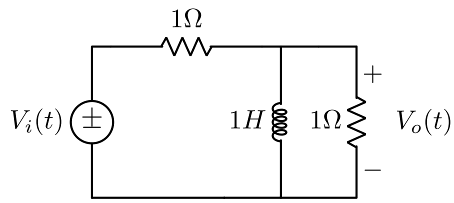
Problem 1

1. Determine the transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ of the network below



Given: As per diagram

Solution:
Using mesh analysis:

@Loop 1:

$$V_i - R_1 i_1 - L\frac{di_1}{dt} + L\frac{di_2}{dt} = 0$$

$$V_i(s) - R_1 I_1(s) - LsI_1(s) + LsI_2(s) = 0$$

$$V_i(s) = (R_1 + Ls)I_1(s) - LsI_2(s) \Rightarrow equation\ 1$$

@Loop 2:

$$-R_2 i_2 - L\frac{di_2}{dt} + L\frac{di_1}{dt} = 0$$

$$R_2 I_2(s) - LsI_2(s) - LsI_1(s) = 0$$

$$(R_1 + Ls)I_2(s) - LsI_1(s) = 0$$

$$LsI_1(s) = (R_1 + Ls)I_2(s)$$

$$I_1(s) = \left(\frac{R_2 + Ls}{Ls}\right) I_2(s) \Rightarrow equation\ 2$$

Substituting equation 2 to 1

$$V_i(s) = (R_1 + Ls)\left(\frac{R_2 + Ls}{Ls}\right) I_2(s) - LsI_2(s)$$

$$V_o = R_2 i_2 \rightarrow V_o(s) = R_2 I_2(s)$$

$$I_2(s) = \frac{V_o(s)}{R_2}$$

$$V_i(s) = (R_1 + Ls)\left(\frac{R_2 + Ls}{Ls}\right)\frac{V_o(s)}{R_2} - Ls\frac{V_o(s)}{R_2}$$

Substituting the values

$$V_i(s) = (S+1)\left(\frac{S+1}{S}\right)V_o(s) - SV_o(s)$$

$$V_i(s) = \left[\frac{S^2 + 2S + 1}{S} - S\right]V_o(s)$$

$$V_i(s) = \left[\frac{S^2 + 2S + 1}{S} - \frac{S^2}{S}\right]V_o(s)$$

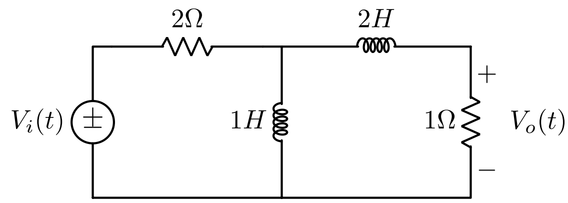
$$V_i(s) = \frac{2S + 1}{S}V_o(s)$$

$$\frac{V_i(s)}{V_o(s)} = \frac{2S + 1}{S}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{S}{2S + 1}}$$

Problem 2

1. Determine the transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ of the network below



Given: As per diagram

Solution:
Using mesh analysis:

@Loop 1:

$$V_i - R_1 i_1 - L\frac{di_1}{dt} + L\frac{di_2}{dt} = 0$$

$$V_i(s) - 2I_1(s) - sI_1(s) + sI_2(s) = 0$$

$$V_i(s) = (s+2)I_1(s) - sI_2(s) \Rightarrow \textit{equation } 1$$

@Loop 2:

$$L\frac{di_1}{dt} - L\frac{di_2}{dt} - L\frac{di_2}{dt} - R_2 i_2 = 0$$

$$sI_1(s) - sI_2(s) - 2sI_2(s)1I_2(s) = 0$$

$$sI_1(s) = (3s+1)I_2(s)$$

$$I_1(s) = \left(\frac{3s+1}{s}\right) I_2(s) \Rightarrow \textit{equation } 2$$

$$V_o = R_2 i_2 \rightarrow V_o(s) = 1I_2(s)$$

$$I_2(s) = \frac{V_o(s)}{1}$$

Substituting equation 2 to 1

$$V_i(s) = (s+2)\left(\frac{3s+1}{s}\right) I_2(s) - sI_2(s)$$

$$V_i(s) = (s+2)\left(\frac{3s+1}{s}\right) \frac{V_o(s)}{1} - s\frac{V_o(s)}{1}$$

$$V_i(s) = (s+2)\left(\frac{3s+1}{s}\right) V_o(s) - sV_o(s)$$

$$V_i(s) = \left(\frac{3s^2+7s+2}{s} - s\right) V_o(s)$$

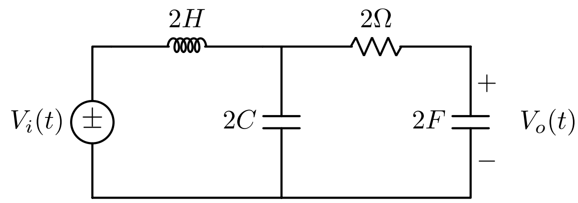
$$V_i(s) = \left(\frac{3s^2+7s+2}{s} - \frac{s^2}{s}\right) V_o(s)$$

$$V_i(s) = \frac{2s^2+7s+2}{s} V_o(s)$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{s}{2s^2+7s+2}}$$

Problem 3

1. Determine the transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ of the network below



Given: As per diagram

Solution:
Using mesh analysis:

@Loop 1:

$$V_i - 2\frac{di_1}{dt} - \frac{1}{2}\int i_1 \quad dt + \frac{1}{2}\int i_2 \quad dt = 0$$

$$V_i(s) - 2sI_1(s) - \frac{1}{2s}I_1(s) + \frac{1}{2s}I_2(s) = 0$$

$$V_i(s) = (2s + \frac{1}{2s})I_1(s) - \frac{1}{2s}I_2(s) \Rightarrow equation\ 1$$

@Loop 2:

$$\frac{1}{2}\int i_1 \quad dt - \frac{1}{2}\int i_2 \quad dt - 2i_2 - \frac{1}{2}\int i_2 \quad dt = 0$$

$$\frac{1}{2s}I_1(s) - \frac{1}{2s}I_2(s) - 2I_2(s) - \frac{1}{2s}I_2(s) = 0$$

$$\frac{1}{2s}I_1(s) = \frac{1}{2s} + 2I_2(s) = 0$$

$$I_1(s) = (4s+1)I_2(s) \Rightarrow equation\ 2$$

$$i_2 = 2\frac{dV_o}{dt}$$

$$I_2(s) = 2sV_o(s)$$

Substituting equation 2 to 1

$$V_i(s) = (2s + \frac{1}{2s})(4s+1)I_2(s) - \frac{1}{2s}I_2(s)$$

$$V_i(s) = (2s + \frac{1}{2s})(4s+1)2sV_o(s) - \frac{1}{2s}2sV_o(s)$$

$$V_i(s) = \left(\frac{(4s^2+1)(4s+1)}{2s}\right)2sV_o(s) - \frac{1}{2s}2sV_o(s)$$

$$V_i(s) = \left(\frac{(16s^3+4s^2+4s+1)}{2s} - \frac{1}{2s}\right)2sV_o(s)$$

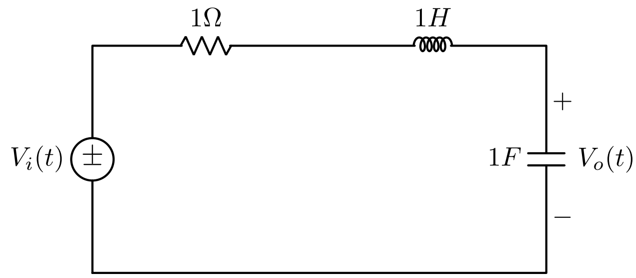
$$V_i(s) = \left(8s^2+2s+2\right)2sV_o(s)$$

$$V_i(s) = 2s\left(8s^2+2s+2\right)V_o(s)$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{2s\,(8s^2+2s+2)}}$$

Problem 4

Determine the transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ of the network below:



Given: As per diagram

Solution:

Using voltage divider in s-domain:

$$Z_R = 1\Omega$$

$$Z_L = sL = s$$

$$Z_C = \frac{1}{sC} = \frac{1}{s}$$

Total impedance:

$$Z_{total} = Z_R + Z_L + Z_C$$

$$= 1 + s + \frac{1}{s}$$

Transfer function:

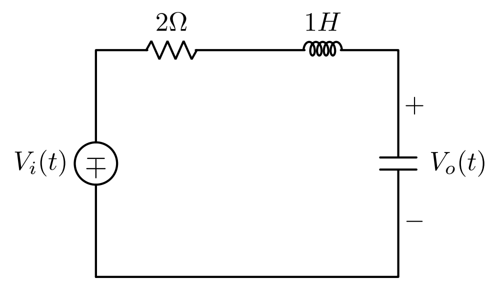
$$\frac{V_o(s)}{V_i(s)} = \frac{Z_C}{Z_{total}}$$

$$= \frac{\frac{1}{s}}{1 + s + \frac{1}{s}}$$

$$= \frac{1}{s + s^2 + 1}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + s + 1}}$$

Problem 5: Transfer Function of an RLC Network
Find the transfer function $\frac{V_o(s)}{V_i(s)}$ *for the following circuit:*



Given: As per diagram
Solution:

1. **Impedance Calculation:**

$$Z_R = 2\,\Omega, \quad Z_L = sL = s, \quad Z_C = \frac{1}{sC} = \frac{2}{s}$$

2. **Voltage Divider Rule:**

$$V_o(s) = V_i(s) \cdot \frac{Z_C}{Z_R + Z_L + Z_C}$$

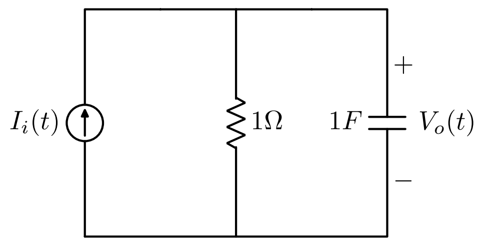
3. **Transfer Function:**

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{2}{s}}{2 + s + \frac{2}{s}} = \frac{2}{s^2 + 2s + 2}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{2}{s^2 + 2s + 2}}$$

Problem 6: Transfer Function of a Parallel RC Network

Analyze the parallel RC network to find $\frac{V_o(s)}{I_i(s)}$*:*



Given: As per diagram

Solution:

1. **Admittance Calculation:**
$$Y_R = \frac{1}{R} = 1\,S, \quad Y_C = sC = s$$

2. **Total Admittance:**
$$Y_{\text{total}} = Y_R + Y_C = 1 + s$$

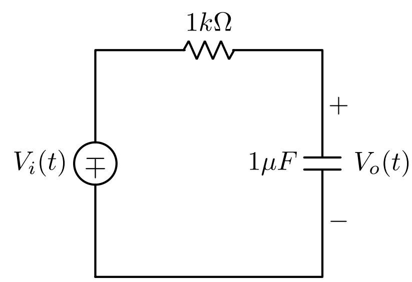
3. **Impedance (Z):**
$$Z = \frac{1}{Y_{\text{total}}} = \frac{1}{s+1}$$

4. **Transfer Function:**
$$\frac{V_o(s)}{I_i(s)} = Z = \frac{1}{s+1}$$

$$\boxed{\frac{V_o(s)}{I_i(s)} = \frac{1}{s+1}}$$

Problem 7: Transfer Function of an RC Network

Determine $G(s) = \frac{V_o(s)}{V_i(s)}$ *for the following RC circuit:*



Given:

- $R = 1\,k\Omega = 1000\,\Omega$
- $C = 1\,\mu F = 10^{-6}\,F$

Solution:

1. **Impedance Calculation:**

$$Z_R = R = 1000\,\Omega, \quad Z_C = \frac{1}{sC} = \frac{1}{10^{-6}s}$$

2. **Voltage Divider Rule:**

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_C}{Z_R + Z_C} = \frac{1/sC}{R + 1/sC}$$

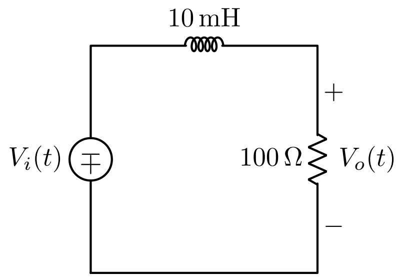
3. **Simplification:**

$$G(s) = \frac{1}{RCs + 1} = \frac{1}{0.001s + 1}$$

$$\boxed{G(s) = \frac{1}{0.001s + 1}}$$

Problem 8

Find $G(s) = \dfrac{V_o(s)}{V_i(s)}$ for this RL network:



Given:

- Inductance: $L = 10\,\text{mH} = 0.01\,\text{H}$
- Resistance: $R = 100\,\Omega$

Step-by-Step Solution:

1. The total impedance in the s-domain is:

$$Z_{\text{total}} = Z_L + Z_R = sL + R = 0.01s + 100$$

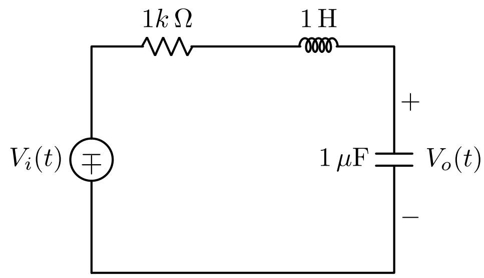
2. The output voltage $V_o(s)$ is across the resistor, so use voltage divider:

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_R}{Z_{\text{total}}} = \frac{100}{0.01s + 100}$$

$$\boxed{G(s) = \frac{100}{0.01s + 100}}$$

Problem 9

Analyze this RLC bandpass filter:



Given:

- $R = 1000\,\Omega$
- $L = 1\,\mathrm{H}$
- $C = 1\,\mu\mathrm{F} = 1 \times 10^{-6}\,\mathrm{F}$

Step-by-Step Solution:

1. Convert all components to s-domain impedances:

$$Z_R = R = 1000, \quad Z_L = sL = s, \quad Z_C = \frac{1}{sC} = \frac{1}{10^{-6}s}$$

2. Total impedance in series:

$$Z_{\text{total}} = Z_R + Z_L + Z_C = 1000 + s + \frac{1}{10^{-6}s}$$

3. Voltage across the inductor (band-pass behavior):

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_L}{Z_{\text{total}}} = \frac{s}{1000 + s + \frac{1}{10^{-6}s}}$$

4. Multiply numerator and denominator by $10^{-6}s$ to simplify:

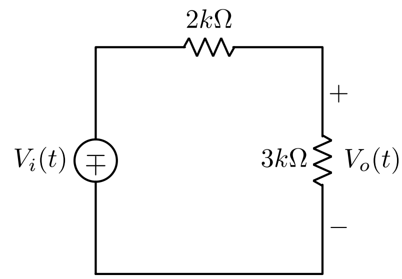
$$G(s) = \frac{10^{-6}s^2}{0.001s^2 + 10^{-6}s + 1}$$

5. Multiply numerator and denominator by 10^6 to eliminate decimals:

$$G(s) = \frac{s^2}{0.001s^2 + s + 1000}$$

$$\boxed{G(s) = \frac{s}{0.001s^2 + s + 1000}}$$

Problem 10
Find: The transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ for this voltage divider:



Given:

- $R_1 = 2k\Omega$ — resistor connected to the voltage source
- $R_2 = 3k\Omega$ — resistor connected to ground, across which output is measured

Solution:
This is a **simple resistive voltage divider**, where two resistors are in series, and the output voltage is taken across R_2. The Laplace domain doesn't change resistor behavior, so the analysis remains the same as in the time domain.

—

Step 1: Voltage Divider Rule
The voltage divider rule states:

$$V_o(s) = V_i(s) \cdot \frac{R_2}{R_1 + R_2}$$

—

Step 2: Derive Transfer Function
The transfer function is:

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1 + R_2}$$

—

Step 3: Substitute Values

$$G(s) = \frac{3000}{2000 + 3000} = \frac{3000}{5000} = \frac{3}{5}$$

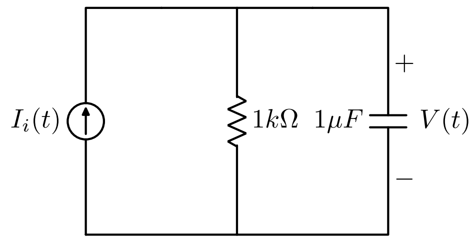
—

$$\boxed{G(s) = 0.6}$$

This means the output voltage is 60% of the input, due to the resistor ratio. This is a purely resistive (frequency-independent) voltage divider.

Problem 11

Find: $G(s) = \dfrac{V(s)}{I_i(s)}$ for this parallel RC network



Given: Resistance $R = 1k\Omega$, Capacitance $C = 1\mu F$

Solution:

This is a **parallel RC network**, where a resistor R and capacitor C are connected in parallel, and a current source $I_i(t)$ is supplying current into the parallel branches.

—

Step 1: Write the impedance of each element in the Laplace domain

- Resistor: $Z_R = R$ - Capacitor: $Z_C = \dfrac{1}{sC}$

—

Step 2: Find the equivalent impedance of the parallel combination

For two components in parallel, the total impedance is:

$$Z_{eq} = \left(\frac{1}{R} + sC\right)^{-1}$$

—

Step 3: Use Ohm's Law in the s-domain

The voltage across both elements is the same and is given by:

$$V(s) = I_i(s) \cdot Z_{eq}$$

Thus, the transfer function is:

$$\frac{V(s)}{I_i(s)} = Z_{eq} = \frac{1}{\frac{1}{R} + sC}$$

—

Step 4: Substitute the given values

$$\frac{V(s)}{I_i(s)} = \frac{1}{\frac{1}{1000} + s \cdot 1 \times 10^{-6}} = \frac{1}{0.001 + 0.000001s} = \frac{1}{0.001s + 1}$$

Now multiply numerator and denominator by 1000 to express in a simplified form:

$$\frac{V(s)}{I_i(s)} = \frac{1000}{0.001s + 1}$$

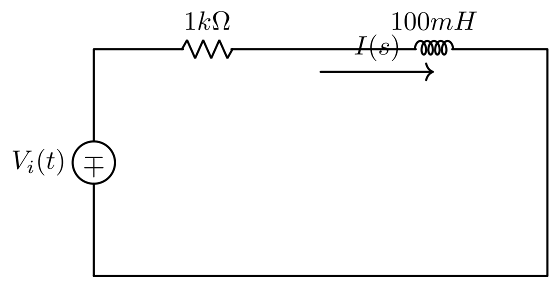
—

$$\boxed{\frac{V(s)}{I_i(s)} = \frac{1000}{0.001s + 1}}$$

This is the transfer function of the parallel RC network. It shows a low-pass filter behavior, where the capacitor shunts high frequencies, reducing the output voltage at high s.

Problem 12

Find: $\dfrac{I(s)}{V_i(s)}$ for this series RL circuit

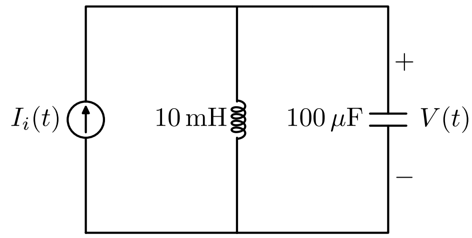


Solution:

$$\frac{I(s)}{V_i(s)} = \frac{1}{R + sL} = \boxed{\frac{1}{0.1s + 1000}}$$

Problem 13

Find: The transfer function $G(s) = \dfrac{V(s)}{I_i(s)}$ for the parallel LC tank circuit shown below.



Given:

- Inductance $L = 10\,\text{mH} = 0.01\,\text{H}$
- Capacitance $C = 100\,\mu\text{F} = 100 \times 10^{-6}\,\text{F}$

Solution:

—

Step 1: Convert to Laplace Domain

In the Laplace domain: - The inductor becomes $\dfrac{1}{sL}$ - The capacitor becomes $\dfrac{1}{sC}$ in impedance, but as admittance (conductance), it's sC

These two elements are in **parallel**, so their **equivalent admittance** is:

$$Y(s) = \frac{1}{sL} + sC$$

Then the **impedance** is:

$$Z(s) = \frac{1}{Y(s)} = \frac{1}{\frac{1}{sL} + sC}$$

—

Step 2: Derive Transfer Function

$$G(s) = \frac{V(s)}{I_i(s)} = Z(s) = \frac{1}{\frac{1}{sL} + sC}$$

—

Step 3: Plug in Component Values

$$L = 0.01\,\text{H}, \quad C = 100 \times 10^{-6}\,\text{F}$$

So:

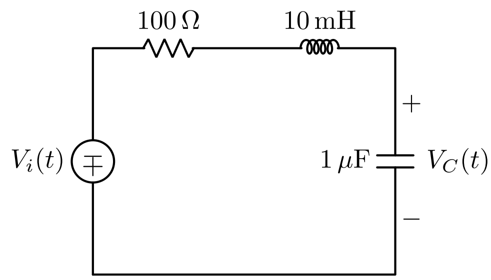
$$G(s) = \frac{1}{\frac{1}{0.01s} + 100 \times 10^{-6}s} = \frac{1}{\frac{1}{0.01s} + 10^{-4}s}$$

Multiply numerator and denominator by $0.01s$ to simplify:

$$G(s) = \frac{0.01s}{1 + 0.000001s^2} = \boxed{\frac{0.01s}{10^{-6}s^2 + 1}}$$

Problem 14

Find: The transfer function $G(s) = \dfrac{V_C(s)}{V_i(s)}$ for the series RLC circuit shown below.



Given:

- Resistance $R = 100\,\Omega$
- Inductance $L = 10\,\mathrm{mH} = 0.01\,\mathrm{H}$
- Capacitance $C = 1\,\mu\mathrm{F} = 1 \times 10^{-6}\,\mathrm{F}$

Solution:

—

Step 1: Laplace Domain Representation

- The resistor stays as R - The inductor becomes sL - The capacitor becomes $\dfrac{1}{sC}$
The circuit is a **series RLC**, and we're asked to find the voltage across the capacitor. This is a **voltage divider**, where the output is across the capacitor:

$$G(s) = \frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

—

Step 2: Plug in Component Values

$$R = 100, \quad L = 0.01, \quad C = 1 \times 10^{-6}$$

Substitute:

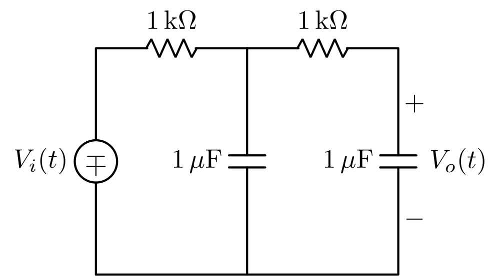
$$G(s) = \frac{1/(s \cdot 10^{-6})}{100 + 0.01s + 1/(s \cdot 10^{-6})}$$

Multiply numerator and denominator by $s \cdot 10^{-6}$ to eliminate fractions:

$$G(s) = \frac{1}{10^{-6}s^2 + 10^{-4}s + 1} = \boxed{\frac{1}{10^{-8}s^2 + 10^{-4}s + 1}}$$

Problem 15

Find: The transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ for this RC ladder network.



Given: Two identical stages of an RC low-pass filter, each with:

- $R = 1\,\mathrm{k}\Omega = 1000\,\Omega$
- $C = 1\,\mu\mathrm{F} = 1 \times 10^{-6}\,\mathrm{F}$

Solution:

—

Step 1: Understand the System

This is a cascade of **two identical RC low-pass filters**. Each stage consists of a resistor in series followed by a capacitor to ground. The transfer function of a single RC low-pass filter is:

$$H(s) = \frac{1}{RCs + 1}$$

When two identical RC filters are connected in series, the total transfer function is the **product** of the individual transfer functions:

$$G(s) = H(s) \cdot H(s) = \left(\frac{1}{RCs + 1}\right)^2$$

—

Step 2: Substitute Values

Given: - $R = 1000\,\Omega$ - $C = 1 \times 10^{-6}\,F$

$$RC = 1000 \times 1 \times 10^{-6} = 0.001$$

So:

$$G(s) = \left(\frac{1}{0.001s + 1}\right)^2 = \frac{1}{(0.001s + 1)^2}$$

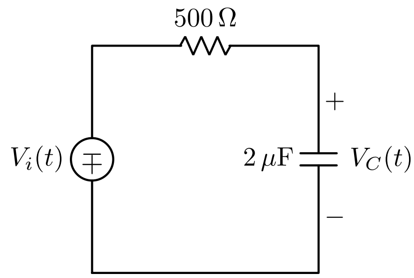
—

Final Answer:

$$\boxed{G(s) = \frac{1}{(0.001s + 1)^2}}$$

Problem 16

Find: $G(s) = \dfrac{V_C(s)}{V_i(s)}$ for this series RC circuit.



Given: $R = 500\,\Omega$, $C = 2\,\mu\text{F}$

Solution:

This is a basic voltage divider in the Laplace domain:

$$G(s) = \frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{sRC + 1}$$

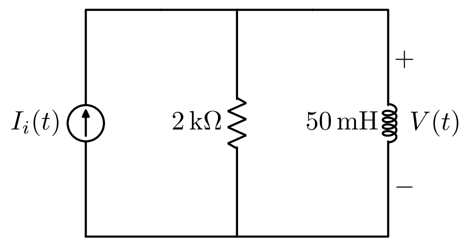
Substitute:

$$RC = 500 \times 2 \times 10^{-6} = 0.001$$

$$\boxed{G(s) = \frac{1}{0.001s + 1}}$$

Problem 17

Find: $G(s) = \dfrac{V(s)}{I_i(s)}$ for this parallel RL circuit.



Given: $R = 2\,\mathrm{k\Omega}$, $L = 50\,\mathrm{mH}$

Solution:

This is a voltage across a parallel RL network:

$$G(s) = \frac{V(s)}{I_i(s)} = \left(\frac{1}{\frac{1}{R} + \frac{1}{sL}}\right) = \frac{RsL}{R + sL}$$

Substitute:

$$R = 2000, \quad L = 0.05$$

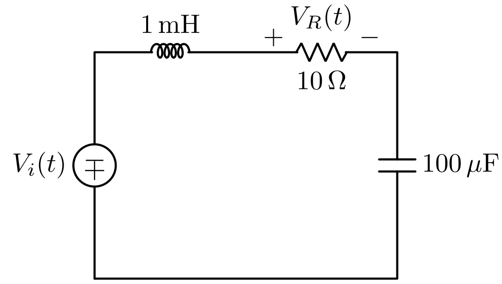
$$G(s) = \frac{2000 \cdot 0.05s}{2000 + 0.05s} = \frac{100s}{2000 + 0.05s}$$

Multiply numerator and denominator by 20 to simplify:

$$\boxed{G(s) = \frac{2000s}{s + 40000}}$$

Problem 18

Find: $G(s) = \dfrac{V_R(s)}{V_i(s)}$ for this series RLC band-pass filter.



Given: $R = 10\,\Omega$, $L = 1\,\text{mH}$, $C = 100\,\mu\text{F}$

Solution:

This is a **band-pass filter**, and the voltage across the resistor is:

$$G(s) = \frac{V_R(s)}{V_i(s)} = \frac{R}{sL + R + \frac{1}{sC}}$$

Substitute:

$$L = 0.001, \quad R = 10, \quad C = 100 \times 10^{-6}$$

$$\frac{1}{sC} = \frac{1}{100 \times 10^{-6}} = \frac{1}{10^{-4}s}$$

Then:

$$G(s) = \frac{10}{0.001s + 10 + \frac{1}{0.0001s}} = \frac{10}{0.001s + 10 + \frac{1}{0.0001s}}$$

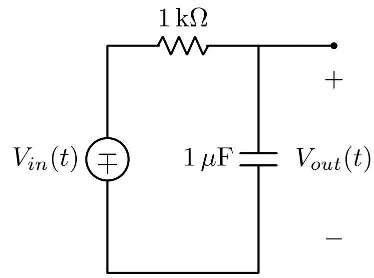
Multiply numerator and denominator by $0.0001s$:

$$G(s) = \frac{0.001s}{10^{-8}s^2 + 0.001s + 1}$$

$$\boxed{G(s) = \frac{0.001s}{10^{-8}s^2 + 0.001s + 1}}$$

Problem 19

Find: $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$ for this RC low-pass filter.



Given: $R = 1\,\text{k}\Omega$, $C = 1\,\mu\text{F}$

Solution:

This is a **low-pass filter**, and the output is across the capacitor.

Impedance of the capacitor:

$$Z_C = \frac{1}{sC}$$

Total impedance in series:

$$Z_{\text{total}} = R + \frac{1}{sC}$$

By the voltage divider rule:

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{Z_C}{R + Z_C} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{sRC + 1}$$

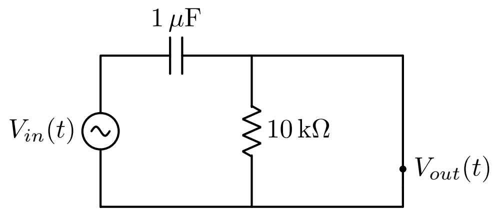
Substitute:

$$R = 1000, \quad C = 1 \times 10^{-6} \Rightarrow RC = 0.001$$

Final answer:

$$\boxed{G(s) = \frac{1}{0.001s + 1}}$$

Problem 20
Find: $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$ for this CR high-pass filter.



Given: $C = 1\,\mu\text{F}$, $R = 10\,\text{k}\Omega$
Solution:
This is a **high-pass filter**, and the output is taken across the resistor.
The impedance of the capacitor is:

$$Z_C = \frac{1}{sC}$$

Using voltage divider rule:

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{sRC + 1}$$

Substitute values:

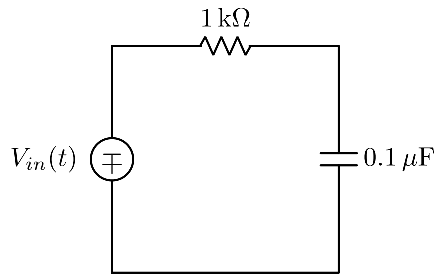
$$R = 10^4, \quad C = 1 \times 10^{-6} \Rightarrow RC = 0.01$$

Final answer:

$$\boxed{G(s) = \frac{0.01s}{0.01s + 1}}$$

Problem 21

Find: $G(s) = \dfrac{V_R(s)}{V_{in}(s)}$ for this series RC circuit (output across resistor)



Given: $R = 1\,\text{k}\Omega$, $C = 0.1\,\mu\text{F}$

Solution:

This is a **high-pass filter** where output is measured across the resistor.

Impedance of the capacitor:

$$Z_C = \frac{1}{sC}$$

Using the voltage divider rule:

$$G(s) = \frac{V_R(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{sRC + 1}$$

Substitute:

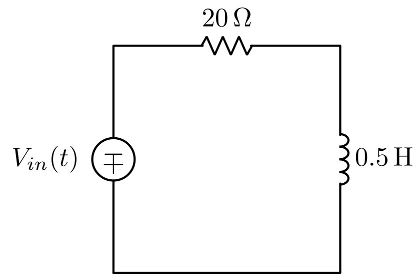
$$R = 1000, \quad C = 0.1 \times 10^{-6} = 1 \times 10^{-7}, \quad RC = 0.0001$$

Final answer:

$$\boxed{G(s) = \frac{0.0001s}{0.0001s + 1}}$$

Problem 22

Find: $G(s) = \dfrac{V_R(s)}{V_{in}(s)}$ for this series RL circuit (output across resistor)



Given: $R = 20\,\Omega$, $L = 0.5\,\mathrm{H}$

Solution:

This is a **high-pass filter**, and output is measured across the resistor.

Impedance of the inductor:

$$Z_L = sL$$

Using voltage division:

$$G(s) = \frac{V_R(s)}{V_{in}(s)} = \frac{R}{R + sL}$$

Substitute the given values:

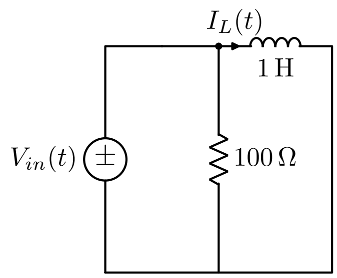
$$R = 20, \quad L = 0.5$$

Final answer:

$$\boxed{G(s) = \frac{20}{20 + 0.5s}}$$

Problem 23

Find: $G(s) = \dfrac{I_L(s)}{V_{in}(s)}$ for a parallel RL circuit



Given: $R = 100\,\Omega$, $L = 1\,\mathrm{H}$

Solution:

For the parallel RL circuit:

1. Total impedance:

$$Z_{total}(s) = R \parallel sL = \frac{R \cdot sL}{R + sL}$$

2. Current through inductor:

$$I_L(s) = \frac{V_{in}(s)}{sL}$$

3. Transfer function:

$$G(s) = \frac{I_L(s)}{V_{in}(s)} = \frac{1}{sL} \cdot \frac{R}{R + sL}$$

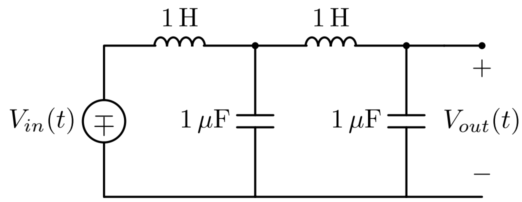
4. Substitute values:

$$G(s) = \frac{100}{100s + s^2}$$

$$\boxed{G(s) = \frac{100}{s^2 + 100s}}$$

Problem 24

Find: $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$ for this 2-stage LC ladder network.



Given: Two LC stages with $L = 1\,\text{H}$, $C = 1\,\mu\text{F}$

Solution:

Using mesh analysis for the 2-stage LC ladder:

For Mesh 1:

$$V_{in} = sL_1 I_1 + \frac{1}{sC_1}(I_1 - I_2)$$

$$= sI_1 + \frac{10^6}{s}(I_1 - I_2)$$

For Mesh 2:

$$0 = \frac{1}{sC_1}(I_2 - I_1) + sL_2 I_2 + \frac{1}{sC_2} I_2$$

$$= \frac{10^6}{s}(I_2 - I_1) + sI_2 + \frac{10^6}{s} I_2$$

Solving the equations:

$$V_{in} = \left(s + \frac{10^6}{s}\right) I_1 - \frac{10^6}{s} I_2$$

$$0 = -\frac{10^6}{s} I_1 + \left(s + \frac{2 \times 10^6}{s}\right) I_2$$

Output voltage:

$$V_{out} = \frac{10^6}{s} I_2$$

The transfer function becomes:

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\left(\frac{10^6}{s}\right)^2}{\left(s + \frac{10^6}{s}\right)\left(s + \frac{2\times 10^6}{s}\right) - \left(\frac{10^6}{s}\right)^2}$$

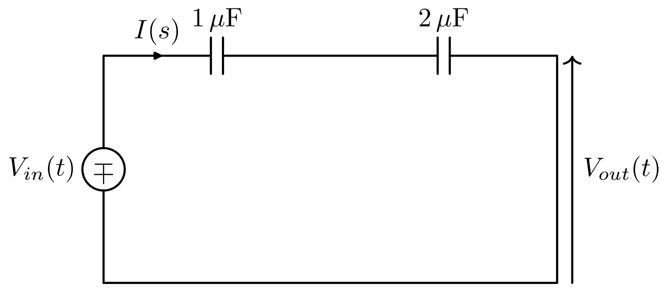
After simplification:

$$G(s) = \frac{10^{12}}{s^4 + 3 \times 10^6 s^2 + 10^{12}}$$

$$\boxed{G(s) = \frac{10^{12}}{s^4 + 3 \times 10^6 s^2 + 10^{12}}}$$

Problem 25

Find: $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$, where V_{out} is the voltage across C_2.



