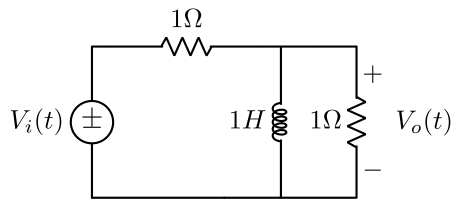
Problem 1

Determine the transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ of the network below



Given: As per diagram

Solution:

Using mesh analysis:

@Loop 1:

$$V_i - R_1 i_1 - L\frac{di_1}{dt} + L\frac{di_2}{dt} = 0$$

$$V_i(s) - R_1 I_1(s) - LsI_1(s) + LsI_2(s) = 0$$

$$V_i(s) = (R_1 + Ls)I_1(s) - LsI_2(s) \quad \textbf{(equation 1)}$$

@Loop 2:

$$-R_2 i_2 - L\frac{di_2}{dt} + L\frac{di_1}{dt} = 0$$

$$R_2 I_2(s) - LsI_2(s) - LsI_1(s) = 0$$

$$(R_2 + Ls)I_2(s) = LsI_1(s)$$

$$I_1(s) = \left(\frac{R_2 + Ls}{Ls}\right) I_2(s) \quad \textbf{(equation 2)}$$

Substitute equation 2 into equation 1:

$$V_i(s) = (R_1 + Ls)\left(\frac{R_2 + Ls}{Ls}\right) I_2(s) - LsI_2(s)$$

Now since $V_o = R_2 i_2 \Rightarrow V_o(s) = R_2 I_2(s)$, and so:

$$I_2(s) = \frac{V_o(s)}{R_2}$$

Substitute into the expression:

$$V_i(s) = (R_1 + Ls)\left(\frac{R_2 + Ls}{Ls}\right)\frac{V_o(s)}{R_2} - Ls\frac{V_o(s)}{R_2}$$

Substituting component values:

$$V_i(s) = (s + 1)\left(\frac{s + 1}{s}\right) V_o(s) - sV_o(s)$$

$$V_i(s) = \left[\frac{s^2+2s+1}{s} - s\right] V_o(s)$$

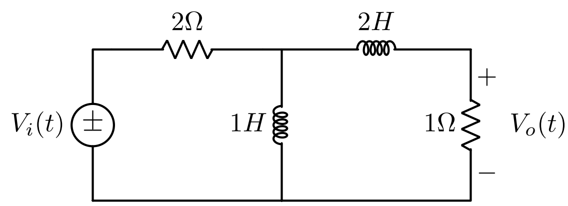
$$V_i(s) = \left[\frac{s^2+2s+1-s^2}{s}\right] V_o(s) = \frac{2s+1}{s} V_o(s)$$

$$\frac{V_i(s)}{V_o(s)} = \frac{2s+1}{s}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{s}{2s+1}}$$

Problem 2

Determine the transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ of the network below



Given: As per diagram

Solution:

Mesh Analysis

Using mesh analysis with two loops:

Loop 1 Equation

$$\begin{aligned}
V_i - R_1 i_1 - L_1\frac{di_1}{dt} + L_2\frac{di_2}{dt} &= 0 \\
V_i(s) - 2I_1(s) - 2sI_1(s) + sI_2(s) &= 0 \\
V_i(s) &= (2s+2)I_1(s) - sI_2(s) \quad \textbf{(Equation 1)}
\end{aligned}$$

Loop 2 Equation

$$\begin{aligned}
L_1\frac{di_1}{dt} - L_1\frac{di_2}{dt} - L_2\frac{di_2}{dt} - R_2 i_2 &= 0 \\
2sI_1(s) - 2sI_2(s) - sI_2(s) - I_2(s) &= 0 \\
2sI_1(s) &= (3s+1)I_2(s) \\
I_1(s) &= \left(\frac{3s+1}{2s}\right) I_2(s) \quad \textbf{(Equation 2)}
\end{aligned}$$

Output Voltage

$$\begin{aligned}
V_o &= R_2 i_2 \\
V_o(s) &= 1 \cdot I_2(s) \\
I_2(s) &= V_o(s)
\end{aligned}$$

Substitution and Solution

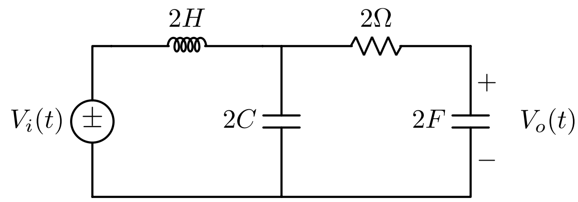
Substitute Equation 2 into Equation 1:

$$
\begin{aligned}
V_i(s) &= (2s+2)\left(\frac{3s+1}{2s}\right)V_o(s) - sV_o(s) \\
&= \left(\frac{6s^2+8s+2}{2s} - s\right)V_o(s) \\
&= \left(\frac{6s^2+8s+2-2s^2}{2s}\right)V_o(s) \\
&= \left(\frac{4s^2+8s+2}{2s}\right)V_o(s)
\end{aligned}
$$

$$
\boxed{\frac{V_o(s)}{V_i(s)} = \frac{2s}{4s^2+8s+2} = \frac{s}{2s^2+4s+1}}
$$

Problem 3

Determine the transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ of the network below



