10/2/2020

UNIT 1

MATHEMATICAL LOGIC

$(p \vee q) \equiv (q \vee p)$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$p \vee (q \vee r) \equiv (p \vee q) \vee r$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

$\sim(p \rightarrow q) \equiv p \wedge \sim q$ or $\sim p \vee q$

Suppose $p \rightarrow q$ is given proposition

i. $q \rightarrow p$ → converse

ii. $\sim p \rightarrow \sim q$ → inverse

iii. $\sim q \rightarrow \sim p$ → Contrapositive.

Conjuctive simplification

$(p \wedge q) \Rightarrow p$

Disjunctive Amplification

$p \Rightarrow (p \vee q)$

p, q, r

$$\frac{\begin{array}{c} p \rightarrow q \\ q \rightarrow r \end{array}}{p \rightarrow r}$$

$p \wedge (p \rightarrow q) \Rightarrow q$

$(p \rightarrow q) \wedge \sim q \Rightarrow \sim p$

1. Let p and q be primitive stmts for which $p \to q$ is false. Determine the truth values of the foll compound proposition.

i. $p \wedge q$ ii. $\sim p \vee q$ iii. $\sim q \to \sim p$

p	q	$p \to q$	$p \wedge q$	$\sim p \vee q$	$\sim q \to \sim p$
T	T				
T	F	F	F	F	F
F	T				
F	F				

2. PT $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	Given P
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

3. $[p \to (q \wedge r)] \Leftrightarrow [(p \to q) \wedge (p \to r)]$ using truth table

Sol^n.

p	q	r	$q \wedge r$	$p \to q \wedge r$	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Hence verified

11/2/2020

1." If x is not a real no then it is not a rational no. and not an irrational no.

$\sim [\sim p \to (\sim q \wedge \sim r)] \Rightarrow \sim p \wedge \sim (\sim q \wedge \sim r)$

Sol^n: p : x is a real no.

q : x is a rational no.

r : x is a irrational no.

$\Rightarrow \sim p \wedge (q \vee r)$

i.e. x is not a real no and it is a rational no. or it is an irrational no.

2. ~~consider~~ $(p \vee q) \wedge \sim(\sim p \wedge q) \Leftrightarrow p$

sol^n: consider $(p \vee q) \wedge \sim(\sim p \wedge q)$

$(p \vee q) \wedge (p \vee \sim q)$ De Morgan's.

$p \vee (q \wedge \sim q)$

$p \vee F_0$

p

3. PT $[(p \to q) \wedge \sim q] \Rightarrow \sim p$

sol^n: By Modus Tollen's ~~ief~~

if $p \to q$ is true and q is false then p is false

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

4. If I study p then I don't fail an exam q

If I don't fail in exam q, my father r gifts me 2 wheeler

Soln. Rule of syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r. \end{array}$$

5. If you invest in the stock market p then you will get rich q

If you get rich q then you will be happy r.

6. Prove the following argument is valid

$$\begin{array}{l} p \to r \\ q \to r \\ \hline (p \vee q) \to r \end{array}$$

soln: $(p \to r) \wedge (q \to r) \Rightarrow (p \vee q) \to r$

Consider $(p \to r) \wedge (q \to r)$

$p \wedge (\sim p \vee q) \vee$

$(p \wedge p \rightarrow q) \Rightarrow q$

$q \wedge \sim(p \wedge q)$

$(q \wedge \sim p) \wedge (q \wedge \sim q)$

$F \quad \wedge F$

$(\sim p \wedge p) \vee (p \wedge q)$

$F_0 \vee (p \wedge q)$

$p \wedge (\sim p \vee q) \vee (q \wedge \sim(p \wedge q)) \Leftrightarrow q$

$\vee \quad F_0$

$p \wedge \sim(p \rightarrow q)$

19-2-2020

Prove that the following arguments are valid

1. Smoking is healthy
If smoking is healthy then cigarettes are prescribed by doctors

---

$\therefore$ Cigarettes are prescribed by doctors

soln. $p \rightarrow$ Smoking is healthy
$q \rightarrow$ Cigarettes are prescribed by doctors.

