



Red-Black Tree

Balanced Binary Search Tree

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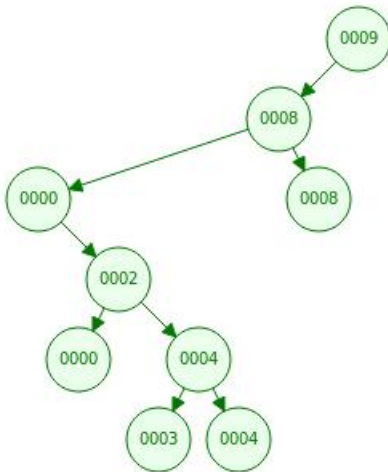
Rotations

Remove and Search



Previous in BST classes...

Let's consider adding these numbers: 9, 8, 8, 0, 2, 0, 4, 4, 3.





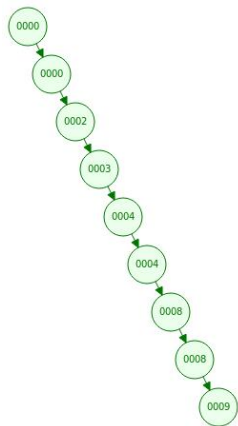
Troubles start to show up...

What if we add the same numbers in descending order?

Low efficiency:

- Search;
- Insert;
- Delete;

It runs $O(n)$!

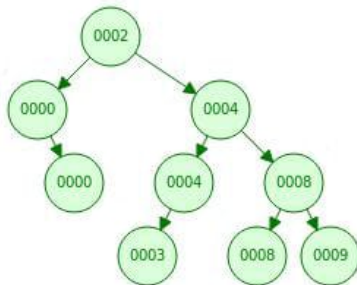




AVL tree

What do we need? **BALANCE!**
And here it comes: **AVL Tree!**

- 4 kinds of rotations (L-L, R-R, R-L, L-R);
- Balance Factor: 0, -1 or 1;
- Searching in AVL is close to $O(\log n)$.



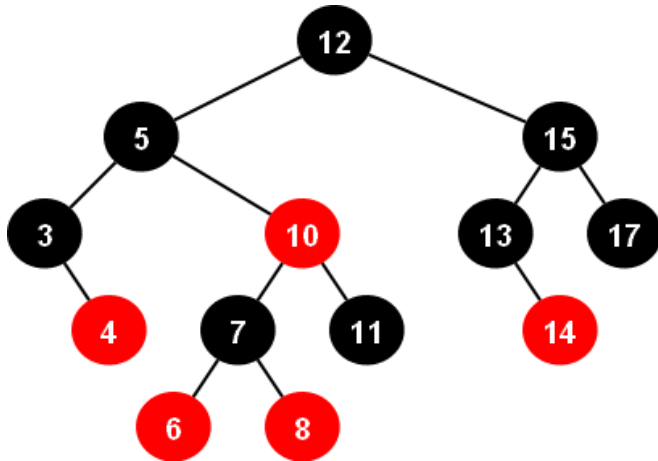


But...

AVL needs too many
rotations!!!



The Red Black-Tree





Definition

- A Binary Search Tree with an extra bit to hold the color:

RED = 1

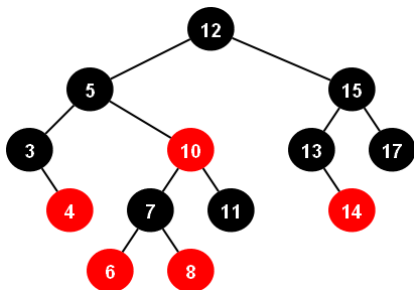
BLACK = 0

- It ensures the tree remains balanced.



