Time Series

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Outline

Macroeconomic data

- Volatility clustering
- Seasonal adjustments
- Data revisions and sources
- Unemployment rate
- Labor force participation rate

Uses of time series econometrics

Time series analysis is a highly valuable skill in industry:

- Forecasting (eg interest rates, returns to a stock, GDP growth)
- Dynamic treatment effects: impact of a policy after 1 month, 3 months, 2 years...
- Dynamic system analysis

But, we will have to deal with problems like correlation over time.

Time series data

- Generally similar to panel data, but with only one unit
- Takes the form $\{Y_t\}_{1<=t<=T}$ with covariates $\{X_t\}_{1<=t<=T}$
- For now, we assume timing is evenly spaced and there are no missing periods

Lags

- Lags are very important in time series analysis
- X_{t-1} is often a very good predictor of X_t
- More generally, the j^{th} lag of Y_t is Y_{t-j}
- We might take a difference: $\Delta_j Y_t = Y_t - Y_{t-j}$

Log variables

- Oftentimes we will use *log* variables instead of the raw values
- Two benefits:
  - Makes exponential growth linear
  - Differences in logs are (approximately) equal to the percentage change:

    $$\log(1+\alpha) \approx \alpha$$

    So long as α is “small”
  - So, $\log X_t - \log X_{t-1} = \log \dfrac{X_t}{X_{t-1}}$ is roughly the percentage change in X from $t-1$ to t

Autocorrelation

- Just like with panel data, we will often have correlation between lagged values
- First autocovariance of Y_t: $cov(Y_t, Y_{t-1})$
- First autocorrelation of Y_t: $corr(Y_t, Y_{t-1}) \equiv \rho_1$
- Can calculate for any lag j in the same way and get the j^{th} serial correlation coefficient

Forecasting and stationarity

- We will often be interested in making an out-of-sample (OOS) forecast of our variable
- This is similar to what we did before trying to target the oracle prediction
- We no longer care about causal interpretation of coefficients, OVB, etc.; we just want to get an accurate OOS prediction
- This requires that our data be *stationary* (or jointly stationary)
  - Intuitively, our out-of-sample data needs to be similar to our sample Data
  - Technically, we need the joint distribution across time to be independent of the time period we are looking at

Forecasting error

- An s periods ahead forecast, using estimated coefficients based on $t = 1, \ldots, T$, is:

$$\hat{Y}_{T+s|T}$$

- The forecast error is:

$$Y_{T+s} - \hat{Y}_{T+s|T}$$

- And we might evaluate our model based on the mean squared forecasting error:

$$MSFE = \mathbb{E}\left[\left(Y_{T+s} - \hat{Y}_{T+s|T}\right)^2\right]$$

Autoregression

- Regress Y_t on its past (lagged values)
- p^{th} order autoregression is Y_t on $Y_{t-1}, \ldots, Y_{t-p}$, denoted AR(p):

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \cdots + \beta_p Y_{t-p} + u_t$$

- Not causal!
- Can easily test of value in period $t - s$ is useful with t-test or multiple periods with F-test
- For the AR(1) model $Y_t = \beta Y_{t-1} + u_t$, where $|\beta| < 1$ and $u = 0$, covariance and correlation between t and $t - s$ are easy:

$$\text{corr}(Y_t, Y_{t-s}) = \beta^s$$

Moving averages

- Moving average model:
$$Y_t = \epsilon_t + \theta \epsilon_{t-1}$$
- Make substitutions:
$$Y_t = \epsilon_t + \theta Y_{t-1} + \theta^2 Y_{t-2} + \dots$$
- Some nice properties:
    - $Var(Y_t) = \sigma^2(1+\theta^2)$
    - $cov(Y_t, Y_{t-1}) = \theta\sigma^2$
    - $cov(Y_t, Y_{t-j}) = 0 \quad \forall j > 0$
- unfortunately, we can't estimate with OLS; have to use NLS
- Box-pierce test for serial correlation: H_0 is the series is white noise

Covariates

- We can add in other variables (and their lags) to form an *autoregressive distributed lag* (ADL) model:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \cdots + \beta_p Y_{t-p} + \beta_1 X_{t-1} + \cdots + \beta_p X_{t-r} + u_t$$

with p lags of Y and r lags of X, denoted $ADL(p, r)$.