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$$

By Modus Pones the argument is valid

5. Continued.
p : invest in stock market
q : you will be rich
r : you will be happy.

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Rule
By Law of Syllogism.

2. Prove that $(p \vee q) \wedge \sim(\sim p \wedge q) \Longleftrightarrow p$

Sol^n. $(p \vee q) \wedge (\sim p \vee \sim q)$

$\Longleftrightarrow p \vee (q \wedge \sim q)$ Distribution Law

$\Longleftrightarrow p \vee F_0$ Identity Law

$\Longleftrightarrow p$ Domination.

3. Prove that $[p \vee (q \vee r) \wedge \sim q] \Rightarrow p \vee r$.

Sol^n. Consider $p \vee (q \vee r) \wedge \sim q$

$\cancel{\Longleftrightarrow p \vee (q\, r \vee q) \wedge \sim q}$

$\cancel{p \vee r}$

$\Rightarrow p \vee \underline{(q \vee r) \wedge \sim q}$

$\Rightarrow p \vee r$. Rule of disjunction syllogism.

4. Simplify : $(p \vee q) \wedge (\sim p \wedge q)$

Sol^n: Consider $(p \vee q) \wedge (\sim p \wedge q)$

$p \wedge (\sim p \wedge q)$ Disjunctive Amplification

$(p \wedge \sim p) \wedge q$ - Associative Law.

$= F \wedge q = F_0$.

5. $p \vee \sim(p \wedge q)$

Sol^n. Consider

$$p \vee \sim(p \wedge q)$$

$$\Rightarrow p \vee \sim p$$

$$\Rightarrow T_0$$

6. PT $(p \to q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim(q \vee p)$

Sol^n.

Consider

$$(p \to q) \wedge \underline{[\sim q \wedge (r \vee \sim q)]}$$

$(p \to q) \wedge \sim q$ Absorption

7. PT the following stmt.s are valid

i. $\sim p \to q$

$\sim q$

$\overline{p}$

By Modus Tollen's

$p \to q$

$\sim q$

$\overline{\sim p.}$

According to Modus Tollen's the given stmt is valid.

8. $$\begin{array}{l} p \to r \\ q \to r \\ \hline (p \vee q) \to r. \end{array}$$

consider : $(p \to r) \wedge (q \to r)$

$\Leftrightarrow (\sim p \vee r) \wedge (\sim q \vee r)$

$\Leftrightarrow (r \vee \sim p) \wedge (r \vee \sim q)$

$\Leftrightarrow r \vee (\sim p \wedge \sim q)$

$\Leftrightarrow \sim(p \vee q) \vee r.$

$\Leftrightarrow (p \vee q) \longrightarrow r.$

9. $$\begin{array}{l} p \to (q \to r) \\ \sim q \to \sim p \\ p \\ \hline r \end{array}$$

soln : $p \to (q \to r) \wedge (\sim q \to \sim p) \wedge p.$

$p \to (q \to r) \wedge p \wedge (\sim q \to \sim p)$

$p \wedge (p \to (q \to r)) \wedge (\sim q \to \sim p)$

$(q \to r) \wedge (p \to q)$

$(p \to q) \wedge (q \to r) \Leftrightarrow p \to r \Leftrightarrow \sim p \vee r \Rightarrow r \vee \sim p$

$\Rightarrow r.$

3.3.2020

2. If all triangles are right angled, then no triangle is equiangular

Soln: $p(x)$: x is right angled

$q(x)$: x is equiangular

$\{\forall x \in T, p(x) \rightarrow \forall x \in T, \sim q(x).\}$

negation of conditional is

$\sim\{\forall x \in T, p(x) \rightarrow \forall x \in T, \sim q(x)\}$

$\sim(p \rightarrow q) \Rightarrow \cancel{p \rightarrow \sim q} \quad p \wedge \sim q$

$\cancel{\{\forall x \in T, p(x)\} \wedge \sim\{\forall x \in T, \sim q(x)\}}$

$\sim$ of $\forall \rightarrow \exists$

$\therefore \forall x \in T, p(x) \wedge \exists x \in T, q(x)$

"All triangles are right angled and some are equiangular.

