A.2. *Simulated Likelihood Calculation.* In this section, we describe the simulation steps that are required to construct the simulated likelihood function.

Step 1. Simulate $\{K_t^m, h_t^m, C_t^m, A_t^m\}_{t=t_0}^T$ starting from the initial period as follows:

1. Draw the true initial human capital $K_{t_0}^m$.

   First, draw the initial period measurement error ξ_0 and then, derive

$$K_{t_0}^m = \frac{K_{t_0}^D}{\xi_0}$$

2. Draw the true initial asset $A_{t_0}^m$.

   If the initial asset is observed, then draw the measurement error ξ_{3,t_0}, and derive

$$A_{t_0}^m = A_{t_0}^D - \xi_{3,t_0}$$

   If the initial asset is not observable, then draw $A_{t_0}^m$ from $N(\bar{A}, V_{\bar{A}})$.

3. Draw the taste shock

$$\ln(\varepsilon_2) \sim N(\frac{1}{2}\sigma_2^2, \sigma_2)$$

   and solve for the optimal consumption and labor supply. That is,

$$\begin{aligned}\{C_{t_0}^m, h_{t_0}^m\} = \arg\max_{\{C_{t_0}, h_{t_0}\}} \{ & u(C_{t_0}, t_0) - v(h_{t_0}, \varepsilon_{2,t_0}) + \beta E_{t_0} V_{t_0+1}[(1+r)A_{t_0} \\ & + K_{t_0}^m h_{t_0} - C_{t_0}, \hat{K}_{t_0+1}, \varepsilon_{t_0+1}]\}\end{aligned}$$

   subject to

$$\begin{aligned}A_{t_0+1} &= (1+r)A_{t_0}^m + K_{t_0}^m h_{t_0} - C_{t_0} \\ \hat{K}_{t_0+1} &= g(h_{t_0}, K_{t_0}^m, t_0)\end{aligned}$$

   Notice that we already have the polynomial approximation of the emax function

$$V^E(A_{t_0+1}, \hat{K}_{t_0+1}) = E_{t_0} V_{t_0+1}(A_{t_0+1}, \hat{K}_{t_0+1}, \varepsilon_{1,t_0+1}, \varepsilon_{2,t_0+1})$$

   from the DP solution, which we will use in this case.

4. Draw the human capital shock ε_{1,t_0+1}, and derive the next period state variables.

   That is,

$$\begin{aligned}A_{t_0+1}^m &= (1+r)A_{t_0}^m + K_{t_0}^m h_{t_0}^m - C_{t_0}^m \\ \hat{K}_{t_0+1} &= g(h_{t_0}^m, K_{t_0}^m, t_0) \\ K_{t_0+1} &= \hat{K}_{t_0} \varepsilon_{1,t_0+1}\end{aligned}$$

5. Now, repeat (3) and (4) until the end period T to derive the sequence of variables $\{K_t^m, h_t^m, C_t^m, A_t^m\}_{t=t_0}^T$.

Step 2. Given the simulated sequence of variables $\{K_t^m, h_t^m, C_t^m, A_t^m\}_{t=t_0}^T$, we then derive the measurement error. Then, we calculate the log likelihood increment for person i at the m^{th} simulation draw as follows.

Let us denote

$$\begin{aligned}\xi_0^m &= \log K_{t_0}^D - \log K_{t_0}^m \\ \xi_{1,t}^m &= \log K_t^D - \log h_t^D - \log K_t^m - \log h_t^m \\ \xi_{2,t}^m &= h_t^D - h_t^m \\ \xi_{3,t}^m &= A_t^D - A_t^m\end{aligned}$$

If the initial wage at period to is available in the data, then we construct the initial wage measurement error $\tilde{\xi}_0^m$ and the log density of $\tilde{\xi}_0^m$ becomes part of the log likelihood increment of person i. If wage and hours data for person i at period $t > t_0$ is available, then we derive the labor income measurement error $\tilde{\xi}_1^m$, and the log density of $\tilde{\xi}_1^m$, becomes part of the log likelihood increment for person i at period t. On the other hand, if either wage or hours xor both for person i at period $t > t_0$ are not available, then the log likelihood increment for person i at period t does not contain any wage data K_t^D. Similarly, if hours data for person i at period t is available, then we derive the hours measurement error $\tilde{\xi}_2^m$, and the log density of $\tilde{\xi}_2^m$ becomes part of the log likelihood increment for person i at period t. On the other hand, if hours data are not available for person i at period t, then the log likelihood increment for person i at period t does not contain any hours information. Construction of the log likelihood increment for assets for person i at period t is done similarly.

Then, the log likelihood increment for person i is

$$\begin{aligned}
l_i^m = & \sum_{t=t_0+1}^{T}\left[\frac{(\tilde{\xi}_{1,t}^m+\frac{1}{2}\sigma_1)^2}{-2\sigma_1^2}-\log\sigma_1-(\log K_t^D+\log h_t^d)\right] I(K_t^D, h_t^D \text{ observable })\\
& +\sum_{t=t_0}^{T}\left[\frac{(\tilde{\xi}_{2,t}^m)^2}{-2\sigma_1^2}-\log\sigma_1\right] I(h_t^D \text{ observable })\\
& +\sum_{t=t_0+1}^{T}\left[\frac{(\tilde{\xi}_{3,t}^m)^2}{-2\sigma_3^2}-\log\sigma_3\right] I(A_t^D \text{ observable })\\
& +\left[\frac{(A_{t_0}^m-\bar{A})^2}{-2\sigma_{\bar{A}}^2}-\log\sigma_{\bar{A}}\right] I(A_t^D \text{ observable })\\
& +\left[\frac{(\tilde{\xi}_0^m+\frac{1}{2}\sigma_0)^2}{-2\sigma_0^2}-\log\sigma_0-\log K_{t_0}^D\right] I(K_t^D, h_t^D \text{ observable })
\end{aligned}$$

We set the starting time t_0 such that both $K_{t_0}^D$ and $h_{t_0}^D$ are observable.

Step 3. We repeat the simulation and likelihood increment calculation for $m = 1,\dots,M$ and derive the simulated log likelihood increment for individual i as follows:

$$l_i = \log\left[\sum_{m=1}^{M}\exp(l_i^m)\right]$$

The total log likelihood is

$$l = \sum_{i=1}^{N} l_i$$