Red-Black Tree

Balanced Binary Search Tree

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https://github.com/ferreiraluana/Data-Structure-Project

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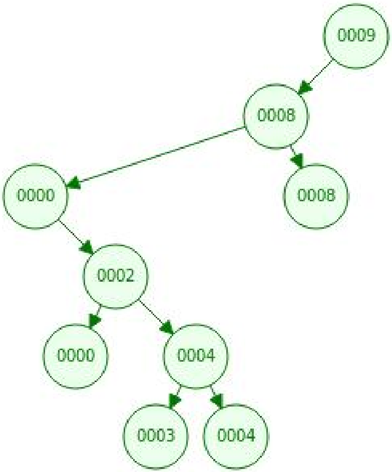
Outline

Previous in BST classes...

Let's consider adding theses numbers: 9, 8, 8, 0, 2, 0, 4, 4, 3.



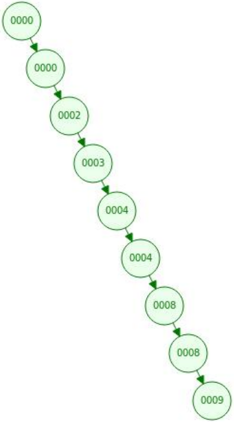
Troubles start to show up...

What if we add the same numbers in descending order?

Low efficiency:

- Search;
- Insert;
- Delete;

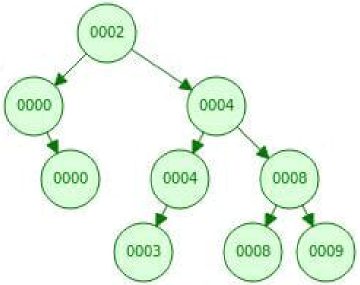
It runs $O(n)$!



AVL tree

What do we need? BALANCE!
And here it comes: AVL Tree!

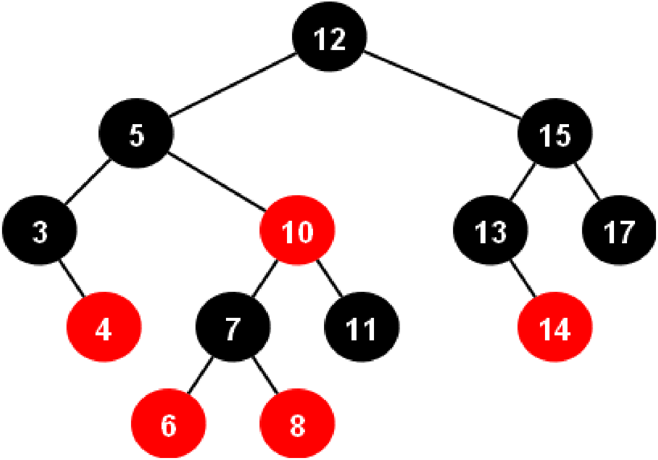
- 4 kinds of rotations (L-L, R-R, R-L, L-R);
- Balance Factor: 0, -1 or 1;
- Searching in AVL is close to O(log n).



But...

AVL needs too many rotations!!!

The Red-Black Tree



Definition

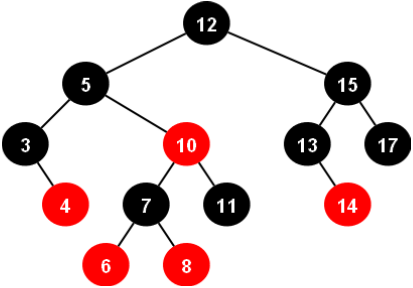
- A Binary Search Tree with an extra bit to hold the color:

$$\mathbf{RED = 1}$$
$$\mathbf{BLACK = 0}$$

- It ensures the tree remains balanced.

Red-Black Structure

```
struct redblack
{
    int item;
    int color;
    redblack *left;
    redblack *right;
}
```



Advantages of Red-Black

1. Rotations run in $O(1)$;
2. Searching, insertion and deletion run in $O(\log n)$;
3. In remotion, the RB tree rotates once (with single or double rotation), while the AVL tree can rotate $\log n$ times;

Properties

- Every node is either red or black;
- The root and leaves **(NIL's)** are black;
- If a node is **red**, then its parent is **black**;
- For each node, every path from the node to the descendant leaves contains the same number of **BLACK** nodes.