Given: As per diagram

Solution:

Using mesh analysis:

@Loop 1:

$$V_i - 2\frac{di_1}{dt} - \frac{1}{2}\int i_1\,dt + \frac{1}{2}\int i_2\,dt = 0$$

$$V_i(s) - 2sI_1(s) - \frac{1}{2s}I_1(s) + \frac{1}{2s}I_2(s) = 0$$

$$V_i(s) = \left(2s + \frac{1}{2s}\right)I_1(s) - \frac{1}{2s}I_2(s) \quad \textbf{(equation 1)}$$

@Loop 2:

$$\frac{1}{2}\int i_1\,dt - \frac{1}{2}\int i_2\,dt - 2i_2 - \frac{1}{2}\int i_2\,dt = 0$$

$$\frac{1}{2s}I_1(s) - \frac{1}{2s}I_2(s) - 2I_2(s) - \frac{1}{2s}I_2(s) = 0$$

$$\frac{1}{2s}I_1(s) = \left(\frac{1}{s} + 2\right)I_2(s)$$

$$I_1(s) = (4s+1)I_2(s) \quad \textbf{(equation 2)}$$

From the capacitor on the output:

$$i_2 = 2\frac{dV_o}{dt} \Rightarrow I_2(s) = 2sV_o(s)$$

Substitute equation 2 into equation 1:

$$V_i(s) = \left(2s + \frac{1}{2s}\right)(4s+1)I_2(s) - \frac{1}{2s}I_2(s)$$

$$V_i(s) = \left(\frac{16s^3 + 4s^2 + 4s + 1}{2s} - \frac{1}{2s}\right)2sV_o(s)$$

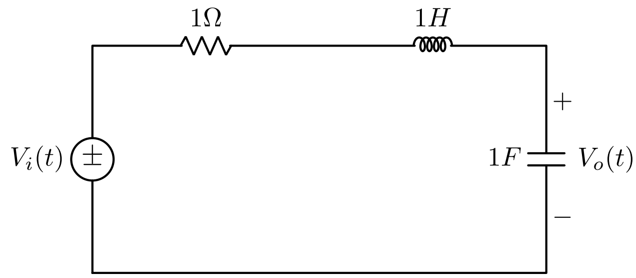
$$V_i(s) = \left(8s^2 + 2s + 2\right)2sV_o(s)$$

$$V_i(s) = 2s(8s^2 + 2s + 2)V_o(s)$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{2s(8s^2+2s+2)}}$$

Problem 4

Determine the transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ of the network below:



Given: As per diagram

Solution:

Using voltage divider in the s-domain:

$$Z_R = 1\Omega, \quad Z_L = sL = s, \quad Z_C = \frac{1}{sC} = \frac{1}{s}$$

Total impedance:

$$Z_{total} = Z_R + Z_L + Z_C = 1 + s + \frac{1}{s}$$

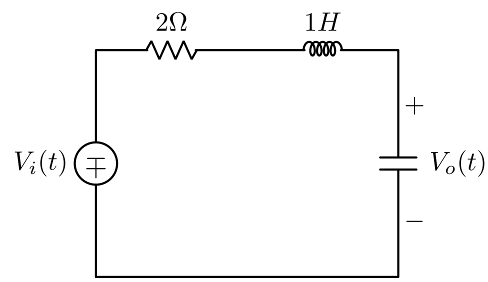
Transfer function:

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_C}{Z_{total}} = \frac{\frac{1}{s}}{1 + s + \frac{1}{s}} = \frac{1}{s + s^2 + 1}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + s + 1}}$$

Problem 5

Find the transfer function $\frac{V_o(s)}{V_i(s)}$ *for the following circuit:*



Given: As per diagram

Solution:

1. **Impedance Calculation:**

$$Z_R = 2\,\Omega, \quad Z_L = sL = s, \quad Z_C = \frac{1}{sC} = \frac{2}{s}$$

2. **Voltage Divider Rule:**

$$V_o(s) = V_i(s) \cdot \frac{Z_C}{Z_R + Z_L + Z_C}$$

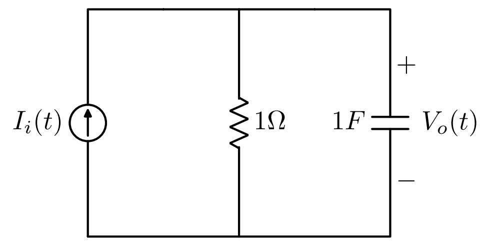
3. **Transfer Function:**

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{2}{s}}{2 + s + \frac{2}{s}} = \frac{2}{s^2 + 2s + 2}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{2}{s^2 + 2s + 2}}$$

