F	F	T	T ✓	T	T ✓

$\therefore$ Given argument is invalid ($\because$ By truth table)

5) If Ravi studies then he will pass in DMS paper
If Ravi does not play cricket, then he will study
Ravi failed in DMS paper

$\therefore$ Ravi played cricket

Sol^n p: Ravi will study
q: Ravi will pass in DMS paper
r: Ravi does not play cricket.
$\sim q$: Ravi ~~will~~ failed DMS paper.
$\sim r$: Ravi played cricket.
$\therefore$

$\therefore$ given argument can be written as

$$\begin{array}{l} p \longrightarrow q \\ r \longrightarrow p \\ \sim q \\ \hline \therefore \sim r \\ \hline \end{array} \equiv \begin{array}{l} r \longrightarrow p \\ p \longrightarrow q \\ \sim q \\ \hline \sim r \\ \hline \end{array}$$ ($\because$ Commutative law)

$$\equiv \begin{array}{l} r \longrightarrow q \\ \sim q \\ \hline \therefore \sim r \\ \hline \end{array}$$ (Rule of syllogism)

$\therefore$ By Modus Tollen's given argument is valid

Quantifiers

Universal quantifiers ← "for all", "for every", "for any", "for each"

Existential quantifiers ← "for some", "there exists"

The words or phrases "for all", "for every", "for any", "for each, "for Some", "there exists" in the proposition are associated with an idea of a quantity

Such words are called quantifiers

For example:

1) For every integer x, x^2 is non-negative integers.

2) There exists a real number whose square is equal to itself.

3) For all integer x, $(x+2)$ is also integer

Universal Quantifiers.

The words "for all", "for any", "for each" are called the universal quantifiers

Remark:— 1) The symbol $\forall$ is used to denote the word for all, for every, for each and for any

2) The symbol $\forall$ is universal quantifiers

Existential Quantifiers

The words "for some", "there exists" are called existential quantifiers

Remark:-

1) The symbol $\exists$ is used to denoted words "for some" & "there exists"
2) The symbol $\exists$ is an existential quantifier

Examples
1) For all matrix $A \in M$, A is a square matrix symbolically written as

$$\forall A \in M, P(A)$$

where the proposition $P(A)$: A is a square matrix

2) There exists an integer x, x is equal to zero symbolically written as

$$\exists x \in \mathbb{Z}, P(x)$$

where the proposition $P(x)$: x is equal to zero

Symbols and its notations

1) $\forall$ — for all, for every, for each, for any

2) $\exists$ — there exists, for some

3) $\in$ - belongs to

4) $\ni$ - such that

5) $\therefore$ — therefore

Quantified statement

A proposition involving the universal or the existential quantifiers is called quantified statement

Truth value of Quantified statements

The following rules are employed for determining the truth value of quantified statements.

Rule 1 :— The statement " $\forall x \in S, p(x)$" is true only when $p(x)$ is true for all $x \in S$

Rule 2:- The statement "$\exists x \in S, p(x)$" is false only when, for each $x \in S$, $p(x)$ is false

Rule 3: - Rule of Universal specification

If the statement $P(x)$ is known to be true for all value of x choosen from universal set S, and if $a \in S$, then $P(a)$ is true.

Rule 4:- Rule of Universal Generalization

If the statement $p(x)$ is proved to be true for any arbitrary value of x choosen from universal set S, then quantified statement "$\forall x \in S, p(x)$" is true

Logical Equivalence of Quantified statement.

For any 2 open statements $p(x)$ and $q(x)$, we have following results where x is an element of universal set S.

1) $\forall x \in S, [p(x) \land q(x)] \equiv [\forall x \in S, p(x)] \land [\forall x \in S, q(x)]$

2) $\exists x \in S, [p(x) \vee q(x)] \equiv [\exists x \in S, p(x)] \vee [\exists x \in S, q(x)]$

3) $\exists x \in S, [p(x) \rightarrow q(x)] \equiv \exists x \in S, [\sim p(x) \vee q(x)]$

$(\because p \rightarrow q \equiv \sim p \vee q)$

4) $\forall x \in S, \sim p(x) \equiv$ for no $x \in S$, $p(x)$

Rule for the negation of Quantified Statements

To form a negation of quantified statement, change the universal quantifier to existential quantifier and vice-versa and replace the statement by its negation

// ~~ex~~ $\sim$(For all $x \in \mathbb{Z}$, x is an odd number)

$\sim$ (For all $x \in \mathbb{Z}$, x^2 is a ~~$~~ +ve integer)

$\equiv$ There exists $x \in \mathbb{Z}$, x^2 is is not a +ve integer

For example:

1) $\sim[\forall x \in S, p(x)] \equiv \exists x \in S, \sim p(x)$

2) $\sim[\exists x \in S, q(x)] \equiv \forall x \in S, \sim q(x)$

Date
Page

Examples

(1) For all integer set, let

$p(x)$: $x > 0$
$q(x)$: x is even
$r(x)$: x is perfect square
$s(x)$: x is divisible by 3
$t(x)$: x is divisible by 7

Write down the following quantified statements in symbolic form

i) At least one integer is even
ii) There exists a positive integer that is even
iii) Some even integers are divisible by 3
iv) If x is even and perfect square, then x is divisible by 3
v) If x is odd or it is divisible by 7, then x is divisible by 3

Solⁿ : By using the defⁿ of quantifiers, we have

i) $\exists x \in \mathbb{Z}, \; q(x)$
ii) $\exists x \in \mathbb{Z}, \; [p(x) \wedge q(x)]$

(iii) $\exists x \in \mathbb{Z}, [q(x) \wedge s(x)]$

(iv) $\forall x \in \mathbb{Z}, [\{q(x) \wedge r(x)\} \longrightarrow s(x)]$

(v) $\forall x \in \mathbb{Z}, [\{\sim q(x) \vee t(x)\} \longrightarrow s(x)]$

2) Consider the open statements $p(x), q(x), r(x), s(x)$ and $t(x)$ from the above example. Express each of the following symbolic statements in words and indicate the truth value

(i) $\forall x \in \mathbb{Z}, [r(x) \longrightarrow s(x)]$, (ii) $\exists x \in \mathbb{Z}, [s(x) \wedge \sim q(x)]$
(iii) $\forall x \in \mathbb{Z}, \sim r(x)$ (iv) $\forall x \in \mathbb{Z}, [r(x) \vee t(x)]$

Solⁿ. Given statements are
$p(x): x > 0$; $q(x)$: x is even
$r(x)$: x is perfect square ; $s(x)$: x is divisible by 3
$t(x)$: x is divisible by 7

(i) $\forall x \in \mathbb{Z}, [r(x) \longrightarrow s(x)]$

For all integer value x is perfect square then x is divisible by 3
The given ~~statement~~ statement is false as 4 does not divisible by 3.

Date
Page

(ii) $\exists x \in \mathbb{Z}, [s(x) \wedge \sim q(x)]$
There exists integer x, x is divisible by 3 and x is odd
The given statement is True as $\exists$ an integer 9 which is divisble by 3 and 9 is odd

(iii) $\forall x \in \mathbb{Z}, \sim r(x)$

For all integer x, x is not a perfect square
$\therefore$ The given statement is false as 4 is a perfect which is a perfect square

(iv) $\forall x \in \mathbb{Z}, [r(x) \vee t(x)]$
For all integers x, x is a perfect square or x is divisible by ~~of~~ 7

$\therefore$ The given statement is false as 3 is an integer which is neither a perfect square nor divisible by 7

3) Write down the following prepositions in symbolic form and ~~its~~ find its negation.

3. Write down the following propositions in symbolic form and find its negation

(i) " All integers are rational numbers and some rational numbers are not ~~figure~~ integers ".

(ii) " If all triangle are right angled, then no triangle is equilateral ".

Sol^n (i) Given statement is

" All integers are rational numbers and some rational ~~no~~ numbers are not integers ".

$p(x)$: x is a rational numbers.

$q(x)$: x is an integer

then $\mathbb{Z}$ is a set of integers ; $\mathbb{Q}$ set of ~~x~~ rational numbers.

$\therefore$ Symbolic form of above statement is

$$[\forall x \in \mathbb{Z}, p(x)] \wedge [\exists x \in \mathbb{Q}, \sim q(x)]$$

Negation of the above statement is

$$\sim \{[\forall x \in \mathbb{Z}, p(x)] \wedge [\exists x \in \mathbb{Q}, \sim q(x)]\}$$

$$\equiv \sim [\forall x \in \mathbb{Z}, p(x)] \vee \cancel{\sim p(x)} \sim [\exists x \in \mathbb{Q}, \sim q(x)]$$

($\because$ By De-Morgan's law)

$\equiv [\exists x \in \mathbb{Z}, \sim p(x)] \vee [\forall x \in \mathbb{Q}, q(x)]$

∴ By its defn of negatn quantified statm)

∴ In words we can write
"There exists an integer x, x is not a rational number or for all rational numbers x, x is an integer".