Prove the following

1. $\forall x, p(x) \rightarrow \exists x, p(x)$

2. $\forall x [p(x) \vee q(x)] \rightarrow \forall x\, p(x) \vee \exists x\, q(x)$

1. Consider

$\forall x . p(x) \Rightarrow p(x)$ is true for every $x \in S$

$\Rightarrow p(a)$ is true for some $a \in S$

$\Rightarrow p(a)$ is true for some $x = a \in S$.

$\Rightarrow p(x)$ is true for some x

$\Rightarrow \exists x, p(x)$.

2. Consider

$\forall x [p(x) \vee q(x)]$

$\Rightarrow p(x)$ or $q(x)$ is true for every $x \in S$

$\Rightarrow \{p(x)$ is true for every $x \in S\} \vee \{q(x)$ is true for every $x \in S\}$

$\Rightarrow \{p(x)$ is true for every $x \in S\} \vee \{q(a)$ is true for some $a \in S\}$

$\Rightarrow \{ p(x) \text{ is true for every } x \in S \} \vee$
$\{ q(x) \text{ is true for some } x \in S \}$

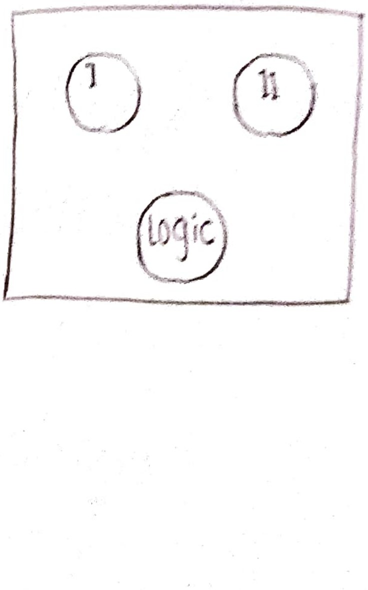
$\Rightarrow \cancel{p(x)} \{\forall x, p(x)\} \vee \{\exists x, q(x)\}$

Find whether the following argument is valid.

1. No engineering student of I ~~and~~ or II Sem studies logic.
   Anil is an engg. student who studies logic

   ∴ Anil is not in II Semester.

Soln: $p(x)$: x is in I Sem



$q(x)$: x is in II Sem

$r(x)$: x studies logic.

$$\forall x, \{p(x) \vee q(x)\} \rightarrow \sim r(x)$$
$$r(a)$$
$$\therefore \sim q(a)$$

Consider

$$\forall x, [\{p(x) \vee q(x)\} \rightarrow \sim r(x)] \wedge r(a) \Rightarrow \sim q(a)$$

Consider

$\forall x \{[p(x) \vee q(x)] \rightarrow \sim r(x)\} \wedge r(a)$

$\Rightarrow \{[p(a) \vee q(a)] \rightarrow \sim r(a)\} \wedge r(a)$

$\Rightarrow \{\sim[\sim r(a)] \rightarrow \sim[p(a) \vee q(a)]\} \wedge r(a)$ Contropositive

$\Rightarrow [r(a) \rightarrow \sim\{p(a) \vee q(a)\}] \wedge r(a)$

$\Rightarrow r(a) \wedge \{r(a) \rightarrow \sim[p(a) \vee q(a)]\}$ commutati

$p \wedge (p \rightarrow q) \Rightarrow q$ Modus Pones.

$\Rightarrow \sim(p(a) \vee q(a))$

$\Rightarrow \sim p(a) \wedge \sim q(a)$

$\Rightarrow \sim q(a) \wedge \sim p(a)$

$\Rightarrow \sim q(a)$ Conjunctive simplification.