Red-Black Structure

```
struct redblack
{
    int item;
    int color;
    redblack *left;
    redblack *right;
}
```





Advantages of Red-Black

- ① Rotations run in $O(1)$;
- ② Searching, insertion and deletion run in $O(\log n)$;
- ③ In remotion, the RB tree rotates once (with single or double rotation), while the AVL tree can rotate $\log n$ times;



Properties

- Every node is either red or black;
- The root and leaves (**NIL's**) are black;
- If a node is **red**, then its parent is black;
- For each node, every path from the node to the descendant leaves contains the same number of **BLACK** nodes.



Properties

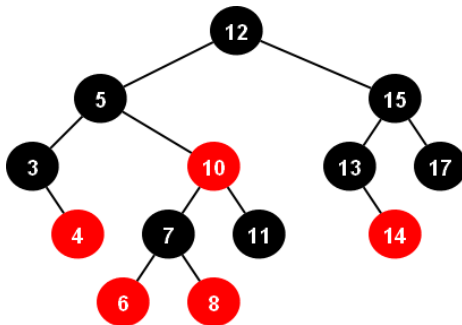
- There can't be two consecutive red nodes in a path from the root to a sub-tree;
- The properties are checked every time a operation is done in the RB tree;
- In case some property is not satisfied, rotations and color flips are done.



Black Height

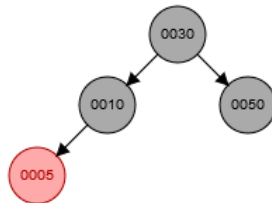
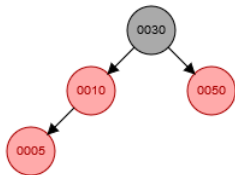
It is the number of **BLACK** nodes found until any descendant node. A red-black tree with n keys has height:

$$h \leq 2\log(n + 1)$$



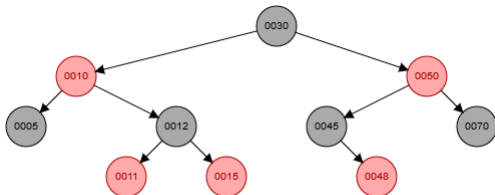
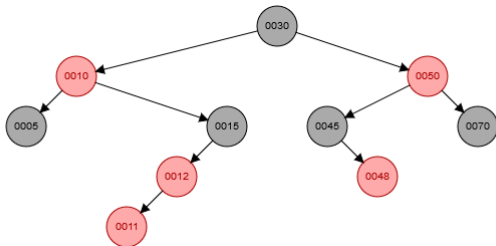


Color Flip



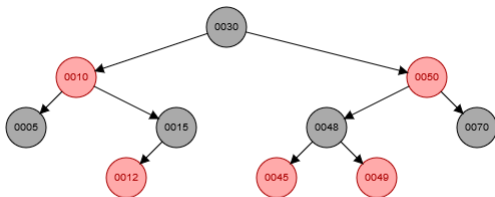
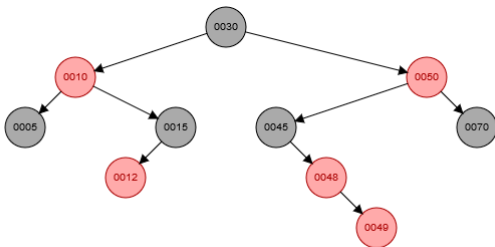


Right Rotation





Left Rotation





Abstract Data Type

```
btree* create_empty_btree();  
btree* rotate_left(btree *bt);  
btree* rotate_right(btree *bt);  
btree* move_left_red(btree *bt);  
btree* move_right_red(btree *bt);  
btree* balance_factor(btree *bt);  
btree* add_bt(btree *bt, int value);  
btree* add_arv(btree *bt, int value);  
btree* remove_bt(btree *bt, int value);  
btree* remove_arv(btree *bt, int value);  
int search(btree *bt, int value, int *flag);  
int color(btree *bt);  
void print_pre_order(btree *bt);  
void color_swap(btree *bt);
```