Given: Two capacitors in series: $C_1 = 1\,\mu\text{F}$, $C_2 = 2\,\mu\text{F}$

Solution:

In the s-domain, the impedance of a capacitor is:

$$Z_C = \frac{1}{sC}$$

So the total impedance:

$$Z_{\text{eq}} = \frac{1}{sC_1} + \frac{1}{sC_2}$$

Using the voltage divider rule:

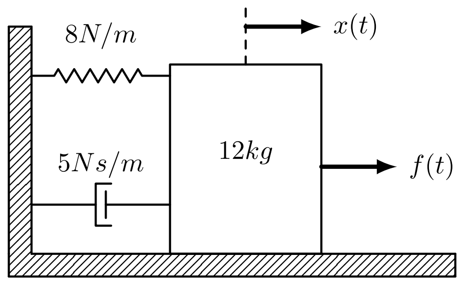
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{sC_2}}{\frac{1}{sC_1} + \frac{1}{sC_2}} = \frac{C_1}{C_1 + C_2}$$

Substitute:

$$C_1 = 1\,\mu\text{F}, \quad C_2 = 2\,\mu\text{F} \Rightarrow \frac{C_1}{C_1 + C_2} = \frac{1}{3}$$

$$\boxed{\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{3}}$$

Translational
Problem 26
Find the transfer function $\dfrac{X(s)}{F(s)}$



Solution:

Equation of Motion:

$$M\frac{d^2x(t)}{dt^2} + fv\frac{dx(t)}{dt} + Kx(t) = f(t)$$

Substitute the Given:

$$12\frac{d^2x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 8x(t) = f(t)$$

Laplace Transform:

(Consider that the Initial Condition is 0)

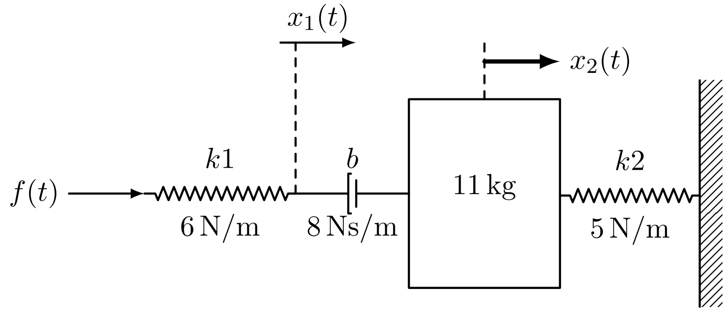
$$12s^2X(s) + 5sX(s) + 8x(s) = F(s)$$

Transfer function $\dfrac{X(s)}{F(s)}$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{12s^2 + 5s + 8}}$$

Problem 27

Find the transfer function $\dfrac{X_2(s)}{F(s)}$



Solution:

Equation for $X_1(t)$:

$$m_1 x_1(t) = f(t) - k_1 x_1(t) - b[x_1(t) - x_2(t)]$$

Simplify:

$$11x_1(t) = f(t) - 6x_1(t) - 8[x_1(t) - x_2(t)]$$

Equation for $X_2(t)$:

$$m_2 x_2(t) = b[x_1(t) - x_2(t)] - k_2 x_2(t)$$

Simplify:

$$11x_2(t) = 8[x_1(t) - x_2(t)] - 5x_2(t)$$

Apply the Laplace Transform

For M1

$$11s^2 x_1(s) = f(s) - 6x_1(s) - 8[sx_1(s) - sx_2(s)]$$

Simplifying

$$11s^2 x_1(s) + 8sx_1(s) + 6x_1(s) - 8sx_2(s) = F(s)$$

This becomes

$$(11s^2 + 8s + 6)x_1(s) - 8sx_2(s) = F(s)$$

For M2

$$11sx_2(s) = 8[sx_1(s) - sx_2(s)] - 5x_2(s)$$

Simplifying

$$11sx_2(s) + 8sx_2(s) + 5sx_2(s) = 8sx_2(s)$$

This becomes

$$(11s^2 + 8s + 5)x_2(s) = 8sx_1(s)$$

Solve for $x_1(s)$ and $x_1(s)$

From the second equation for $x_2(s)$

$$x_1(s) = \frac{(11s^2 + 8s + 5)}{8s}x_2(s)$$

Substitute into the first equation

$$(11s^2 + 8s + 5)\frac{(11s^2 + 8s + 5)}{8s}x_2(s) - 8sx_2(s) = F(s)$$

Multiply out the terms

$$\frac{(11s^2 + 8s + 5)(11s^2 + 8s + 5)}{8s}x_2(s) - 8sx_2(s) = F(s)$$

Simplify the expression

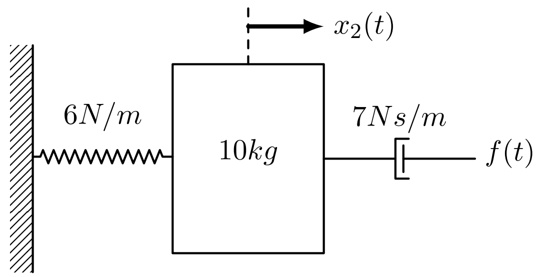
$$x_2(s)\left[\frac{(11s^2 + 8s + 5)(11s^2 + 8s + 5)}{8s} - 8s\right] = F(s)$$

Thus, the Transfer Function is

$$\boxed{G(s) = \frac{X_2(s)}{F(s)} = \frac{8s}{(11s^2 + 8s + 5)(11s^2 + 8s + 5) - 64s^2}}$$

Problem 28

Find the transfer function $\dfrac{X_2(s)}{F(s)}$



Solution:

Formulate the Equation of Motion

$$mx_2(t) + cx_2(t) + kx_2(t) = f(t)$$

Substitute the Values

$$10x_2(t) + 7x_2(t) + 6x_2(t) = f(t)$$

Take the Laplace Transform

$$10s^2x_2(s) + 7sx_2(s) + 6x_2(s) = f(s)$$

Solve for $\dfrac{X_2(s)}{F(s)}$

Rearrange the Equation

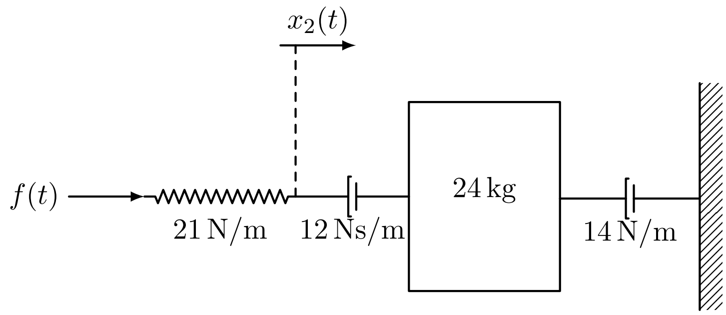
$$x_2(s)(10s^2 + 7s + 6) = F(s)$$

Transfer Function:

$$\boxed{G(s) = \frac{X_2(s)}{F(s)} = \frac{1}{10s^2 + 7s + 6}}$$

Problem 29

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Solution:

Formulate the Equation of Motion

$$mx_2(t) = f(t) - c_1x_2(t) - kx_2(t) - c_2x_2(t)$$

Simplify the Equation

$$mx_2(t) + (c_1 + c_2)x_2(t) + kx_2(t) = f(t)$$

Substitute the Values

$$24x_2(t) + (12 + 4)x_2(t) + 21x_2(t) = f(t)$$

Simplify

$$24x_2(t) + 26x_2(t) + 21x_2(t) = f(t)$$

Take the Laplace Transform

$$24s^2x_2(s) + 26sx_2(s) + 21x_2(s) = f(s)$$

Solve for $\dfrac{X_2(s)}{F(s)}$

Rearrange the Equation

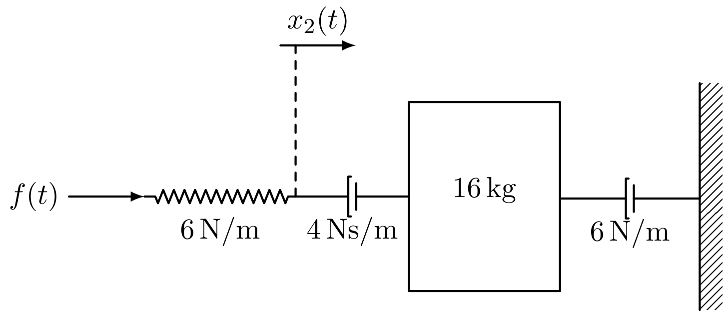
$$x_2(s)(24s^2 + 26s + 21) = F(s)$$

Transfer Function:

$$\boxed{G(s) = \frac{X_2(s)}{F(s)} = \frac{1}{24s^2 + 26s + 21}}$$

Problem 30

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Solution:

Formulate the Equation of Motion

$$mx_2(t) = f(t) - c_1x_2(t) - kx_2(t) - c_2x_2(t)$$

Simplify the Equation

$$mx_2(t) + (c_1 + c_2)x_2(t) + kx_2(t) = f(t)$$

Substitute the Values

$$16x_2(t) + (4 + 6)x_2(t) + 6x_2(t) = f(t)$$

Simplify

$$16x_2(t) + 10x_2(t) + 6x_2(t) = f(t)$$

Take the Laplace Transform

$$16s^2x_2(s) + 10sx_2(s) + 6x_2(s) = f(s)$$

Solve for $\dfrac{X_2(s)}{F(s)}$

Rearrange the Equation

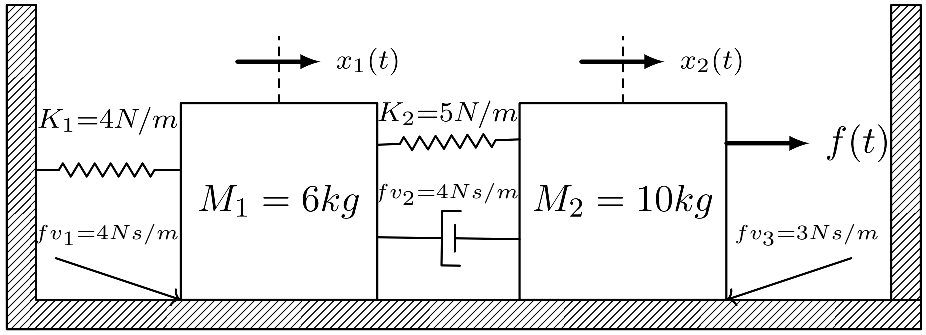
$$x_2(s)(16s^2 + 10s + 6) = F(s)$$

Transfer Function:

$$\boxed{G(s) = \frac{X_2(s)}{F(s)} = \frac{1}{16s^2 + 10s + 6}}$$

Problem 31

Find the transfer function $G(s) = \dfrac{X_1(s)}{F(s)}$



Solution:

Derive the Equations of Motion:

Equation of Motion for M1:

$$M_1\ddot{x}_1(t) = -f_{v1}\dot{x}_1(t) - k_1x_1(t) + f_{v2}[\dot{x}_2(t) - \dot{x}_1(t)] + k_2[x_2(t) - x_1(t)]$$

Substitute the Values:

$$6\ddot{x}_1(t) = -4\dot{x}_1(t) - 4x_1(t) + 4[\dot{x}_2(t) - \dot{x}_1(t)] + 5[x_2(t) - x_1(t)]$$

Simplifying:

$$6\ddot{x}_1(t) = -(4+5)x_1(t) + 5x_2(t) - (4+4)\dot{x}_1(t) + 4\dot{x}_2(t)$$

$$6\ddot{x}_1(t) = -9x_1(t) + 5x_2(t) - 8\dot{x}_1(t) + 4\dot{x}_2(t)$$

Equation of Motion for M2:

$$M_2\ddot{x}_2(t) = k_2[x_1(t) - x_2(t)] + f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] - f_{v3}\dot{x}_2(t) + F(t)$$

Substituting the Given Values:

$$10\ddot{x}_2(t) = 5[x_1(t) - x_2(t)] + 4[\dot{x}_1(t) - \dot{x}_2(t)] - 3\dot{x}_2(t) + F(t)$$

Simplifying:

$$10\ddot{x}_2(t) = 5x_1(t) - 5x_2(t) + 4\dot{x}_1(t) - (4+3)\dot{x}_2(t) + F(t)$$

$$10\ddot{x}_2(t) = 5x_1(t) - 5x_2(t) + 4\dot{x}_1(t) - 7\dot{x}_2(t) + F(t)$$

Laplace Transform:

For M1:

$$6s^2 X_1(s) = -9X_1(s) + 5X_2(s) - 8sX_1(s) + 4sX_2(s)$$

Simplifying:

$$(6s^2 + 8s + 9)X_1(s) = (5 + 4s)X_2(s)$$

For M2:

$$10s^2 X_2(s) = 5X_1(s) - 5X_2(s) + 4sX_1(s) - 7sX_2(s) + F(s)$$

Simplifying:

$$(10s^2 + 7s + 5)X_2(s) = (5 + 4s)X_1(s) + F(s)$$

Solving for the Transfer Function:

Substitute $X_2(s)$ from the Second Equation into the First Equation:

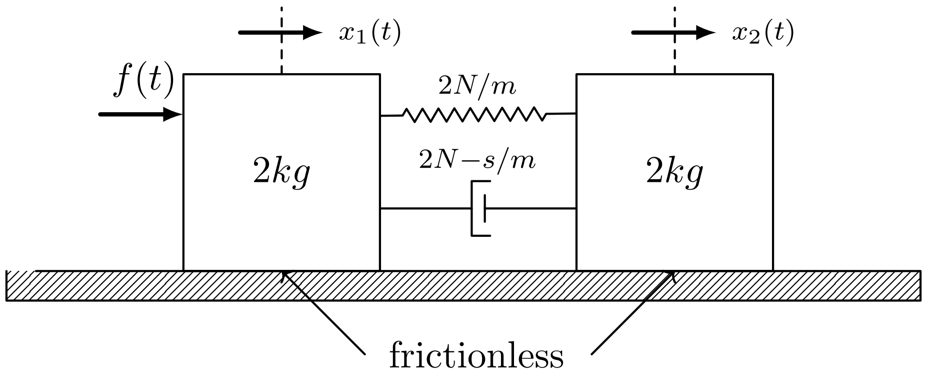
$$(6s^2 + 8s + 9)X_1(s) = (5 + 4s)\left[\frac{(5 + 4s)X_1(s) + F(s)}{10s^2 + 7s + 5}\right]$$

Final Transfer Function:

$$\boxed{\frac{X_1(s)}{F(s)} = \frac{5 + 4s}{(6s^2 + 8s + 9)(10s^2 + 7s + 5) - (5 + 4s)^2}}$$

Problem 32

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Solution:

Equation for Mass 1:

Using Newton's second law:

$$M_1\ddot{x}_1(t) = f(t) - B[\dot{x}_1(t) - \dot{x}_2(t)] - K[x_1(t) - x_2(t)]$$

Taking the Laplace Transform (with zero initial conditions):

$$M_1 s^2 X_1(s) = F(s) - Bs[X_1(s) - X_2(s)] - K[X_1(s) - X_2(s)]$$

Substitute the Values:

$$2s^2 X_1(s) = F(s) - 2s[X_1(s) - X_2(s)] - 2[X_1(s) - X_2(s)]$$

$$2s^2 X_1(s) = F(s) - (2s + 2)[X_1(s) - X_2(s)]$$

Equation for Mass 2:

Using Newton's second law:

$$M_2\ddot{x}_2(t) = B[\dot{x}_1(t) - \dot{x}_2(t)] - K[x_1(t) - x_2(t)]$$

Taking the Laplace Transform (with zero initial conditions):

$$M_2 s^2 X_2(s) = Bs[X_1(s) - X_2(s)] - K[X_1(s) - X_2(s)]$$

Substitute the Values:

$$2s^2 X_2(s) = 2s[X_1(s) - X_2(s)] + 2[X_1(s) - X_2(s)]$$

$$2s^2 X_2(s) = (2s + 2)[X_1(s) - X_2(s)]$$

Solve the System of Equations:

$$X_1(s) - X_2(s) = \frac{2s^2 X_2(s)}{2s+2}$$

$$X_1(s) = X_2(s) + \frac{s^2 X_2(s)}{s+1}$$

$$X_1(s) = \frac{X_2(s)(s^2+s+1)}{s+1}$$

Substitute into the First Equation:

Substitute this Expression for $X_1(s)$ into the First Equation:

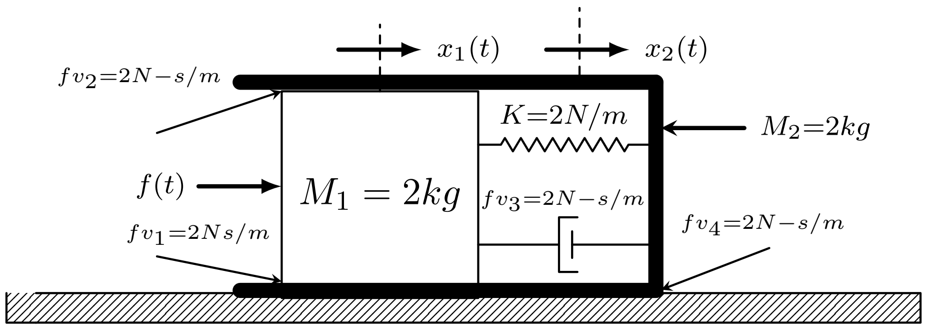
$$2s^2 \frac{X_2(s)(s^2+s+1)}{s+1} = F(s) - (2s+2)\left[\frac{X_2(s)(s^2+s+1)}{s+1} - X_2(s)\right]$$

Final Transfer Function:

$$\boxed{G(s) = \frac{s+1}{2s^4+4s^3+4s^2+2s}}$$

Problem 33

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$ for the translational mechanical system shown in the figure



Solution:

Equations of motion for each mass in the Laplace Domain:

For M_1:

$$M_1\ddot{x}_1(t) = F(t) - f_{v1}\dot{x}_1(t) - f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] - K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_1 s^2 X_1(s) = F(s) - f_{v1} s X_1(s) - f_{v2}[X_1(s) - X_2(s)] - K[X_1(s) - X_2(s)]$$

Substitute the values:

$$2s^2 X_1(s) = F(s) - 2sX_1(s) - 2s[X_1(s) - X_2(s)] - 2[X_1(s) - X_2(s)]$$

Simplify the terms:

$$2s^2 X_1(s) = F(s) - 4sX_1(s) + 2sX_2(s) - 2X_1(s) + 2X_2(s)$$

$$2s^2 X_1(s) = F(s) - (2s+2)X_1(s) + (2s+2)X_2(s)$$

Rearrange to:

$$(2s^2 + 2s + 2)X_1(s) - (2s+2)X_2(s) = F(s)$$

For M_2:

$$M_2\ddot{x}_2(t) = -f_{v3}\dot{x}_2(t) + f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] + K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_2sX_2(s) = -f_{v3}sX_2(s) + f_{v2}[X_1(s) - X_2(s)] + K[X_1(s) - X_2(s)]$$

Substitute the terms:

$$2s^2X_2(s) = -2sX_2(s) + 2s[X_1(s) - sX_2(s)] + 2[X_1(s) - X_2(s)]$$

Simplify the terms:

$$2s^2X_2(s) = -2sX_2(s) + 2sX_1(s) - 2sX_2(s) + 2X_1(s) - 2X_2(s)$$

$$2s^2X_2(s) = 2sX_1(s) + 2X_1(s) - (2s + 2)X_2(s)$$

Rearrange to:

$$(2s^2 + 2s + 2)X_2(s) = (2s + 2)X_1(s)$$

Solving the System of Equations

Solve equation (2) for $X_1(s)$:

$$X_1(s) = \frac{(2s^2 + 2s + 2)X_2(s)}{2s + 2}$$

Substitute this into equation (1):

$$(2s^2 + 2s + 2)\left[\frac{(2s^2 + 2s + 2)X_2(s)}{2s + 2}\right] - (2s + 2)X_2(s) = F(s)$$

Multiply out:

$$\frac{(2s^2 + 2s + 2)^2X_2(s)}{2s + 2} - (2s + 2)X_2(s) = F(s)$$

Multiply both sides by $2s + 2$ to eliminate the denominator:

$$(2s^2 + 2s + 2)^2X_2(s) - (2s + 2)^2X_2(s) = (2s + 2)F(s)$$

Factor out $X_2(s)$ on the left-hand side:

$$[(2s^2 + 2s + 2)^2 - (2s + 2)^2]X_2(s) = (2s + 2)F(s)$$

Simplify the terms:

$$(2s^2 + 2s + 2)^2 = 4s^4 + 8s^3 + 12s^2 + 8s + 4$$

$$(2s + 2)^2 = 4s^2 + 8s + 4$$

Subtract the two:

$$(4s^4 + 8s^3 + 12s^2 + 8s + 4) - (4s^2 + 8s + 4) = 4s^4 + 8s^3 + 8s^2$$

So:

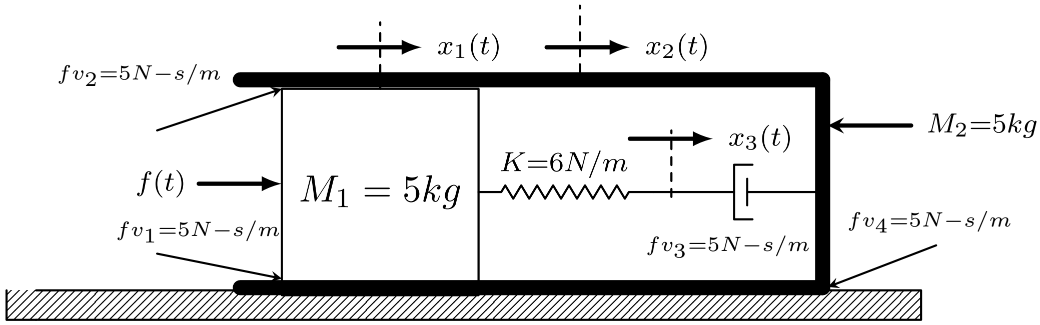
$$(4s^4 + 8s^3 + 8s^2)X_2(s) = (2s + 2)F(s)$$

Final Transfer Function:

$$\boxed{G(s) = \frac{2s + 2}{4s^4 + 8s^3 + 8s^2}}$$

Problem 34

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$ for the translational mechanical system shown in the figure



Solution:

Define the Forces Acting on Each Mass:

For M_1:

$$M_1\ddot{x}_1(t) = F(t) - f_{v1}\dot{x}_1(t) - f_{v2}\dot{x}_2(t) - K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_1 s^2 X_1(s) = F(s) - f_{v1}sX_1(s) - f_{v2}sX_2(s) - K[X_1(s) - X_2(s)]$$

Substitute the values:

$$5s^2X_1(s) = F(s) - 5sX_1(s) - 5s[X_1(s) - X_2(s)] - 6[X_1(s) - X_2(s)]$$

Simplify the terms:

$$5s^2X_1(s) = F(s) - 10sX_1(s) - 6X_1(s) + 6X_2(s)$$

For M_2:

$$M_2\ddot{x}_2(t) = -f_{v3}\dot{x}_2(t) - f_{v4}\dot{x}_2(t) + K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_2sX_2(s) = -f_{v3}sX_2(s) - f_{v2}sX_2 + K[X_1(s) - X_2(s)]$$

Substitute the terms:

$$5s^2X_2(s) = -5sX_2(s) - 5sX_2(s) - 5sX_2 + 6[X_1(s) - X_2(s)]$$

Simplify the terms:

$$5s^2X_2(s) = 6X_1(s) - (6 + 15s)X_2(s)$$

Solve the Equations:

Solve equation (2) for $X_1(s)$:

$$X_1(s) = \frac{5s^2X_2(s) + (6 + 15s)X_2(s)}{6}$$

$$X_1(s) = \frac{X_2(s)(5s^2 + 15s + 6)}{6}$$

Substitute $X_1(s)$ into equation (1):

$$\frac{5s^2X_2(s)(5s^2 + 15s + 6)}{6} = F(s) - 10s\frac{X_2(s)(5s^2 + 15s + 6)}{6} - 6\frac{X_2(s)(5s^2 + 15s + 6)}{6} + 6X_2(s)$$

Simplifying and solving the algebra, the resulting transfer function $G(s) = \frac{X_2(s)}{F(s)}$ will involve terms of the form:

$$G(s) = \frac{6}{D(s)}$$

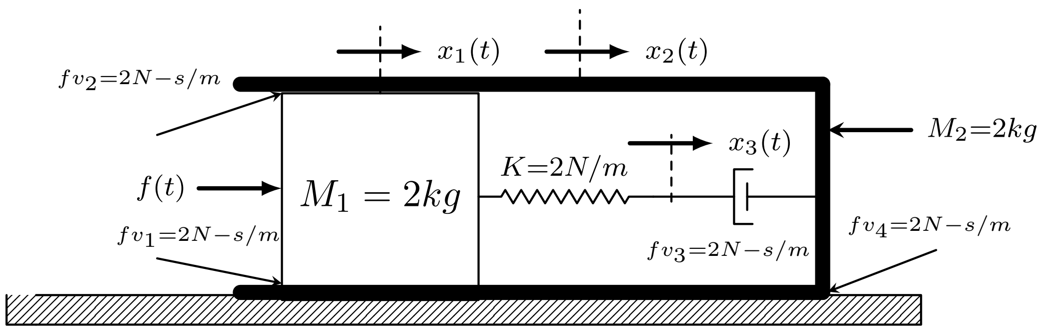
Where $D(s)$ is a polynomial depending on s.

Final Transfer Function:

$$\boxed{G(s) = \frac{6}{25s^4 + 125s^3 + 225s^2 + 195s + 72}}$$

Problem 35

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Solution:

Define the Forces Acting on Each Mass:

For M_1:

$$M_1\ddot{x}_1(t) = F(t) - f_{v1}\dot{x}_1(t) - f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] - K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_1s^2X_1(s) = F(s) - f_{v1}sX_1(s) - f_{v2}[X_1(s) - X_2(s)] - K[X_1(s) - X_2(s)]$$

Substitute the values:

$$2s^2X_1(s) = F(s) - 2sX_1(s) - 2s[X_1(s) - X_2(s)] - 2[X_1(s) - X_2(s)]$$

Simplify the terms:

$$2s^2X_1(s) = F(s) - 4sX_1(s) + 2sX_2(s) - 2X_1(s) + 2X_2(s)$$

$$2s^2X_1(s) = F(s) - (2s+2)X_1(s) + (2s+2)X_2(s)$$

For M_2:

$$M_2\ddot{x}_2(t) = -f_{v3}\dot{x}_2(t) + f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] + K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_2sX_2(s) = -f_{v3}sX_2(s) + f_{v2}[X_1(s) - X_2(s)] + K[X_1(s) - X_2(s)]$$

Substitute the terms:

$$2s^2X_2(s) = -2sX_2(s) + 2s[X_1(s) - sX_2(s)] + 2[X_1(s) - X_2(s)]$$

Simplify the terms:

$$2s^2X_2(s) = -2sX_2(s) + 2sX_1(s) - 2sX_2(s) + 2X_1(s) - 2X_2(s)$$

$$2s^2X_2(s) = 2sX_1(s) + 2X_1(s) - (2s+2)X_2(s)$$

Solving the System of Equations

Solve equation (2) for $X_1(s)$:

$$2s^2X_2(s) + (2s+2)X_2(s) = (2s+2)X_1(s)$$

$$X_1(s) = \frac{2s^2X_2(s) + (2s+2)X_2(s)}{2s+2}$$

$$X_1(s) = \frac{X_2(s)(2s^2+2s+2)}{2s+2}$$

Substitute $X_1(s)$ into equation (1):

$$2s^2\frac{X_2(s)(2s^2+2s+2)}{2s+2} = F(s) - (2s+2)\frac{X_2(s)(2s^2+2s+2)}{2s+2} + (2s+2)X_2(s)$$

Cancel $(2s+2)$ from some terms:

$$2s^2(2s^2+2s+2)X_2(s) = F(s) - (2s^2+2s+2)X_2(s) + (2s+2)X_2(s)$$

Simplify Further:

$$(4s^4 + 4s^3 + 4s^2 + 2s^2 + 2s + 2)X_2(s) = F(s)$$

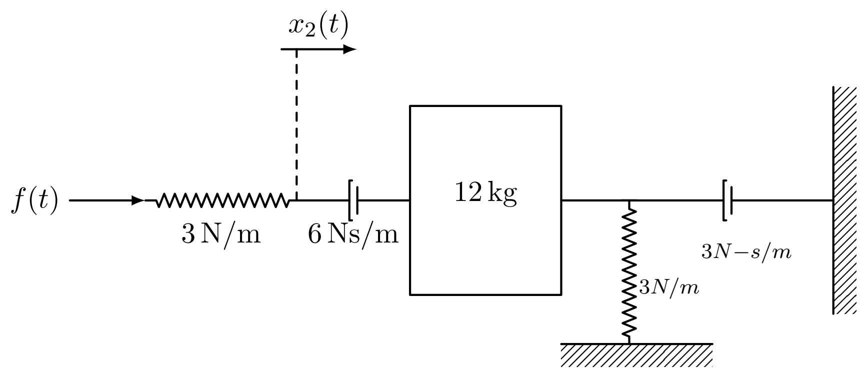
$$(4s^4 + 4s^3 + 6s^2 + 2s + 2)X_2(s) = F(s)$$

Final Transfer Function:

$$\boxed{G(s) = \frac{1}{4s^4 + 4s^3 + 6s^2 + 2s + 2}}$$

Problem 36

Find the transfer function $G(s) = \dfrac{X(s)}{F(s)}$ of the given mechanical system using translational mechanical system



Solution:

Force Balance on M:

$$M\ddot{x}(t) + (b_1 + b_2)\dot{x}(t) + (k_1 + k_2)x(t) = f(t)$$

Laplace Transform:

$$Ms^2X(s) + (b_1 + b_2)sX(s) + (k_1 + k_2)X(s) = F(s)$$

Substitute the Given Values:

$$12s^2X(s) + (6 + 3)sX(s) + (3 + 3)X(s) = F(s)$$

Simplify:

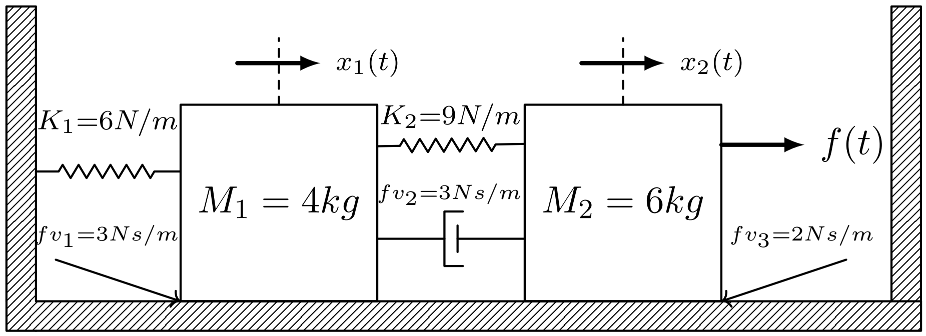
$$12s^2X(s) + 9sX(s) + 6X(s) = F(s)$$

Transfer function solving for $\dfrac{X(s)}{F(s)}$:

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{12s^2 + 9s + 6}}$$

Problem 37

Find the transfer function, $\dfrac{X_1(s)}{F(s)}$



Solution:

Equation of Motion:

For M_1:

$$M_1\ddot{x}_1(t) = F_{v1}\dot{x}_1(t) - k_1x_1(t) + F_{v2}[\dot{x}_2(t) - \dot{x}_1(t)] + k_2[x_2(t) - x_1(t)]$$

Substitute :

$$4\ddot{x}_1(t) = -3\dot{x}_1(t) - 6x_1(t) + 3[\dot{x}_2(t) - \dot{x}_1(t)] + 9[x_2(t) - x_1(t)]$$

Simplify:

$$4\ddot{x}_1(t) = -6\dot{x}_1(t) - 15x_1(t) + 3\dot{x}_2(t) + 9x_2(t)$$

For M_2:

$$M_2\ddot{x}_2(t) = -F_{v2}[\dot{x}_2(t) - \dot{x}_1(t)] - k_2[x_2(t) - x_1(t)] - f_{v3}\dot{x}_2(t) + F(t)$$

Substitute :

$$6\ddot{x}_2(t) = -3[\dot{x}_2(t) - \dot{x}_1(t)] - 9[x_2(t) - x_1(t)] - 2\dot{x}_2(t) + F(t)$$

Simplify:

$$6\ddot{x}_2(t) = -5\dot{x}_2(t) + 3\dot{x}_1(t) - 9x_2(t) + 9x_1(t) + F(t)$$

Apply the Laplace Transform for M_1 and M_2

For M_1:

$$(4s^2 + 6s + 15)X_1(s) = (3s + 9)X_2(s)$$

For M_2

$$(6s^2 + 5s + 9)X_2(s) = (3s + 9)X_1(s) + F(s)$$

Solve for $G(s) = \dfrac{X_1(s)}{F(s)}$:

$$X_2(s) = \frac{(3s + 9)X_1(s) + F(s)}{6s^2 + 5s + 9}$$

Substitute $X_2(s)$ to $X_1(s)$

$$(4s^2 + 6s + 15)X_1(s) = \frac{(3s + 9)^2X_1(s) + (3s + 9)F(s)}{6s^2 + 5s + 9}$$

$$(4s^2 + 6s + 15)(6s^2 + 5s + 9)X_1(s) = (9s^2 + 54s + 81)X_1(s) + (3s + 9)F(s)$$

Simplify:

$$(24s^4 + 44s^3 + 111s^2 + 135s + 135)X_1(s) = (9s^2 + 54s + 81)X_1(s) + (3s + 9)F(s)$$

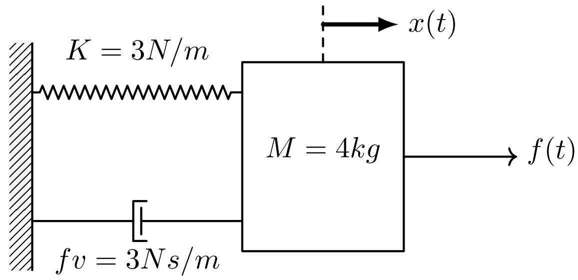
$$(24s^4 + 44s^3 + 102s^2 + 81s + 54)X_1(s) = (3s + 9)F(s)$$

Thus $G(s) = \dfrac{X_1(s)}{F(s)}$

$$\boxed{G(s) = \frac{3s + 9}{24s^4 + 44s^3 + 102s^2 + 81s + 54}}$$

Problem 38

Find the transfer function, $\dfrac{X(s)}{F(s)}$



Solution:

Equation of Motion:

$$M\frac{d^2x(t)}{dt^2} + fv\frac{dx(t)}{dt} + Kx(t) = f(t)$$

Substitute the Given:

$$4\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 3x(t) = f(t)$$

Laplace Transform:

(Consider that the Initial Conditionis 0)

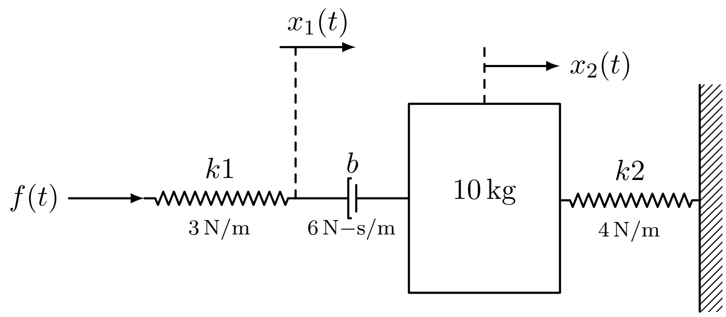
$$4s^2X(s) + 3sX(s) + 3X(s) = F(s)$$

Transfer function $\dfrac{X(s)}{F(s)}$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{4s^2 + 3s + 3}}$$

Problem 39

Find the transfer function $\dfrac{X_2(s)}{F(s)}$



Solution:

Equation for $X_1(t)$:

$$F(t) - k_1x_1(t) - b[\dot{x}_1(t) - \dot{x}_2(t)] = 0$$

Apply the Laplace Transform:

$$F(s) - k_1X_1(s) - b[sX_1(s) - sX_2(s)] = 0$$

Simplify:

$$F(s) = (k_1 + bs)X_1(s) - bsX_2(s)$$

Equation for $X_2(t)$:

$$b[\dot{x}_1(t) - \dot{x}_2(t)] + k_2x_2(t) = M\ddot{x}_2(t)$$

Apply Laplace Transform:

$$b[sX_1(s) - sX_2(s)] + k_2X_2(s) = Ms^2X_2(s)$$

Simplify:

$$bsX_1(s) - (bs + k_2)X_2(s) = Ms^2X_2(s)$$

Solving for $\dfrac{X_2(s)}{F(s)}$:

$X_1(s)$:

$$X_1(s) = \frac{F(s) + bsX_2(s)}{k_1 + bs}$$

Substitute $X_1(s)$ to 2^{nd} Equation:

$$b\left(s\frac{F(s)+bsX_2(s)}{k_1+bs}\right)-(bs+k_2)X_2(s)=Ms^2X_2(s)$$

Simplify:

$$\frac{bsF(s)}{k_1+bs}+\frac{b^2s^2X_2(s)}{k_1+bs}-(bs+k_2)X_2(s)=Ms^2X_2(s)$$

Multiply Both Sides By (k_1+bs):

$$bsF(s)+b^2s^2X_2(s)-(bs+k_2)(k_1+bs)X_2(s)=Ms^2(k_1+bs)X_2(s)$$

Collecting Terms:

$$bsF(s)=[Ms^2(k_1+bs)+(bs+k_2)(k_1+bs)-b^2s^2]X_2(s)$$

Hence:

$$\frac{X_2(s)}{F(s)}=\frac{bs}{Ms^2(k_1+bs)+(bs+k_2)(k_1+bs)-b^2s^2}$$

Substitute the Given Values:

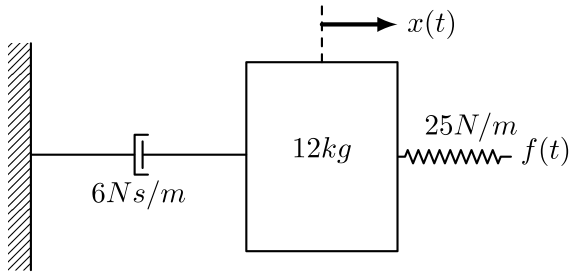
$$\frac{X_2(s)}{F(s)}=\frac{6s}{10s^2(3+6s)+(6s+4)(3+6s)-36s^2}$$

Or

$$\boxed{\frac{X_2(s)}{F(s)}=\frac{6s}{60s^3+30s^2+(36s^2+42s+12)-36s^2}}$$

Problem 40

Find the transfer function, $G(s) = \dfrac{X_1(s)}{F(s)}$ for the translational mechanical system shown in Figure



Solution:

Equation of Motion:

$$M\ddot{x}(t) = F(t) - F_s - F_d$$

Substitute the Given Values:

$$12\ddot{x}(t) = F(t) - 25x(t) - 6\dot{x}(t)$$

Apply the Laplace Transform (zero initial conditions):

$$12s^2X(s) = F(s) - 25X(s) - 6sX(s)$$

Rearrange:

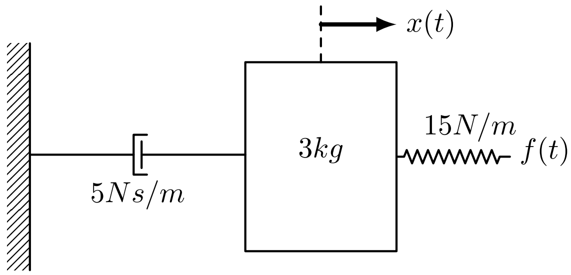
$$X(s)(12s^2 + 6s + 25) = F(s)$$

Final Transfer Function:

$$\boxed{G(s) = \frac{1}{12s^2 + 6s + 25}}$$

Problem 41

Find the transfer function, $G(s) = \dfrac{X_1(s)}{F(s)}$ for the translational mechanical system shown in Figure



Solution:

Equation of Motion:

$$M\ddot{x}(t) = F(t) - F_s - F_d$$

Substitute the Given Values:

$$3\ddot{x}(t) = F(t) - 15x(t) - 5\dot{x}(t)$$

Apply the Laplace Transform (zero initial conditions):

$$3s^2X(s) = F(s) - 15X(s) - 5sX(s)$$

Rearrange:

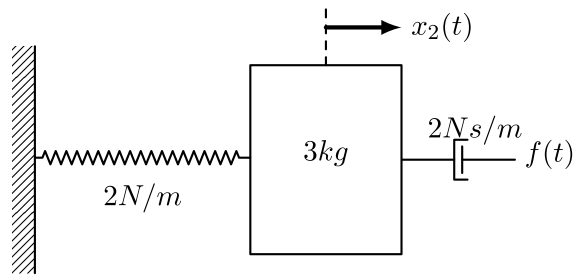
$$X(s)(3s^2 + 5s + 15) = F(s)$$

Final Transfer Function:

$$\boxed{G(s) = \frac{1}{3s^2 + 5s + 15}}$$

Problem 42

For the translational mechanical system shown below, find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Solution:

Equation of Motion:

$$M\ddot{x}_2(t) = F(t) - F_s - F_d$$

Apply Laplace Transform:

$$3s^2X_2(s) = F(s) - 2X_2(s) - 2sX_2(s)$$

Simplify:

$$3s^2X_2(s) + 2sX_2(s) + 2X_2(s) = F(s)$$

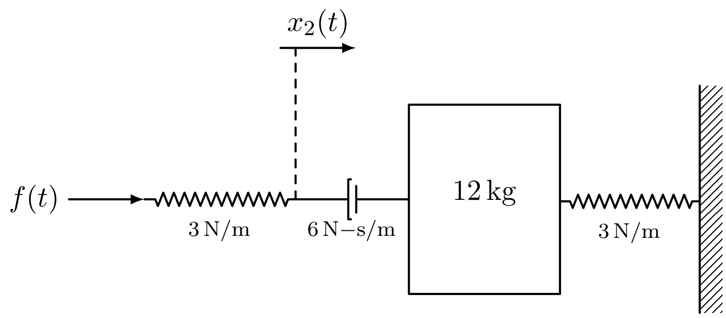
$$X_2(s)(3s^2 + 2s + 2) = F(s)$$

Thus $G(s) = \dfrac{X_2(s)}{F(s)}$

$$\boxed{G(s) = \frac{1}{3s^2 + 2s + 2}}$$

Problem 43

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Solution:

For Mass M:

$$M\ddot{x}_2(t) = F(t) - F_{s1} - F_d - F_{s2}$$

$$12\ddot{x}_2(t) = F(t) - 3[x_2(t) - x_1(t)] - 6\dot{x}_2(t) - 3x_2(t)$$

Simplify:

$$12\ddot{x}_2(t) + 6\dot{x}_2(t) + 6x_2(t) - 3x_1(t) = F(t)$$

Taking Laplace Transform:

$$12s^2X_2(s) + 6sX_2(s) + 6X_2(s) - 3X_1(s) = F(s)$$

Solve for $X_2(s)$:

$$X_2(s)(12s^2 + 6s + 6) - 3X_1(s) = F(s)$$

Relate $X_1(s)$ to $X_2(s)$:

$$3x_1(t) + 6\dot{x}_1(t) = F(t)$$

Laplace Transform:

$$3X_1(s) + 6sX_1(s) = F(s)$$

$$X_1(s)(3 + 6s) = F(s)$$

$$X_1(s) = \frac{F(s)}{3 + 6s}$$

Substitute $X_1(s)$ Back:

$$X_2(s)(12s^2 + 6s + 6) - 3\left(\frac{F(s)}{3 + 6s}\right) = F(s)$$

Solve for Transfer Function:

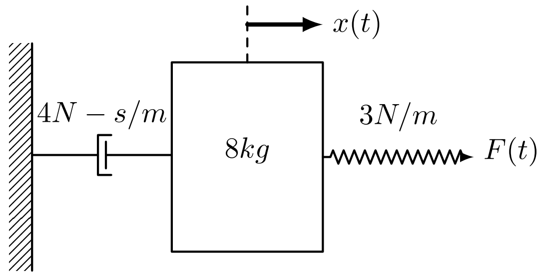
$$X_2(s) = \frac{F(s) + \left(\frac{3F(s)}{3+6s}\right)}{12s^2 + 6s + 6}$$

Thus $G(s) = \dfrac{X_2(s)}{F(s)}$

$$\boxed{G(s) = \frac{6s + 4}{(3 + 6s)(12s^2 + 6s + 6)}}$$

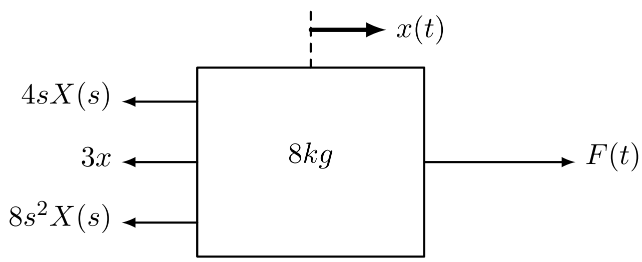
Problem 44

Determine the transfer function $\dfrac{X(s)}{F(s)}$ of the system below



Solution:

Free Body Diagram:



Force Balance:

$$F(t) - 3x(t) - 4\dot{x}(t) - 8\ddot{x}(t) = 0$$

Laplace Transform:

$$F(s) - 3X(s) - 4sX(s) - 8s^2X(s) = 0$$

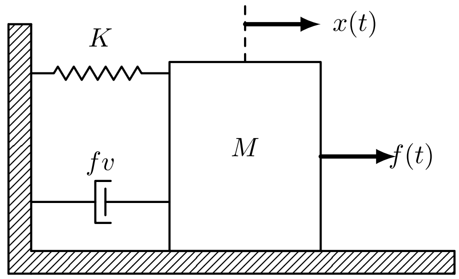
$$F(s) = (8s^2 + 4s + 3)X(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{8s^2 + 4s + 3}}$$

Problem 45

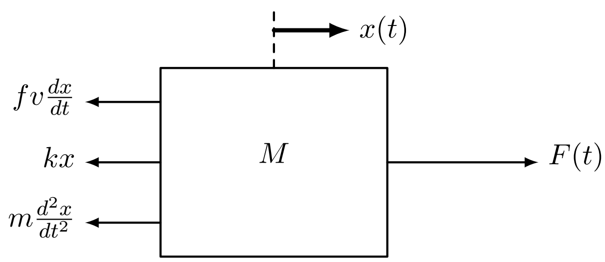
1. Determine the transfer function $\dfrac{X(s)}{F(s)}$ of the system below

$K = 4Nm; fv = 5Nm/s; M = 2kg$



Solution:

FBD of M



$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$f(t) - m\frac{d^2x}{dt^2} - fv\frac{dx}{dt} - kx = 0$

$F(s) - 2s^2X(s) - 5sX(s) - 4X(s) = 0$

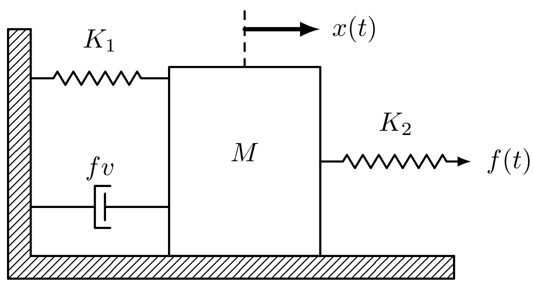
$F(s) = (2s^2 + 5s + 4)X(s)$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{2s^2 + 5s + 4}}$$

Problem 46

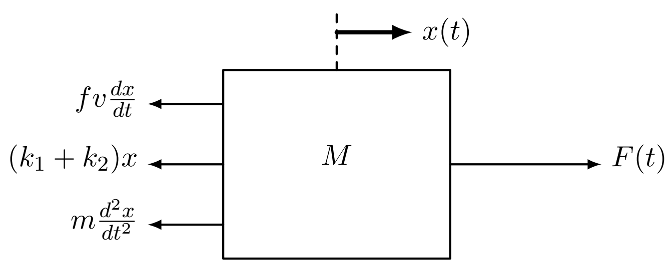
1. Determine the transfer function $\frac{X(s)}{F(s)}$ of the system below

$K_1 = 4Nm; K_2 = 3Nm; fv = 2Nm/s; M = 1kg$



Solution:

FBD of M



$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$f(t) - m\frac{d^2x}{dt^2} - fv\frac{dx}{dt} - (k_1 + k_2)x = 0$

$F(s) - s^2X(s) - 2sX(s) - (4 + 3)X(s) = 0$

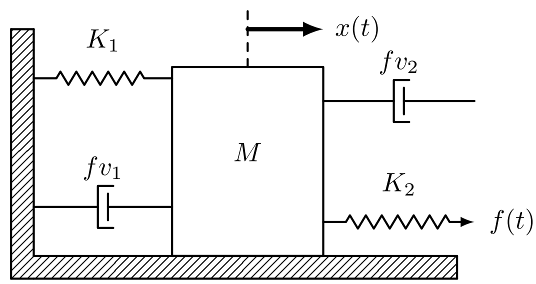
$F(s) = (s^2 + 5s + 7)X(s)$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{s^2 + 5s + 7}}$$

Problem 47

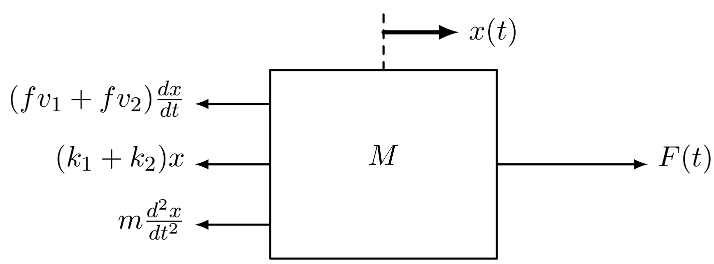
1. Determine the transfer function $\frac{X(s)}{F(s)}$ of the system below

$K_1 = 2Nm; K_2 = 6Nm; fv_1 = 4Nm/s; fv_2 = 2Nm/s; M = 5kg$



Solution:

FBD of M



$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$f(t) - m\frac{d^2x}{dt^2} - (fv_1 + fv_2)\frac{dx}{dt} - (k_1 + k_2)x = 0$

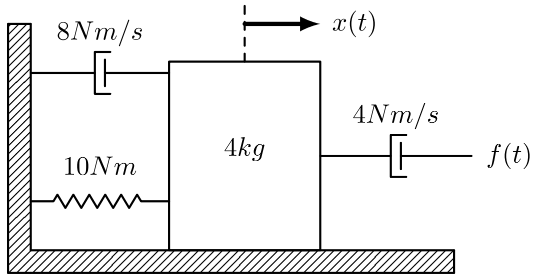
$F(s) - 5s^2X(s) - (4+2)sX(s) - (2+6)X(s) = 0$

$F(s) = (5s^2 + 6s + 8)X(s)$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{5s^2 + 6s + 8}}$$

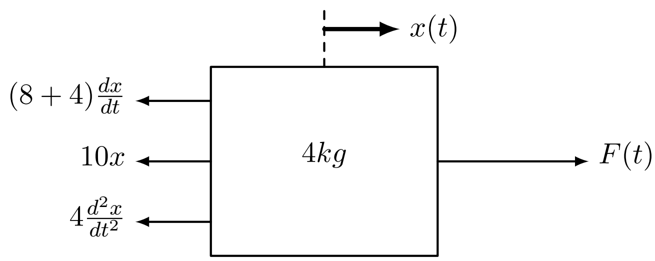
Problem 48

Determine the transfer function $\dfrac{X(s)}{F(s)}$ of the system below



Solution:

FBD @ $4kg$ mass



$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$f(t) - 4\frac{d^2x}{dt^2} - 12\frac{dx}{dt} - 10x = 0$

$F(s) - 4s^2X(s) - 12sX(s) - 10X(s) = 0$

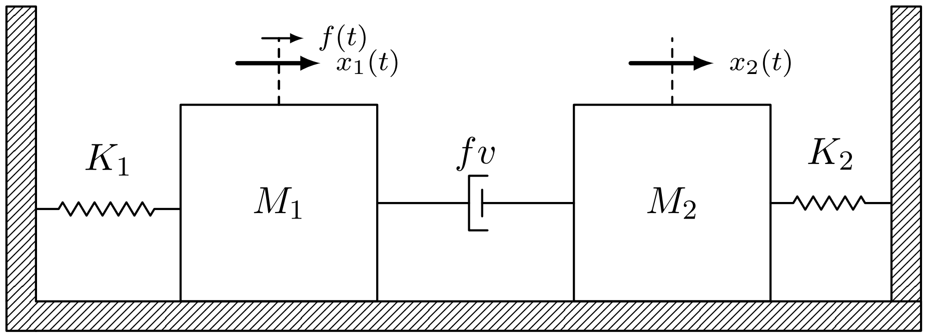
$F(s) = (4s^2 + 12s + 10)X(s)$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{4s^2 + 12s + 10}}$$

Problem 49

Determine the transfer function $\dfrac{X_2(s)}{F(s)}$ of the system below

$K_1 = 3Nm; K_2 = 6Nm; fv = 6Nm/s; M_1 = M_2 = 3kg$



Solution:

FBD of M_1

$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$f(t) + 6\frac{dx_2}{dt} - 3\frac{d^2x_1}{dt^2} - 6\frac{dx_1}{dt} - 3x_1 = 0$

$F(s)+6sX_2(s)-3s^2X_1(s)-6sX_1(s)-3X_1(s) = 0$

$F(s) = (3s^2 + 6s + 3)X_1(s) - 6sX_2(s) \rightarrow eq.1$

FBD of M_2

$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$6\frac{dx_1}{dt} - 3\frac{d^2x_2}{dt^2} - 6\frac{dx_2}{dt} - 6x_2 = 0$

$6sX_1(s) - 3s^2X_2(s) - 6sX_2(s) - 6X_2(s) = 0$

$6sX_1(s) = (3s^2 + 6s + 6)X_2(s)$

$X_1(s) = \dfrac{3s^2 + 6s + 6}{6s}X_2(s) \rightarrow eq.2$

Substitute eq.2 to eq.1

$$F(s) = (3s^2 + 6s + 3)\left(\frac{3s^2 + 6s + 6}{6s}X_2(s)\right) - 6sX_2(s)$$

$$F(s) = X_2(s)\left(\frac{9s^4 + 36s^3 + 63s^2 + 54s + 18}{6s} - 6s\right)$$

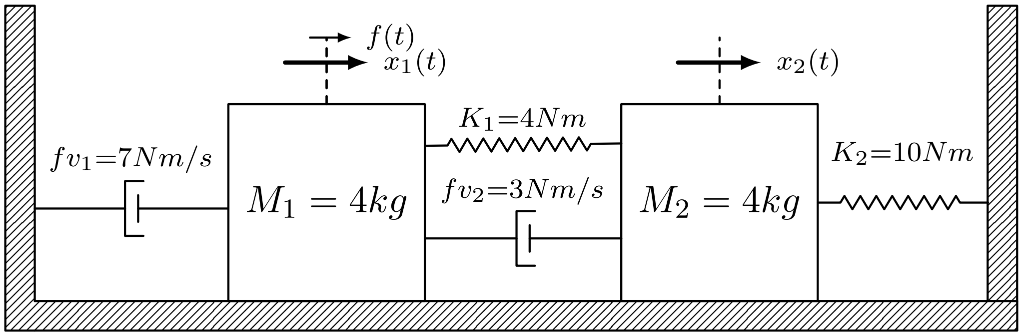
$$F(s) = X_2(s)\left(\frac{9s^4 + 36s^3 + 63s^2 + 54s + 18 - 36s^2}{6s}\right)$$

$$\frac{F(s)}{X_2(s)} = \frac{9s^4 + 36s^3 + 27s^2 + 54s + 18}{6s}$$

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{6s}{9s^4 + 36s^3 + 27s^2 + 54s + 18}}$$

Problem 50

Determine the transfer function $\dfrac{X_2(s)}{F(s)}$ of the system below



Solution:

FBD of M_1

$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$f(t) + 3\frac{dx_2}{dt} + 4x_2 - 4\frac{d^2x_1}{dt^2} - 10\frac{dx_1}{dt} - 4x_1 = 0$

$F(s) + 3sX_2(s) + 4X_2(s) - 4s^2X_1(s) - 10sX_1(s) - 4X_1(s) = 0$

$F(s) = (4s^2 + 10s + 4)X_1(s) - (3s + 4)X_2(s) \rightarrow eq.1$

FBD of M_2

$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$3\frac{dx_1}{dt} + 4x_1 - 4\frac{d^2x_2}{dt^2} - 3\frac{dx_2}{dt} - 14x_2 = 0$

$3sX_1(s) + 4X_1(s) - 4s^2X_2(s) - 3sX_2(s) - 14X_2(s) = 0$

$(3s + 4)X_1(s) = (4s^2 + 3s + 14)X_2(s)$

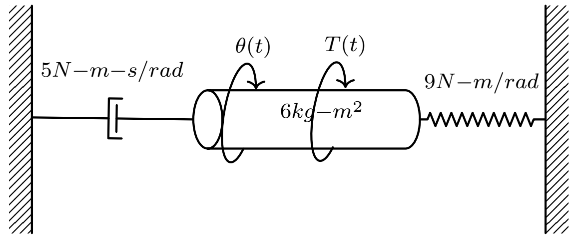
$X_1(s) = \dfrac{4s^2 + 3s + 14}{3s + 4}X_2(s) \rightarrow eq.2$

Substitute eq.2 to eq.1

$$F(s) = (4s^2 + 10s + 4)\left(\frac{4s^2 + 3s + 14}{3s + 4}X_2(s)\right) - (3s + 4)X_2(s)$$
$$F(s) = X_2(s)\left(\frac{16s^4 + 52s^3 + 93s^2 + 138s + 40}{3s + 4}\right)$$
$$\frac{F(s)}{X_2(s)} = \frac{16s^4 + 52s^3 + 93s^2 + 138s + 40}{3s + 4}$$
$$\boxed{\frac{X_2(s)}{F(s)} = \frac{3s + 4}{16s^4 + 52s^3 + 93s^2 + 138s + 40}}$$

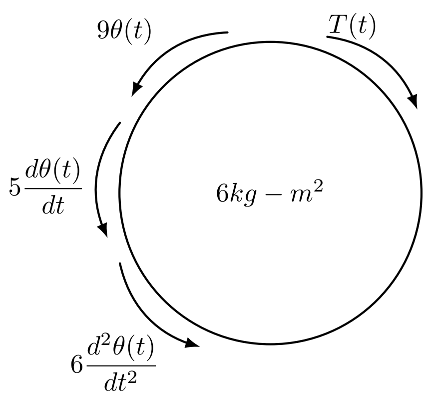
Problem 51

1. Determine the transfer function $\dfrac{\theta(s)}{T(s)}$ of the system below



Solution:

FBD @ 6 $kg - m^2$ inertia



$\sum T = 0 \quad \oplus$

$$T(t) - 6\frac{d^2\theta(t)}{dt^2} - 5\frac{d\theta(t)}{dt} - 9\theta(t) = 0$$

$$T(s) - 6s^2\theta(s) - 5s\theta(s) - 9\theta(s) = 0$$

$$(6s^2 + 5s + 9)\theta(s) = T(s)$$

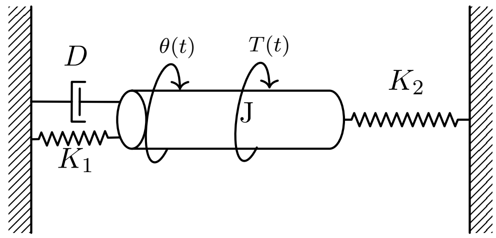
$$\frac{T(s)}{\theta(s)} = (6s^2 + 5s + 9)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{1}{6s^2 + 5s + 9}}$$

Problem 52

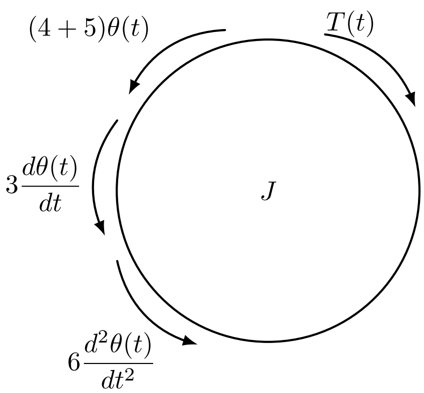
1. Determine the transfer function $\dfrac{\theta(s)}{T(s)}$ of the system below

$D = 3N-m-s/rad; K_1 = 4N-m/rad; K_2 = 5N-m/rad; J = 6kg-m^2$



Solution:

FBD of J inertia



$\sum T = 0 \quad \oplus$

$$T(t) - 6\frac{d^2\theta(t)}{dt^2} - 3\frac{d\theta(t)}{dt} - (4+5)\theta(t) = 0$$

$$T(s) - 6s^2\theta(s) - 3s\theta(s) - 9\theta(s) = 0$$

$$(6s^2 + 3s + 9)\theta(s) = T(s)$$

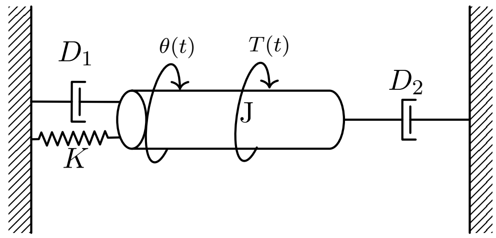
$$\frac{T(s)}{\theta(s)} = (6s^2 + 3s + 9)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{1}{6s^2 + 3s + 9}}$$

Problem 53

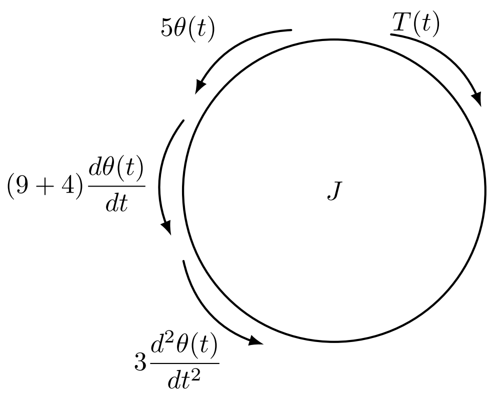
1. Determine the transfer function $\dfrac{\theta(s)}{T(s)}$ of the system below

$D_1 = 9N - m - s/rad; D_2 = 4N - m - s/rad; K = 5N - m/rad; J = 3kg - m^2$



Solution:

FBD of J inertia



$\sum T = 0 \quad \oplus$

$$T(t) - 3\frac{d^2\theta(t)}{dt^2} - (9+4)\frac{d\theta(t)}{dt} - 5\theta(t) = 0$$

$$T(s) - 3s^2\theta(s) - 13s\theta(s) - 5\theta(s) = 0$$

$$(3s^2 + 13s + 5)\theta(s) = T(s)$$

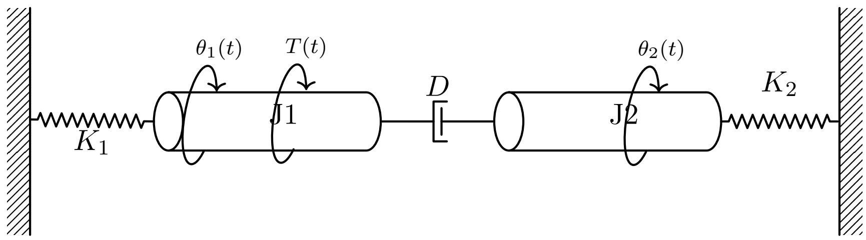
$$\frac{T(s)}{\theta(s)} = (3s^2 + 13s + 5)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{1}{3s^2 + 13s + 5}}$$

Problem 54

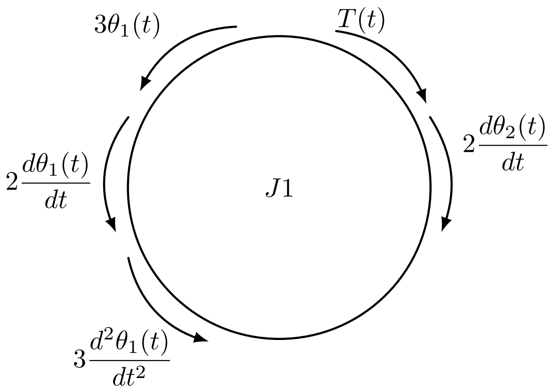
1. Determine the transfer function $\dfrac{\theta_1(s)}{T(s)}$ of the system below

$D = 2N - m - s/rad; K_1 = 3N - m/rad; K_2 = 6N - m/rad; J1 = J2 = 5kg - m^2$



Solution:

FBD of J1 inertia



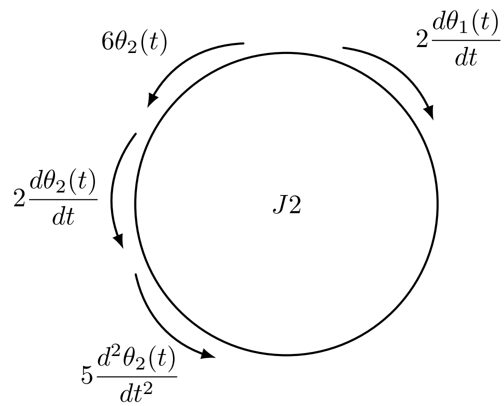
$\sum T = 0 \quad \oplus$

$$T(t) + 2\frac{d\theta_2(t)}{dt} - 5\frac{d^2\theta_1(t)}{dt^2} - 2\frac{d\theta_1(t)}{dt} - 3\theta_1(t) = 0$$

$$T(s) + 2s\theta_2(s) - 5s^2\theta_1(s) - 2s\theta_1(s) - 3\theta_1(s) = 0$$

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J2 inertia



$\sum T = 0 \quad \oplus$

$$2\frac{d\theta_1(t)}{dt} - 5\frac{d^2\theta_2(t)}{dt^2} - 2\frac{d\theta_2(t)}{dt} - 6\theta_2(t) = 0$$

$$2s\theta_1(s) - 5s^2\theta_2(s) - 2s\theta_2(s) - 3\theta_2(s) = 0$$

$$2s\theta_1(s) = (5s^2 + 2s + 3)\theta_2(s)$$

$$\theta_2(s) = \frac{2s}{5s^2 + 2s + 3}\theta_1(s) \rightarrow eq.2$$

Substitute eq.2 to eq.1

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\theta_2(s) = T(s)$$

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\left(\frac{2s}{5s^2 + 2s + 3}\theta_1(s)\right) = T(s)$$

$$(5s^2 + 2s + 3)\theta_1(s)\left(\frac{-4s^2}{5s^2 + 2s + 3}\theta_1(s)\right) = T(s)$$

$$\theta_1(s)\left((5s^2 + 2s + 3) + \frac{-4s^2}{5s^2 + 2s + 3}\right) = T(s)$$

$$\theta_1(s)\left(\frac{-4s^2 + 25s^4 + 20s^3 + 34s^2 + 12s + 9}{5s^2 + 2s + 3}\right) = T(s)$$

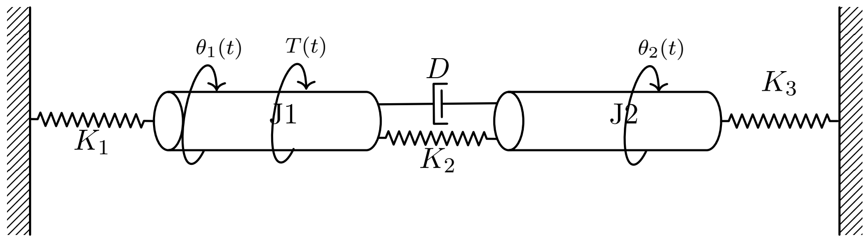
$$\left(\frac{25s^4 + 20s^3 + 30s^2 + 12s + 9}{5s^2 + 2s + 3}\right) = \frac{T(s)}{\theta_1(s)}$$

$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{5s^2 + 2s + 3}{25s^4 + 20s^3 + 30s^2 + 12s + 9}}$$

Problem 55

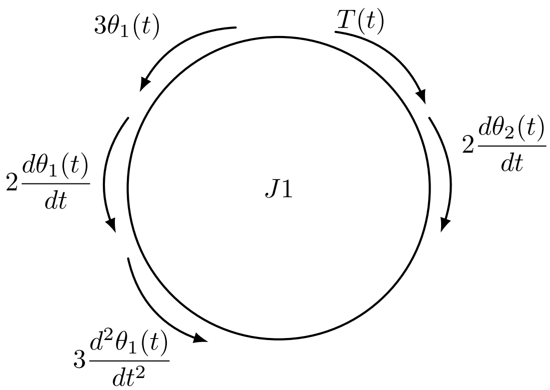
1. Determine the transfer function $\dfrac{\theta_1(s)}{T(s)}$ of the system below

$D = 8N - m - s/rad; K_1 = 5N - m/rad; K_2 = K_3 = 7N - m/rad; J1 = J2 = 7kg - m^2$



Solution:

FBD of J1 inertia



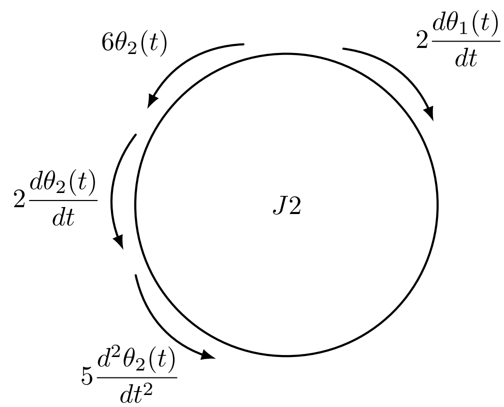
$\sum T = 0 \quad \oplus$

$$T(t) + 2\frac{d\theta_2(t)}{dt} - 5\frac{d^2\theta_1(t)}{dt^2} - 2\frac{d\theta_1(t)}{dt} - 3\theta_1(t) = 0$$

$$T(s) + 2s\theta_2(s) - 5s^2\theta_1(s) - 2s\theta_1(s) - 3\theta_1(s) = 0$$

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J2 inertia



$\sum T = 0 \quad \oplus$

$$2\frac{d\theta_1(t)}{dt} - 5\frac{d^2\theta_2(t)}{dt^2} - 2\frac{d\theta_2(t)}{dt} - 6\theta_2(t) = 0$$

$$2s\theta_1(s) - 5s^2\theta_2(s) - 2s\theta_2(s) - 3\theta_2(s) = 0$$

$$2s\theta_1(s) = (5s^2 + 2s + 3)\theta_2(s)$$

$$\theta_2(s) = \frac{2s}{5s^2 + 2s + 3}\theta_1(s) \rightarrow eq.2$$

Substitute eq.2 to eq.1

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\theta_2(s) = T(s)$$

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\left(\frac{2s}{5s^2 + 2s + 3}\theta_1(s)\right) = T(s)$$

$$(5s^2 + 2s + 3)\theta_1(s)\left(\frac{-4s^2}{5s^2 + 2s + 3}\theta_1(s)\right) = T(s)$$

$$\theta_1(s)\left((5s^2 + 2s + 3) + \frac{-4s^2}{5s^2 + 2s + 3}\right) = T(s)$$

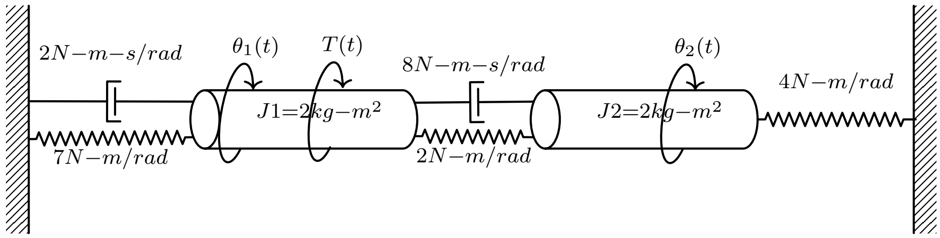
$$\theta_1(s)\left(\frac{-4s^2 + 25s^4 + 20s^3 + 34s^2 + 12s + 9}{5s^2 + 2s + 3}\right) = T(s)$$

$$\left(\frac{25s^4 + 20s^3 + 30s^2 + 12s + 9}{5s^2 + 2s + 3}\right) = \frac{T(s)}{\theta_1(s)}$$