Prove that the following argument is valid.

$\forall x \; [p(x) \rightarrow q(x)]$

$\forall x \; [q(x) \rightarrow r(x)]$

$\forall x \; [p(x) \rightarrow r(x)]$

Sol^n: According to Rule of Syllogism above argument is valid.

Prove the following argument is valid

$$\forall x \; [p(x) \rightarrow \{q(x) \land r(x)\}]$$
$$\forall x \; [p(x) \land s(x)]$$
$$\overline{\hspace{6cm}}$$
$$\therefore \forall x \; [r(x) \land s(x)]$$

Sol^n:

$\Rightarrow \{p(x) \land [p(x) \rightarrow \{q(x) \land r(x)\}]\} \land s(x)$

~~$\{ s(x) \land [\quad]\}$~~

$\Rightarrow \{q(x) \land r(x)\} \land s(x)$ — Modus Pones

$q(x) \land [r(x) \land s(x)]$ — Associative

$\Rightarrow [r(x) \land s(x)] \land q(x)$ — Commutative

$\Rightarrow r(x) \land s(x)$ — Conjunctive Simplification

$\therefore$

Method of Proof :

For any given condition of the form $p \rightarrow q$ the process of establishing that the condition is true by using the ~~p~~ rules or laws of logic constitutes a proof a Condition

* **Direct proof :**

The direct method of proving the condition $p \rightarrow q$ is true is as below.

1. Hypothesis : (Assumption) p is true
2. Analysis : Starting with hypothesis and applying rules of logic infere that q is true
3. Conclusion : $p \rightarrow q$ is true.

This is known as method of direct proof.

Indirect proof :

For the given Condition $p \rightarrow q$ show that the contrapositive $\sim q \rightarrow \sim p$ is true

* Proof by Contradiction.

The steps are as below.

1. Hypothesis : Assume that $p \to q$ is false
   i.e. Assume p is true and q is false
2. Analysis : Starting with the hypothesis that q is false and by applying rules of logic and other known facts prove that p is also false.

   This contradicts the assumption that p is true.
3. Conclusion : Because of the contradiction of the analysis we infere that $p \to q$ is true.

4-3-2020

Problems on Direct proof

1. The square of an odd integer is an odd integer

Soln : If n is an odd integer then n^2 is an odd integer

p : n is odd integer

q : n^2 is odd integer

Direct proof

Assume p is true

i.e n is an odd integer

$\Rightarrow n = 2K+1$, K is an integer

$n^2 = (2K+1)^2$

$= 4K^2 + 4K + 1$

$= 4(K^2+K) + 1$

$\therefore n^2$ is odd integer

$\therefore p \rightarrow q$ is true

2. Prove that for all integers K and l, if K and l are both odd then $K+l$ is even and Kl is odd

Soln:

p : K and l are odd

q : $K+l$ is even ~~and Kl is odd~~

r : Kl is odd.

To prove : $p \rightarrow (q \wedge r)$

Assume p is true

i.e. k and l are odd

$k = 2m+1$, m is integer

$l = 2n+1$, n is integer

Now, $\cancel{2}\, k+l = (2m+1) + \cancel{2m}\, (2n+1)$

$= 2m + 2n + 2$

$= 2(m+n+1)$

$\therefore k+l$ is even

Hence q is true.

Now, $kl = (2m+1)(2n+1)$

$= 4mn + 2m + 2n + 1$

$= 2(mn+m+n) + 1$

$\therefore kl$ is odd.

Hence r is true.

Hence $p \rightarrow (q \wedge r)$ is true.

Problems on Method of Contradiction:

1. If n is an odd integer then $n+11$ is an even integer.

Sol^n.

p : n is an odd integer

q : $n+11$ is an even integer

Contradiction:

Assume $p \to q$ is false.

i.e. p is true and q is false

q is false i.e. $\cancel{n}$ $n+11$ is an odd integer

$n+11 = 2K+1$, K is integer

$n = 2K-10$

$n = 2(K-5)$.