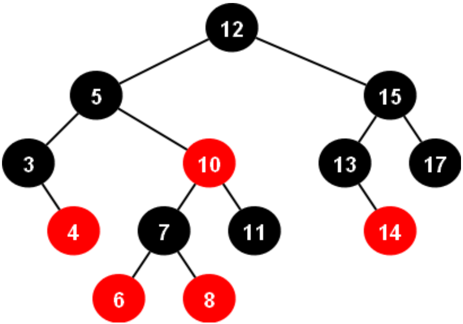
Properties

- There can't be two consecutive red nodes in a path from the root to a sub-tree;
- The properties are checked every time a operation is done in the RB tree;
- In case some property is not satisfied, rotations and color flips are done.

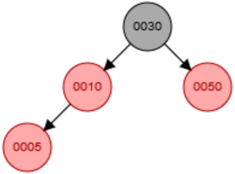
Black Height

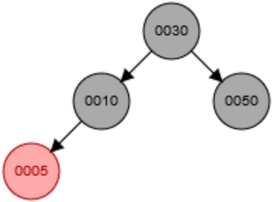
It is the number of **BLACK** nodes found until any descendant node. A red-black tree with n keys has height:

$$h \leq 2log(n+1)$$

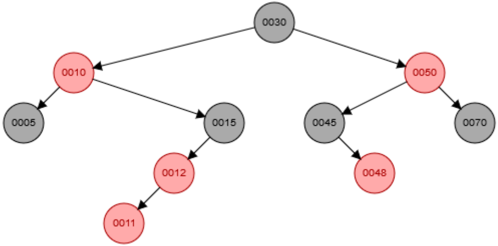


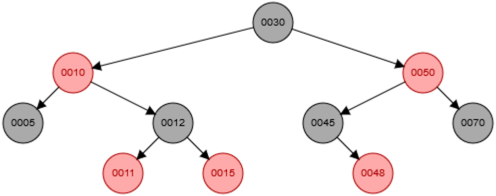
Color Flip



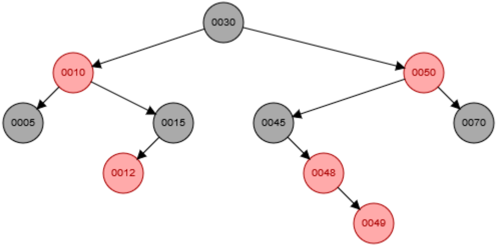


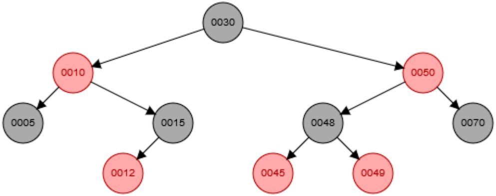
Right Rotation





Left Rotation





Abstract Data Type

```
btree* create_empty_btree();
btree* rotate_left(btree *bt);
btree* rotate_right(btree *bt);
btree* move_left_red(btree *bt);
btree* move_right_red(btree *bt);
btree* balance_factor(btree *bt);
btree* add_bt(btree *bt, int value);
btree* add_arv(btree *bt, int value);
btree* remove_bt(btree *bt, int value);
btree* remove_arv(btree *bt, int value);
int search(btree *bt, int value, int *flag);
int color(btree *bt);
void print_pre_order(btree *bt);
void color_swap(btree *bt);
```

Add

```
btree* add_bt(btree *bt, int value)
{
if(bt == NULL)
{
btree *new_btree = (btree*) malloc ( sizeof(btree) );
if(new_btree == NULL) return NULL;
new_btree → item = value;
new_btree → color = RED;
new_btree → left = NULL;
new_btree → right = NULL;
return new_btree;
}
if(value != bt → item)
{
if(value < bt → item)  bt → left = add_bt(bt → left, value);
else if(value > bt → item)  bt → right = add_bt(bt → right, value);
}
if(color(bt → right) == RED  &&  color(bt → left) == BLACK)  bt = rotate_left(bt);
if(color(bt → left) == RED  &&  color(bt → left → left) == RED)  bt = rotate_right(bt);
if(color(bt → left) == RED  &&  color(bt → right) == RED)
color_swap(bt);
return bt;
}
```