Add

```

btree* add_bt(btree *bt, int value)
{
    if(bt == NULL)
    {
        btree *new_btree = (btree*) malloc ( sizeof(btree) );
        if(new_btree == NULL) return NULL;
        new_btree → item = value;
        new_btree → color = RED;
        new_btree → left = NULL;
        new_btree → right = NULL;
        return new_btree;
    }
    if(value != bt → item)
    {
        if(value < bt → item)  bt → left = add_bt(bt → left, value);
        else if(value > bt → item)  bt → right = add_bt(bt → right, value);
    }
    if(color(bt → right) == RED && color(bt → left) == BLACK)  bt = rotate_left(bt);
    if(color(bt → left) == RED && color(bt → left → left) == RED)  bt = rotate_right(bt);
    if(color(bt → left) == RED && color(bt → right) == RED)
        color_swap(bt);
    return bt;
}

```



Balance Factor

```
btree* balance_factor(btree *bt);  
{  
  if(color(bt → right) == RED)  bt = rotate_left(bt);  
  if(bt → left != NULL  &&  color(bt → right) == RED  &&  
    color(bt → left → left) == RED)  bt = rotate_right(bt);  
  if(color(bt → left) == RED  &&  color(bt → right) ==  
    RED)  color_swap(bt);  
  return bt;  
}
```



Rotate Left and Rotate Right

```
btree* rotate_left(btree *bt)
{
  btree *aux = bt → right;
  bt → right = aux → left;
  aux → left = bt;
  aux → color = bt → color;
  bt → color = RED;
  return aux;
}
```

```
btree* rotate_right(btree *bt)
{
  btree *aux = bt → left;
  bt → left = aux → right;
  aux → right = bt;
  aux → color = bt → color;
  bt → color = RED;
  return aux;
}
```



Move Left Red and Move Right Red

```
btree* move_left_red(btree *bt)
{
    color_swap(bt);
    if(color(bt
    → right → left) == RED)
    {
        bt → right = rotate_right(bt →
        right);
        bt = rotate_left(bt);
        color_swap(bt);
    }
    return bt;
}
```

```
btree* move_right_red(btree *bt)
{
    color_swap(bt);
    if(color(bt
    → left → left) == RED)
    {
        bt = rotate_right(bt);
        color_swap(bt);
    }
    return bt;
}
```



Remove

```

btree* remove_bt(btree *bt, int value)
{
  if(value < bt->item)
  {
    if(color(bt->left) == BLACK && color(bt->left->left) == BLACK) bt =
      move_left_RED(bt);
    bt->left = remove_bt(bt->left, value);
  }
  else {
    if(color(bt->left) == RED) bt = rotate_right(bt);
    if(value == bt->item && bt->right == NULL) { free(bt); return NULL; }
    if(color(bt->right) == BLACK && color(bt->right->left) ==
      BLACK) bt = move_right_RED(bt); if(value == bt->item)
    {
      btree *aux = minor_search(bt->right);
      bt->item = aux->item;
      bt->right = minor_remove(bt->right);
    } else bt->right = remove_bt(bt->right, value);
  }
  return balance_factor(bt);
}

```



Search

```
int search(btree *bt, int value, int *flag)
{
  if(bt != NULL)
  {
    if(bt → item == value)    *flag = 1;
    search(bt → left, value, flag);
    search(bt → right, value, flag);
  }
  return *flag;
}
```



Animation

bit.ly/gifredblack
imgur.com/vV1RDz5



Conclusion

Red-Black Trees can be very useful!

- ① Running time: $O(\log n)$;
- ② Rotations: $O(1)$;



References

- E.Demaine, "**Introduction to algorithms**", Lecture 10, Massachusetts Institute of technology Open Course, 2015;
- S. W. Song, "**Árvore Rubro-Negra**", Estruturas de Dados, Universidade de São Paulo - IME/USP, 2008;
- T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein, "**Introduction to Algorithms**", 2º edition, MIT Press & McGraw-Hill, 2001



Thank you!