$\therefore$ n is an even integer

p ~~is~~ false which contradicts the assumption that p is true

$\therefore$ $p \to q$ is true.

2. For every integer n, if n^2 is odd then n is odd

Sol^n: p : n^2 is odd

q : n is odd

Contradiction

Assume $p \to q$ is false

i.e. p is true and q is false.

q is false i.e. n is even.

$n = 2k$ k is integer

Now $n^2 = (2k)^2$

$n^2 = 4k^2$

$\therefore$ n^2 is even

p is false which contradicts the assumption that p is true

$\therefore$ $p \to q$ is true.

Problems on Indirect Proof

1. The product of 2 even integer is an even integer

Soln If a and b are even integer then ab is an even integer.

p : a and b are even integers.

q : ab is even integer

Indirect proof : $\sim q \rightarrow \sim p$

Assume $\sim q$ is true

i.e. ab is an odd integer

ab is not divisible by 2

then a is not divisible by 2, also b is not divisible by 2

$\Rightarrow$ a is odd integer and b is odd integer

i.e. p is false

$\sim p$ is true

Hence $\sim q \rightarrow \sim p$ is true.

$p \rightarrow q$ is true.

6-3-2020

2 PT for all real no.s x and y, if $x+y >= 100$ then x ~~is~~ ≥ 50 and or $y \geq 50$.

Sol^n: $p : x+y \geq 100$

$q : x \geq 50$ $\quad r : y \geq 50$

Indirect Proof $p \rightarrow (q \vee r)$

$\sim(q \vee r) \rightarrow \sim p$.

i.e. $(\sim q \wedge \sim r) \rightarrow \sim p$.

Assume $(\sim q \wedge \sim r)$ is true.

i.e. $x \leq 50$ and $y \leq 50$

$x + y \leq 100$.

i.e. p is false

$\sim p$ is true.

Hence $\sim(q \vee r) \rightarrow \sim p$

$\therefore$ $p \rightarrow (q \vee r)$ is true

Stirling Number :

If A and B are finite set with Cardinality of A = m and |B| = n where m ≥ n then the number of onto functions from A to B is given by .

$$P(m,n) = \sum_{k=0}^{n} (-1)^k ({}^nC_{n-k}) (n-k)^m$$

~~with~~ and $\frac{P(m,n)}{n!}$ is called the stirling number of Second Kind and is denoted as $S(m,n)$

$$S(m,n) = \frac{1}{n!} \sum_{k=0}^{n} (-1)^k ({}^nC_{n-k})(n-k)^m$$

- The Stirling number represents the no. of ways in which it is possible to assign m distinct objects into n identical places with no place left empty.

★ Note : $S(m,1) = 1$

$S(m,m) = 1$ for $m \geq 1$.

· The no. of possible ways to assign m distinct objects to n identical places with empty places allowed is given by.

