1: The MIT soccer team has 2 games scheduled for one weekend. It has a 0.4 probability of not losing the first game, and a 0.7 probability of not losing the second game, independent of the first. If it does not lose a particular game, the team is equally likely to win or tie, independent of what happens in the other game. The MIT team will receive 2 points for a win, 1 for a tie, and 0 for a loss. Find the PMF of the number of points that the team earns over the weekend.

|  |  |  |  |
| --- | --- | --- | --- |
| *2nd down, 1st across* | *0* | *1* | *2* |
| *0* | *.18* | *.06* | *.06* |
| *1* | *.21* | *.07* | *.07* |
| *2* | *.21* | *.07* | *.07* |

*Therefore:*

***0 = .18***

***1 = .27***

***2 = .34***

***3 = .14***

***4 = .07***

13: A family has 5 natural children and has adopted 2 girls. Each natural child has equal probability of being a girl or a boy, independent of the other children. Find the PMF of the number of girls out of 7 children.

*0 = 1/32*

*1 = 5/32*

*2 = 10/32*

*3 = 10/32*

*4 = 5/32*

*5 = 1/32*

PMF(X+2) = PMF(X) + 2

***2 = 1/32***

***3 = 5/32***

***4 = 10/32***

***5 = 10/32***

***6 = 5/32***

***7 = 1/32***

15: Let *K* be a random variable that takes, with equal probability 1/(2*n*+1), the integer values in the interval [-*n*, *n*]. Find the PMF of the random variable *Y* = ln *X*, where *X* = *a*|*K*|, and *a* is a positive number.

*2/(2n+1) for all but 0; 1/(2n+1) for 0.*

***P(0) = 1/(2n+1)***

***P(k) = 2/(2n+1) where k != 0.***

20: As an advertising campaign, a chocolate factory places golden tickets in some of its candy bars, with the promise that a golden ticket is worth a trip through the chocolate factory, and all the chocolate you can eat for life. If the probability of finding a golden ticket is *p*, find the mean and the variance of the number of candy bars you need to eat to find a ticket.

***Mean: 1/p***

***Variance: (1-p)/p^2***

23:

(a) A fair coin is tossed repeatedly and independently until two consecutive heads or two consecutive tails appear. Find the PMF, the expected value, and the variance of the number of tosses.

*2: 1/2*

*3: 2/4*

*4: 4/8*

*5: 8/16*

*n: (2^(n-1))/(2^n)*

***PMF = 1/2***

***Expected value = 1/0.5 = 2 + 1 = 3***

***Variance = (1-0.5)/0.5^2 = 0.5/0.25 = 2***

(b) Assume now that the coin is tossed until we obtain a tail that is immediately preceded by a head. Find the PMF and the expected value of the number of tosses.

*2: 1/4*

*3: 2/8*

*4: 3/16*

*5: 4/32*

*n: (n-1)/(2^n)*

***PMF = n\*(n-1)/(2^n)***

*Sigma (infinity, n=2): (n-1)/(2^n) = Sigma (infinity, n=2): n/2^n - Sigma (infinity, n=2): ½^n = 1/2*

***Expected value = 1/0.5 = 2***