$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{5s^2 + 2s + 3}{25s^4 + 20s^3 + 30s^2 + 12s + 9}}$$

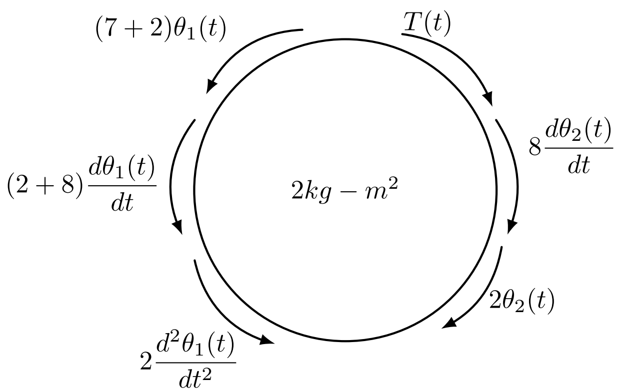
Problem 56

1. Determine the transfer function $\dfrac{\theta_2(s)}{T(s)}$ of the system below



Solution:

FBD of J1 inertia



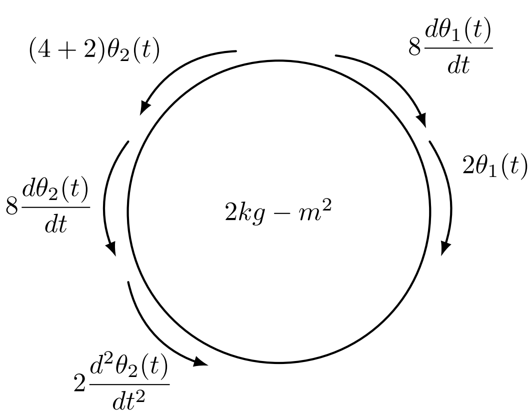
$\sum T = 0 \quad \oplus$

$$T(t) + 8\frac{d\theta_2(t)}{dt} + 2\theta_2(t) - 2\frac{d^2\theta_1(t)}{dt^2} - (2+8)\frac{d\theta_1(t)}{dt} - (7+2)\theta_1(t) = 0$$

$$T(s) + 8s\theta_2(s) + 2\theta_2(s) - 2s^2\theta_1(s) - 10s\theta_1(s) - 9\theta_1(s) = 0$$

$$(2s^2 + 10s + 9)\theta_1(s) - (8s+2)\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J2 inertia



$$\sum T = 0 \quad \oplus$$

$$8\frac{d\theta_1(t)}{dt} + 2\theta_1(t) - 2\frac{d^2\theta_2(t)}{dt^2} - 8\frac{d\theta_2(t)}{dt} - (4+2)\theta_2(t) = 0$$

$$8s\theta_1(s) + 2\theta_1(s) - 2s^2\theta_2(s) - 8s\theta_2(s) - 6\theta_2(s) = 0$$

$$(8s+2)\theta_1(s) = (2s^2 + 8s + 6)\theta_2(s)$$

$$\theta_1(s) = \frac{2s^2 + 8s + 6}{8s+2}\theta_2(s) \rightarrow eq.2$$

Substitute eq.2 to eq.1

$$T(s) = (2s^2 + 10s + 9)\theta_1(s) - (8s+2)\theta_2(s)$$

$$T(s) = (2s^2 + 10s + 9)\left(\frac{2s^2 + 8s + 6}{8s+2}\theta_2(s)\right) - (8s+2)\theta_2(s)$$

$$T(s) = \left(\frac{(2s^2 + 10s + 9)(2s^2 + 8s + 6)}{8s+2}\theta_2(s)\right) - (8s+2)\theta_2(s)$$

$$T(s) = \left(\frac{4s^4 + 36s^3 + 110s^2 + 132s + 54}{8s+2} - (8s+2)\right)\theta_2(s)$$

$$T(s) = \left(\frac{4s^4 + 36s^3 + 110s^2 + 132s + 54 - 64s^2 - 32s - 4}{8s+2}\right)\theta_2(s)$$

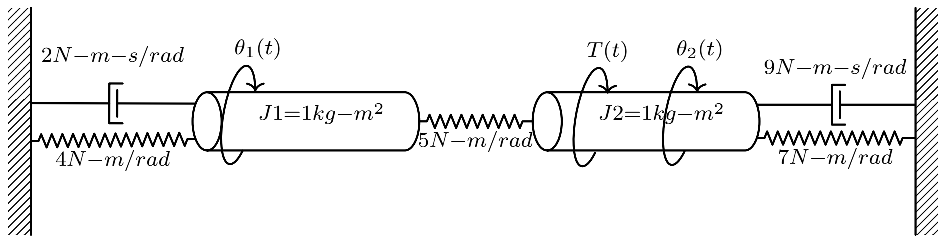
$$T(s) = \left(\frac{4s^4 + 36s^3 + 46s^2 + 100s + 50}{8s+2}\right)\theta_2(s)$$

$$\frac{T(s)}{\theta_2(s)} = \left(\frac{4s^4 + 36s^3 + 46s^2 + 100s + 50}{8s+2}\right)$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{8s+2}{4s^4 + 36s^3 + 46s^2 + 100s + 50}}$$

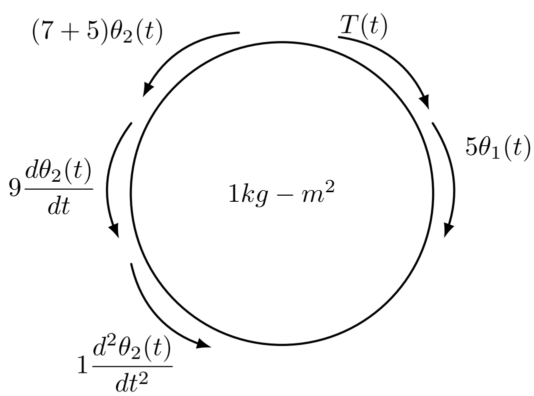
Problem 57

1. Determine the transfer function $\dfrac{\theta_2(s)}{T(s)}$ of the system below



Solution:

FBD of J2 inertia



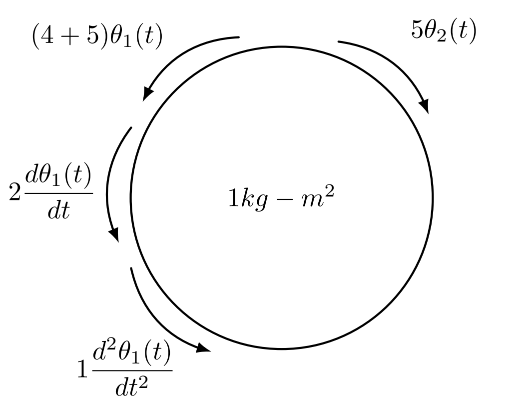
$\sum T = 0 \quad \oplus$

$$T(t) + 5\theta_1(t) - 1\frac{d^2\theta_2(t)}{dt^2} - 9\frac{d\theta_2(t)}{dt} - (7+5)\theta_2(t) = 0$$

$$T(s) + 5\theta_1(s) - s^2\theta_2(s) - 9s\theta_2(s) - 12\theta_2(s) = 0$$

$$-5\theta_1(s) + (s^2 + 9s + 12)\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J1 inertia



$\sum T = 0 \quad \oplus$

$$5\theta_2(t) - 1\frac{d^2\theta_1(t)}{dt^2} - 2\frac{d\theta_1(t)}{dt} - (4+5)\theta_1(t) = 0$$

$$5\theta_1(s) - s^2\theta_1(s) - 2s\theta_1(s) - 9\theta_1(s) = 0$$

$$5\theta_1(s) = (s^2 + 2s + 9)\theta_2(s)$$

$$\theta_1(s) = \frac{s^2 + 2s + 9}{5}\theta_2(s) \rightarrow eq.2$$

Substitute eq.2 to eq.1

$$T(s) = -5\theta_1(s) + (s^2 + 9s + 12)\theta_2(s)$$

$$T(s) = -5\left(\frac{s^2 + 2s + 9}{5}\theta_2(s)\right) + (s^2 + 9s + 12)\theta_2(s)$$

$$T(s) = \left(\frac{(-5)(s^2 + 2s + 9)}{5}\theta_2(s)\right) + (s^2 + 9s + 12)\theta_2(s)$$

$$T(s) = \left(\frac{-5s^2 - 10s - 45}{5} + (s^2 + 9s + 12)\right)\theta_2(s)$$

$$T(s) = \left(\frac{-5s^2 - 10s - 45 + 5s^2 + 45s + 60}{5}\right)\theta_2(s)$$

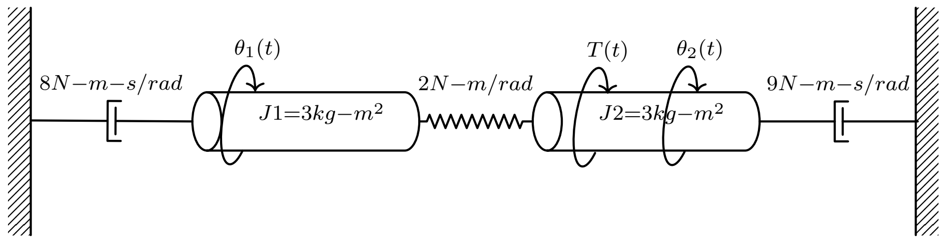
$$T(s) = \left(\frac{35s + 15}{5}\right)\theta_2(s)$$

$$\frac{T(s)}{\theta_2(s)} = \frac{35s + 15}{5}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{5}{35s + 15}}$$

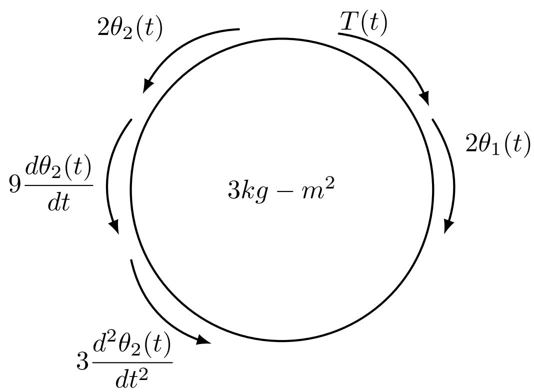
Problem 58

1. Determine the transfer function $\dfrac{\theta_2(s)}{T(s)}$ of the system below



Solution:

FBD of J2 inertia



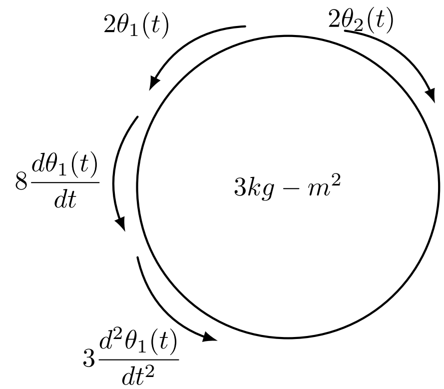
$\sum T = 0 \quad \oplus$

$$T(t) + 2\theta_1(t) - 3\frac{d^2\theta_2(t)}{dt^2} - 9\frac{d\theta_2(t)}{dt} - 2\theta_2(t) = 0$$

$$T(s) + 2\theta_1(s) - 3s^2\theta_2(s) - 9s\theta_2(s) - 2\theta_2(s) = 0$$

$$-2\theta_1(s) + (3s^2 + 9s + 2)\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J1 inertia



$\sum T = 0 \quad \oplus$

$$2\theta_2(t) - 3\frac{d^2\theta_1(t)}{dt^2} - 8\frac{d\theta_1(t)}{dt} - 2\theta_1(t) = 0$$

$$2\theta_2(s) - 3s^2\theta_2(s) - 8s\theta_2(s) - 2\theta_2(s) = 0$$

$$2\theta_1(s) = (3s^2 + 8s + 2)\theta_2(s)$$

$$\theta_1(s) = \frac{3s^2 + 8s + 2}{2}\theta_2(s) \rightarrow eq.2$$

Substitute eq.2 to eq.1

$$T(s) = -2\theta_1(s) + (3s^2 + 9s + 2)\theta_2(s)$$

$$T(s) = -2\left(\frac{3s^2 + 8s + 2}{2}\theta_2(s)\right) + (3s^2 + 9s + 2)\theta_2(s)$$

$$T(s) = \left(\frac{(-2)(3s^2 + 8s + 2)}{2}\theta_2(s)\right) + (3s^2 + 9s + 2)\theta_2(s)$$

$$T(s) = \left(\frac{-6s^2 - 16s - 4}{2} + (3s^2 + 9s + 2)\right)\theta_2(s)$$

$$T(s) = \left(\frac{-6s^2 - 16s - 4 + 6s^2 + 18s + 4}{2}\right)\theta_2(s)$$

$$T(s) = \left(\frac{2s}{2}\right)\theta_2(s)$$

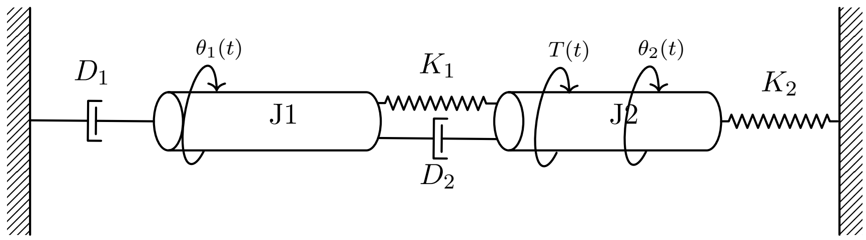
$$\frac{T(s)}{\theta_2(s)} = \frac{2s}{2}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{2}{2s}}$$

Problem 59

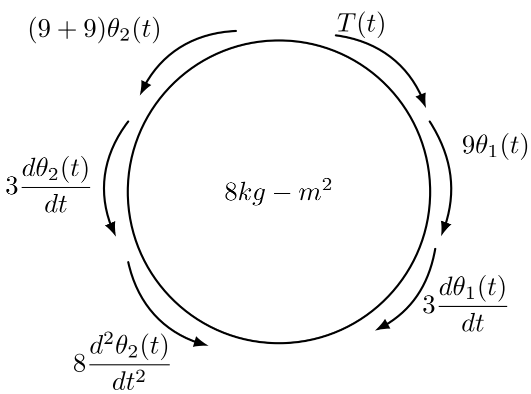
1. Determine the transfer function $\dfrac{\theta_2(s)}{T(s)}$ of the system below

$D_1 = 5N-m-s/rad; D_2 = 3N-m-s/rad; K_1 = K_2 = 9N-m/rad; J1 = J2 = 8kg-m^2$



Solution:

FBD of J2 inertia



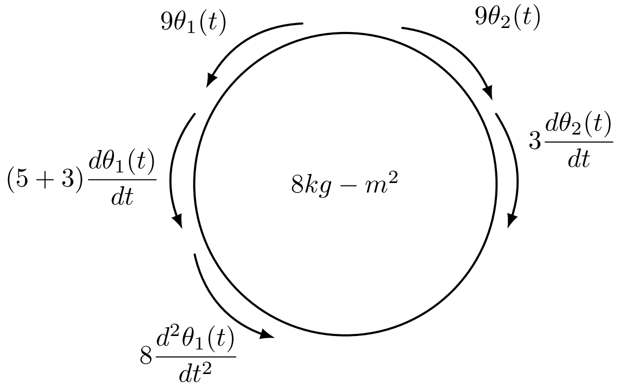
$\sum T = 0 \quad \oplus$

$$T(t) + 9\theta_1(t) + 3\frac{d\theta_1(t)}{dt} - 8\frac{d^2\theta_2(t)}{dt^2} - 3\frac{d\theta_2(t)}{dt} - (9+9)\theta_2(t) = 0$$

$$T(s) + 9\theta_1(s) + 3s\theta_1(s) - 8s^2\theta_2(s) - 3s\theta_2(s) - 18\theta_2(s) = 0$$

$$-(3s+9)\theta_1(s) + (8s^2+3s+18)\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J1 inertia



$\sum T = 0 \quad \oplus$

$$3\frac{d\theta_2(t)}{dt} + 9\theta_2(t) - 8\frac{d^2\theta_1(t)}{dt^2} - (5+3)\frac{d\theta_1(t)}{dt} - 9\theta_1(t) = 0$$

$$3s\theta_2(s) + 9\theta_2(s) - 8s^2\theta_1(s) - 8s\theta_1(s) - 9\theta_1(s) = 0$$

$$(8s^2 + 8s + 9)\theta_1(s) = (3s + 9)\theta_2(s)$$

$$\theta_1(s) = \frac{3s+9}{8s^2+8s+9}\theta_2(s) \rightarrow eq.2$$

Substitute eq.2 to eq.1

$$T(s) = -(3s+9)\theta_1(s) + (8s^2 + 3s + 18)\theta_2(s)$$

$$T(s) = -(3s+9)\left(\frac{3s+9}{8s^2+8s+9}\theta_2(s)\right) + (8s^2 + 3s + 18)\theta_2(s)$$

$$T(s) = \left(\frac{-9s^2 - 54s - 81}{8s^2+8s+9}\theta_2(s)\right) + (8s^2 + 3s + 18)\theta_2(s)$$

$$T(s) = \left(\frac{-9s^2 - 54s - 81}{8s^2+8s+9} + (8s^2 + 3s + 18)\right)\theta_2(s)$$

$$T(s) = \left(\frac{-9s^2 - 54s - 81 + 64s^4 + 88s^3 + 240s^2 + 171s + 162}{8s^2+8s+9}\right)\theta_2(s)$$

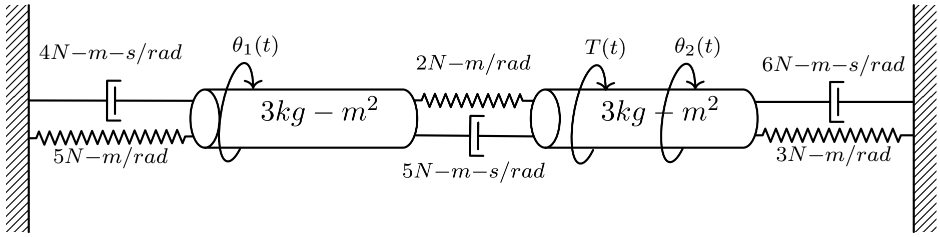
$$T(s) = \left(\frac{64s^4 + 88s^3 + 231s^2 + 117s + 81}{8s^2+8s+9}\right)\theta_2(s)$$

$$\frac{T(s)}{\theta_2(s)} = \frac{64s^4 + 88s^3 + 231s^2 + 117s + 81}{8s^2+8s+9}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{8s^2+8s+9}{64s^4 + 88s^3 + 231s^2 + 117s + 81}}$$

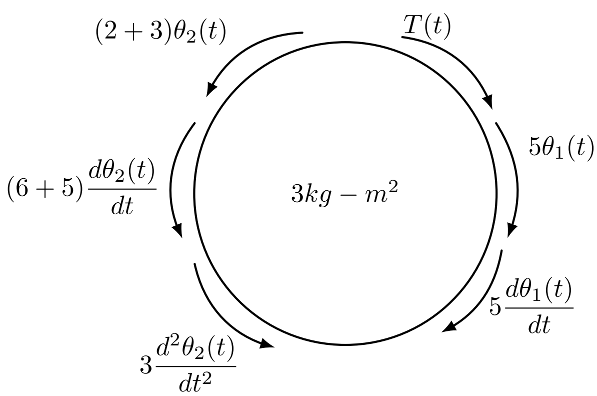
Problem 60

1. Determine the transfer function $\frac{\theta_1(s)}{T(s)}$ of the system below



Solution:

FBD of J2 inertia



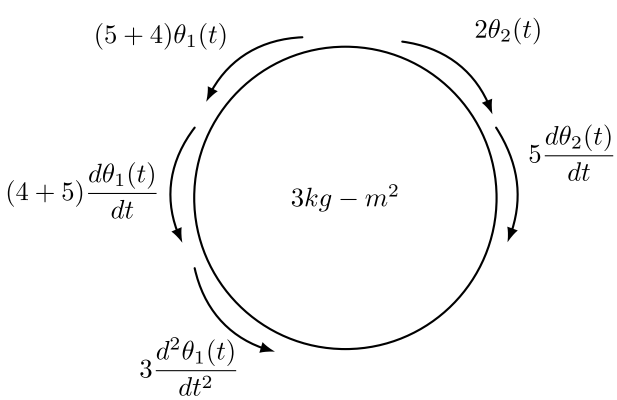
$\sum T = 0 \quad \oplus$

$$T(t) + 5\theta_1(t) + 5\frac{d\theta_1(t)}{dt} - 3\frac{d^2\theta_2(t)}{dt^2} - (6+5)\frac{d\theta_2(t)}{dt} - (2+3)\theta_2(t) = 0$$

$$T(s) + 5\theta_1(s) + 5s\theta_1(s) - 3s^2\theta_2(s) - 11s\theta_2(s) - 5\theta_2(s) = 0$$

$$-(5s+5)\theta_1(s) + (3s^2 + 11s + 5)\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J1 inertia



$\sum T = 0 \quad \oplus$

$$5\frac{d\theta_2(t)}{dt} + 2\theta_2(t) - 3\frac{d^2\theta_1(t)}{dt^2} - (4+5)\frac{d\theta_1(t)}{dt} - (5+4)\theta_1(t) = 0$$

$$5s\theta_2(s) + 2\theta_2(s) - 3s^2\theta_1(s) - 9s\theta_1(s) - 9\theta_1(s) = 0$$

$$(3s^2 + 9s + 9)\theta_1(s) = (5s+2)\theta_2(s)$$

$$\theta_2(s) = \frac{3s^2 + 9s + 9}{5s+2}\theta_1(s) \rightarrow eq.2$$

Substitute eq.2 to eq.1

$$T(s) = -(5s+5)\theta_1(s) + (3s^2 + 11s + 5)\theta_2(s)$$

$$T(s) = -(5s+5)\theta_1(s) + (3s^2 + 11s + 5)\left(\frac{3s^2 + 9s + 9}{5s+2}\theta_1(s)\right)$$

$$T(s) = -(5s+5)\theta_1(s) + \left(\frac{9s^4 + 54s^3 + 129s^2 + 144s + 45}{5s+2}\theta_1(s)\right)$$

$$T(s) = \theta_1(s)\left(\frac{9s^4 + 54s^3 + 129s^2 + 144s + 45}{5s+2} - (5s+5)\right)$$

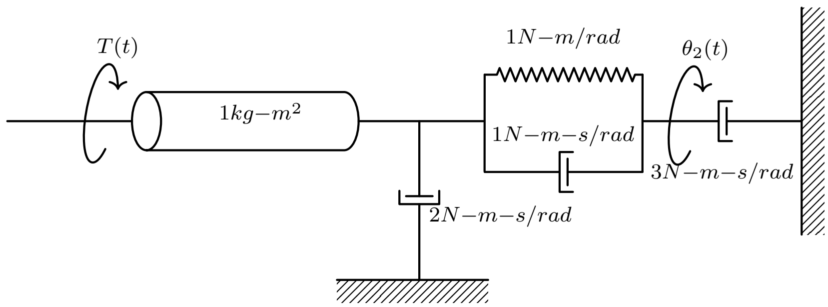
$$T(s) = \theta_1(s)\left(\frac{9s^4 + 54s^3 + 129s^2 + 144s + 45 - 25s^2 - 35s - 10}{5s+2}\right)$$

$$T(s) = \theta_1(s)\left(\frac{9s^4 + 54s^3 + 104s^2 + 109s + 35}{5s+2}\right)$$

$$\frac{T(s)}{\theta_1(s)} = \frac{9s^4 + 54s^3 + 104s^2 + 109s + 35}{5s+2}$$

$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{5s+2}{9s^4 + 54s^3 + 104s^2 + 109s + 35}}$$

Problem 61
Find the transfer function $\theta_2(s)T(s)$



Solution:

Equation of Motion

For θ_1

$$J\frac{d^2\theta_1(t)}{dt^2} + (b_1 + b_2)\frac{d\theta_1(t)}{dt} + k[\theta_1(t) - \theta_2(t)] = T(t)$$

Laplace Domain:

$$Js^2\theta_1(s) + (b_1 + b_2)s\theta_1(s) + k[\theta_1(s) - \theta_2(s)] = T(s)$$

Substitute the Given Values:

$$1s^2\theta_1(s) + (1 + 2)s\theta_1(s) + 1[\theta_1(s) - \theta_2(s)] = T(s)$$

$$s^2\theta_1(s) + 3s\theta_1(s) + \theta_1(s) - \theta_2(s) = T(s)$$

$$(s^2 + 3s + 1)\theta_1(s) - \theta_2(s) = T(s)$$

$$(s^2 + 3s + 1)\theta_1(s) = T(s) + \theta_2(s)$$

Equation of Motion

For θ_2

$$(b_2 + b_3)\frac{d\theta_2(t)}{dt} + k[\theta_2(t) - \theta_1(t)] = 0$$

Laplace Domain:

$$(b_2 + b_3)s\theta_2(s) + k[\theta_2(s) - \theta_1(s)] = 0$$

Substitute the Given Values:

$$(2+3)s\theta_2(s) + 1[\theta_2(s) - \theta_1(s)] = 0$$

$$5s\theta_2(s) + \theta_2(s) - \theta_1(s) = 0$$

$$(5s+1)\theta_2(s) = \theta_1(s)$$

$$\theta_1(s) = (5s+1)\theta_2(s)$$

Substitute θ_1 Into First Equation:

$$(s^2 + 3s + 1)(5s+1)\theta_2(s) = T(s) + \theta_2(s)$$

$$(5s+1)(s^2 + 3s + 1)\theta_2(s) = T(s) + \theta_2(s)$$

$$(5s^3 + 16s^2 + 8s + 1)\theta_2(s) = T(s) + \theta_2(s)$$

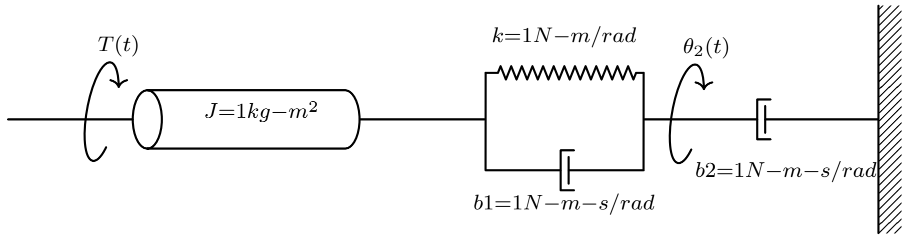
$$(5s^3 + 16s^2 + 8s)\theta_2(s) = T(s)$$

Solve for $G(s) = \dfrac{\theta_2(s)}{T(s)}$:

$$\boxed{G(s) = \frac{\theta_2(s)}{T(s)} = \frac{1}{5s^3 + 16s^2 + 8s}}$$

Problem 62

Find the transfer function $\dfrac{\theta_2(s)}{T(s)}$



Solution:

Force Balance on M

First Mass (Moment of Inertia J):

$$J\frac{d^2\theta_1(t)}{dt^2} + b_1\frac{d\theta_1(t)}{dt} + k[\theta_1(t) - \theta_2(t)] = T(t)$$

Second Mass:

$$J\frac{d^2\theta_2(t)}{dt^2} + b_2\frac{d\theta_2(t)}{dt} + k[\theta_2(t) - \theta_1(t)] = T(t)$$

Laplace Transform Where Initial Condition is 0:

First Inertia:

$$Js^2\theta_1(s) + b_1s\theta_1(s) + k[\theta_1(s) - \theta_2(s)] = T(s)$$

Second Inertia:

$$Js^2\theta_2(s) + b_2s\theta_2(s) + k[\theta_2(s) - \theta_1(s)] = 0$$

Solving for $\theta_2(s)$

From 2^{nd} Equation:

$$(Js^2 + b_2s + k)\theta_2(s) = k\theta_1(s)$$

$$\theta_2(s) = \frac{k\theta_1(s)}{Js^2 + b_2s + k}$$

Substitute it to 1^{st} Equation:

$$Js^2\theta_1(s) + b_1s\theta_1(s) + k[\theta_1(s) - \frac{k\theta_1(s)}{Js^2 + b_2s + k}] = T(s)$$

Simplify:

Factor out $\theta_1(s)$:

$$\theta_1(s)\left(Js^2+b_1s+k-\frac{k^2}{Js^2+b_2s+k}\right)=T(s)$$

$$\theta_1(s)\left[\frac{(Js^2+b_2s+k)(Js^2+b_1s+k)-k^2}{Js^2+b_2s+k}\right]=T(s)$$

$$\theta_1(s)=\frac{T(s)(Js^2+b_2s+k)}{(Js^2+b_2s+k)(Js^2+b_1s+k)-k^2}$$

Substitute θ_2 in Equation:

$$\theta_2(s)=\frac{(k)[T(s)](Js^2+b_2s+k)}{(Js^2+b_2s+k)[(Js^2+b_1s+k)(Js^2+b_2s+k)]-k^2}$$

Simplify:

$$\frac{\theta_2(s)}{T(s)}=\frac{k(Js^2+b_2s+k)}{(Js^2+b_1s+k)(Js^2+b_2s+k)-k^2}$$

Substitute the Given Values:

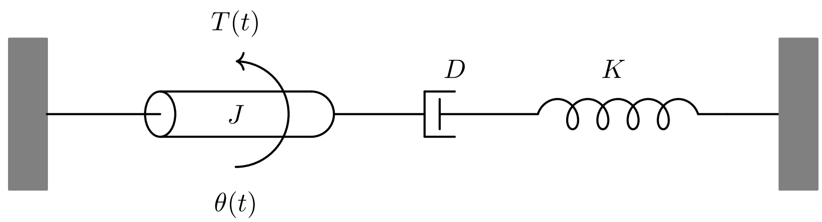
$$\frac{\theta_2(s)}{T(s)}=\frac{s^2+s+1}{(s^2+s+1)^2-1}$$

$$\boxed{\frac{\theta_2(s)}{T(s)}=\frac{s^2+s+1}{s^4+2s^3+s^2}}$$

Problem 63
Given $J = 2\,\text{kg} \cdot \text{m}^2$, $D = 0.5\,\text{N} \cdot \text{m} \cdot \text{s/rad}$, and $K = 3\,\text{N} \cdot \text{m/rad}$, find the transfer function

$$G(s) = \frac{\Theta(s)}{T(s)}$$

for the system below:



Given:
$J = 2\,\text{kg} \cdot \text{m}^2$
$D = 0.5\,\text{N} \cdot \text{m} \cdot \text{s/rad}$
$K = 3\,\text{N} \cdot \text{m/rad}$

Required: G(s)

Solution:
From the diagram, this is a single-degree-of-freedom rotational mechanical system with moment of inertia J, damper D, and torsional spring K.

The equation of motion in the Laplace domain is:

$$(Js^2 + Ds + K)\Theta(s) = T(s)$$

Solving for the transfer function:

$$G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2 + Ds + K}$$

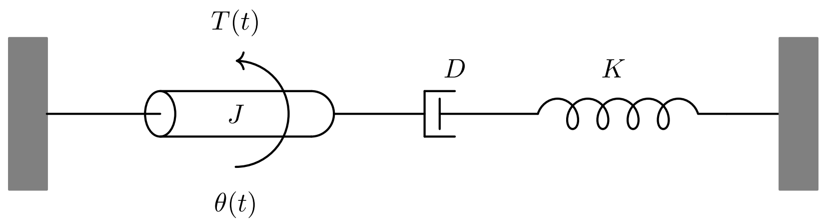
Substitute the given values:

$$G(s) = \frac{1}{2s^2 + 0.5s + 3}$$

Answer:

$$\boxed{G(s) = \frac{1}{2s^2 + 0.5s + 3}}$$

Problem 64



Find the natural frequency ω_n and damping ratio ζ for the above system.

Given: As per diagram

Required: ω_n and damping ratio ζ

Solution:
This system is a standard second-order rotational mechanical model. We compute:

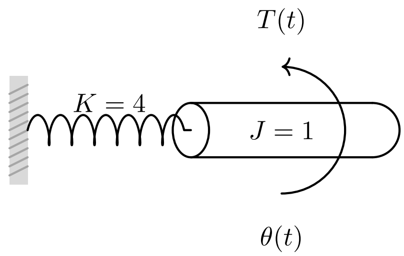
$$\omega_n = \sqrt{\frac{K}{J}} = \sqrt{\frac{3}{2}} \approx 1.225$$

$$\zeta = \frac{D}{2\sqrt{JK}} = \frac{0.5}{2\sqrt{2 \cdot 3}} \approx 0.102$$

Answer:

$$\boxed{\omega_n \approx 1.225,\ \zeta \approx 0.102}$$

Problem 65
A system has $J = 1$, $K = 4$, and no damper. Compute ω_n and comment on damping.



Given:
$J = 1$
$K = 4$

Required: ω_n and damping

Solution:

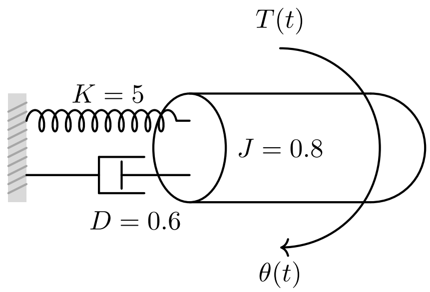
$$\omega_n = \sqrt{\frac{4}{1}} = 2, \quad \zeta = 0 \text{ (undamped)}$$

Answer:

$$\boxed{\omega_n = 2,\ \zeta = 0 \text{ (undamped)}}$$

Problem 66

Given $J = 0.8,\ D = 0.6,\ K = 5$, find the steady-state angular displacement θ_{ss} for a unit step torque input.



Given:

$J = 0.8,\ D = 0.6,\ K = 5$

Required: θ_{ss}

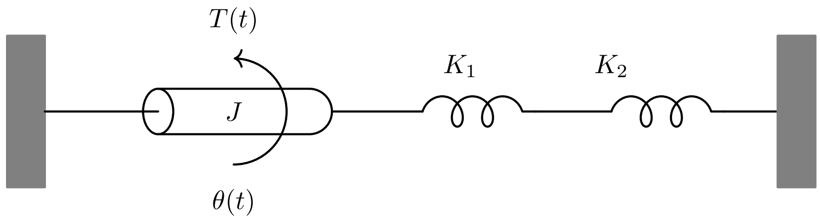
Solution:

$$\theta_{ss} = \frac{1}{K} = \frac{1}{5} = 0.2$$

Answer:

$$\boxed{\theta_{ss} = 0.2 \text{ rad}}$$

Problem 67
Determine the transfer function $G(s)$ for:



with $J = 1$, $K_1 = 3$, $K_2 = 2$.

Given: As per drawing
$J = 1$, $K_1 = 3$, $K_2 = 2$.

Required: G(s)

Solution:

The two torsional springs K_1 and K_2 are connected in series to the mass moment of inertia J. Since they are both acting in parallel on the same shaft, their equivalent stiffness is simply:

$$K = K_1 + K_2 = 3 + 2 = 5$$

The standard transfer function for such a rotational system is:

$$G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2 + K}$$

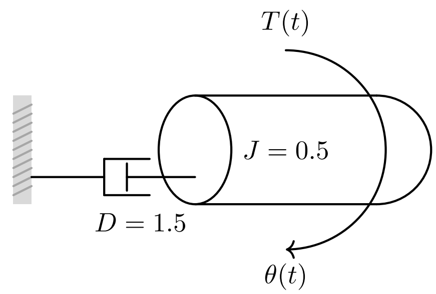
Substituting the values:

$$G(s) = \frac{1}{s^2 + 5}$$

Answer:

$$\boxed{G(s) = \frac{1}{s^2 + 5}}$$

Problem 68
Compute the time constant for a rotational system with $J = 0.5$ and $D = 1.5$ (no spring).