$$P(m) = \sum_{i=1}^{n} S(m,i) \quad , \quad m \geq 1$$

1. Evaluate $S(5,4)$, $S(8,6)$.

Sol^n: i. $m = 5$, $n = 4$

$$S(5,4) = \frac{1}{n!}\sum_{K=0}^{n} (-1)^K \left({}^{n}C_{n-K}\right)(n-K)^m$$

$$= \frac{1}{4!}\Big[(-1)^0\, {}^4C_4 * (4-0)^5 +$$

$$(-1)^1\, {}^4C_3 * (4-1)^5 +$$

$$(-1)^2\, {}^4C_2 * (4-2)^5 +$$

$$(-1)^3\, {}^4C_1 * (4-3)^5 +$$

$$(-1)^4\, {}^4C_0 \times (4-4)^5\Big]$$

$$= \frac{1}{24}\Big[-1 \times 1 \times 1024 + (-1) \times 4 \times 243$$

$$+ 1 \times 6 \times 32 + (-1) \times 4 \times 1 + 0\Big]$$

$$= \frac{1}{24}\Big\{ -\frac{226}{3}$$

2. $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{w, x, y, z\}$

Sol^n. $|A| = 6$ $\quad |B| = 4$

$P(6, 4) = ?$

$$P(m, n) = \sum_{K=0}^{n} (-1)^K \, {}^nC_{n-K} \, (n-K)^m$$

$$= \sum_{K=0}^{4} (-1)$$

$$= \Big[(-1)^0 \, {}^4C_4 \, (4-0)^6 +$$
$$(-1)^1 \, {}^4C_3 \, (4-1)^6 +$$
$$(-1)^2 \, {}^4C_2 \, (4-2)^6 +$$
$$(-1)^3 \, {}^4C_1 \, (4-3)^6$$
$$(-1)^4 \, {}^4C_0 \, (4-4)^6 \Big]$$

$$= \Big[- \cancel{1} \times \cancel{6}\, 1 \times 4096 - 4 \times 729 + 6 \times 64$$
$$- 4 \times 1 + 0 \Big]$$

$$= -6632$$

3. There are 6 programmers who can assist 8 executives in how many ways can the executives be assisted so that each programmer assist at-least one executive.

Soln:

$$P(m,n) = n!\,(S(m,n))$$

$$|m| = 8 \qquad |n| = 6$$

$$P(m,n) = \frac{\cancel{1}\,n!}{\cancel{n!}} \sum_{k=0}^{n} (-1)^k \left({}^nC_{n-k}\right)(n-k)^m$$

$$= 6!\left[(-1)^0\,{}^6C_6\,(6-0)^8 + \right.$$
$$(-1)^1\,{}^6C_5\,(6-\cancel{3}1)^8 +$$
$$(-1)^2\,{}^6C_4\,(6-2)^8 +$$
$$(-1)^3\,{}^6C_3\,(6-3)^8 +$$
$$(-1)^4\,{}^6C_4\,(6-4)^8 +$$
$$(-1)^5\,{}^6C_5\,(6-5)^8 +$$
$$\left.(-1)^6\,{}^6C_6\,(6-6)^8\right]$$

$$= 6!\left[-1\times\right.$$

$$= 191520$$

3. Let $A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3, 4, 5, 6\}$

a. find how many functions are there from A to B

b. How ~~may~~ many of these are one to one

c. How many are onto.

i. find how many functions are there from B to A.

ii. how many of these are onto

Soln.: $|A| = m = 4$.

$|B| = n = 6$

a. The no. of possible functions from A to B

$n^m = 6^4 = 1296$.

b. There will be $\frac{n!}{(n-m)!}$ no. of one to one.

functions from A to B.

$$\frac{6!}{(6-4)!} = 360$$

11-3-2020.

Prove that if 30 dictionaries in a library contain a total of 61327 pages then atleast one of the dictionaries must have atleast 2045 pages.

soln:

Pigeon Hole

If m pigeon occupy n pigeon holes then atleast 1 ~~k~~pigeon hole must contain p+1 or more pigeons. where $p = \left\lfloor \frac{m-1}{n} \right\rfloor$

$$p = \left\lfloor \frac{61327-1}{30} \right\rfloor = \lfloor 2044.2 \rfloor = 2044.$$

Treating pages as pigeons and dictionaries as pigeon holes by pigeon hole principle atleast one of the dictionary must contain p+1 or more pages where $p = 2044$.

ST if any 5 no.s from 1 to 8 are chosen then 2 of them will have their sum = 9

sol^n. Let us consider the following sets

$A_1 = \{1, 8\}$ $A_2 = \{2, 7\}$ $A_3 = \{3, 6\}$

$A_4 = \{4, 5\}$

These are the only sets containing the no.s from 1 to 8 whose sum is 9

Since every no. from 1 to 8 belongs to one of these sets each of the 5 no. chosen must belong to one of the sets. also there are only 4 sets, 2 of the 5 chosen no.s have to belong to the same set.

