Econometrics Discussion Section 2

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Recap

What did we do this week?

- Review of probability and statistics (Stock & Watson chapters 2 and 3)
- Random variables and their first 4 moments
- Marginal and joint distributions
- LLN and CLT
- Properties of estimators: consistency, unbiasedness, normal approximation

Random variables

- A random variable is a *function* from a space of possible outcomes to (usually) the real numbers.
- Example:
    - Roll two dice and add up the numbers
    - Sample space is all the possible rolls of the two dice (how many?)
    - Outcome space is all the possible sums of those rolls (how many?)
    - The sum is a random variable

		FIRST DICE					
		1	**2**	**3**	**4**	**5**	**6**
SECOND DICE	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Mean

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- What is the mean value of the result from rolling one die?
- What is the mean value of the sum of rolling both dice?

Mean

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- What is the mean value of the result from rolling one die?
  - $E[X] = \frac{1}{6} \times 1 + \frac{1}{6} \times 2 + \frac{1}{6} \times 3 + \frac{1}{6} \times 4 + \frac{1}{6} \times 5 + \frac{1}{6} \times 6 = 3.5$
- What is the mean value of the sum of rolling both dice?
  - $E[X] = \frac{1}{36} \times 2 + \frac{2}{36} \times 3 + \frac{3}{36} \times 4 + \frac{4}{36} \times 5 + \frac{5}{36} \times 6 + \frac{6}{36} \times 7 + \frac{5}{36} \times 8 + \frac{4}{36} \times 9 + \frac{3}{36} \times 10 + \frac{2}{36} \times 11 + \frac{1}{36} \times 12 = 7$
  - (Could also write out using $\frac{1}{36}$ weight everywhere but easier to consolidate terms)

Variance

- The second moment of a random variable is its variance
  - Think of this as the mean distance from the mean: it measures the spread of our data
  - $Var(X) = E\left[(X - E[X])^2\right]$
  - Why do we need to square the spread?
- What is the variance of rolling one die?
- What is the variance of the sum of rolling both dice?

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- What is the variance of rolling one die?
  - $Var(X) = \frac{1}{6} \times (1 - 3.5)^2 + \frac{1}{6} \times (2 - 3.5)^2 + \cdots + \frac{1}{6} \times (6 - 3.5)^2 = \frac{35}{12} \approx 2.92$
- Similar idea for the sum of 2 dice.
- Often we will work with the *standard deviation* which is the square of the variance since the units are more interpretable.

Higher moments

- Third moment is *skewness* and tells us how symmetric our distribution is
  - What is the skewness of our example?
- Fourth moment is the *kurtosis* which measures the mass of the tails
  - Gives us an idea of the likelihood of large values

Covariance

- The covariance of two random variables tells us the strength of their *linear* (careful!) relationship
- If two random variables are independent, their covariance is 0
  - What is the covariance in our example?
- Often we will look at the *correlation* instead of the covariance, $\frac{cov(X,Y)}{(var(X)var(Y))^{1/2}}$
  - Unlike covariance, correlation is scaled between -1 and 1 and so is easily interpretable

Rules on joint distributions

- If two variables are independent, $P(X = x, Y = y) = P(X = x) * P(Y = y)$
- Conditional probability: $P(X = x|Y = y) = \frac{P(X=x,Y=y)}{P(Y=y)}$
  - This gives us the marginal distribution
  - What happens if the events are independent?
- Bayes' Rule: $P(X = x|Y = y) = \frac{P(Y=y|X=x)P(X=x)}{P(Y=y)}$
- Law of total probability: $P(X = x) = \sum_y P(X = x, Y = y)$

Second example

- New example:
    - Roll the first dice
    - If first roll ≥ 4 then we roll the second die and observe its value
    - If first roll ≤ 3 then the second value is simply set to 1
- What is the joint probability distribution?
- What is the covariance of the 2 rolls?
- What is the correlation?
- What is the marginal distribution of the two r.v.?