Given: $J = 0.5$ and $D = 1.5$

Required: τ

Solution:

The time constant for a first-order rotational system is:

$$\tau = \frac{J}{D}$$

Substituting the values:

$$\tau = \frac{0.5}{1.5} = \frac{1}{3}$$

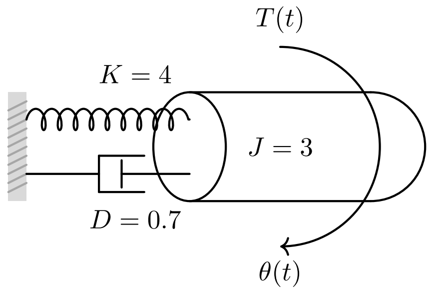
Answer:

$$\boxed{\tau = \frac{1}{3}\ \mathrm{s}}$$

Problem 69
Given:

$$J = 3, \quad D = 0.7, \quad K = 4$$

find the transfer function and identify the damping ratio.



Given: J = 3, D = 0.7, K = 4

Required: transfer function and damping ratio

Solution:

The standard form of the transfer function is:

$$G(s) = \frac{1}{Js^2 + Ds + K}$$

Substituting the values:

$$G(s) = \frac{1}{3s^2 + 0.7s + 4}$$

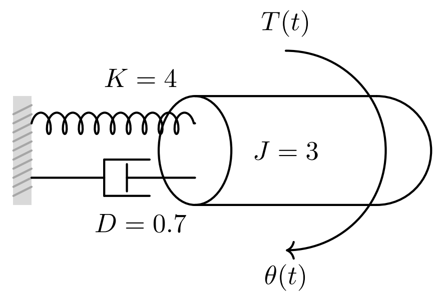
The damping ratio is:

$$\zeta = \frac{D}{2\sqrt{JK}} = \frac{0.7}{2\sqrt{3 \cdot 4}} = \frac{0.7}{2\sqrt{12}} \approx 0.101$$

Answer:

$$\boxed{G(s) = \frac{1}{3s^2 + 0.7s + 4}}, \quad \boxed{\zeta \approx 0.101}$$

Problem 70
Determine θ_{ss} for a unit step torque input in Problem 7.



Given: Refer to problem 7

Required: θ_{ss}

Solution:

For a step input, steady-state displacement is:

$$\theta_{ss} = \lim_{s\to 0} G(s) = \frac{1}{K}$$

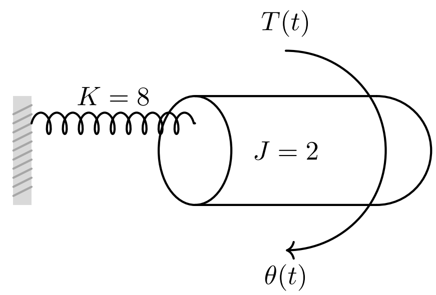
From Problem 7, $K = 4$:

$$\theta_{ss} = \frac{1}{4} = 0.25$$

Answer:

$$\boxed{\theta_{ss} = 0.25 \text{ rad}}$$

Problem 71
For a system with $J = 2$, $K = 8$, find ω_n.



Given: $J = 2$, $K = 8$

Required: ω_n

Solution:
Natural frequency is given by:

$$\omega_n = \sqrt{\frac{K}{J}}$$

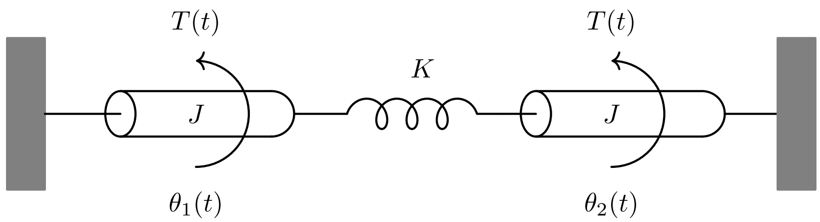
Substitute values:

$$\omega_n = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$$

Answer:

$$\boxed{\omega_n = 2}$$

Problem 72
A shaft connects two disks as shown:



with $J_1 = 1,\ J_2 = 2,\ K = 5$. Find $G(s) = \frac{\Theta_2(s)}{T(s)}$.

Given: As per drawing
$J_1 = 1,\ J_2 = 2,\ K = 5$

Required: $G(s) = \frac{\Theta_2(s)}{T(s)}$

Solution:

The total equivalent moment of inertia is:

$$J = J_1 + J_2 = 1 + 2 = 3$$

The transfer function is:

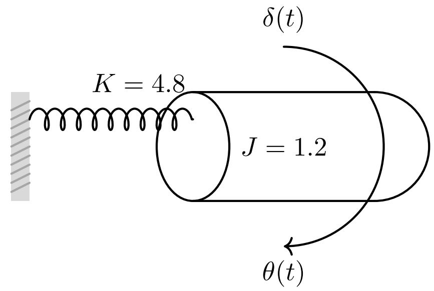
$$G(s) = \frac{1}{Js^2 + K} = \frac{1}{3s^2 + 5}$$

Answer:

$$\boxed{G(s) = \frac{1}{3s^2 + 5}}$$

Problem 73

A rotational system has $J = 1.2$ and $K = 4.8$. If subjected to an impulse torque, find the maximum angular velocity.



Given: $J = 1.2$ and $K = 4.8$

Required: $\omega_{\max}$

Solution:

An impulse torque causes an instantaneous change in angular velocity:

$$\omega_{\max} = \frac{1}{J}$$

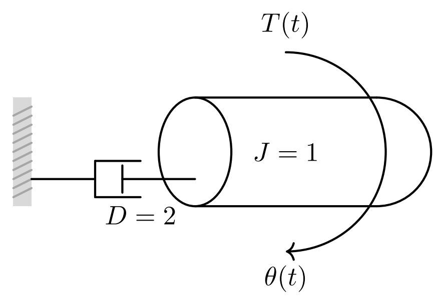
Substitute the given value:

$$\omega_{\max} = \frac{1}{1.2} \approx 0.833$$

Answer:

$$\boxed{\omega_{\max} \approx 0.833 \text{ rad/s}}$$

Problem 74
Determine the settling time (5%) for a system with $J = 1$, $D = 2$.



Given: $J = 1$, $D = 2$, 5% settling time

Required: t_s

Solution:

Time constant is:

$$\tau = \frac{J}{D} = \frac{1}{2} = 0.5$$

The 5% settling time is:

$$t_s = 4\tau = 4 \cdot 0.5 = 2$$

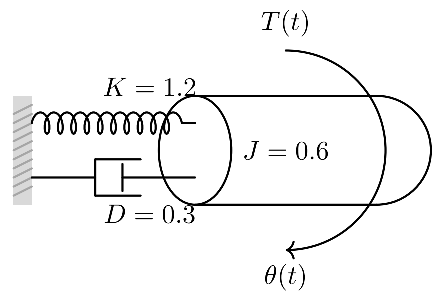
Answer:

$$\boxed{t_s = 2 \text{ s}}$$

Problem 75
Given:

$$J = 0.6, \quad D = 0.3, \quad K = 1.2$$

find the transfer function.



Given: J = 0.6, D = 0.3, K = 1.2

Required: G(s)

Solution:

The transfer function is:

$$G(s) = \frac{1}{Js^2 + Ds + K}$$

Substitute the given values:

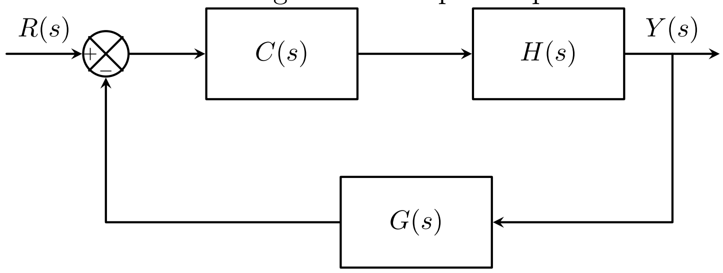
$$G(s) = \frac{1}{0.6s^2 + 0.3s + 1.2}$$

Answer:

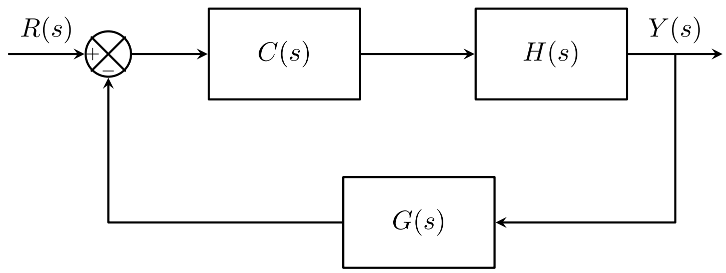
$$\boxed{G(s) = \frac{1}{0.6s^2 + 0.3s + 1.2}}$$

Problem 76

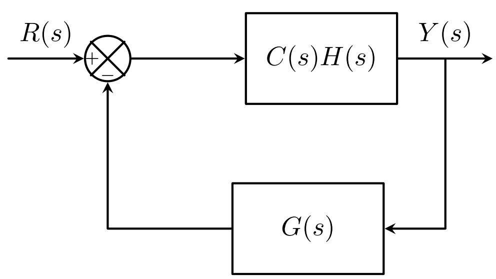
Reduce the block diagram to its open-loop form:



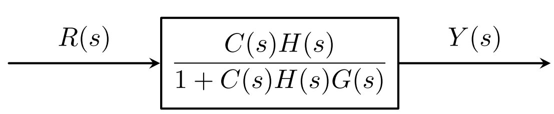
Solution:



Used Cascaded rule of $C(s)$ and $H(s)$

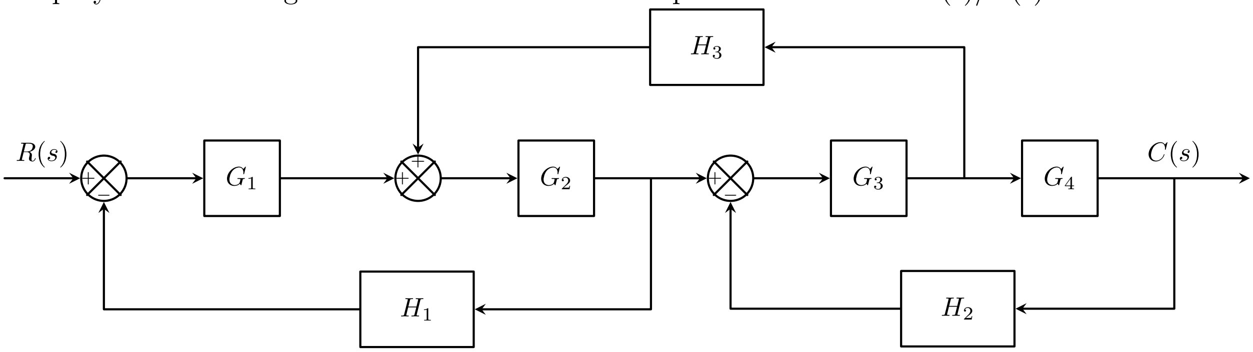


Used Feedback rule of $C(s)H(s)$ and $G(s)$



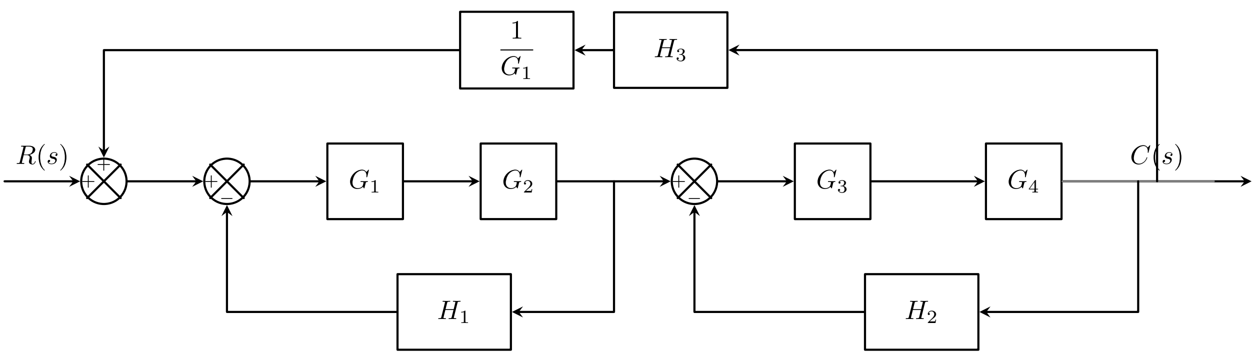
Problem 77

Simplify the block diagram then obtain the close-loop transfer function C(s)/R(s).

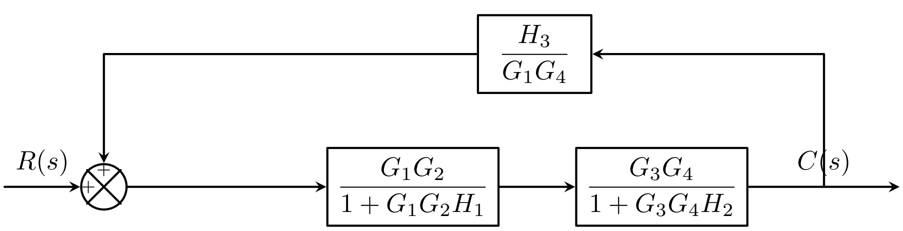


Solution:

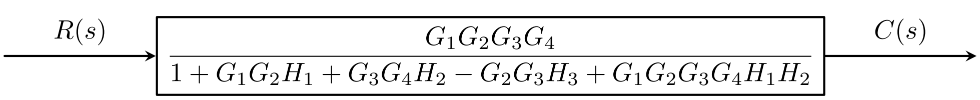
First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4 and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point.



By simplifying each loop, the block diagram can be modified as



Further simplification results in

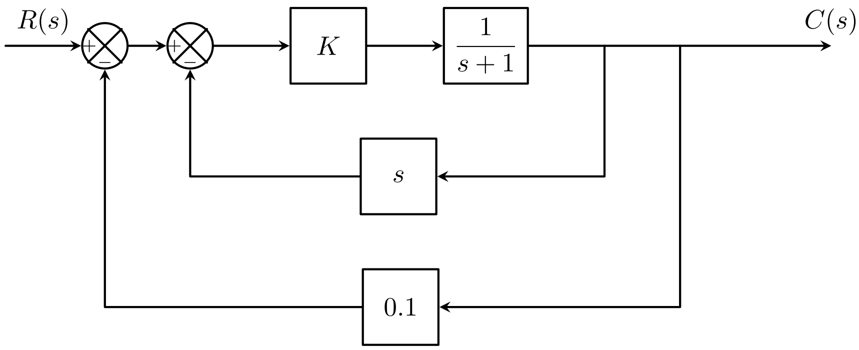


The closed-loop transfer function $C(s)/R(s)$ is obtained as

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_3G_4H_2 - G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}$$

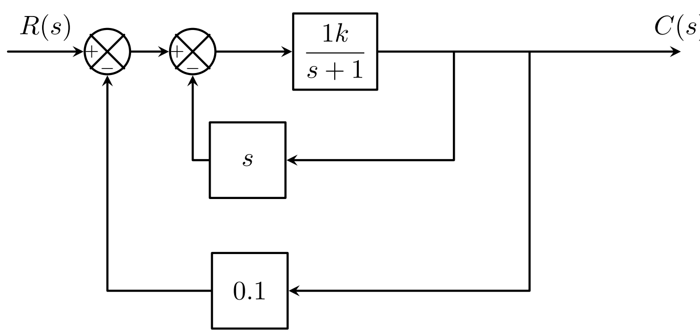
Problem 78

Reduce the given block diagram to canonical form.

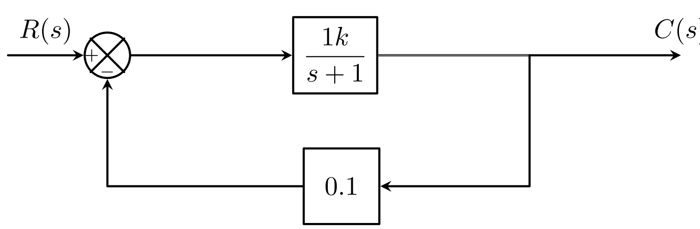


Solution:

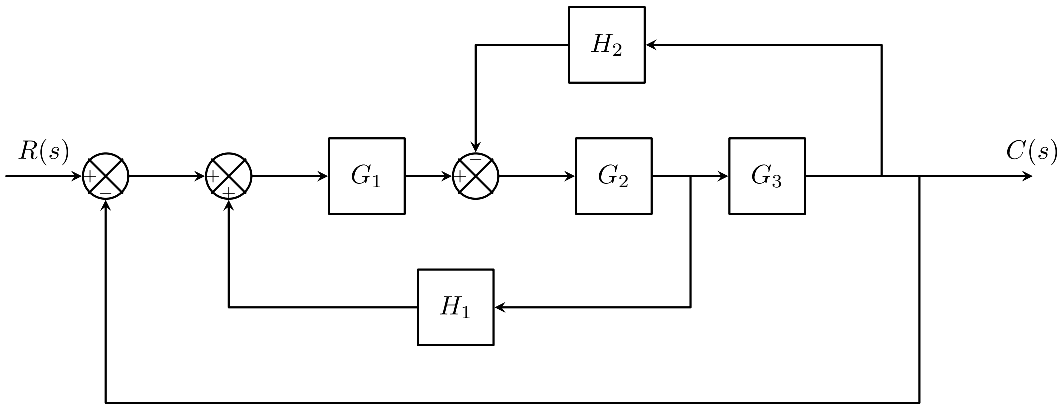
Used Cascaded rule of K and $\dfrac{1}{s+1}$



Used Feedback rule of $\dfrac{1k}{s+1}$ and s

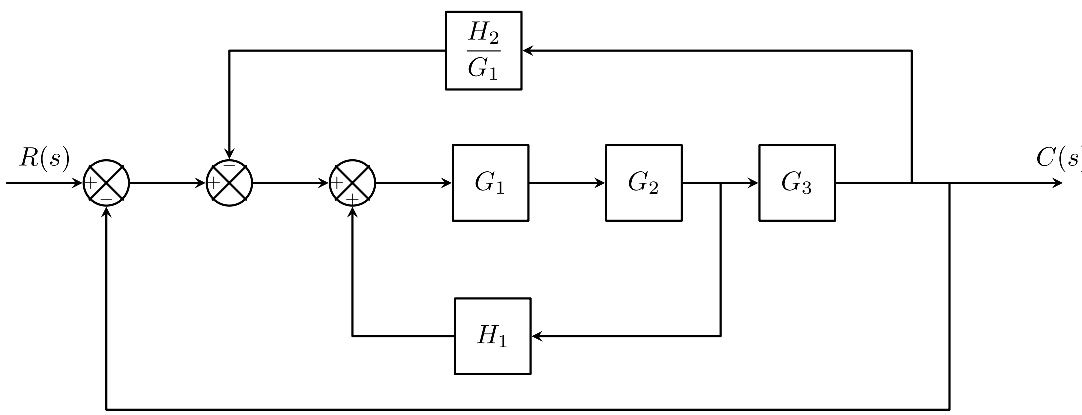


Problem 79
Simplify the block diagram.

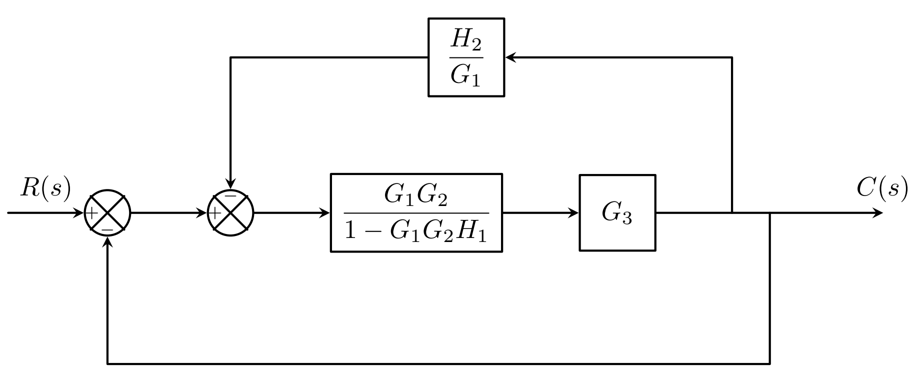


Solution:

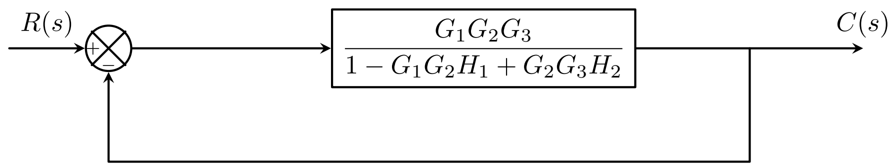
By moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1,we obtain figure



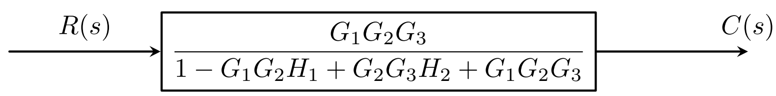
Eliminating the positive feedback loop, we have



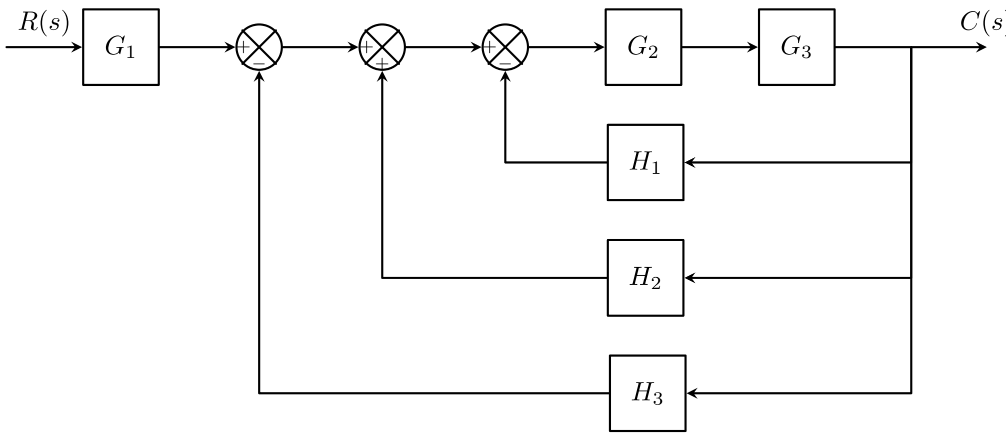
The elimination of the loop containing $\dfrac{H_2}{G_1}$ gives



Finally,eliminating the feedback loop results in

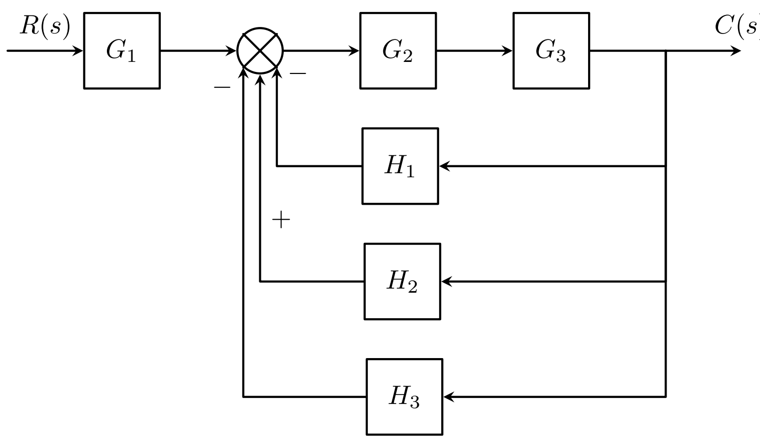


Problem 80
Reduce the Block Diagram.

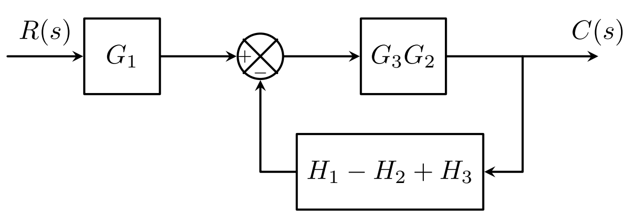


Solution:

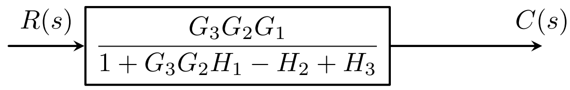
First, the three summing junctions can be collapsed into a single summing junction.



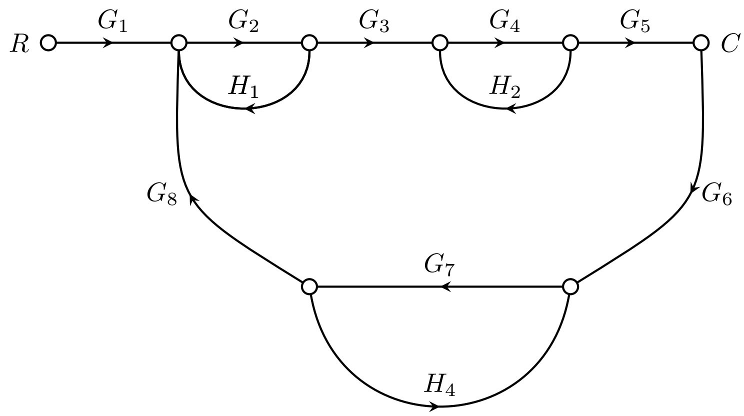
Second,recognize that the three feedback functions, H_1, H_2 and H_3 are connected in parallel.They are fed from a common signal source and their outputs are summed.Also recognize that G_2 and G_3 are connected in cascade.



Finally, the feedback system is reduced and multiplied by G_1 to yield the equivalent transfer function shown in figure



Problem 81
Find the transfer function of the signal flow graph below



Solution:

Forward Path:

$P_1 = G_1G_2G_3G_4G_5$

Loop Gain:
$L_1 = G_2H_1$
$L_2 = G_4H_2$
$L_3 = G_7H_4$
$L_4 = G_2G_3G_4G_5G_6G_7G_7$

2 Non-touching Loops:

$L_1 = (G_2H_1)(G_4H_2)$
$L_2 = (G_2H_1)(G_7H_4)$
$L_3 = (G_4H_2)(G_7H_4)$

3 Non-touching Loops:
$L_1 = (G_2H_1)(G_4H_2)(G_7H_4)$

4 Non-touching Loops:

N/A

Mason's Rule

$$\frac{C}{R} = \frac{\sum T_k\Delta_k}{\Delta}$$

$$\begin{aligned}\Delta = 1 &- (G_2H_1 + G_4H_2 + G_7H_4 + G_2G_3G_4G_5G_6G_7G_8)\\ &+(G_2G_4H_1H_2 + G_2G_7H_1H_4 + G_4G_7H_2H_4)\\ &-(G_2G_4G_7H_1H_2H_4)\end{aligned}$$

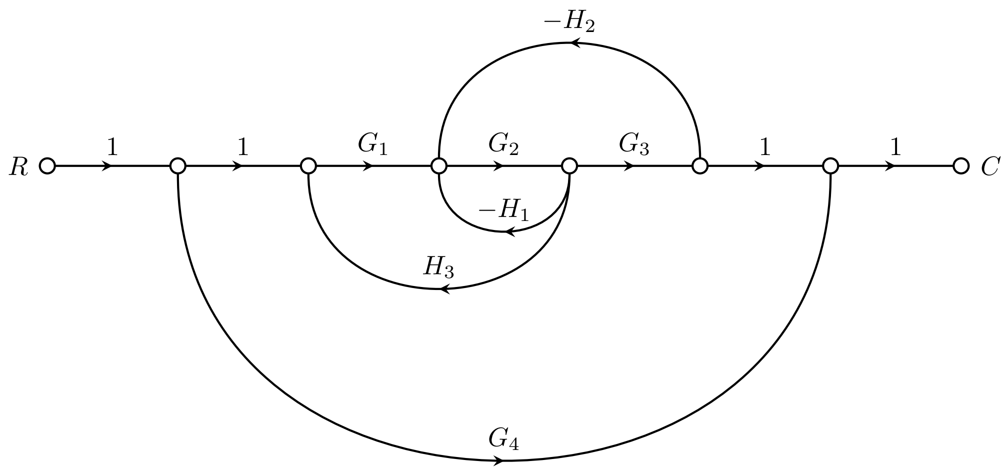
$$\Delta_1 = 1 - G_7H_4$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4G_5(1 - G_7H_4)}{\Delta}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4G_5 - G_1G_2G_3G_4G_5G_7H_4}{\begin{aligned}&1 - G_2H_1 - G_4H_2 - G_7H_4 - G_2G_3G_4G_5G_6G_7G_8+\\ &G_2G_4H_1H_2 + G_2G_7H_1H_4 + G_4G_7H_2H_4\\ &-G_2G_4G_7H_1H_2H_4\end{aligned}}$$

Problem 82

For the system shown,obtain the closed loop transfer function



Solution:

Forward Path:

$P_1 = G_1G_2G_3$
$P_2 = G_4$

Loop Gain:
$L_1 = -G_2H_1$
$L_2 = G_1G_2H_1$
$L_3 = -G_2G_3H_2$

2 Non-touching Loops:

N/A

Mason's Rule

$$\frac{C}{R} = \frac{\sum T_k \Delta_k}{\Delta}$$

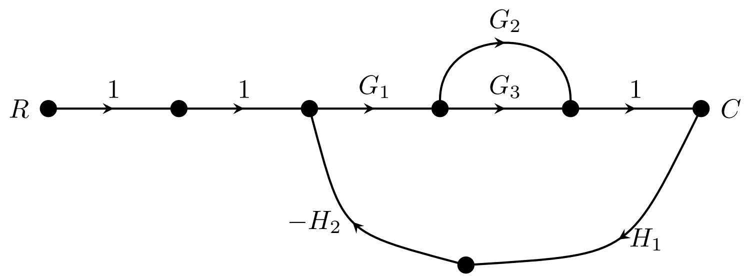
$$\Delta = 1 - (-G_2H_1 + G_1G_2H_1 - G_2G_3H_2)$$

$\Delta_1 = 1 \qquad \Delta_2 = 1$

$$\frac{C}{R} = \frac{G_1G_2G_3(1) + G_4(1)}{1 + G_2H_1 - G_1G_2H_1 + G_2G_3H_2}$$

$$\frac{C}{R} = \frac{G_1G_2G_3 + G_4}{1 + G_2H_1 - G_1G_2H_1 + G_2G_3H_2}$$

Problem 83
Find the transfer function of the signal flow graph below



Solution:

Forward Path:

$P_1 = G_1 G_2$
$P_2 = G_1 G_3$

Loop Gain:
$L_1 = -G_1 G_3 H_1 H_2$
$L_2 = -G_1 G_2 H_1 H_2$

2 Non-touching Loops:
N/A

Mason's Rule

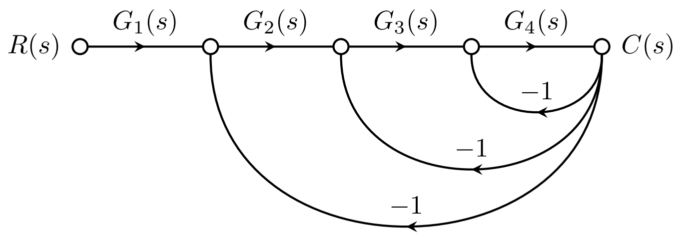
$$\frac{C}{R} = \frac{\sum T_k \Delta_k}{\Delta}$$

$$\Delta = 1 - (-G_1 G_3 H_1 H_2 - G_1 G_2 H_1 H_2)$$
$$\Delta_1 = 1 \qquad \Delta_2 = 1$$
$$\frac{C}{R} = \frac{G_1 G_2 (1) + G_1 G_3 (1)}{1 + G_1 G_3 H_1 H_2 + G_1 G_2 H_1 H_2}$$

$$\frac{C}{R} = \frac{G_1 G_2 + G_1 G_3}{1 + G_1 G_3 H_1 H_2 + G_1 G_2 H_1 H_2}$$

Problem 84

Find the transfer function of the signal flow graph below



Solution:

Forward Path:

$P_1 = G_1G_2G_3G_4$

Loop Gain:
$L_1 = -G_2G_3G_4$
$L_2 = -G_3G_4$
$L_3 = -G_4$

2 Non-touching Loops:

N/A

Mason's Rule

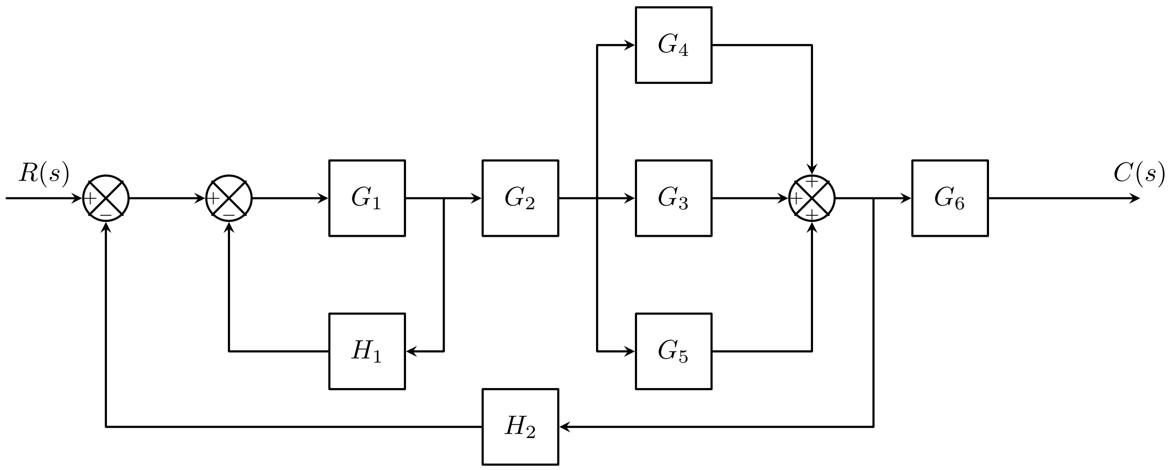
$$\frac{C}{R} = \frac{\sum T_k\Delta_k}{\Delta}$$

$\Delta = 1 - (-G_2G_3G_4 - G_3G_4 - G_4)$
$\Delta_1 = 1$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4}{1 + G_2G_3G_4 + G_3G_4 + G_4}$$

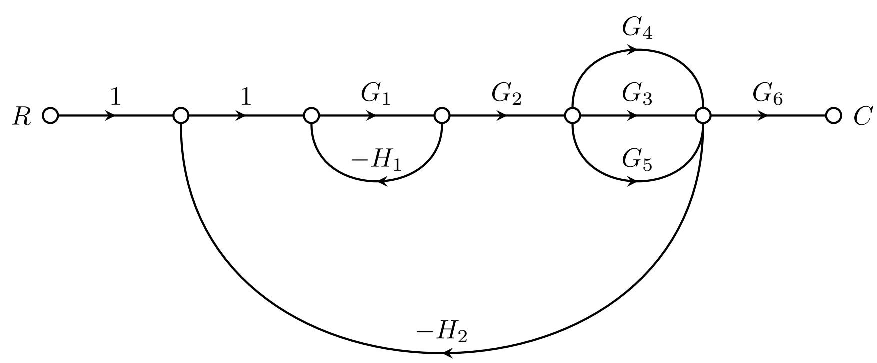
Problem 85

Find the transfer function of the system shown by a block diagram



Solution:

Converting into SFG



Forward Path:

$P_1 = G_1G_2G_3G_6$

$P_2 = G_1G_2G_4G_6$

$P_3 = G_1G_2G_5G_6$

Loop Gain:

$L_1 = -G_1H_1$

$L_2 = -G_1G_2G_3H_2$

$L_3 = -G_1G_2G_4H_2$

$L_4 = -G_1G_2G_5H_2$

2 Non-touching Loops:

N/A

Mason's Rule

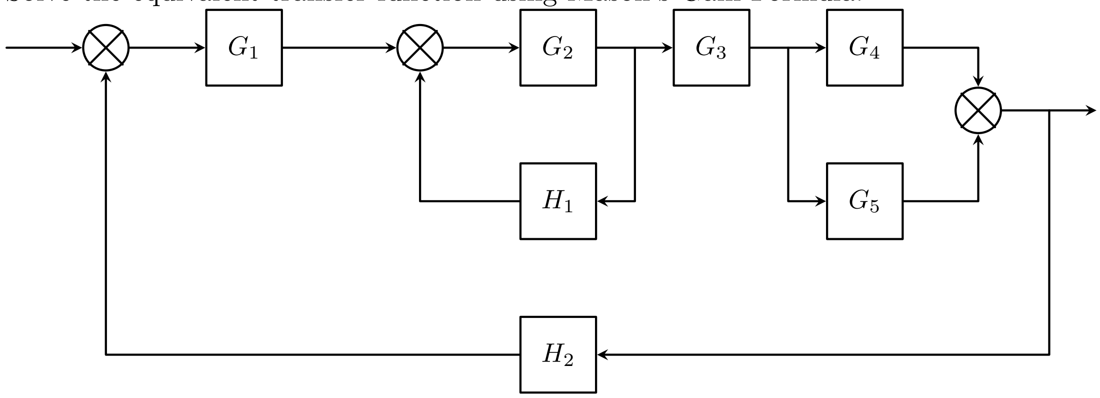
$$\frac{C}{R} = \frac{\sum T_k \Delta_k}{\Delta}$$

$$\Delta = 1 - (-G_1H_1 - G_1G_2G_3H_2 - G_1G_2G_4H_2 - G_1G_2G_5H_2)$$

$$\Delta_1 = 1 \qquad \Delta_2 = 1 \qquad \Delta_3 = 1$$

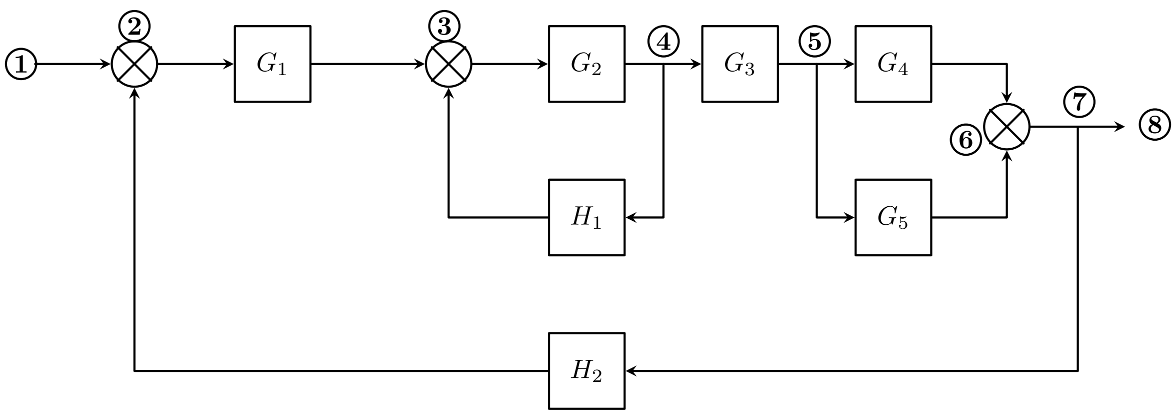
$$\frac{C}{R} = \frac{G_1G_2G_3G_6 + G_1G_2G_4G_6 + G_1G_2G_5G_6}{1 + G_1H_1 + G_1G_2G_3H_2 + G_1G_2G_4H_2 + G_1G_2G_5H_2}$$

Problem 86
Solve the equivalent transfer function using Mason's Gain Formula:

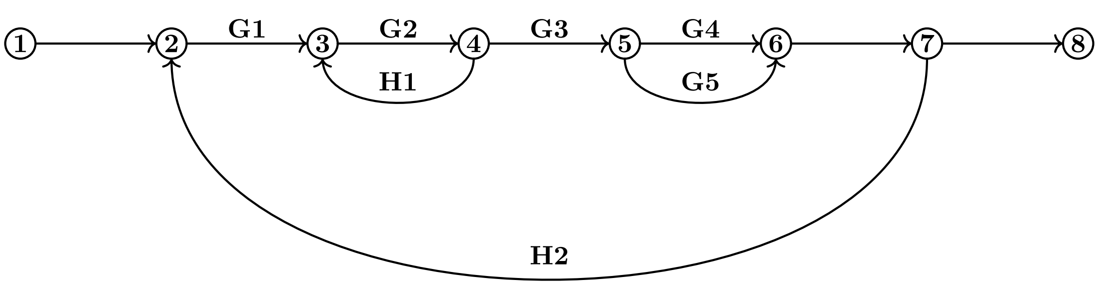


Solution:

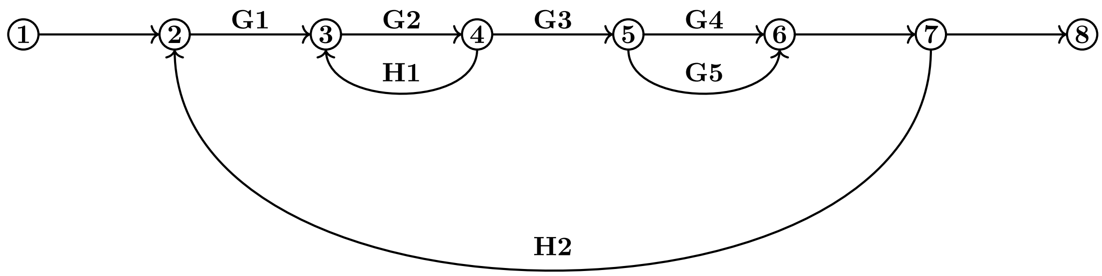
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify forward path and all loops



$$F_1 = G_2G_3G_4$$

$$\Delta_1 = 0$$

$$F_2 = G_2G_3G_5$$

$$\Delta_2 = 0$$
$$L_1 = G_2H_1$$

$$L_2 = G_1G_2G_3G_4H_2$$

$$L_3 = G_1G_2G_3G_5H_2$$

Step 4. Use the Mason's Gain Formula (positive loop technique)

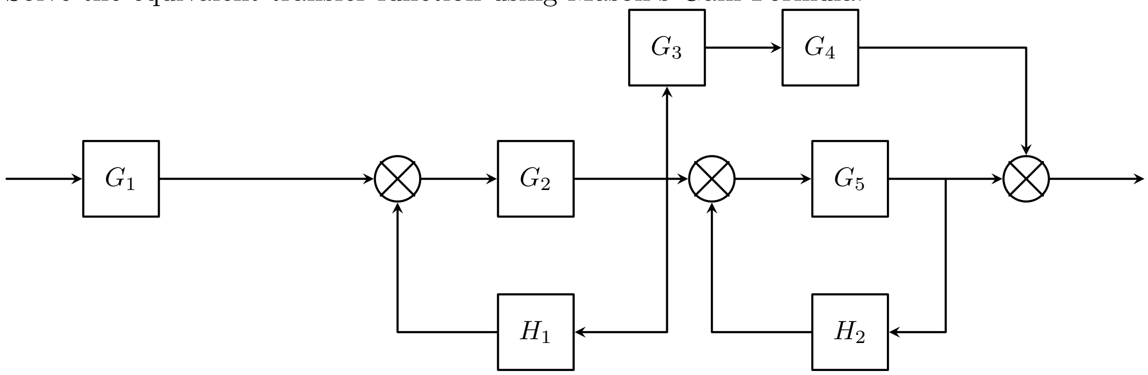
$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

($*$) means not touching each other

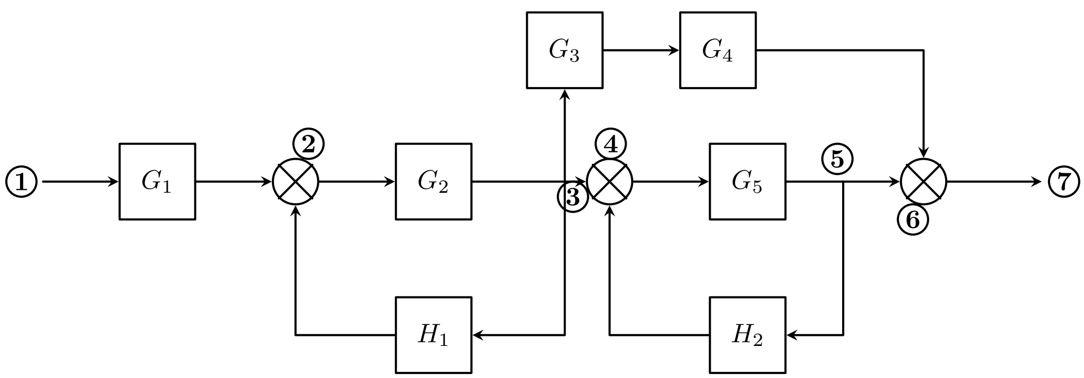
$$TF = \frac{G_1G_2G_3G_4(1-0) + G_1G_2G_3G_5(1-0)}{1 + (G_2H_1 + G_1G_2G_3G_4H_2 + G_1G_2G_3G_5H_2) + (0) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_3G_4 + G_1G_2G_3G_5}{1 + G_2H_1 + G_1G_2G_3G_4H_2 + G_1G_2G_3G_5H_2}}$$

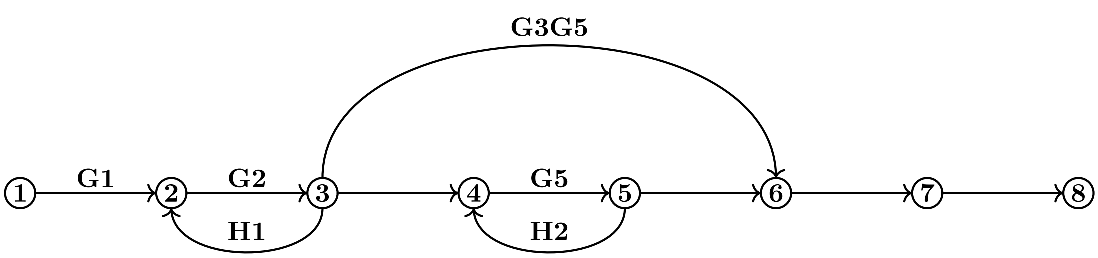
Problem 87
Solve the equivalent transfer function using Mason's Gain Formula:



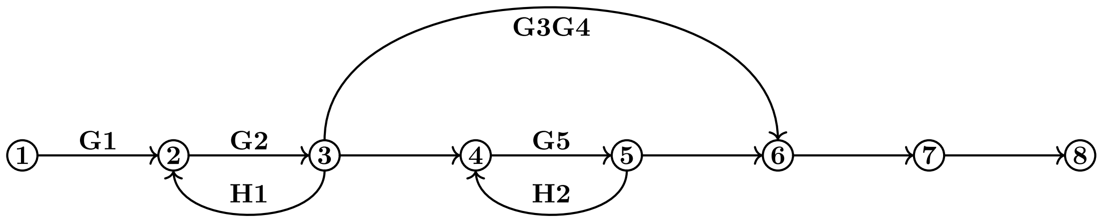
Solution:
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



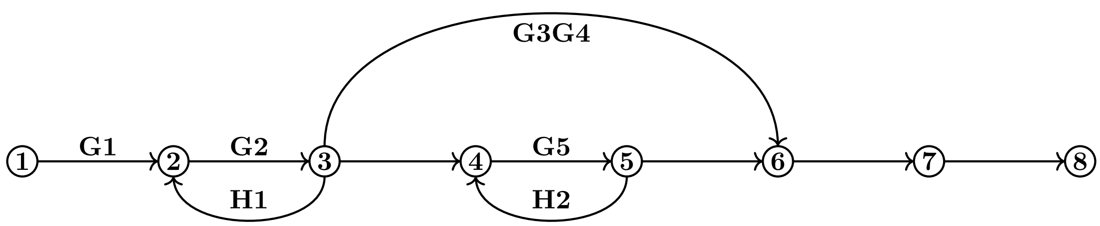
$$F_1 = G_1G_2G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_2G_3G_4$$

$$\Delta_2 = G_5H_2$$

Step 4. Identify all loops



$$L_1 = G_2H_1$$

$$L_2 = G_5H_1$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

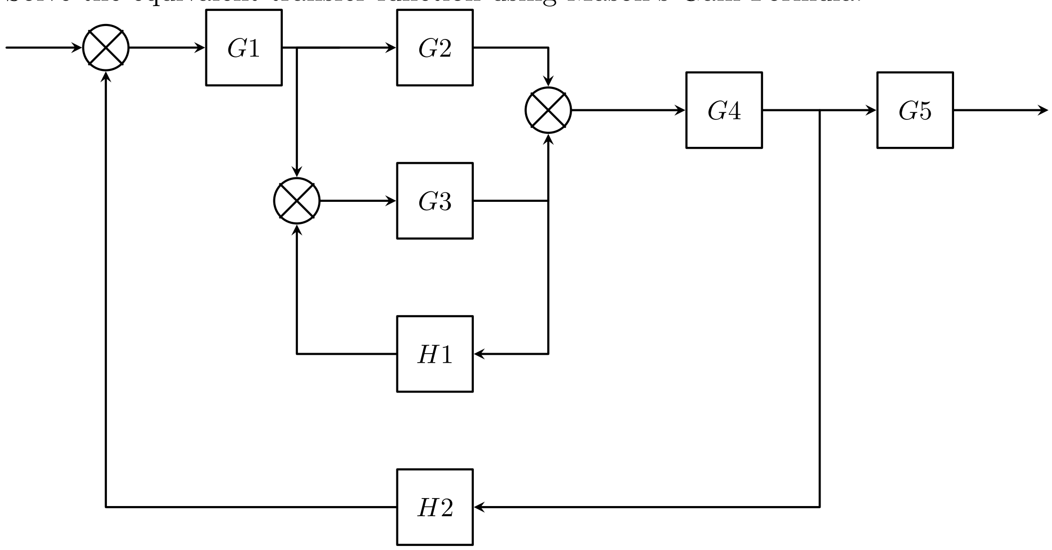
$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

($*$) means not touching each other

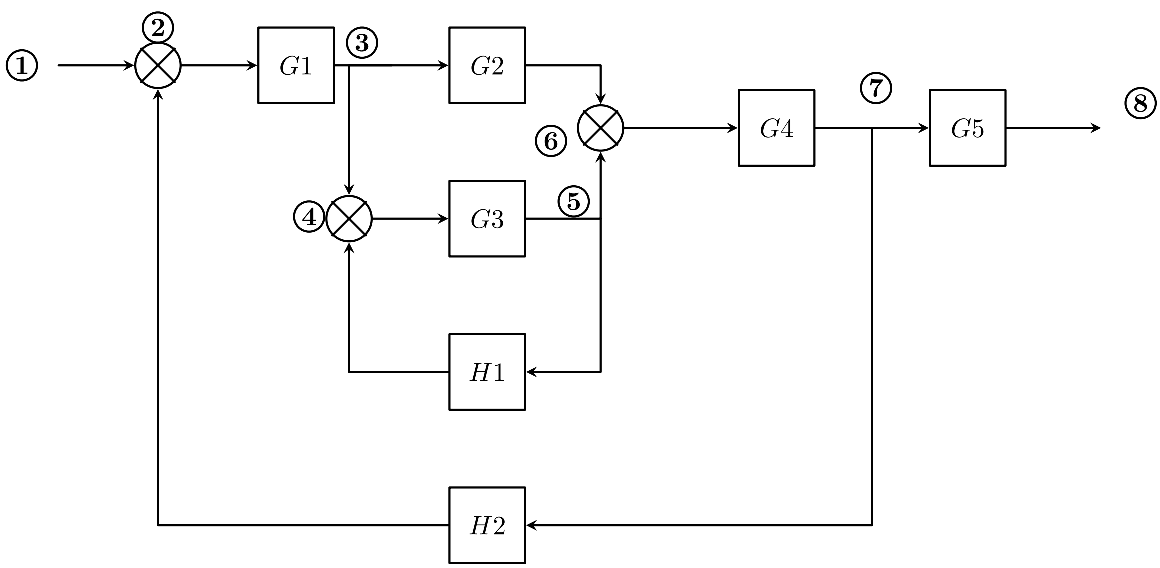
$$TF = \frac{G_1G_2G_5(1+0) + G_1G_2G_3G_4(1+G_5H_2)}{1 + (G_2H_1 + G_5H_2) + (G_2G_5H_1H_2) + (0) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_5 + G_1G_2G_3G_4 + G_1G_2G_3G_4G_5H_2}{1 + G_2H_1 + G_5H_2 + G_2G_5H_1H_2}}$$

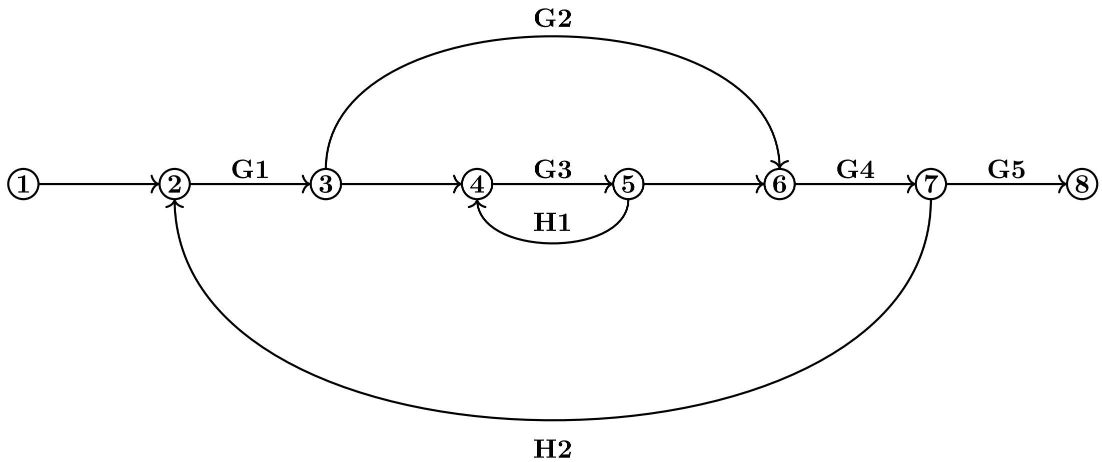
Problem 88
Solve the equivalent transfer function using Mason's Gain Formula:



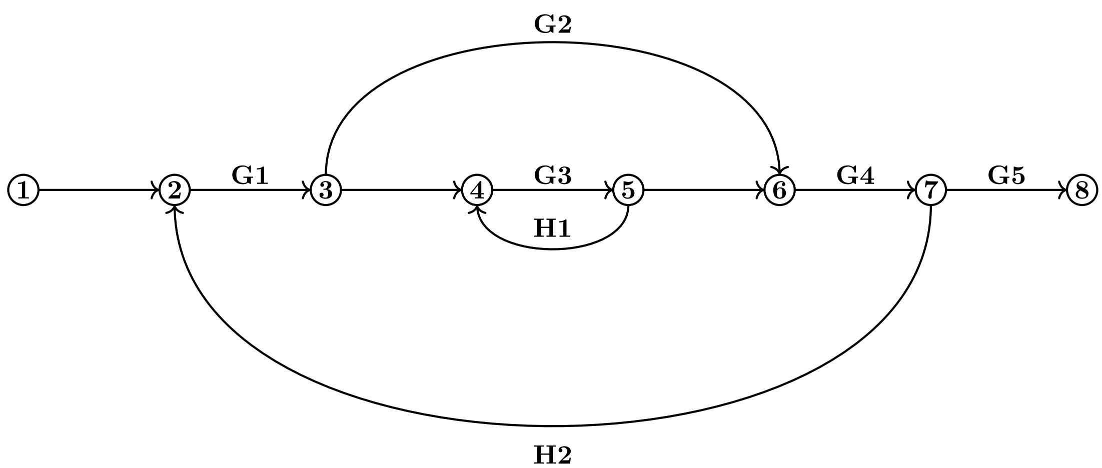
Solution:
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



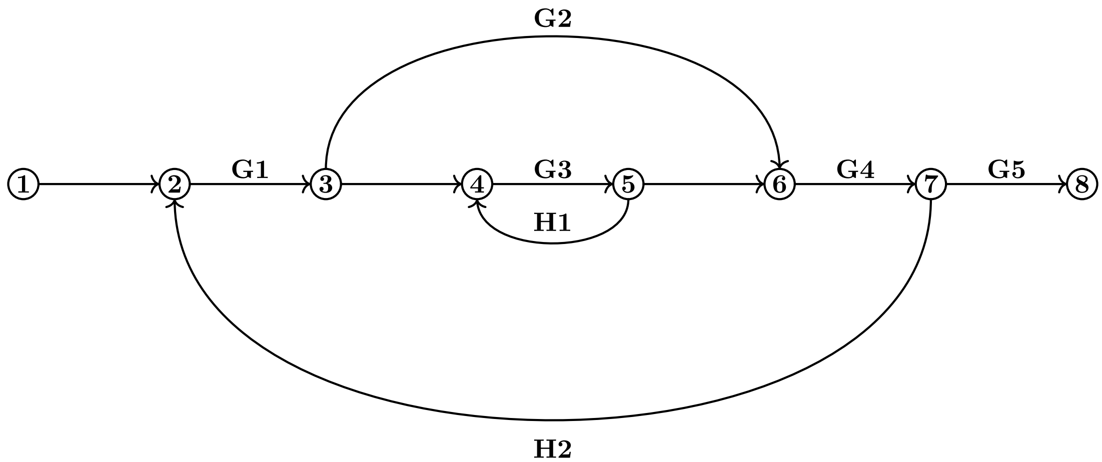
$$F_1 = G_1G_3G_4G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_2G_4G_5$$

$$\Delta_2 = G_3H_1$$

Step 4. Identify all loops



$$L_1 = G_3H_1$$

$$L_2 = G_1G_3G_4H_2$$

$$L_3 = G_1G_2G_4H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

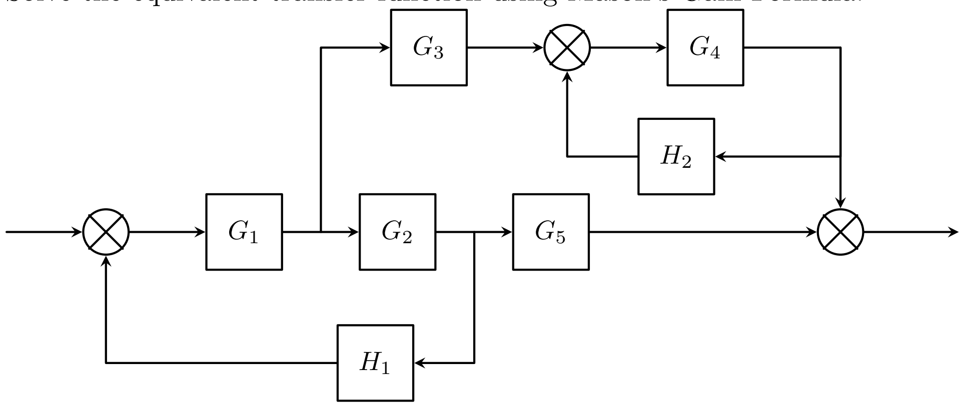
$(*)$ means not touching each other

$$TF = \frac{G_1G_3G_4G_5(1+0) + G_1G_2G_4G_5(1 + G_3H_1)}{1 + (G_3H_1 + G_1G_3G_4H_2 + G_1G_2G_4H_2) + (G_3H_1 \cdot G_1G_2G_4H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_3G_4G_5 + G_1G_2G_4G_5 + G_1G_2G_3G_4G_5H_1}{1 + G_3H_1 + G_1G_3G_4H_2 + G_1G_2G_4H_2 + G_1G_2G_3G_4H_1H_2}}$$

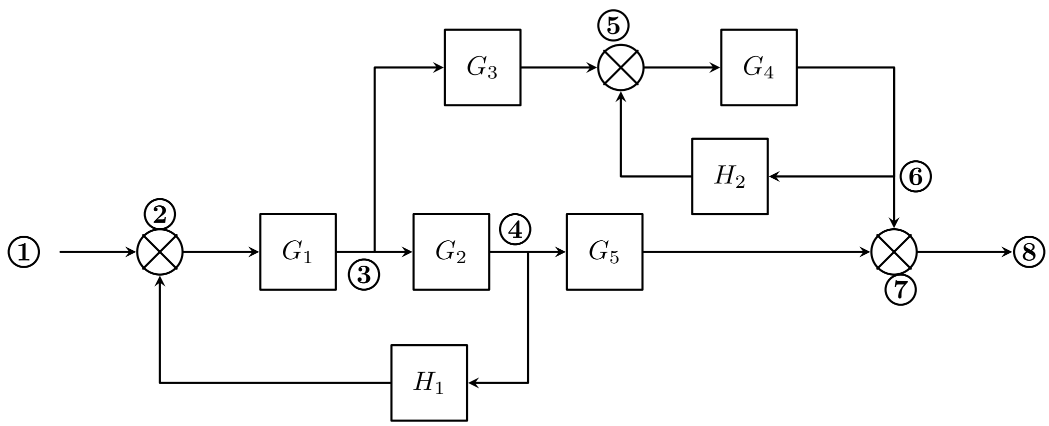
Problem 89

Solve the equivalent transfer function using Mason's Gain Formula:

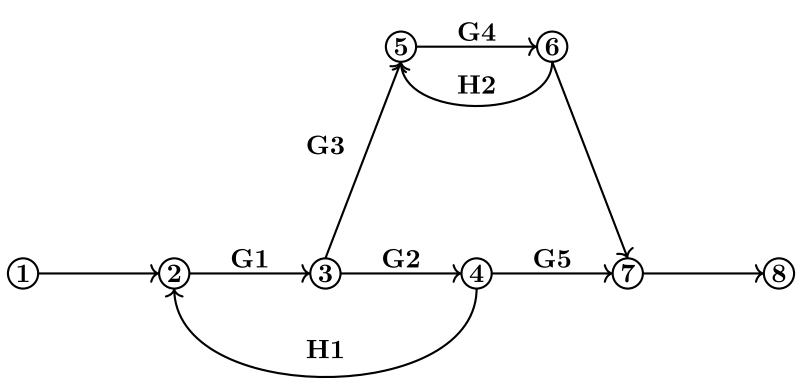


Solution:

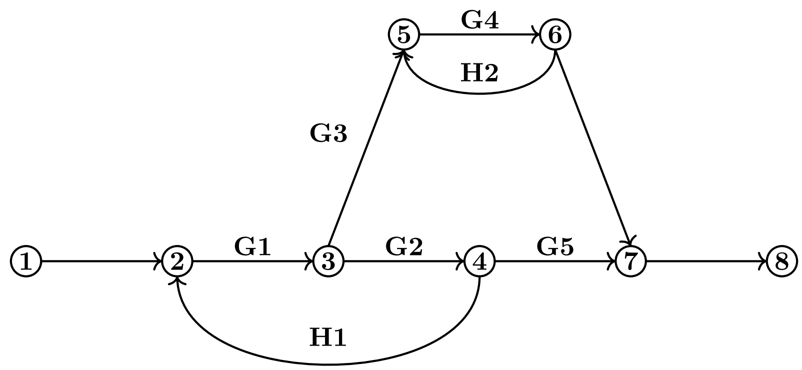
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



$$F_1 = G_1G_2G_5$$

$$\Delta_1 = G_4H_2$$

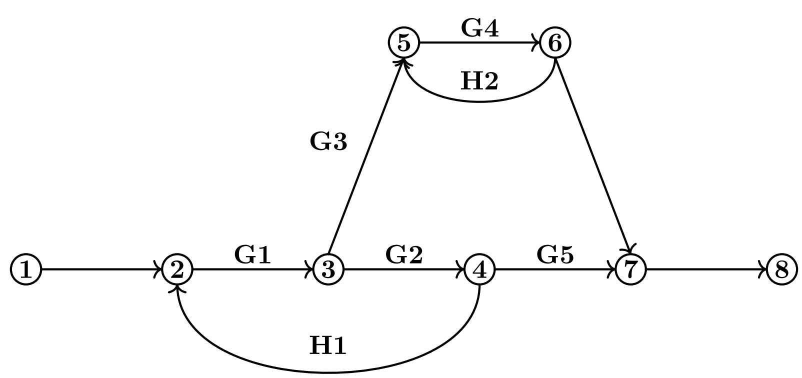
$$F_2 = G_1G_3G_4$$

$$\Delta_2 = 0$$

$$L_1 = G_1G_2H_1$$

$$L_2 = G_4H_2$$

Step 4. Identify all loops



Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

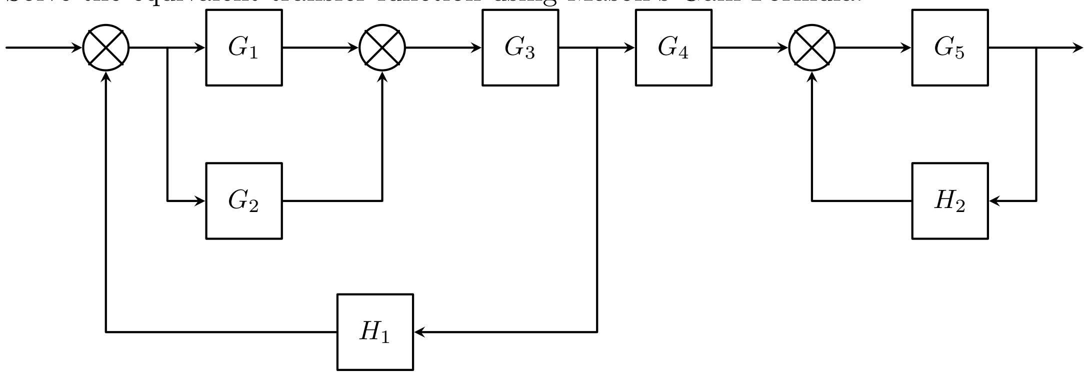
(∗) means not touching each other

$$TF = \frac{G_1G_2G_5(1 + G_4H_2) + G_1G_3G_4(1 + 0)}{1 + (G_1G_2H_1 + G_4H_2) + (G_1G_2H_1 \cdot G_4H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_5 + G_1G_3G_4 + G_1G_2G_4G_5H_2}{1 + G_1G_2H_1 + G_4H_2 + G_1G_2G_4H_1H_2}}$$

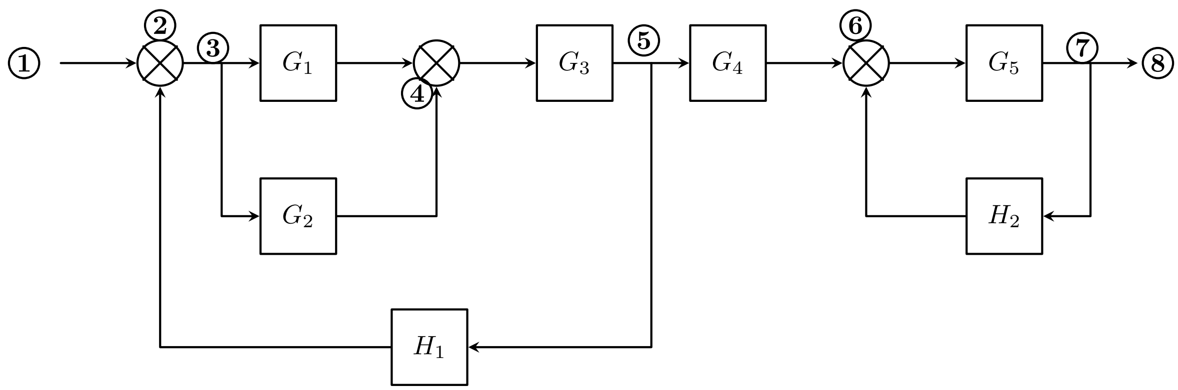
Problem 90

Solve the equivalent transfer function using Mason's Gain Formula:

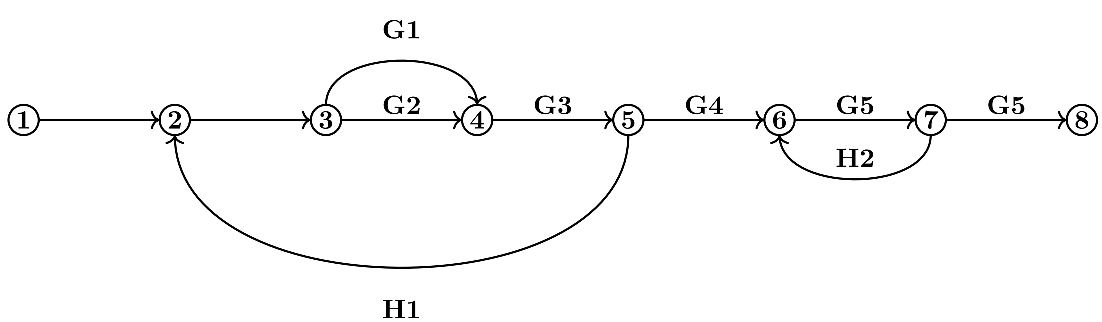


Solution:

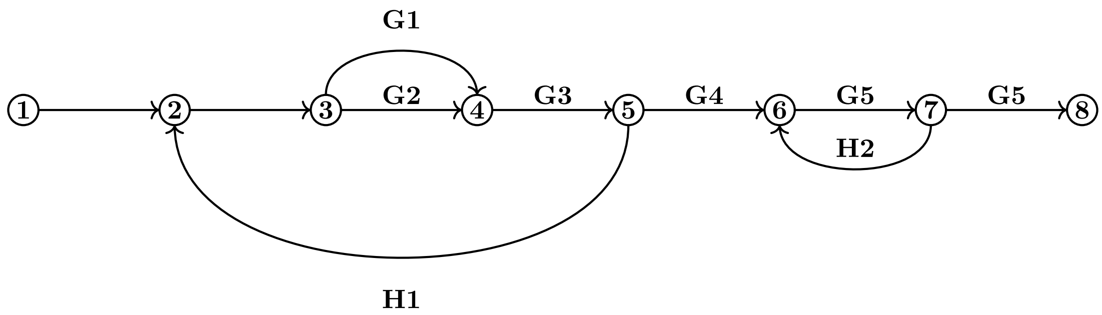
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



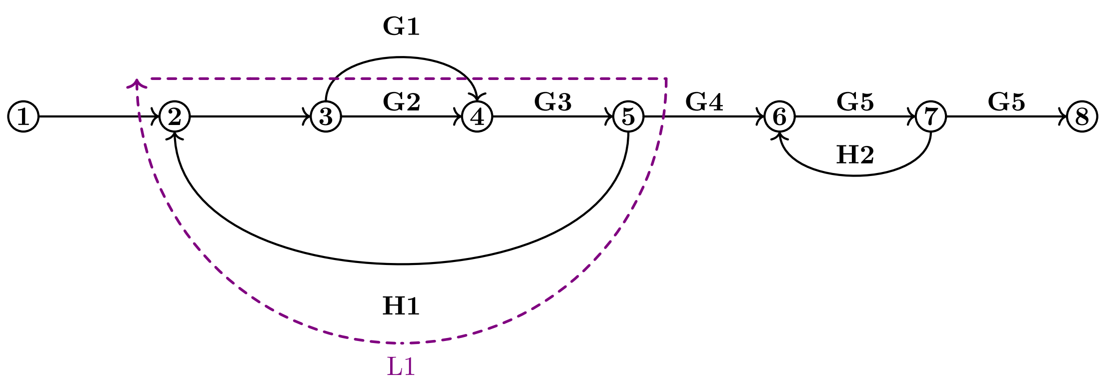
$$F_1 = G_1 G_3 G_4 G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_2 G_3 G_4 G_5$$

$$\Delta_2 = 0$$

Step 4. Identify all loops



$$L_1 = G_2 G_3 H_1$$

$$L_2 = G_1 G_3 H_1$$

$$L_3 = G_5 H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

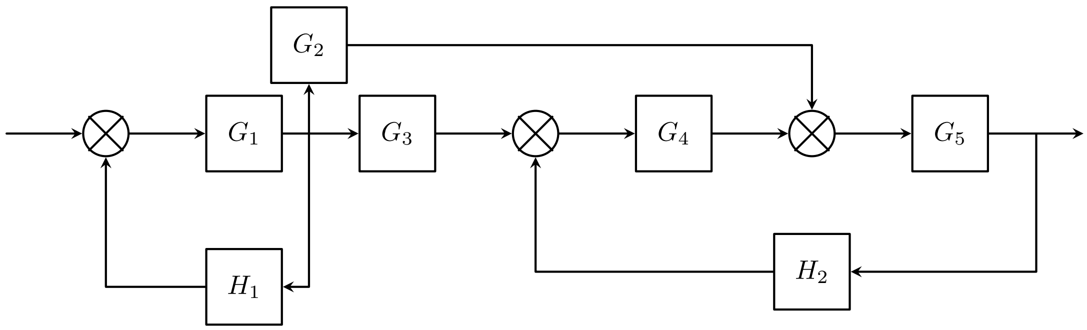
$(*)$ means not touching each other

$$TF = \frac{G_1G_3G_4G_5(1+0) + G_2G_3G_4G_5(1+0)}{1 + (G_2G_3H_1 + G_1G_3H_1 + G_5H_2) + (G_2G_3H_1 \cdot G_5H_2 + G_1G_3H_1 \cdot G_5H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_3G_4G_5 + G_2G_3G_4G_5}{1 + G_1G_3H_1 + G_2G_3H_1 + G_5H_2 + G_1G_3G_5H_1H_2 + G_2G_3G_5H_1H_2}}$$