Forecast error

- Sometimes we look at the root mean squared forecast error, which gives us a "typical" error:

$$RMSFE = \sqrt{E\left[\left(Y_{T+1} - \hat{Y}_{T+1|T}\right)^2\right]}$$

- Again, MSFE has two components:

$$MSFE = E\left(u_{T+1}\right)^2 + E\left[\left(\hat{\beta}_0 - \beta_0\right) + \left(\hat{\beta}_1 - \beta_1\right)Y_T + \left(\hat{\beta}_2 - \beta_2\right)X_T\right]^2$$

1. Unavoidable oracle uncertainty (first term)
2. Uncertainty from estimating coefficients (second term)

Forecast error

Three methods for estimating the MSFE:

1. If number of regressors is small, use the in-sample regression error, $\widehat{MSFE}_{SER} = s^2_{\hat{u}} = \frac{SSR}{T-p-1}$ which ignores the uncertainty from $\hat{\beta}$
2. If errors are homoscedastic, we can use this approximation for the full MSFE:

$$\widehat{MSFE}_{FPE} = \left(\frac{T+p+1}{T}\right) s^2_{\hat{u}} = \left(\frac{T+p+1}{T-p-1}\right)\left(\frac{SSR}{T}\right)$$

3. Method with fewest assumptions: pseudo out of sample. Create a rolling estimate through $t = s$, calculate error for $t = s+1, \ldots, T$, repeat; $\widehat{MSFE}_{\text{POOS}} = \frac{1}{P}\sum_{s=T-P+1}^{T} \tilde{u}^2_s$

Number of lags

- How can we choose number of lags p?
- We can use a bunch of t or F tests... but model will usually be too large
- *Information criterion* helps us select by allowing more bias but lower variance (kind of like with the shrinkage estimators)

$$\text{AIC}(p) = \ln\left(\frac{\text{SSR}(p)}{T}\right) + (p+1)\frac{2}{T}$$
$$\text{BIC}(p) = \ln\left(\frac{\text{SSR}(p)}{T}\right) + (p+1)\frac{\ln T}{T}$$

- BIC is consistent; AIC is not but may be good if you think more lags are more important
- Can generalize to ADL model

Misc. forecast topics

- Forecast evaluation
- Combining forecasts
- Vector autoregression (VAR):

$$\begin{aligned} Y_t &= \beta_{01} + \beta_{11} Y_{t-1} + \ldots + \beta_{1p} Y_{t-p} + \gamma_{11} X_{t-1} + \ldots + \gamma_{1p} X_{t-p} + u_{1t} \\ X_t &= \beta_{02} + \beta_{21} Y_{t-1} + \ldots + \beta_{2p} Y_{t-p} + \gamma_{21} X_{t-1} + \ldots + \gamma_{2p} X_{t-p} + u_{2t} \end{aligned}$$

- Granger causality
- Direct vs. iterative prediction

Asset returns

- Two features:
  1. Volatility clustering
  2. Fat tails
- ARCH model:

$$\begin{aligned} Y_t &= \mu + u_t \\ u_t &\sim N\left(0, \sigma_t^2\right) \\ \sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 \end{aligned}$$

- Generalized ARCH (GARCH):

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \phi_1 \sigma_{t-1}^2$$

- These capture both of our desired features

Stationarity

- All these models are assuming *stationarity*
- First threat is a *trend*, a persistent long-term movement which might be deterministic or stochastic
  - $Y_t = \beta_0 + Y_{t-1} + u_t$ where u_t is serially correlated and β_0 gives the drift
- If Y_t is a random walk, then ΔY_t is stationary
- Trends may cause:
  1. Coefficients biased to 0; bad forecast
  2. t-test doesn't work
  3. Spurious correlation between 2 random walks

Dickey-Fuller Test

- Dickey-Fuller test has H_0 of random walk vs. H_A of stationarity
- Use DF table for critical value
  - Depends if model has intercept only or also has a time trend
- If DF fails to reject and has a unit root, use first differences model

Breaks

- Second threat are structural breaks where the model itself (coefficients) changes over time
- If we suspect a break at a specific point we can test this using dummies for before and after the break

$$\begin{aligned} Y_t = {} & \beta_0 + \beta_1 Y_{t-1} + \delta_1 X_{t-1} \\ & + \gamma_0 D_t(\tau) + \gamma_1 \left[D_t(\tau) \times Y_{t-1} \right] + \gamma_2 \left[D_t(\tau) \times X_{t-1} \right] + u_t \end{aligned}$$

- Use a Chow test statistic on γ
- If we don't know when the break happens we can test dates on $\underline{t} <= t <= \bar{t}$ and take the largest Chow statistic (QLR statistic)
- The more QLR stats we compute, the larger the critical value