Problem 6

Analyze the parallel RC network to find $\frac{V_o(s)}{I_i(s)}$:



Given: As per diagram

Solution:

1. **Admittance Calculation:**

$$Y_R = \frac{1}{R} = 1\,S, \quad Y_C = sC = s$$

2. **Total Admittance:**

$$Y_{\text{total}} = Y_R + Y_C = 1 + s$$

3. **Impedance (Z):**

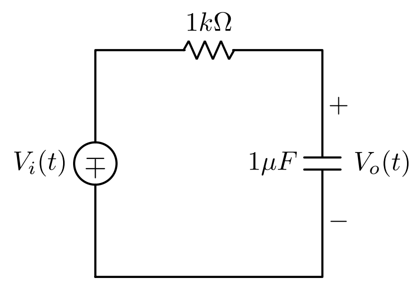
$$Z = \frac{1}{Y_{\text{total}}} = \frac{1}{s+1}$$

4. **Transfer Function:**

$$\frac{V_o(s)}{I_i(s)} = Z = \frac{1}{s+1}$$

$$\boxed{\frac{V_o(s)}{I_i(s)} = \frac{1}{s+1}}$$

Problem 7
Determine $G(s) = \frac{V_o(s)}{V_i(s)}$ *for the following RC circuit:*



Given:

- $R = 1\,k\Omega = 1000\,\Omega$
- $C = 1\,\mu F = 10^{-6}\,F$

Solution:

1. **Impedance Calculation:**

$$Z_R = R = 1000\,\Omega, \quad Z_C = \frac{1}{sC} = \frac{1}{10^{-6}s}$$

2. **Voltage Divider Rule:**

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_C}{Z_R + Z_C} = \frac{1/sC}{R + 1/sC}$$

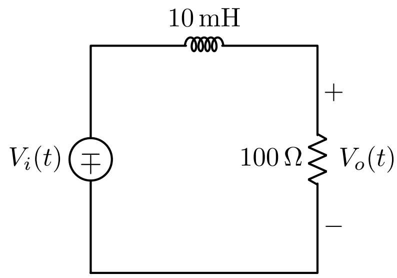
3. **Simplification:**

$$G(s) = \frac{1}{RCs + 1} = \frac{1}{0.001s + 1}$$

$$\boxed{G(s) = \frac{1}{0.001s + 1}}$$

Problem 8

Find $G(s) = \dfrac{V_o(s)}{V_i(s)}$ for this RL network:



Given:

- Inductance: $L = 10\,\text{mH} = 0.01\,\text{H}$
- Resistance: $R = 100\,\Omega$

Solution:

1. The total impedance in the s-domain is:

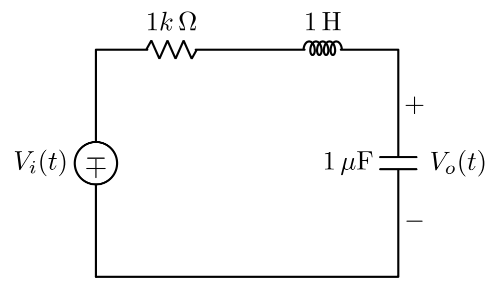
$$Z_{\text{total}} = Z_L + Z_R = sL + R = 0.01s + 100$$

2. The output voltage $V_o(s)$ is across the resistor, so use voltage divider:

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_R}{Z_{\text{total}}} = \frac{100}{0.01s + 100}$$

$$\boxed{G(s) = \frac{100}{0.01s + 100}}$$

Problem 9
Analyze this RLC bandpass filter:



Given:

- $R = 1000\,\Omega$
- $L = 1\,\text{H}$
- $C = 1\,\mu\text{F} = 1 \times 10^{-6}\,\text{F}$

Solution:

1. Convert all components to s-domain impedances:

$$Z_R = R = 1000, \quad Z_L = sL = s, \quad Z_C = \frac{1}{sC} = \frac{1}{10^{-6}s}$$

2. Total impedance in series:

$$Z_{\text{total}} = Z_R + Z_L + Z_C = 1000 + s + \frac{1}{10^{-6}s}$$

3. Voltage across the inductor (band-pass behavior):

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_L}{Z_{\text{total}}} = \frac{s}{1000 + s + \frac{1}{10^{-6}s}}$$