Balance Factor

btree* balance_factor(btree *bt);
{
if(color(bt $\rightarrow$ $right) == RED) \quad bt = rotate\_left(bt)$;
if(bt $\rightarrow$ $left! = NULL \quad \&\& \quad color(bt \rightarrow right) == RED \quad \&\&$ color(bt $\rightarrow$ $left \rightarrow left) == RED) \quad bt = rotate\_right(bt)$;
if(color(bt $\rightarrow$ $left) == RED \quad \&\& \quad color(bt \rightarrow right) == RED) \quad color\_swap(bt)$;
return bt;
}

Rotate Left and Rotate Right

btree* rotate_left(btree *bt)
{
btree *aux $= \mathrm{bt} \rightarrow right$;
bt $\rightarrow right = aux \rightarrow left$;
aux $\rightarrow left = bt$;
aux $\rightarrow color = bt \rightarrow color$;
bt $\rightarrow color = RED$;
return aux;
}

btree* rotate_right(btree *bt)
{
btree *aux $= \mathrm{bt} \rightarrow left$;
bt $\rightarrow left = aux \rightarrow right$;
aux $\rightarrow right = bt$;
aux $\rightarrow color = bt \rightarrow color$;
bt $\rightarrow color = RED$;
return aux;
}

Move Left Red and Move Right Red

```
btree* move_left_red(btree *bt)
{
color_swap(bt);
if(color(bt->right->left) == RED)
{
bt->right = rotate_right(bt->right);
bt = rotate_left(bt);
color_swap(bt);
}
return bt;
}
```

```
btree* move_right_red(btree *bt)
{
color_swap(bt);
if(color(bt->left->left) == RED)
{
bt = rotate_right(bt);
color_swap(bt);
}
return bt;
}
```

Remove

btree* remove_bt(btree *bt, int value)
{
if(value $<$ bt $\rightarrow$ $item$)
{
if(color(bt $\rightarrow$ $left$) $== BLACK \quad \&\& \quad color(bt \rightarrow left \rightarrow left) == BLACK) \quad bt = move\_left\_RED(bt)$;
bt $\rightarrow$ $left = remove\_bt(bt \rightarrow left, value)$; }
else {
if(color(bt $\rightarrow$ $left$) $== RED) \quad bt = rotate\_right(bt)$;
if(value $==$ bt $\rightarrow$ $item \quad \&\& \quad bt \rightarrow right == NULL) \quad \{free(bt); returnNULL; \}$
if(color(bt $\rightarrow$ $right$) $== BLACK \quad \&\& \quad color(bt \rightarrow right \rightarrow left) == BLACK) \quad bt = move\_right\_RED(bt); if(value == bt \rightarrow item)$
{
btree *aux $=$ minor_search(bt $\rightarrow$ $right$);
bt $\rightarrow$ $item = aux \rightarrow item$;
bt $\rightarrow$ $right = minor\_remove(bt \rightarrow right)$;
} else bt $\rightarrow$ $right = remove\_bt(bt \rightarrow right, value)$;
}
return balance_factor(bt);
}

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Search

int search(btree *bt, int value, int *flag)
{
if(bt $!=$ NULL)
{
if(bt $\rightarrow item == value) \quad *flag = 1;$
search(bt $\rightarrow left, value, flag);$
search(bt $\rightarrow right, value, flag);$
}
return *flag;
}

Animation

bit.ly/gifredblack
imgur.com/vV1RDz5

Conclusion

Red-Black Trees can be very useful!

1. Running time: $O(\log n)$;
2. Rotations: $O(1)$;

References

- E.Demaine, **"Introduction to algorithms"**, Lecture 10, Massachusetts Institute of technology Open Course, 2015;
- S. W. Song, **"Árvore Rubro-Negra"**, Estruturas de Dados, Universidade de São Paulo - IME/USP, 2008;
- T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein, **"Introduction to Algorithms"**, 2º edition, MIT Press & McGraw-Hill, 2001

Thank you!