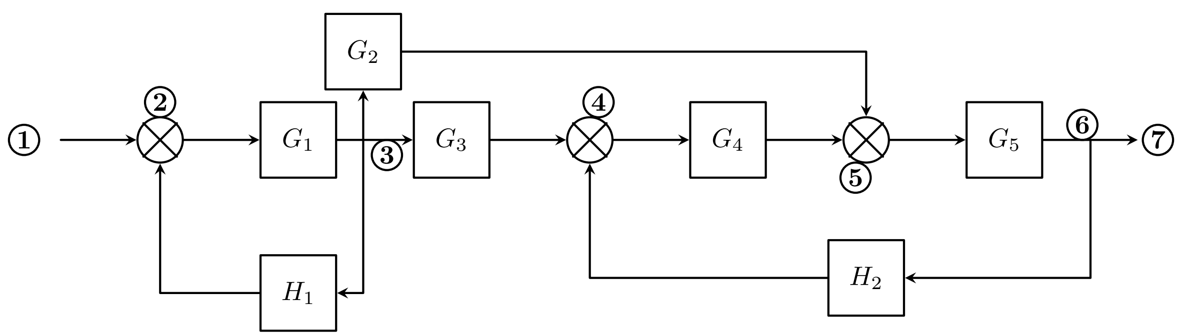
Problem 91

Solve the equivalent transfer function using Mason's Gain Formula:

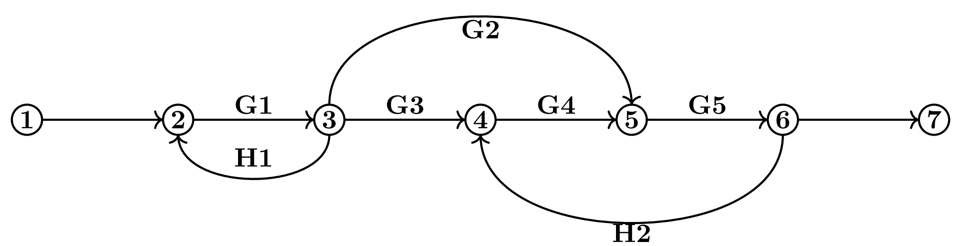


Solution:

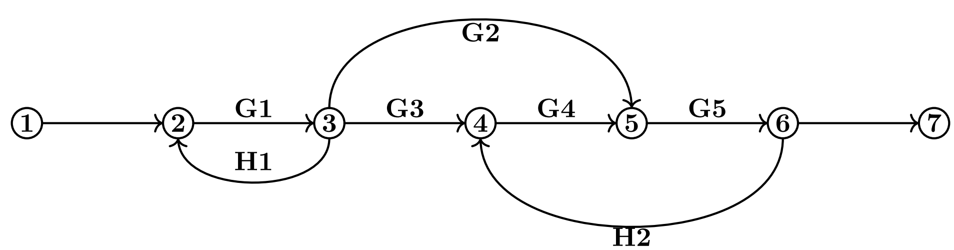
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



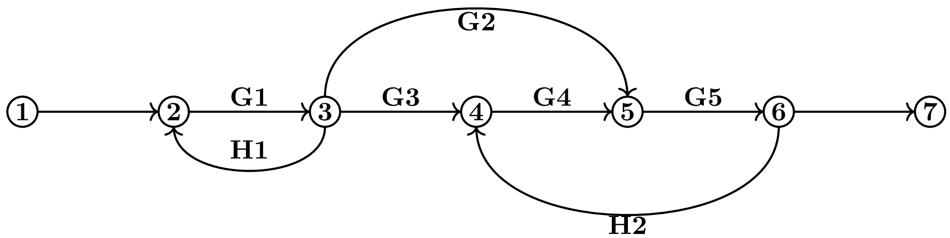
$$F_1 = G_1G_2G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_3G_4G_5$$

$$\Delta_2 = 0$$

Step 4. Identify all loops



$$L_1 = G_1H_1$$

$$L_2 = G_4G_5H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

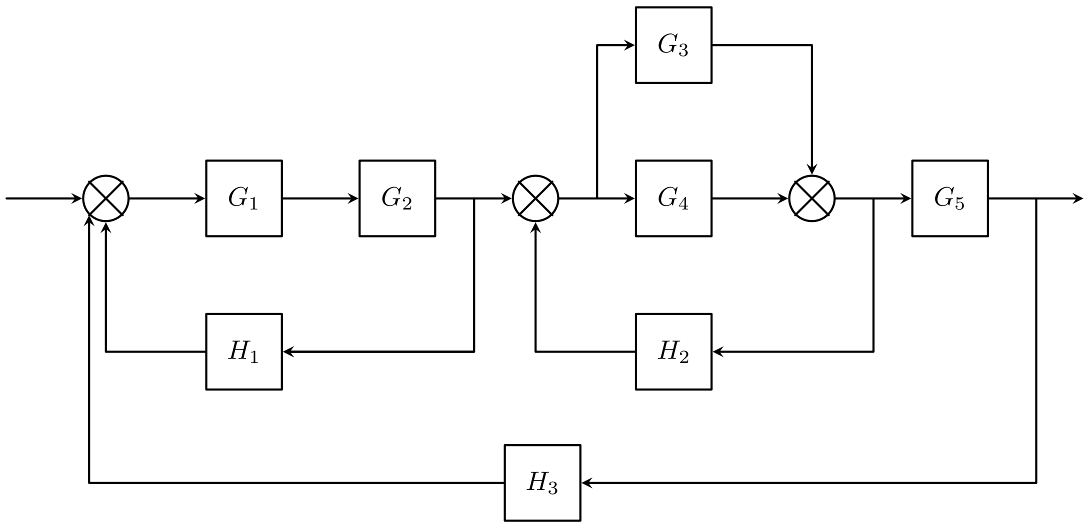
$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_5(1+0) + G_1G_3G_4G_5(1+0)}{1 + (G_1H_1 + G_4G_5H_2) + (G_1H_1 \cdot G_4G_5H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_5 + G_1G_3G_4G_5}{1 + G_1H_1 + G_4G_5H_2 + G_1G_4G_5H_1H_2}}$$

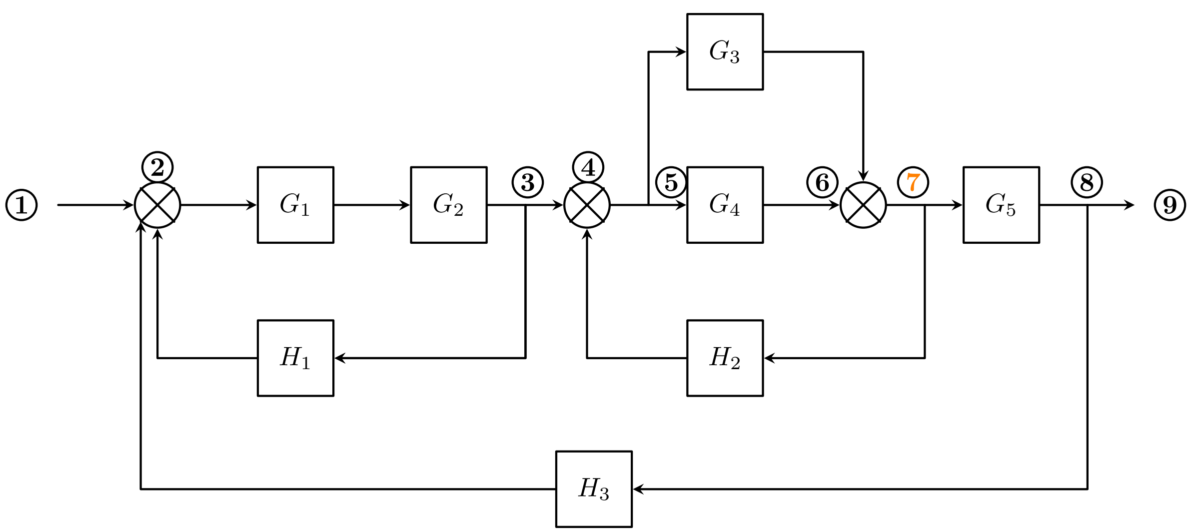
Problem 92

Solve the equivalent transfer function using Mason's Gain Formula:

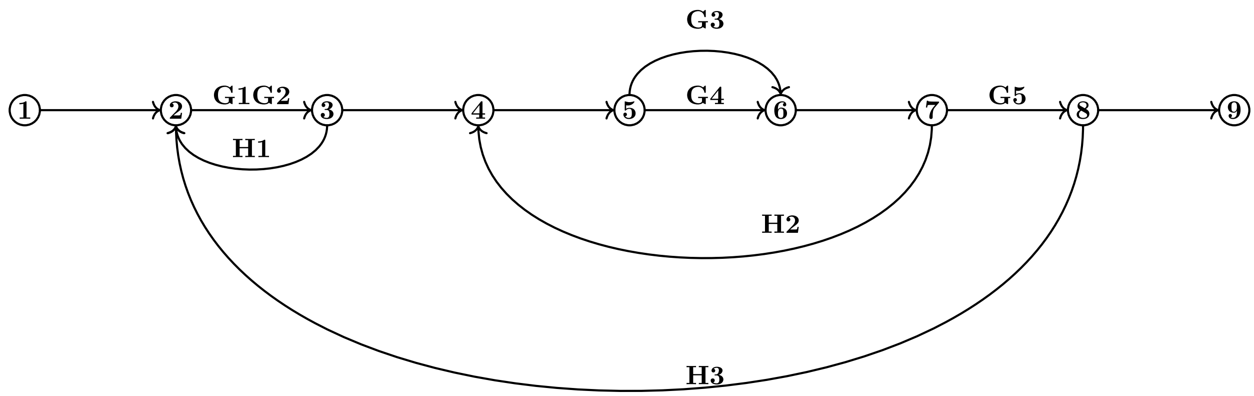


Solution:

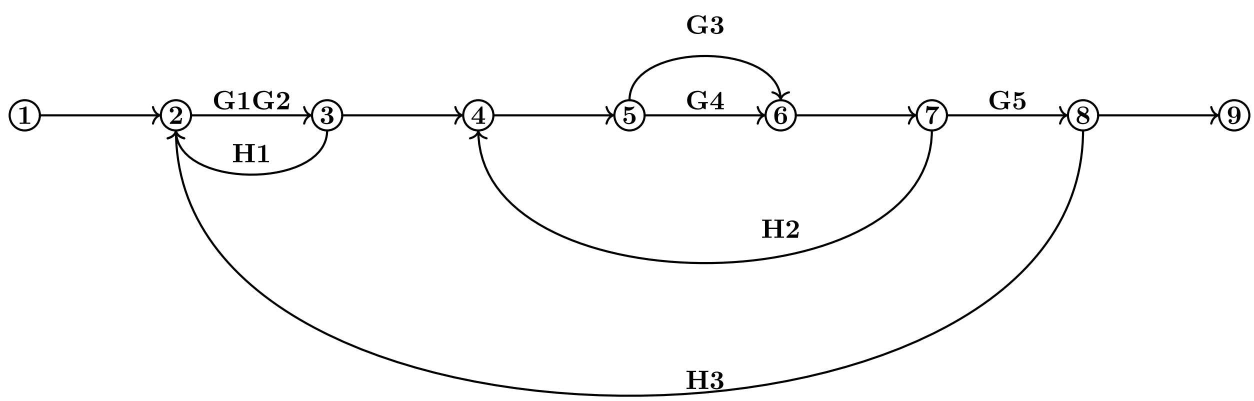
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



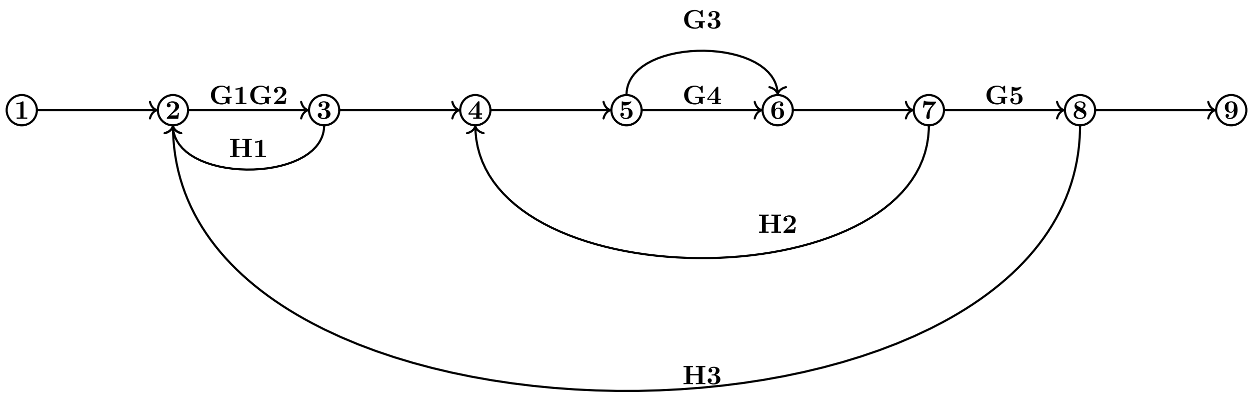
$$F_1 = G_1G_2G_3G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_2G_4G_5$$

$$\Delta_2 = 0$$

Step 4. Identify all loops



$$L_1 = G_1G_2H_1$$

$$L_2 = G_3H_2$$

$$L_3 = G_4H_2$$

$$L_4 = G_1G_2G_3G_5H_3$$

$$L_5 = G_1G_2G_4G_5H_3$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

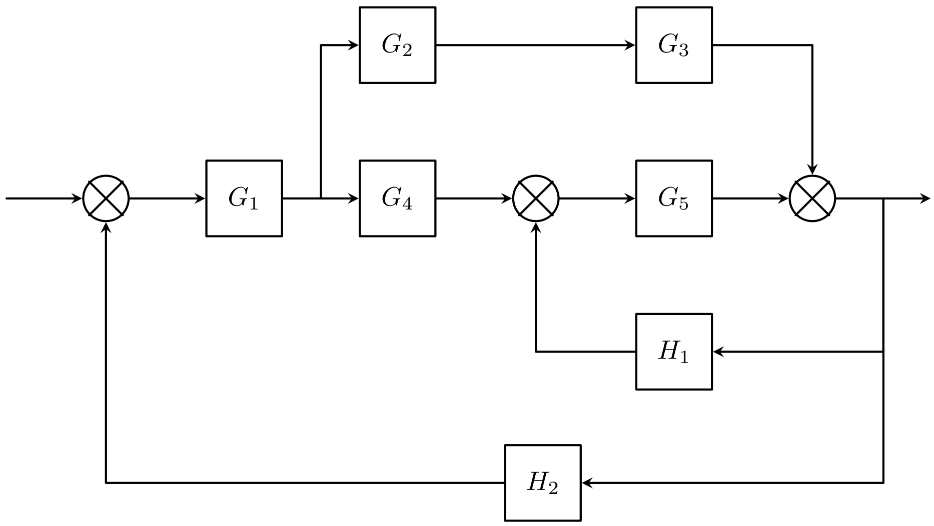
($*$) means not touching each other

$$TF = \frac{G_1G_2G_3(1+0) + G_1G_2G_4G_5(1+0)}{1 + (G_1G_2H_1 + G_3H_2 + G_4H_2 + G_1G_2G_3G_5H_3 + G_1G_2G_4G_5H_3) + (G_1G_2H_1 \cdot G_3H_2 + G_1G_2H_1 \cdot G_4H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_3G_5 + G_1G_2G_4G_5}{1 + G_1G_2H_1 + G_3H_2 + G_4H_2 + G_1G_2G_3G_5H_3 + G_1G_2G_4G_5H_3 + G_1G_2G_3H_1H_2 + G_1G_2G_4H_1H_2}}$$

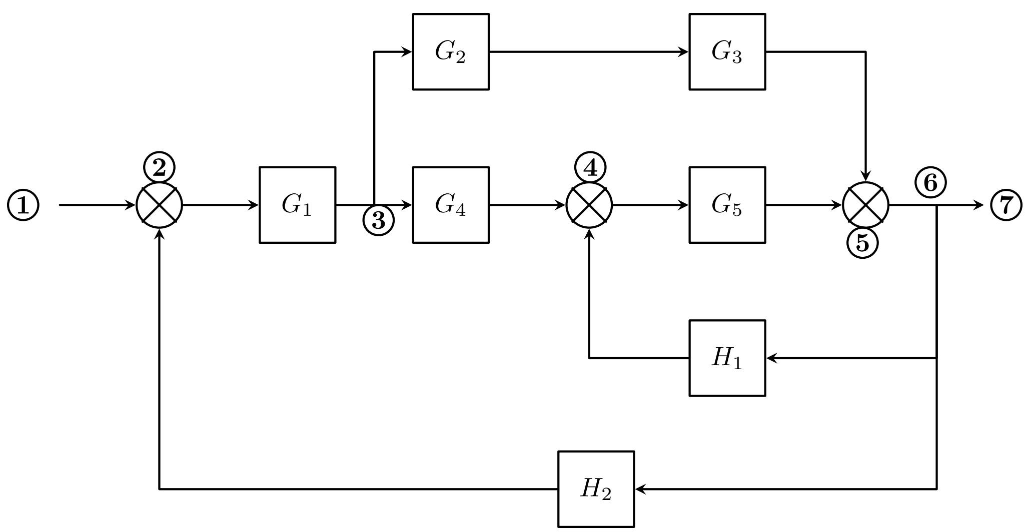
Problem 93

Solve the equivalent transfer function using Mason's Gain Formula:

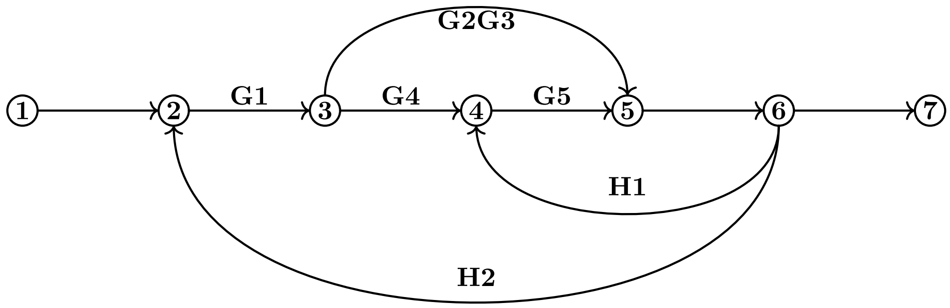


Solution:

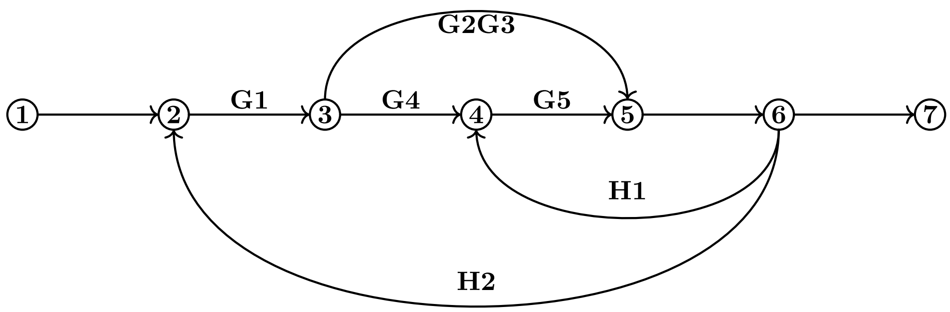
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



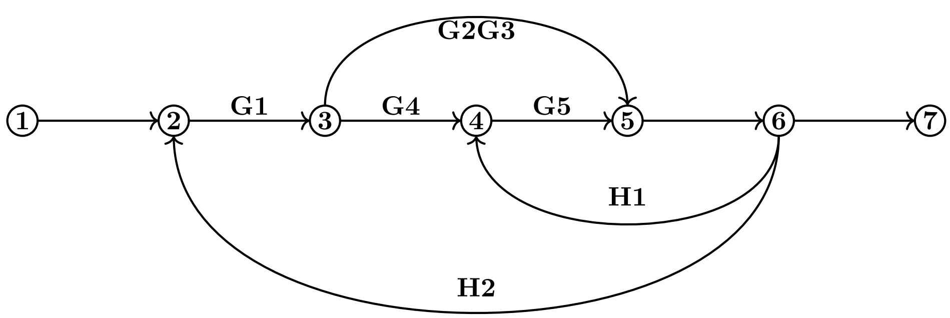
$$F_1 = G_1G_2G_3$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_4G_5$$

$$\Delta_2 = 0$$

Step 4. Identify all loops



$$L_1 = G_5H_1$$

$$L_2 = G_1G_4G_5H_2$$

$$L_3 = G_1G_2G_3H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

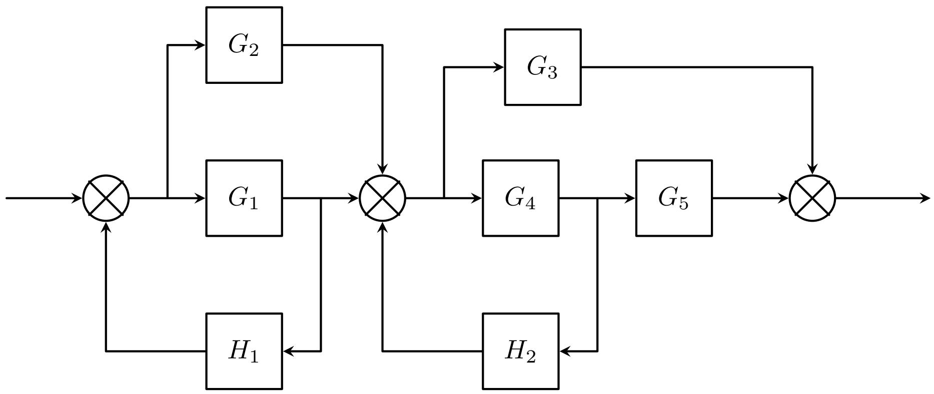
$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_3(1+0) + G_1G_4G_5(1+0)}{1 + (G_5H_1 + G_1G_4G_5H_2 + G_1G_2G_3H_2) + (0) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_3 + G_1G_4G_5}{1 + G_5H_1 + G_1G_4G_5H_2 + G_1G_2G_3H_2}}$$

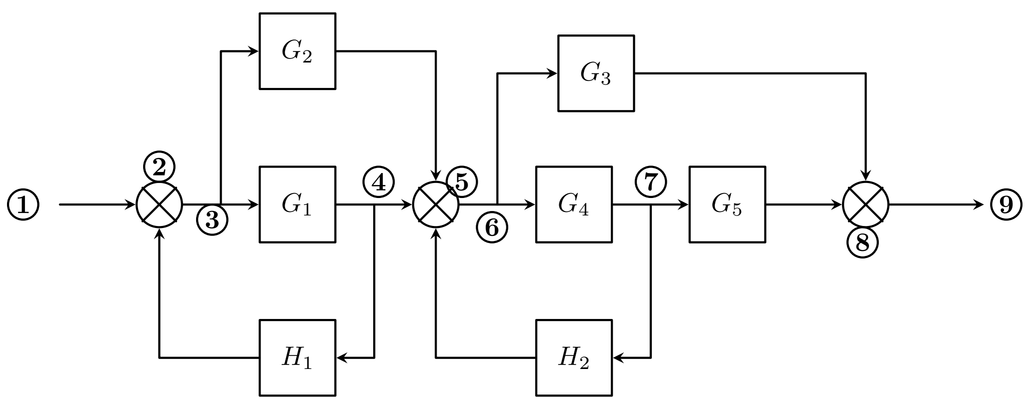
Problem 94

Solve the equivalent transfer function using Mason's Gain Formula:

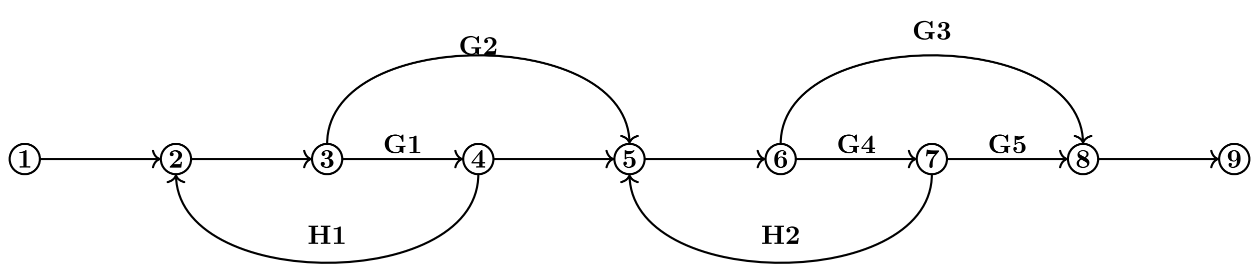


Solution:

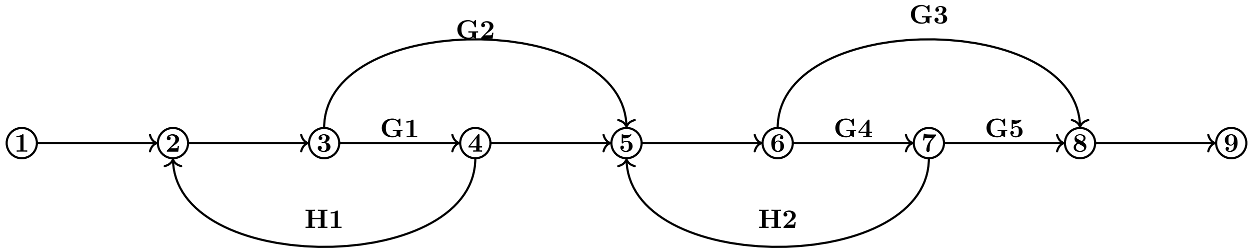
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them if it exist



$$F_1 = G_1G_4G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_2G_4G_5$$

$$\Delta_2 = 0$$

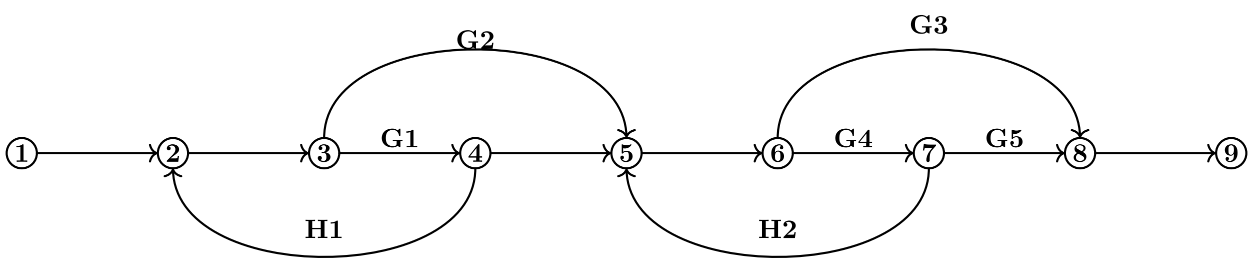
$$F_3 = G_1G_3$$

$$\Delta_3 = 0$$

$$F_4 = G_2G_3$$

$$\Delta_4 = 0$$

Step 4. Identify all loops



$$L_1 = G_1H_1$$

$$L_2 = G_4H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

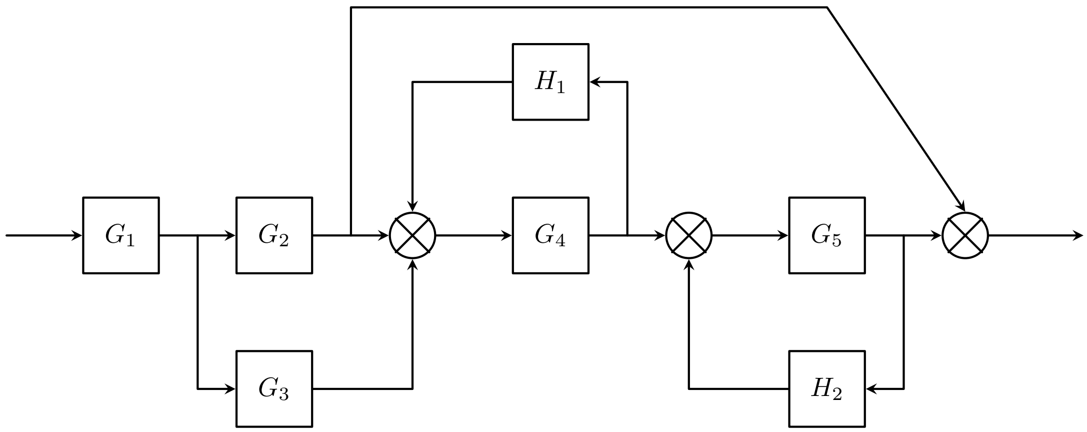
$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

$(*)$ means not touching each other

$$TF = \frac{G_1G_4G_5(1+0) + G_2G_4G_5(1+0) + G_1G_3(1+0) + G_2G_3(1+0)}{1 + (G_1H_1 + G_4H_2) + (G_1H_1 \cdot G_4H_2) + (0) + ...}$$

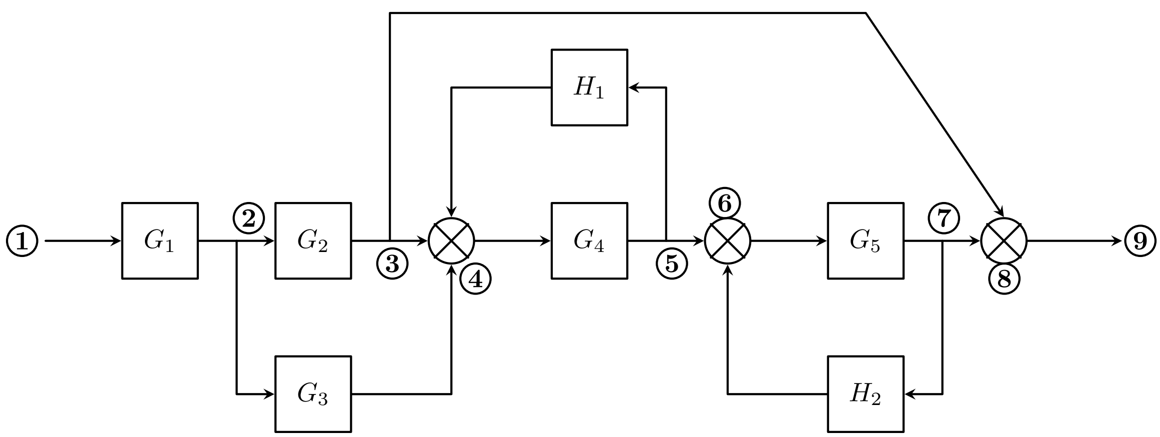
$$\boxed{TF = \frac{G_1G_4G_5 + G_2G_4G_5 + G_1G_3 + G_2G_3}{1 + G_1H_1 + G_4H_2 + G_1G_4H_1H_2}}$$

Problem 95
Solve the equivalent transfer function using Mason's Gain Formula:

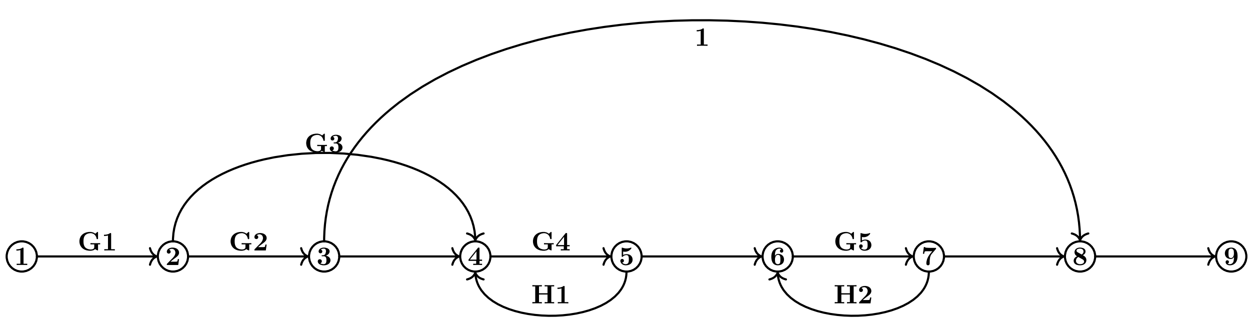


Solution:

Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.