4. Multiply numerator and denominator by $10^{-6}s$ to simplify:

$$G(s) = \frac{10^{-6}s^2}{0.001s^2 + 10^{-6}s + 1}$$

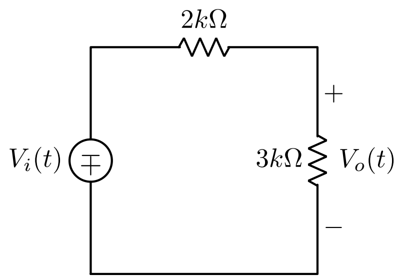
5. Multiply numerator and denominator by 10^6 to eliminate decimals:

$$G(s) = \frac{s^2}{0.001s^2 + s + 1000}$$

$$\boxed{G(s) = \frac{s}{0.001s^2 + s + 1000}}$$

Problem 10

Find: The transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ for this voltage divider:



Given:

- $R_1 = 2k\Omega$ — resistor connected to the voltage source
- $R_2 = 3k\Omega$ — resistor connected to ground, across which output is measured

Solution:

Step 1: Voltage Divider Rule
The voltage divider rule states:

$$V_o(s) = V_i(s) \cdot \frac{R_2}{R_1 + R_2}$$

Step 2: Derive Transfer Function
The transfer function is:

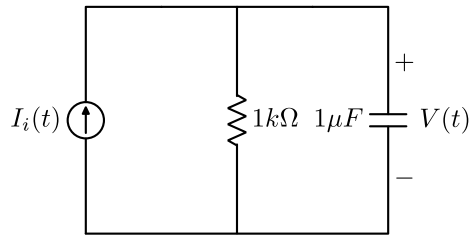
$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1 + R_2}$$

Step 3: Substitute Values

$$G(s) = \frac{3000}{2000 + 3000} = \frac{3000}{5000} = \frac{3}{5}$$

$$\boxed{G(s) = 0.6}$$

Problem 11
Find: $G(s) = \dfrac{V(s)}{I_i(s)}$ for this parallel RC network



Given: As per diagram

Solution:

Write the impedance of each element in the Laplace domain
Resistor: $Z_R = R$ Capacitor: $Z_C = \dfrac{1}{sC}$
For two components in parallel, the total impedance is:

$$Z_{eq} = \left(\frac{1}{R} + sC\right)^{-1}$$

Use Ohm's Law in the s-domain
The voltage across both elements is the same and is given by:

$$V(s) = I_i(s) \cdot Z_{eq}$$

Thus, the transfer function is:

$$\frac{V(s)}{I_i(s)} = Z_{eq} = \frac{1}{\frac{1}{R} + sC}$$

Substitute the given values

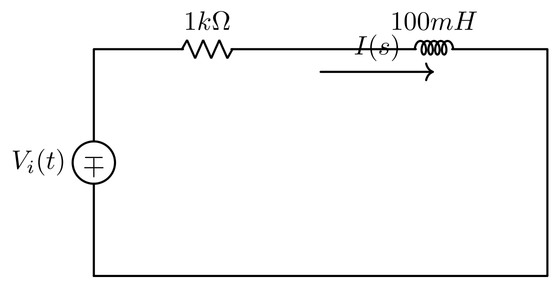
$$\frac{V(s)}{I_i(s)} = \frac{1}{\frac{1}{1000} + s \cdot 1 \times 10^{-6}} = \frac{1}{0.001 + 0.000001s} = \frac{1}{0.001s + 1}$$

$$\frac{V(s)}{I_i(s)} = \frac{1000}{0.001s + 1}$$

$$\boxed{\frac{V(s)}{I_i(s)} = \frac{1000}{0.001s + 1}}$$

Problem 12

Find: $\dfrac{I(s)}{V_i(s)}$ for this series RL circuit



Given: As per diagram

Solution:

Circuit Analysis in Time Domain

$$V_i(t) = V_R(t) + V_L(t)$$

Formulating the Differential Equation

Substituting the component relations into KVL gives:

$$V_i(t) = R \cdot i(t) + L\frac{di(t)}{dt}$$

For our specific circuit:

- Resistance $R = 1k\Omega = 1000\Omega$
- Inductance $L = 100mH = 0.1H$

Thus:

$$V_i(t) = 1000 \cdot i(t) + 0.1\frac{di(t)}{dt}$$

Laplace Transformation

Taking the Laplace transform of both sides (assuming zero initial conditions):

$$\mathcal{L}\{V_i(t)\} = \mathcal{L}\{1000 \cdot i(t) + 0.1\frac{di(t)}{dt}\}$$

Using linearity property and derivative property of Laplace transforms:

$$V_i(s) = 1000I(s) + 0.1[sI(s) - i(0^+)]$$

Assuming initial current $i(0^+) = 0$:

$$V_i(s) = 1000I(s) + 0.1sI(s)$$

Solving for Transfer Function

Factor out $I(s)$:

$$V_i(s) = (1000 + 0.1s)I(s)$$

Now solve for the transfer function:

$$\frac{I(s)}{V_i(s)} = \frac{1}{1000 + 0.1s}$$

Final Form

We can rewrite the denominator for clarity:

$$\frac{I(s)}{V_i(s)} = \frac{1}{0.1s + 1000}$$

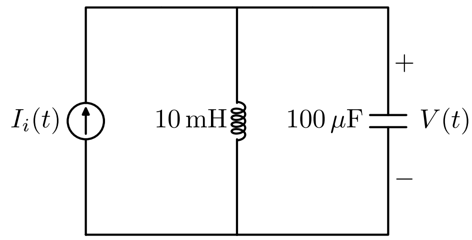
Or alternatively, multiplying numerator and denominator by 10:

$$\frac{I(s)}{V_i(s)} = \frac{10}{s + 10000}$$

$$\boxed{\frac{I(s)}{V_i(s)} = \frac{1}{0.1s + 1000}}$$

Problem 13

Find: The transfer function $G(s) = \dfrac{V(s)}{I_i(s)}$ for the parallel LC tank circuit shown below.



Given: As per diagram

Solution:

Convert to Laplace Domain
Two elements are in parallel:

$$Y(s) = \frac{1}{sL} + sC$$

Then the **impedance** is:

$$Z(s) = \frac{1}{Y(s)} = \frac{1}{\frac{1}{sL} + sC}$$

Derive Transfer Function

$$G(s) = \frac{V(s)}{I_i(s)} = Z(s) = \frac{1}{\frac{1}{sL} + sC}$$

Plug in Component Values

$$L = 0.01\,\text{H}, \quad C = 100 \times 10^{-6}\,\text{F}$$

So:

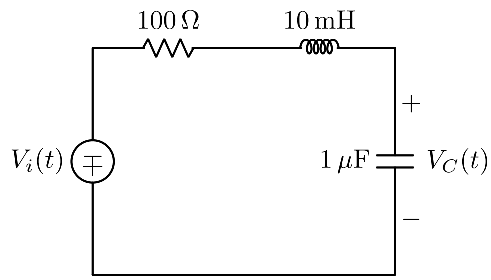
$$G(s) = \frac{1}{\frac{1}{0.01s} + 100 \times 10^{-6}s} = \frac{1}{\frac{1}{0.01s} + 10^{-4}s}$$

Multiply by $0.01s$ to simplify:

$$G(s) = \frac{0.01s}{1 + 0.000001s^2} = \boxed{\frac{0.01s}{10^{-6}s^2 + 1}}$$

Problem 14

Find: The transfer function $G(s) = \dfrac{V_C(s)}{V_i(s)}$ for the series RLC circuit shown below.



Given: As per diagram

Solution:

Laplace Domain Representation

The resistor stays as R The inductor becomes sL The capacitor becomes $\dfrac{1}{sC}$

$$G(s) = \frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

Plug in Component Values

$$R = 100, \quad L = 0.01, \quad C = 1 \times 10^{-6}$$

Substitute:

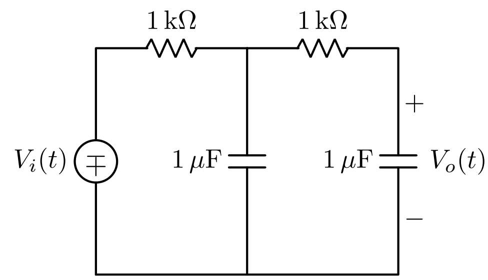
$$G(s) = \frac{1/(s \cdot 10^{-6})}{100 + 0.01s + 1/(s \cdot 10^{-6})}$$

Multiply by $s \cdot 10^{-6}$:

$$G(s) = \frac{1}{10^{-6}s^2 + 10^{-4}s + 1} = \boxed{\frac{1}{10^{-8}s^2 + 10^{-4}s + 1}}$$

Problem 15

Find: The transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$ for this RC ladder network.



Given: Two identical stages of an RC low-pass filter, each with:

- $R = 1\,\text{k}\Omega = 1000\,\Omega$
- $C = 1\,\mu\text{F} = 1 \times 10^{-6}\,\text{F}$

Solution:
Step 1: Understand the System

$$H(s) = \frac{1}{RCs + 1}$$

Two identical RC filters are connected in series:

$$G(s) = H(s) \cdot H(s) = \left(\frac{1}{RCs + 1}\right)^2$$

Step 2: Substitute Values
Given: - $R = 1000\,\Omega$ - $C = 1 \times 10^{-6}\,F$

$$RC = 1000 \times 1 \times 10^{-6} = 0.001$$

So:

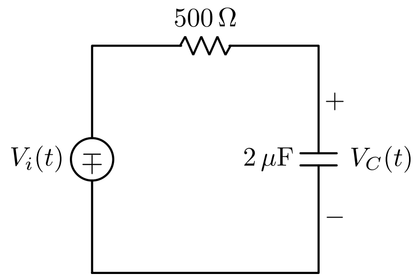
$$G(s) = \left(\frac{1}{0.001s + 1}\right)^2 = \frac{1}{(0.001s + 1)^2}$$

—
Final Answer:

$$\boxed{G(s) = \frac{1}{(0.001s + 1)^2}}$$

Problem 16

Find: $G(s) = \dfrac{V_C(s)}{V_i(s)}$ for this series RC circuit.