And these 2 no.s have their sum = 9

Principle of Inclusion and Exclusion:

If S is a finite set A and B are subsets of S then $|A \cup B| = |A| + |B| - |A \cap B|$

$\bar{A} \cap \bar{B} = |\overline{A \cup B}| = S - |A \cup B|$

$= S - \{|A| + |B| - |A \cap B|\}$

$= S - |A| - |B| + |A \cap B|$ —— 2

One 1 is known as addition principle and 2 is known as principle and exclusion principle

For n sets $|\bar{A}_1 \cap \bar{A}_2 \ldots \cap \bar{A}_n| = S - \Sigma |A_i| +$

$\Sigma |A_i \cap A_j| -$

$\Sigma |A_i \cap A_j \cap A_k| +$

$(-1)^n (A_1 \cap A_2 \cap A_3 \ldots \cap A_n)$

1 A computer company requires 30 programmers to handle system programming jobs and 40 programmers for applicat^n programming jobs. If the company appoints 55 programmers to carry out these jobs i] how many of these performs jobs of both type

soln:

ii. how many handle only system programing job

ii. how many handle only application programming job.

soln $|S| = 55 = |A \cup B|$

$|A| = 30 \qquad |B| = 40$

i. $|A \cap B| = -|A \cup B| + |A| + |B|$

$= -55 + 30 + 40$

$= 15$

ii $|A| - |A \cap B| = 30 - 15 = 15$

iii. $|B| - |A \cap B| = 40 - 15 = 25$

2. 30 cars are assembled in a factory, the options available are a music system, AC and power windows. It is known that 15 of the cars have music system, 8 AC, and 6 have power window further 3 have all the options

Determine how many cars do not have any options at all

12-3-2020:

sol^n:

$|S| = 30$

$|A_1| = 15$

$|A_2| = 8$

$|A_3| = 6$.

$|A_1 \cap A_2 \cap A_3| = 3$.

To find: $|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3|$

$|\bar{A} \cap \bar{B} \cap \bar{C}| = |S| - [\Sigma |A_i|] + [\Sigma |A_i \cap A_j|] - |A_1 \cap A_2 \cap A_3|$

$= 30 - 29 + \Sigma |A_i \cap A_j| - 3$

$= \Sigma A_i \cap A_j - 2$ ——— 1.

We know

$A_1 \cap A_2 \cap A_3$ is a subset of $A_i \cap A_j$ for $i, j = 1, 2, 3$

ie. Each of $|A_i \cap A_j|$ which are 3 in number is greater than or equal to $|A_1 \cap A_2 \cap A_3|$

$\therefore \sum |A_i \cap A_j| >= 3|A_1 \cap A_2 \cap A_3|$

$\geq 3 \times 3$

$\therefore \sum |A_i \cap A_j| \geq 9$ —— 2.

using 2 in 1

$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| \geq 9 - 2$

$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| \geq 7$

Hence there will greater than or equal to 7 cars which do not have any options.

3. In how many ways can the 26 letters of the english alphabet be permutted so that none of the patterns CAR, DOG, PUN and BYTE occurs

Soln: Let S denote the set of all the permutations of 26 letters i.e.

$|S| = 26!$

Let A_1 be the set of all the permutations in which CAR appears., A_2 is the set of all permutations in which DOG III'ly A_3 and A_4 has all the permutations in which PUN and BYTE appears

The set A_1 has the cardinality $|A_1| = 24!$

The word CAR consists of 3 letters which form a single block.

set A_1 consists of all permutations which contain this single block and 23 remaining letters.

III'y $|A_2| = 24!$

$|A_3| = 24!$

$|A_4| = 23!$

$|A_1 \cap A_2| = (26 - 6 + 2)!$

$= 22!$

$|A_1 \cap A_3| = |A_2 \cap A_3| = 22!$

Also we can find that $|A_1 \cap A_2| = |A_1 \cap A_3| = |A_2 \cap A_3| =$ is $22!$.