		FIRST DICE						
		1	2	3	4	5	6	Marginal:
SECOND DICE	1	0.17	0.17	0.17	0.03	0.03	0.03	0.58
	2	0	0	0	0.03	0.03	0.03	0.08
	3	0	0	0	0.03	0.03	0.03	0.08
	4	0	0	0	0.03	0.03	0.03	0.08
	5	0	0	0	0.03	0.03	0.03	0.08
	6	0	0	0	0.03	0.03	0.03	0.08
Marginal:		0.17	0.17	0.17	0.17	0.17	0.17	

Normal distribution

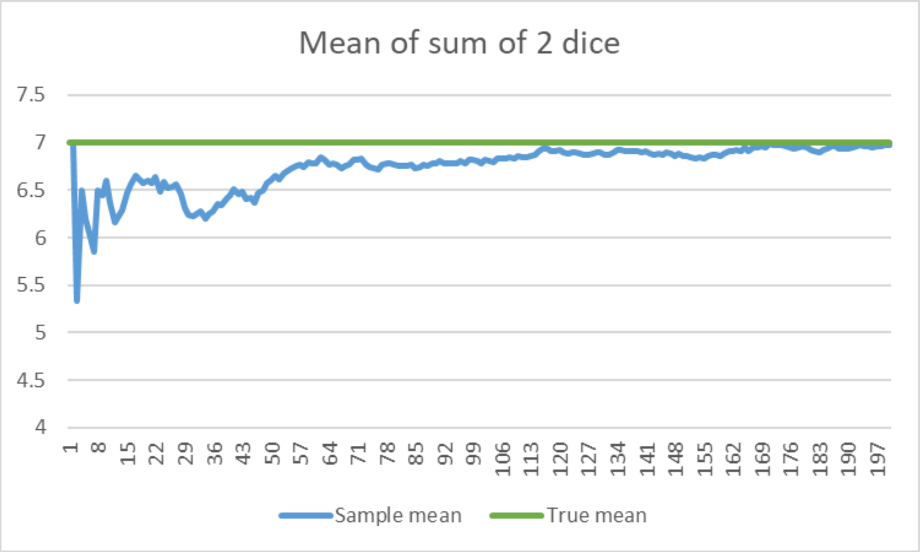
- Two parameters: mean μ and variance σ^2
- We can *standardize* a normal random variable X:
  - $Z = \frac{X-\mu}{\sigma}$ so $Z \sim N(0, 1)$
  - Careful to divide by standard deviation and not by variance!
- This will be helpful in hypothesis testing because we can consider how unlikely a given value z is to be drawn from a N(0,1) distribution

Estimation

- We will often be interested in estimating the mean of a distribution
- Example: wait time in Brody Cafe
- Natural estimator is a sample mean:
  - Take a survey of people as they walk out of Brody and ask how long they waited
  - Average the responses
- Sample mean $\bar{Y}$ is a random variable since we taking a random sample and thus $\bar{Y}$ has a sampling distribution
  - What can we say about it?

Law of Large Numbers

- $E[\bar{Y}] = \mu_Y$
- This means that $\bar{Y}$ is an *unbiased* estimator of μ_Y
- As our sample size grows, the sample mean will converge to the true mean
- $var(\bar{Y}) = \frac{\sigma_Y^2}{n}$ so that the variance of our estimator is decreasing as our sample gets larger
- So as our sample size grows, the sample mean will converge to the true mean: $\bar{Y}$ is a *consistent* estimator of μ_Y (LLN)


Mean of sum of 2 dice
7.5
7
6.5
6
5.5
5
4.5
4
1 8 15 22 29 36 43 50 57 64 71 78 85 92 99 106 113 120 127 134 141 148 155 162 169 176 183 190 197
Sample mean
True mean

Central Limit Theorem

- Even better: as $n \to \infty$, $\bar{Y}$ becomes normal ie $\bar{Y} \sim N(\mu_Y, \frac{\sigma_Y^2}{n})$
- This means that we can use the normal distribution to make inferences about the sample mean
- To make it easy, we can standardize the sample mean: $Z = \frac{\bar{Y} - \mu_Y}{\sigma_Y / \sqrt{n}} \sim N(0, 1)$
- We will use the sample variance as an estimator for the population variance, just like we do for mean (but we will need to correct a small bias)

Hypothesis testing

- We can use the normal approximation to perform a *hypothesis test*
  - One-sided or two-sided
- Intuition: assuming the true mean is some value $\mu_{Y,0}$, how likely is it that we would observe the sample mean $\bar{Y}$?
  - If it is "very" unlikely, we will reject the null
  - If it is "reasonably likely" then we fail to reject the null
- p-value: probability of a test statistic at least as unlikely as the one you observe (under the null)

Some other terminology

- **Type-1 error**: reject a true null hypothesis
  - **Size** is probability of a type-1 error
- **Type-2 error**: fail to reject a false hypothesis
  - **Power** is probability of a type-2 error
- Which of these two mistakes is worse?