Given: As per diagram

Solution:

This is a basic voltage divider in the Laplace domain:

$$G(s) = \frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{sRC + 1}$$

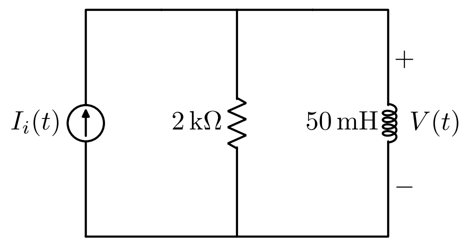
Substitute:

$$RC = 500 \times 2 \times 10^{-6} = 0.001$$

$$\boxed{G(s) = \frac{1}{0.001s + 1}}$$

Problem 17

Find: $G(s) = \dfrac{V(s)}{I_i(s)}$ for this parallel RL circuit.



Given: As per diagram

Solution:

This is a voltage across a parallel RL network:

$$G(s) = \frac{V(s)}{I_i(s)} = \left(\frac{1}{\frac{1}{R} + \frac{1}{sL}}\right) = \frac{RsL}{R + sL}$$

Substitute:

$$R = 2000, \quad L = 0.05$$

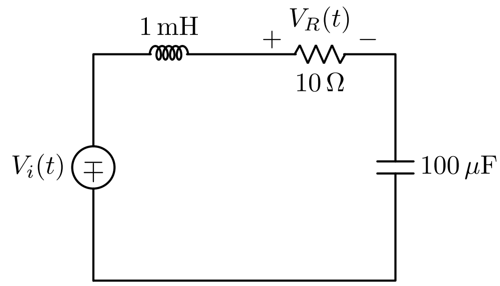
$$G(s) = \frac{2000 \cdot 0.05s}{2000 + 0.05s} = \frac{100s}{2000 + 0.05s}$$

Multiply numerator and denominator by 20 to simplify:

$$\boxed{G(s) = \frac{2000s}{s + 40000}}$$

Problem 18

Find: $G(s) = \dfrac{V_R(s)}{V_i(s)}$ for this series RLC band-pass filter.



Given: As per diagram

Solution:

This is a **band-pass filter**, and the voltage across the resistor is:

$$G(s) = \frac{V_R(s)}{V_i(s)} = \frac{R}{sL + R + \frac{1}{sC}}$$

Substitute:

$$L = 0.001, \quad R = 10, \quad C = 100 \times 10^{-6}$$

$$\frac{1}{sC} = \frac{1}{100 \times 10^{-6}} = \frac{1}{10^{-4}s}$$

Then:

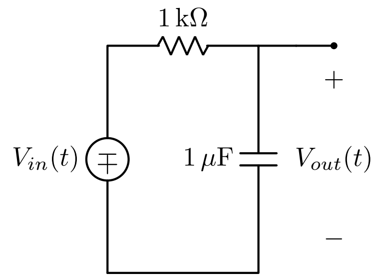
$$G(s) = \frac{10}{0.001s + 10 + \frac{1}{0.0001s}} = \frac{10}{0.001s + 10 + \frac{1}{0.0001s}}$$

Multiply numerator and denominator by $0.0001s$:

$$G(s) = \frac{0.001s}{10^{-8}s^2 + 0.001s + 1}$$

$$\boxed{G(s) = \frac{0.001s}{10^{-8}s^2 + 0.001s + 1}}$$

Problem 19
Find: $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$ for this RC low-pass filter.



Given: As per diagram

Solution:

Impedance of the capacitor:

$$Z_C = \frac{1}{sC}$$

Total impedance in series:

$$Z_{\text{total}} = R + \frac{1}{sC}$$

By the voltage divider rule:

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{Z_C}{R + Z_C} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{sRC + 1}$$

Substitute:

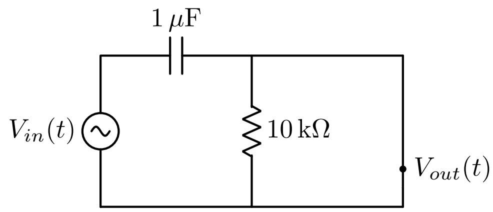
$$R = 1000, \quad C = 1 \times 10^{-6} \Rightarrow RC = 0.001$$

Final answer:

$$\boxed{G(s) = \frac{1}{0.001s + 1}}$$

Problem 20

Find: $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$ for this CR high-pass filter.