$|A_1 \cap A_4| = (26 - 7 + 2)! = 21!$

III'y $|A_2 \cap A_4| = |A_3 \cap A_4| = 21!$

$|A_1 \cap A_2 \cap A_3| = (26-9+3)! = 20!$

$|A_2 \cap A_3 \cap A_4| = (26-10+3)! = 19!$

$|A_1 \cap A_3 \cap A_4| = 19!$

$|A_1 \cap A_2 \cap A_4| = 19!$

$|A_1 \cap A_2 \cap A_3 \cap A_4| = (26-13+4)! = 17!$

We need to find

$$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4| = |S| - \sum |A_i| + \sum |A_i \cap A_j| - \sum |A_i \cap A_j \cap A_k| + |A_1 \cap A_2 \cap A_3 \cap A_4|$$

$$\cancel{= 26! - 71! + 108! - 77! + 17}.$$

4. Find the no. of non-negative integer solution of the equation $x_1 + x_2 + x_3 + x_4 = 18$ under the condition $x_i \leqslant 7$ for $i = 1, 2, 3, 4$ '

Note:

$$C(n+r-1, r) = \frac{(n+r-1)!}{r!\,(n-1)!}$$

represents the no. of non negative integer solu^n of the equ^n $x_1+x_2+x_3+\ldots+x_n=r$.

sol^n Let S denote the set of all non negative integer solution of the given equ^n.

The number of such solution is

$$C(4+18-1,18) = C(21,18)$$

$$\therefore |S| = C(21,18)$$

Let A_1 be the subset of S containing non negative integer sol^n of the given equ^n with the condition $x_1 > 7$ $x_2, x_3, x_4 > 0$

$A_1 = \{(x_1, x_2, x_3, x_4) \in S \mid x_1 > 7\}$

$A_2 : \{(x_1, x_2, x_3; x_4) \in S \mid x_2 > 7\}$

$A_3 : \{(x_1, x_2, x_3, x_4) \in S \mid x_3 > 7\}$

$A_4 = \{ \qquad \in S \mid x_4 > 7\}$

We now require to find cardinality of

$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4|$

Let us set $y_1 = x_1 - 8$ but $x_1 > 7$

$\Rightarrow x_1 \geq 8$

$\therefore y \geq 0$

i.e. We shall have

$$x_1 + x_2 + x_3 + x_4 = 18$$

$$y_1 + x_2 + x_3 + x_4 = 10$$

The no. of such non-negative integer solutions will be

$$C(4+10-1, 10) = C(13, 10)$$

$$|A_1| = C(13, 10).$$

$|||^{ly}$ because of symmetry

$$|A_2| = |A_3| = |A_4| = C(13, 10)$$

Now let us take

$y_1 = x_1 - 8$ $\quad$ $y_2 = x_2 - 8$

$y_1 \geq 0$ $\quad$ $y_2 \geq 0$

$$y_1 + y_2 + x_3 + x_4 = 2$$

$\therefore$ The no. of such non-negative integer sol^{n}s will be

$$C(4+2-1, 2) = C(5, 2)$$

$$|A_1 \cap A_2| = C(5, 2)$$

By symmetry $|A_1 \cap A_3| = |A_1 \cap A_4| = |A_2 \cap A_3| = |A_2 \cap A_4|$
$= |A_3 \cap A_4| = C(5,2)$

In the given equat^n more than 2 x_i's cannot be greater than 7 simultaneously
hence

$$|A_1 \cap A_2 \cap A_3| = |A_2 \cap A_3 \cap A_4| = |A_1 \cap A_3 \cap A_4| =$$
$$|A_1 \cap A_2 \cap A_4| = |A_1 \cap A_2 \cap A_3 \cap A_4|$$
$$= 0$$

$$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4| = |S| - \Sigma|A_i| + \Sigma|A_i \cap A_j| - \Sigma|A_i \cap A_j \cap A_k|$$
$$+ |A_1 \cap A_2 \cap A_3 \cap A_4|$$
$$= C(21,18) + 6(C(5,2))$$
$$- 4C(13,10)$$