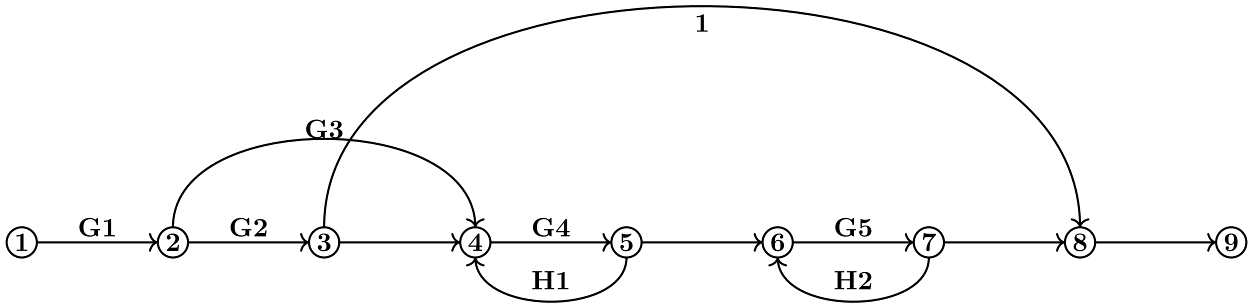


Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there

exist)



$$F_1 = G_1G_2G_4G_5$$

$$\Delta_1 = 0$$

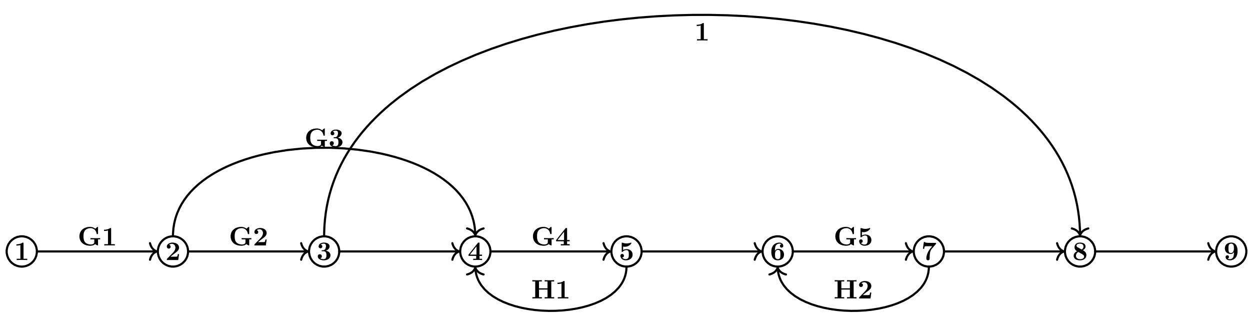
$$F_2 = G_1G_2$$

$$\Delta_2 = G_4H_1 \ \& \ G_5H_2$$

$$F_3 = G_1G_3G_4G_5$$

$$\Delta_3 = 0$$

Step 4. Identify all loops



$$L_1 = G_4H_1$$

$$L_2 = G_5H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

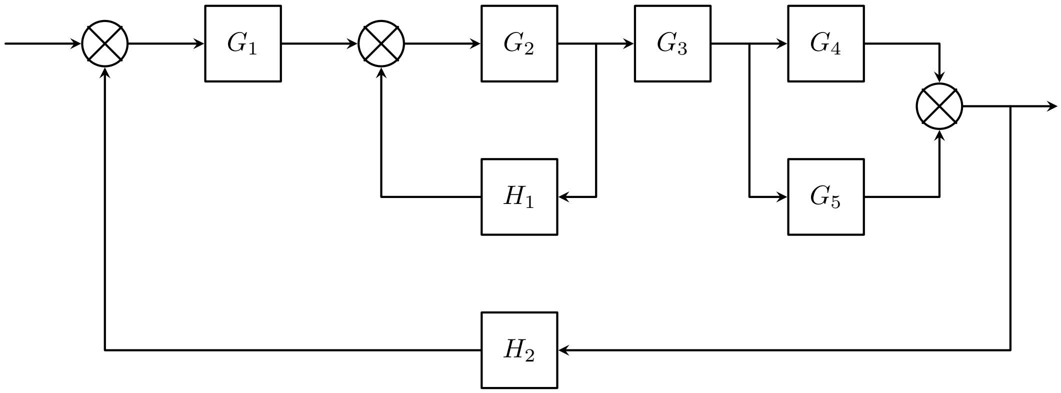
$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_4G_5(1+0) + G_1G_2(1 + G_4H_1 + G_5H_2 + G_4G_5H_1H_2) + G_1G_3G_4G_5}{1 + (G_4H_1 + G_5H_2) + (G_4H_1 \cdot G_5H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_4G_5 + G_1G_2 + G_1G_2G_4H_1 + G_1G_2G_5H_2 + G_1G_2G_4G_5H_1H_2 + G_1G_3G_4G_5}{1 + G_4H_1 + G_5H_2 + G_4G_5H_1H_2}}$$

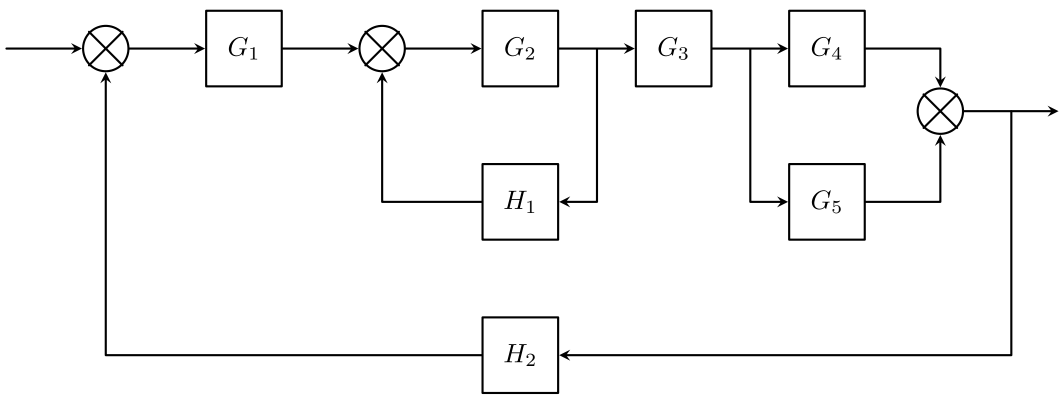
Problem 96

Simplify the following Block Diagram

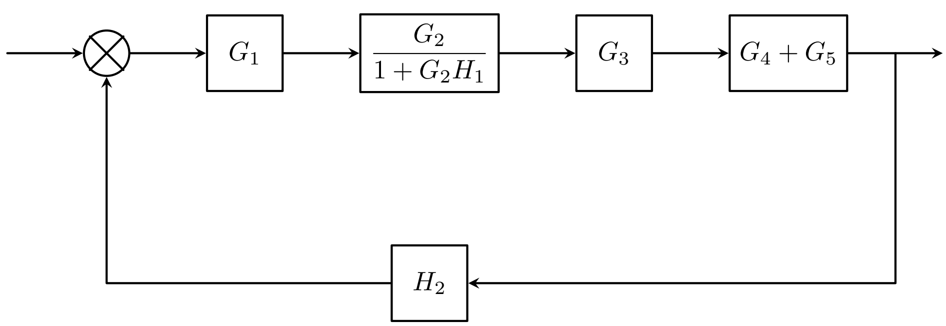


Solution:

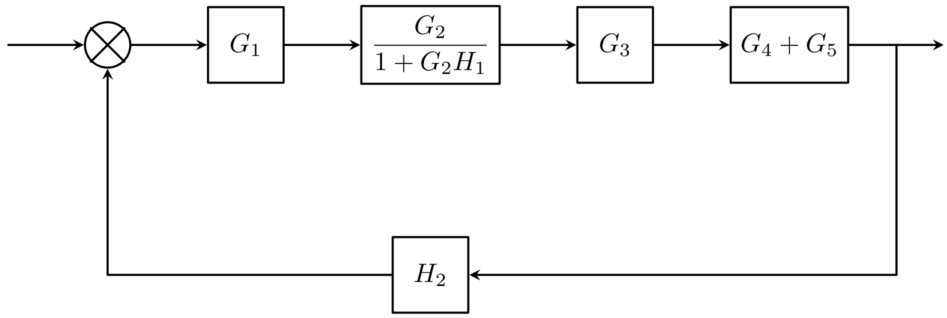
Step 1



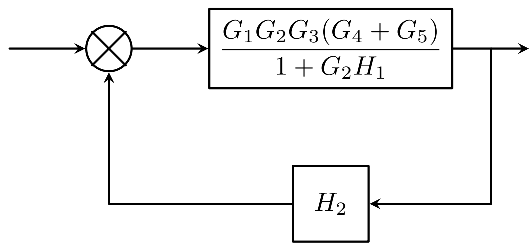
Simplify



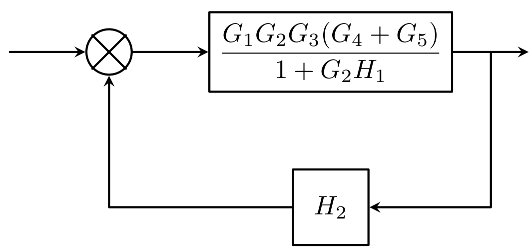
Step 2



Simplify



Step 3



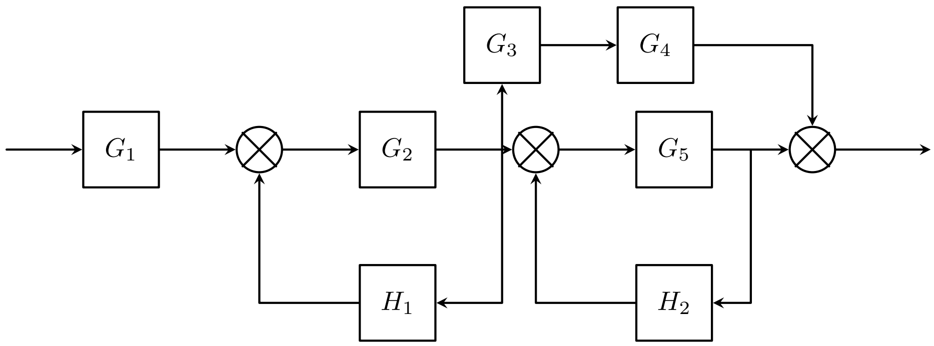
Simplify (Final Transfer Function)

$$TF = \frac{\left[\dfrac{G_1G_2G_3(G_4+G_5)}{1+G_2H_1}\right]}{1+\left[\dfrac{G_1G_2G_3(G_4+G_5)}{1+G_2H_1}\right]H_2}$$

$$\boxed{TF = \frac{G_1G_2G_3(G_4+G_5)}{1+G_2H_1+G_1G_2G_3(G_4+G_5)+H_2}}$$

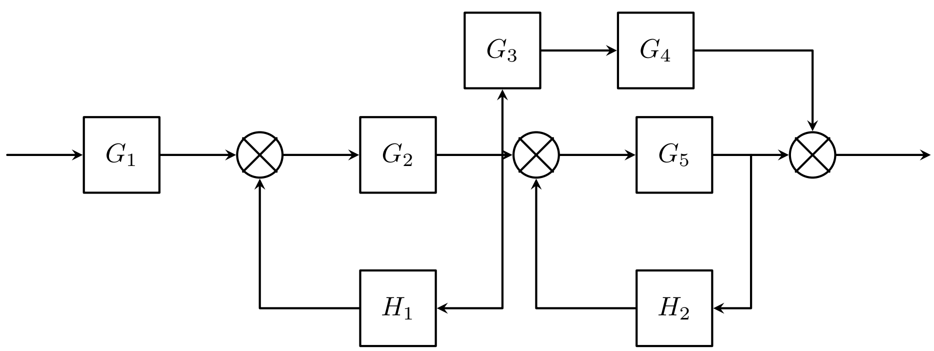
Problem 97

Simplify the following Block Diagram

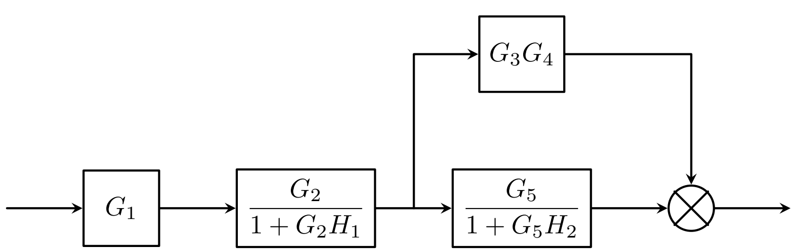


Solution:

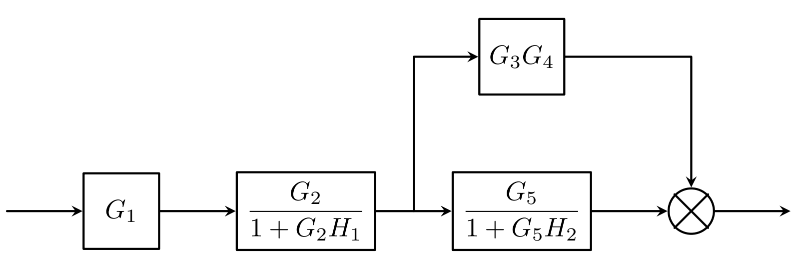
Step 1



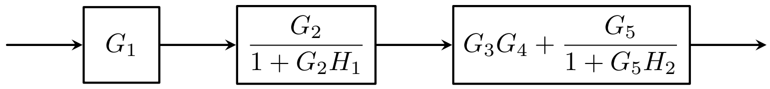
Simplify



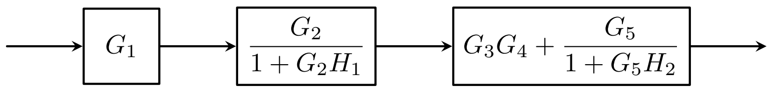
Step 2



Simplify



Step 3

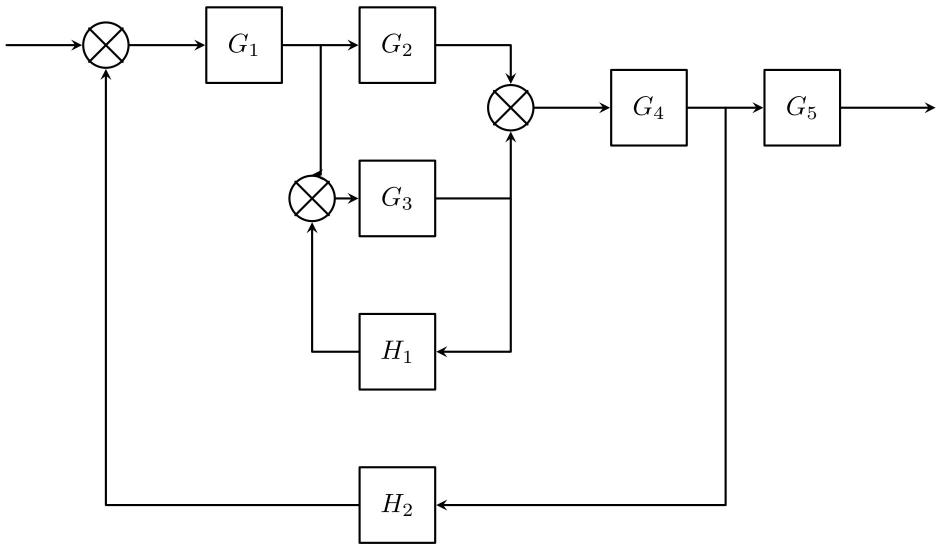


Simplify (Final Transfer Function)

$$TF = (G_1)\left(\frac{G_2}{1+G_2H_1}\right)\left(G_3G_4 + \frac{G_5}{1+G_5H_2}\right)$$

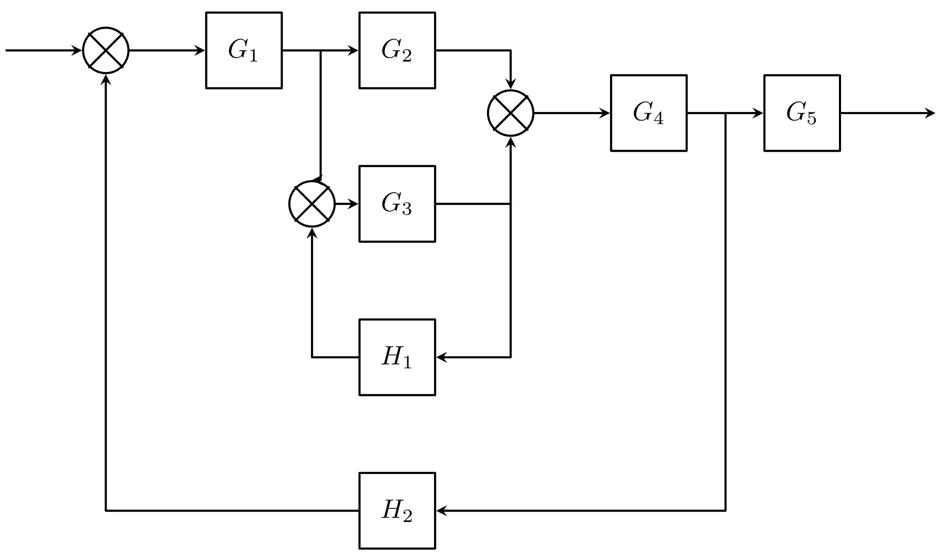
$$\boxed{TF = \frac{G_1G_2G_5 + G_1G_2G_3G_4 + G_1G_2G_3G_4G_5H_2}{1+G_2H_1+G_5H_2+G_2G_5H_1H_2}}$$

Problem 98
Simplify the following Block Diagram using Block Diagram Algebra

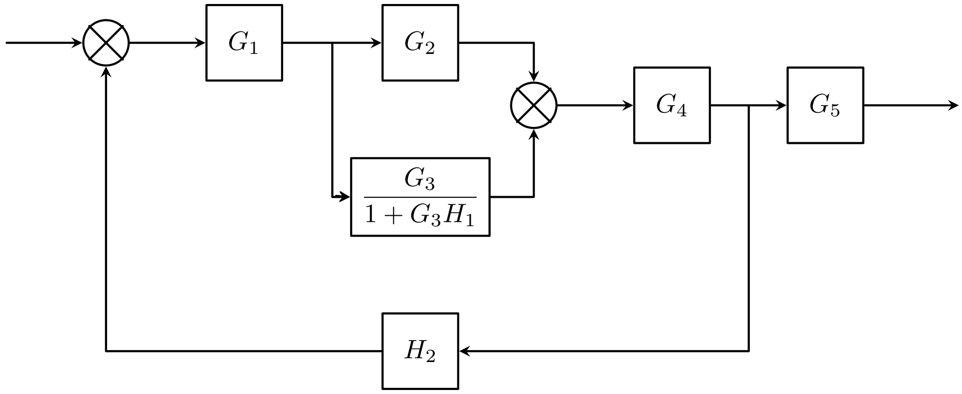


Solution:

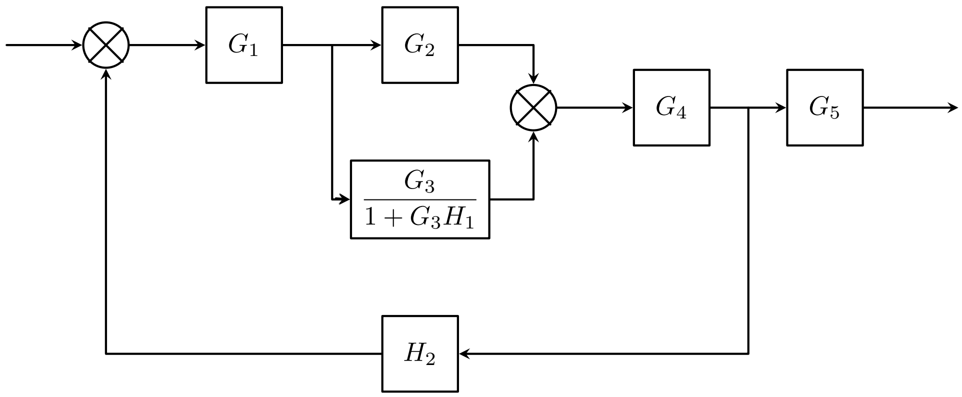
Step 1



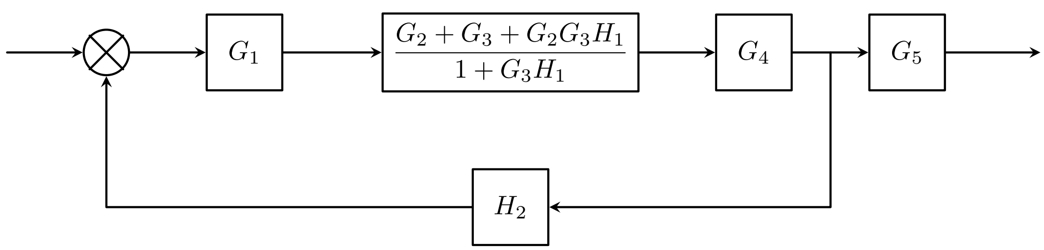
Simplify



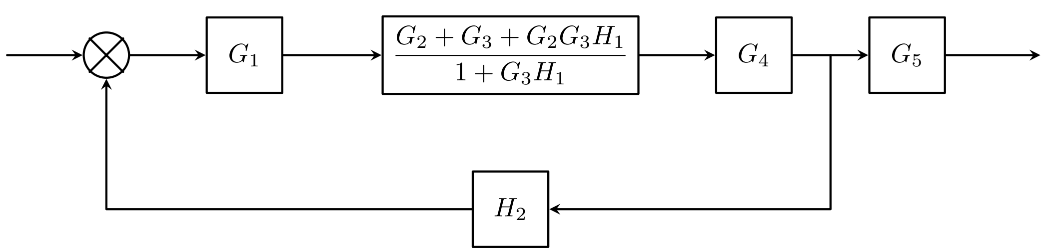
Step 2



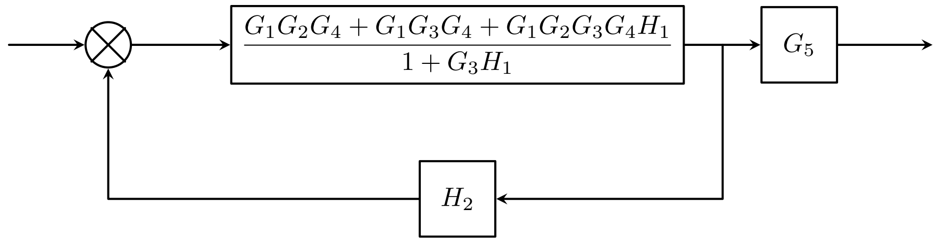
Simplify



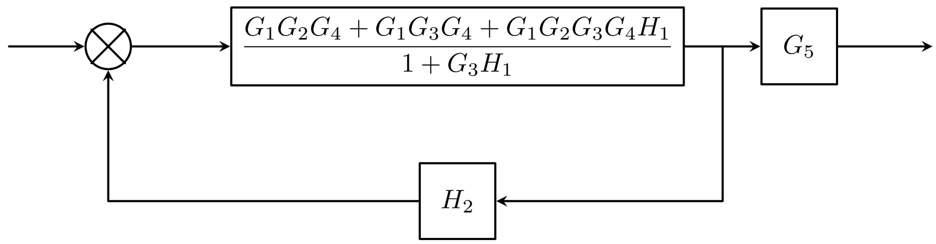
Step 3



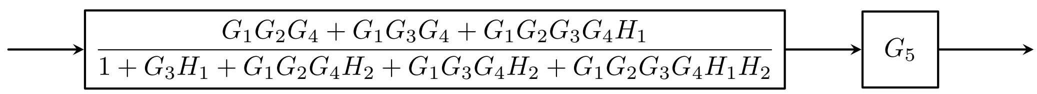
Simplify



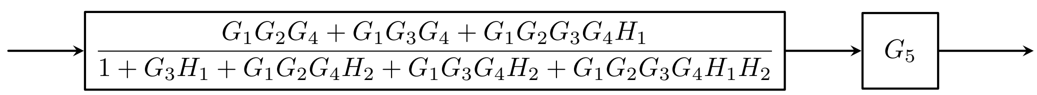
Step 4



Simplify



Step 5



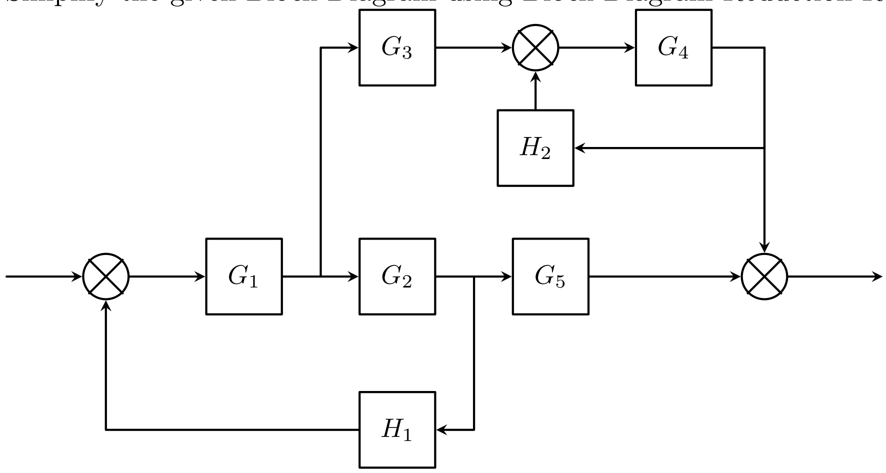
Simplify (Final Transfer Function)

$$TF = \left(\frac{G_1G_2G_4 + G_1G_3G_4 + G_1G_2G_3G_4H_1}{1 + G_3H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2 + G_1G_2G_3G_4H_1H_2}\right)(G_5)$$

$$\boxed{TF = \frac{G_1G_2G_4G_5 + G_1G_3G_4G_5 + G_1G_2G_3G_4G_5H_1}{1 + G_3H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2 + G_1G_2G_3G_4H_1H_2}}$$

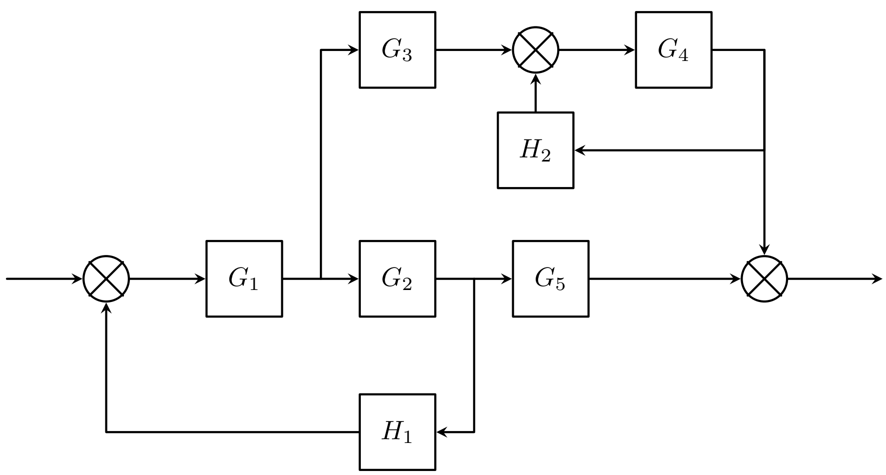
Problem 99

Simplify the given Block Diagram using Block Diagram Reduction Rules.

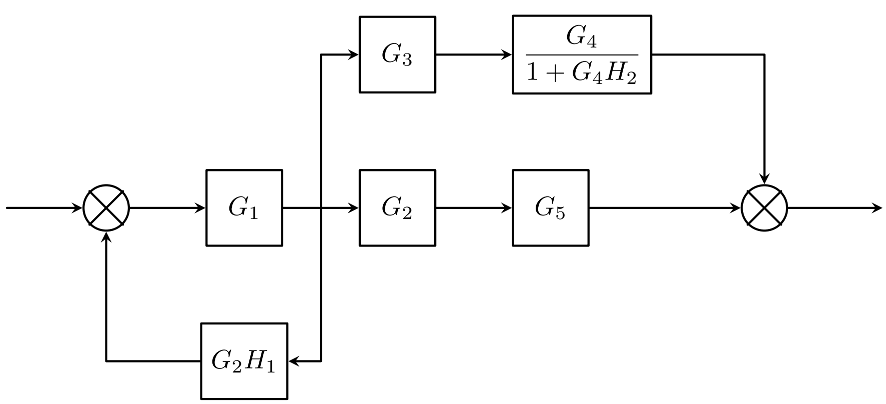


Solution:

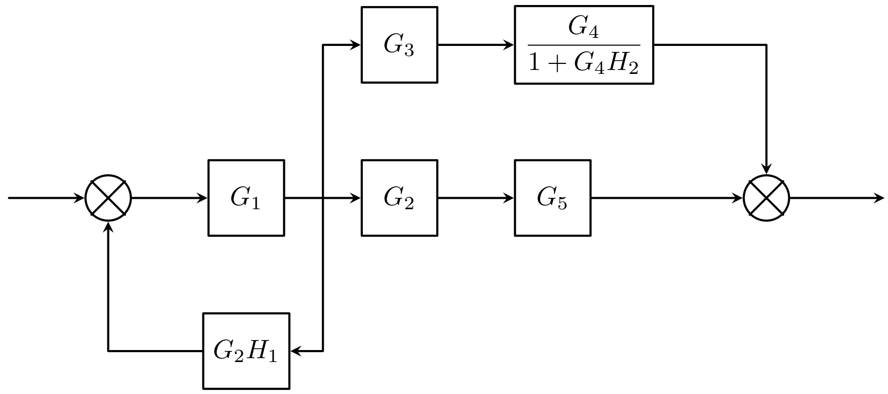
Step 1



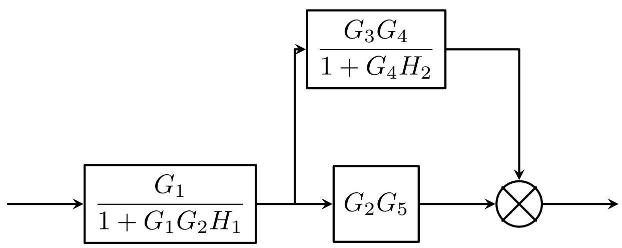
Simplify



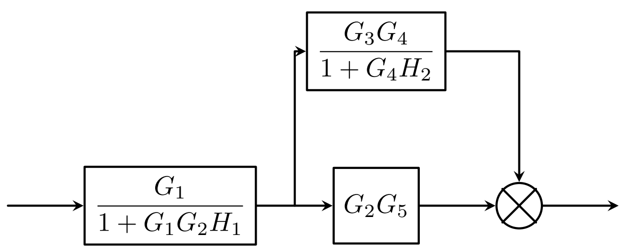
Step 2



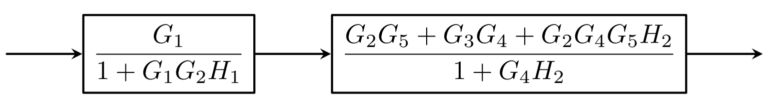
Simplify



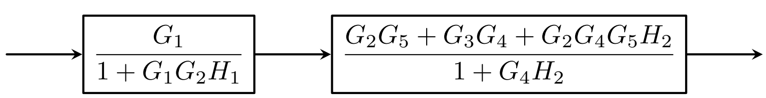
Step 3



Simplify



Step 4



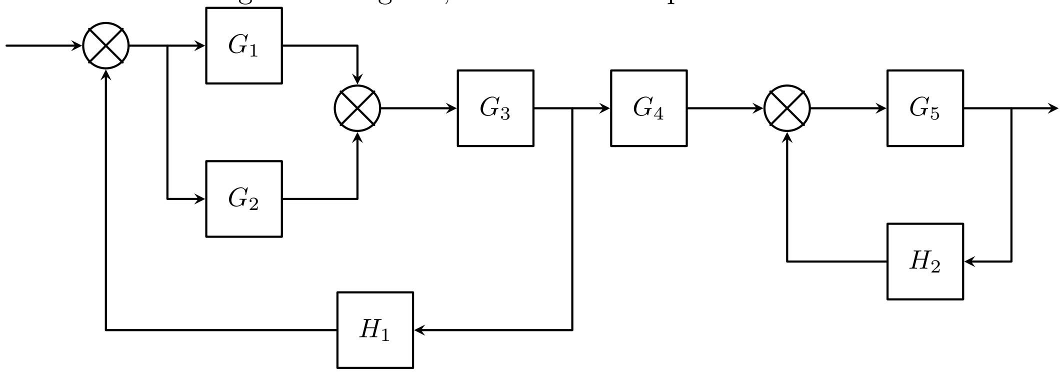
Simplify (Final Transfer Function)

$$TF = \left(\frac{G_1}{1+G_1G_2H_1}\right)\left(\frac{G_2G_5+G_3G_4+G_2G_4G_5H_2}{1+G_4H_2}\right)$$

$$\boxed{TF = \frac{G_1G_2G_5+G_1G_3G_4+G_1G_2G_4G_5H_2}{1+G_1G_2H_1+G_4H_2+G_1G_2G_4H_1H_2}}$$

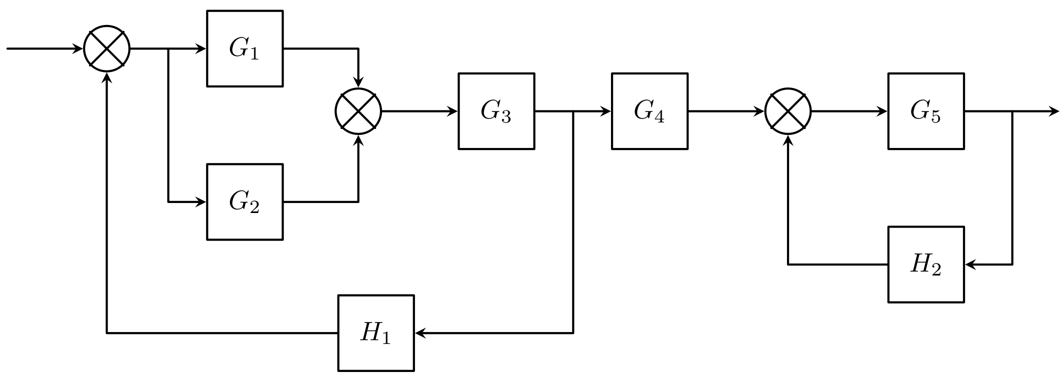
Problem 100

Given the following block diagram, determine the equivalent transfer function.

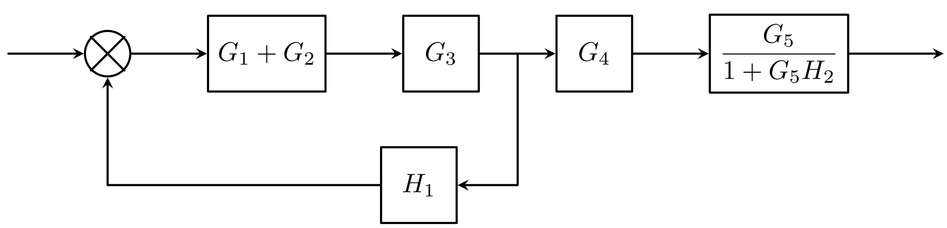


Solution:

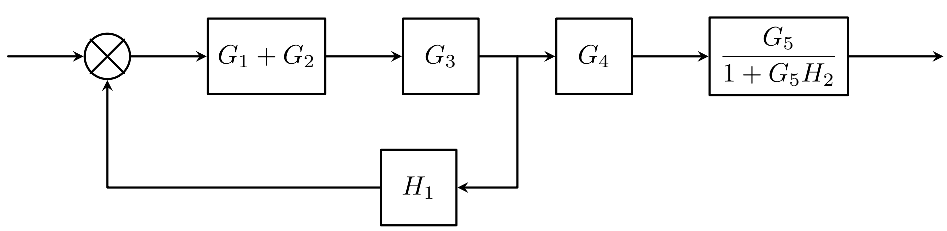
Step 1



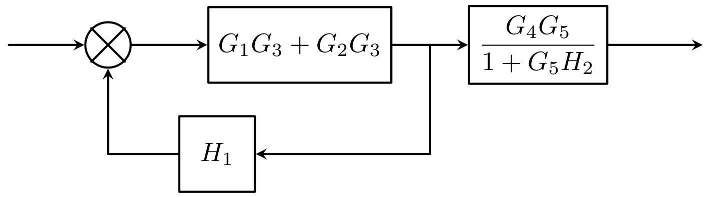
Simplify



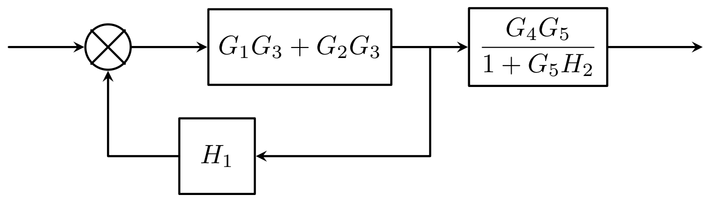
Step 2



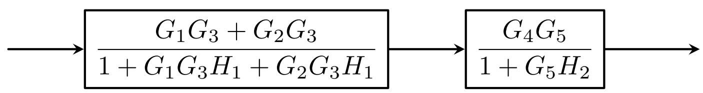
Simplify



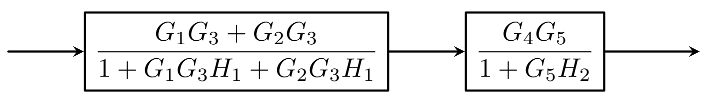
Step 3



Simplify



Step 4



Simplify (Final Transfer Function)

$$TF = \left(\frac{G_1G_3 + G_2G_3}{1+G_1G_3H_1+G_2G_3H_1}\right)\left(\frac{G_4G_5}{1+G_5H_2}\right)$$

$$\boxed{TF = \frac{G_1G_3G_4G_5 + G_2G_3G_4G_5}{1+G_1G_3H_1+G_2G_3H_1+G_5H_2+G_1G_3G_5H_1H_2+G_2G_3G_5H_1H_2}}$$