(ii) Given statement is

"If all triangle are right angled, then no triangle is equilateral"

$\therefore$ $p(x)$: x is a right angled triangle
$q(x)$: x is equilateral triangle

T - is a set of all triangles
$\therefore$ Symbolic statement of the above is

$$\{\forall x \in T, p(x)\} \longrightarrow \{\forall x \in T, \sim q(x)\}$$

Negation of the above statement is

$$\sim[\{\forall x \in T, p(x)\} \longrightarrow \{\forall x \in T, \sim q(x)\}]$$

$\therefore \sim(p \to q) \equiv p \wedge \sim q$

$$\equiv \{\forall x \in T, p(x)\} \wedge \{\forall x \in T, \sim q(x)\}$$

$$(\because \sim(p \longrightarrow q) \equiv p \wedge \sim q)$$

$$\equiv \{\forall x \in T, p(x)\} \wedge \{\exists x \in T, q(x)\}$$

{$\because$ defn of negation of quantified statement}

In words we write as follows

Date
Page

For all triangle x, x is a right angled triangle and for some triangle x, x is equilateral triangle.

(4) Negation and simplify each of the following

(i) $\exists x \in s, \; [p(x) \vee q(x)]$,

(ii) $\forall x \in s, \; [p(x) \wedge \sim q(x)]$

(iii) $\forall x \in s, \; [p(x) \longrightarrow q(x)]$

(iv) $\exists x \in s, \; [\{p(x) \vee q(x)\} \longrightarrow r(x)]$

Sol^n (i) $\exists x \in s, \; [p(x) \vee q(x)]$

$\therefore$ Negation of it is

$$\sim [\exists x \in s, \{p(x) \vee q(x)\}] \equiv \forall x \in s, \; \sim\{p(x) \vee q(x)\}$$

$$\equiv \forall x \in s, \; [\sim p(x) \wedge \sim q(x)]$$

($\because$ By De morgan laws)

(ii) $\forall x \in s, \; [p(x) \wedge \sim q(x)]$

$\therefore$ Negation of it is

~~$\sim [p(x) \wedge$~~

$$\sim [\forall x \in s, \{p(x) \wedge \sim q(x)\}] \equiv \exists x \in s, \; \sim\{p(x) \wedge \sim q(x)\}$$

$\equiv \exists x \in S, [\sim p(x) \vee q(x)]$
($\because$ De morgan's law)

(iii) $\forall x \in S; [p(x) \longrightarrow q(x)]$
$\therefore$ Negation of it is

$\sim [\forall x \in S, \{p(x) \longrightarrow q(x)\}]$

$\equiv \exists x \in S, \sim \{p(x) \longrightarrow q(x)\}$

$\equiv \exists x \in S, p(x) \wedge \sim q(x)$
($\because \sim(p \longrightarrow q) \equiv p \wedge \sim q$)

(iv) $\exists x \in S, [\{p(x) \vee q(x)\} \longrightarrow r(x)]$
$\therefore$ Negation of it is

$\cancel{\sim [\exists x \in S, \{(p}$
$\sim [\exists x \in S, \{(p(x) \vee q(x)) \longrightarrow r(x)\}]$

$\equiv \forall x \in S, \sim [\{p(x) \vee q(x) \longrightarrow r(x)\}]$
$\equiv \forall x \in \sim, [\{p(x) \vee q(x)\} \wedge \sim r(x)]$

Ex 5 Consider the following open statements with the set of real numbers

$p(x): |x| > 3$; $q(x): x > 3$

Find the truth value of the quantified statement

$\forall x \in R, [p(x) \longrightarrow q(x)]$

Also write, converse, inverse and contrapositive of the statements and find their truth values

Soln Given open statements are

$p(x): |x| > 3$; $q(x): x > 3$

$p(x): x < -3$ and $x > 3$, $q(x): x > 3$

OR

$p(x): (-\infty, -3)$ and $(3, \infty)$, $q(x): x > 3$

Given Statements is

$\forall x \in \mathbb{R}, [p(x) \longrightarrow q(x)]$

if $x = -4$, then

$p(-4) = |-4| = 4 > 3$ is true

but $q(-4) = -4 > 3$ is false

$\therefore p \longrightarrow q$

$p(x) \longrightarrow q(x)$ is false as p is true & q is false

$\therefore$ Given Statement is false

Converse of the statement

$\forall x \in \mathbb{R}, [q(x) \longrightarrow p(x)]$

In words real number x, if $x \geq 3$, then $|x| > 3$. This statement is true (obvious)

Inverse of the stat^n

$\forall x_2 \in \mathbb{R}, [\sim p(x) \longrightarrow \sim q(x)]$

In words

"For every real number x, if $|x| \leq 3$, then $x < 3$". This statement is true

(OR Converse and inverse are logically equivalent)

Contrapositive of the statement

$\forall x \in \mathbb{R}, [\sim q(x) \longrightarrow \sim p(x)]$

In words

"For every real number x, if $x < 3$, then $|x| \leq 3$

This statement is false ($\because$ take $x = -4$

$-4 < 3$ but

$|-4| = 4 \not\leq 3$)

OR (Conditional and contrapositive are logically equivalent).