Econometrics Discussion Section 2

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Omitted Variable Bias

- In economics, we often want to determine the impact of X on Y
  - But X is not the only thing going on! Usually many variables are effecting Y
  - If we don't include these other variables in our model, we will get biased estimates of the effect of X on Y, a problem referred to as *omitted variable bias*
- Let's think about what this does to our estimate with a silly example
  - We are an AC company, and we want to know what causes people to buy more AC units
  - Our statistician tells us that the number of swimming pool accidents is a good predictor of AC units sold
  - We run a regression of AC units sold on swimming pool accidents and find a positive relationship
  - Should we conclude that swimming pool accidents cause people to buy more AC units?

Omitted Variable Bias

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- Of course not! The omitted variable here is temperature. When it's hot, people buy more AC units and more people go swimming
- Intuitively, what direction do we expect the bias in the effect of swimming pool accidents to be?

Omitted Variable Bias

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- Of course not! The omitted variable here is temperature. When it's hot, people buy more AC units and more people go swimming
- Intuitively, what direction do we expect the bias in the effect of swimming pool accidents to be?
- It should bias the estimate *upwards*: we are attributing the effect of temperature to swimming pool accidents and thus making swimming pool accidents look more important than they are

Omitted Variable Bias

- We have omitted variable bias when we have a variable Z that is correlated with X and is a determinant of Y
  - Note: if Z is a determinant of Y but is not correlated with X, then it is not a problem!
  - These conditions mean that our OLS assumption of $E(u|X) = 0$ is violated
- Formula for OVB:

$$\hat{\beta} \rightarrow_p \beta + \frac{\sigma_u}{\sigma_X}\rho_{Xu}$$

  - The intuition from our example shows up in the correlation term

Causal effects

- We can use OLS to summarize a relationship without attaching any directionality
    - In this case we need to be careful in our language: ”a change in X is associated with a change in Y”
- Usually economists want to be able to say something *causal*
- Ideal is a randomized controlled trial (RCT): some group gets the treatment, some group doesn’t, and we compare outcomes between the two
    - Observational data usually differs from this in important ways

Solution to OVB

- An RCT eliminates OVB because we randomly assign the treatment, which will then not be correlated with any other variables!
  - This is usually not possible in economics — this is why economics is hard!
- Cross-tabulation eg run the regression on a subsample of your population where there is no OVB problem (but other issues emerge)
- Try and include omitted variables (obviously) in a multivariable regression

Multivariate regression

- Same logic as before:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

- Estimators derived in same way as before (just using matrices)
- Before we looked at R^2 as a measure of fit, but now we have to be careful: adding in more variables on the RHS will always increase R^2
- Adjusted R^2 is a better measure of fit which includes a degrees-of-freedom correction to penalize for adding in more variables
- Add one more assumption our previous 3 from the single-variable case: no perfect multicolinearity

Perfect multicolinearity

- This is when one of your RHS variables is a perfect linear combination of others
- Why is this a problem?

Perfect multicolinearity

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- Why is this a problem?
- Think about basic algebra:
    - $y = a * x$
    - What is a?

Perfect multicolinearity

- $y = a * x$
- What is a?
- $a = \frac{y}{x}$
- But now what if I give you:
  - $y = b * x + c * x$
  - What are a and b?

Perfect multicolinearity

- But now what if I give you:
  - $y = b * x + c * x$
  - What are b and c?
- This question has no unique answer! Any combination of b and c such that $a = b + c$ will work (an infinite number)
- This is what's going on when their is perfect multicolinarity: our coefficient estimates β are not identified/unique

Binary variables

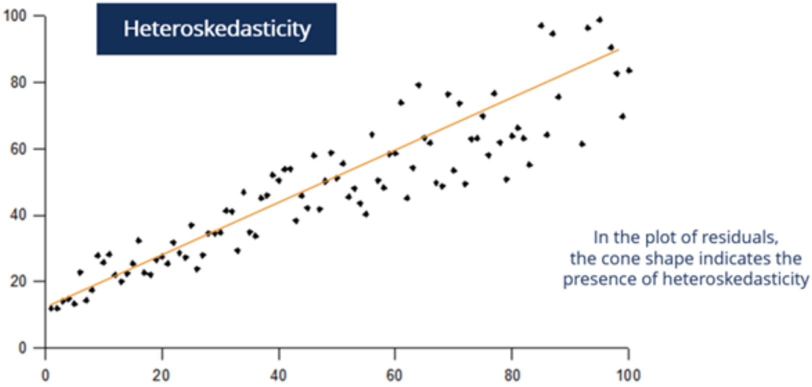
- For this reason we need to be careful when we have binary (0/1) variables in our regression (or any kind of categorical variable)
- We either need to leave one out (the omitted group) or we need to drop our intercept
- Either is fine, just changes interpretation of the coefficient estimates

Control variables

- Usually, we cannot just include the omitted variables
- Instead, look for control variable(s) W which are correlated with the omitted variables
- Helps us address the OVB problem and get the causal impact of a variable X (→ conditional mean independence holds)
- Example: impact of years of higher education on wages
  - Problem: ability is correlated with both! (In which direction is OVB?)
  - Solution: control for ability, eg use parental income, quality of local high schools, etc.

Residuals

- Usually, we cannot just include the omitted variables
- Instead, look for control variable(s) W which are correlated with the omitted variables
- Helps us address the OVB problem and get the causal impact of a variable X ($\rightarrow$ conditional mean independence holds)
- Example: relationship between GPA and job performance
  - Problem: ability is correlated with both! (In which direction is OVB?)
  - Solution: control for ability, eg use parental income, quality of local high schools, etc.



Heteroskedasticity

In the plot of residuals, the cone shape indicates the presence of heteroskedasticity

Hypothesis testing

- We already know about the hypothesis test on a single coefficient
- What about a joint hypothesis test? Ie $H_0: \ \beta_1 = \beta_2 = 0$?
- We want to test these hypotheses jointly, and will do so using an F-statistic which follows a χ^2_q distribution where q is degrees of freedom
- Intuition: look at the fit under the null and alternate hypotheses; if fit under the null is much worse than under the alternate, reject.
- Tests of equality between coefficients, eg $H_0: \ \beta_1 = \beta_2$
- Confidence set for coefficients: all combos not rejected under a given test