Given: As per diagram

Solution:

The impedance of the capacitor is:

$$Z_C = \frac{1}{sC}$$

Using voltage divider rule:

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{sRC + 1}$$

Substitute values:

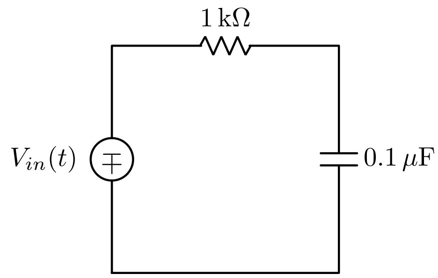
$$R = 10^4, \quad C = 1 \times 10^{-6} \Rightarrow RC = 0.01$$

Final answer:

$$\boxed{G(s) = \frac{0.01s}{0.01s + 1}}$$

Problem 21

Find: $G(s) = \dfrac{V_R(s)}{V_{in}(s)}$ for this series RC circuit (output across resistor)



Given: As per diagram

Solution:

This is a **high-pass filter** where output is measured across the resistor.
Impedance of the capacitor:

$$Z_C = \frac{1}{sC}$$

Using the voltage divider rule:

$$G(s) = \frac{V_R(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{sRC + 1}$$

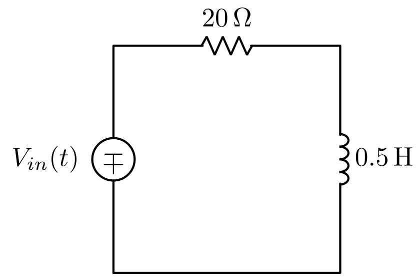
Substitute:

$$R = 1000, \quad C = 0.1 \times 10^{-6} = 1 \times 10^{-7}, \quad RC = 0.0001$$

Final answer:

$$\boxed{G(s) = \frac{0.0001s}{0.0001s + 1}}$$

Problem 22
Find: $G(s) = \dfrac{V_R(s)}{V_{in}(s)}$ for this series RL circuit (output across resistor)



Given: As per diagram

Solution:

This is a **high-pass filter**, and output is measured across the resistor.
Impedance of the inductor:

$$Z_L = sL$$

Using voltage division:

$$G(s) = \frac{V_R(s)}{V_{in}(s)} = \frac{R}{R + sL}$$

Substitute the given values:

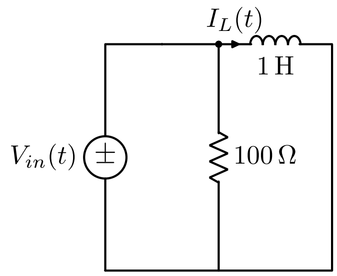
$$R = 20, \quad L = 0.5$$

Final answer:

$$\boxed{G(s) = \frac{20}{20 + 0.5s}}$$

Problem 23

Find: $G(s) = \dfrac{I_L(s)}{V_{in}(s)}$ for a parallel RL circuit



Given: As per diagram

Solution:

For the parallel RL circuit:

1. Total impedance:

$$Z_{total}(s) = R \parallel sL = \frac{R \cdot sL}{R + sL} = \frac{100s}{100 + s}$$

2. Current through inductor:

$$I_L(s) = \frac{V_{in}(s)}{sL} = \frac{V_{in}(s)}{s}$$

3. Transfer function derivation:

$$G(s) = \frac{I_L(s)}{V_{in}(s)} = \frac{1}{sL} \cdot \frac{R}{R + sL} = \frac{1}{s} \cdot \frac{100}{100 + s}$$

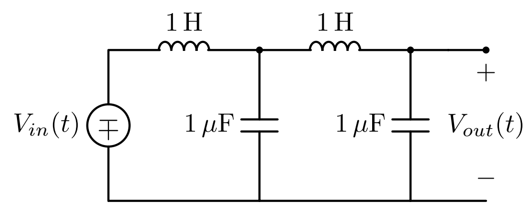
4. Final transfer function:

$$G(s) = \frac{100}{s(100 + s)} = \frac{100}{s^2 + 100s}$$

$$\boxed{G(s) = \frac{100}{s^2 + 100s}}$$

Problem 24

Find: $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$ for this 2-stage LC ladder network.



Given: As per diagram

Solution:

Using mesh analysis for the 2-stage LC ladder:

Mesh 1 Analysis

$$V_{in} = sL_1 I_1 + \frac{1}{sC_1}(I_1 - I_2)$$

Substituting component values:

$$V_{in} = sI_1 + \frac{10^6}{s}(I_1 - I_2)$$

Mesh 2 Analysis

$$0 = \frac{1}{sC_1}(I_2 - I_1) + sL_2 I_2 + \frac{1}{sC_2} I_2$$

Substituting component values:

$$0 = \frac{10^6}{s}(I_2 - I_1) + sI_2 + \frac{10^6}{s} I_2$$

System of Equations

Rewriting in matrix form:

$$V_{in} = \left(s + \frac{10^6}{s}\right) I_1 - \frac{10^6}{s} I_2$$

$$0 = -\frac{10^6}{s} I_1 + \left(s + \frac{2 \times 10^6}{s}\right) I_2$$

Output Voltage

$$V_{out} = \frac{10^6}{s} I_2$$

Transfer Function Derivation

Solving the system for the transfer function:

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\left(\frac{10^6}{s}\right)^2}{\left(s + \frac{10^6}{s}\right)\left(s + \frac{2\times10^6}{s}\right) - \left(\frac{10^6}{s}\right)^2}$$

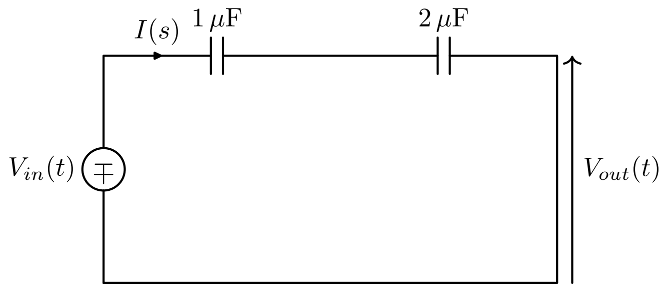
After simplification:

$$G(s) = \frac{10^{12}}{s^4 + 3 \times 10^6 s^2 + 10^{12}}$$

$$\boxed{G(s) = \frac{10^{12}}{s^4 + 3 \times 10^6 s^2 + 10^{12}}}$$

Problem 25

Find: $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$, where V_{out} is the voltage across C_2.



Given: As per diagram

Solution:

In the s-domain, the impedance of a capacitor is:

$$Z_C = \frac{1}{sC}$$

So the total impedance:

$$Z_{\text{eq}} = \frac{1}{sC_1} + \frac{1}{sC_2}$$

Using the voltage divider rule:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{sC_2}}{\frac{1}{sC_1} + \frac{1}{sC_2}} = \frac{C_1}{C_1 + C_2}$$

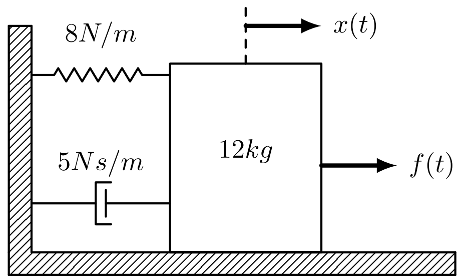
Substitute:

$$C_1 = 1\,\mu\text{F}, \quad C_2 = 2\,\mu\text{F} \Rightarrow \frac{C_1}{C_1 + C_2} = \frac{1}{3}$$

$$\boxed{\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{3}}$$

Problem 26

Find the transfer function $\dfrac{X(s)}{F(s)}$



Given: As per diagram

Solution:

Equation of Motion:

$$M\frac{d^2x(t)}{dt^2} + fv\frac{dx(t)}{dt} + Kx(t) = f(t)$$

Substitute the Given:

$$12\frac{d^2x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 8x(t) = f(t)$$

Laplace Transform:

(Consider that the Initial Condition is 0)

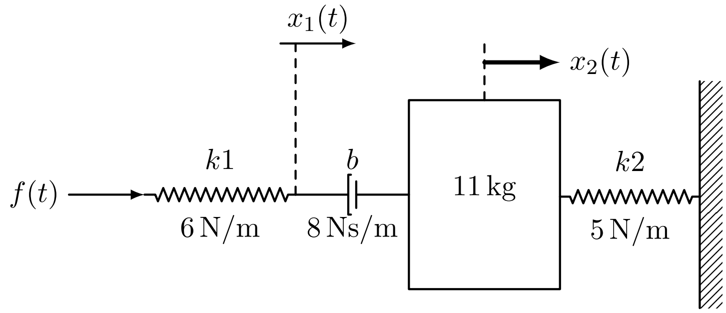
$$12s^2X(s) + 5sX(s) + 8x(s) = F(s)$$

Transfer function $\dfrac{X(s)}{F(s)}$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{12s^2 + 5s + 8}}$$

Problem 27

Find the transfer function $\dfrac{X_2(s)}{F(s)}$



Given: As per diagram

Solution:

Equation for $X_1(t)$:

$$m_1 x_1(t) = f(t) - k_1 x_1(t) - b[x_1(t) - x_2(t)]$$

Simplify:

$$11x_1(t) = f(t) - 6x_1(t) - 8[x_1(t) - x_2(t)]$$

Equation for $X_2(t)$:

$$m_2 x_2(t) = b[x_1(t) - x_2(t)] - k_2 x_2(t)$$

Simplify:

$$11x_2(t) = 8[x_1(t) - x_2(t)] - 5x_2(t)$$

Apply the Laplace Transform

For M1

$$11s^2 x_1(s) = f(s) - 6x_1(s) - 8[sx_1(s) - sx_2(s)]$$

Simplifying

$$11s^2 x_1(s) + 8sx_1(s) + 6x_1(s) - 8sx_2(s) = F(s)$$

This becomes

$$(11s^2 + 8s + 6)x_1(s) - 8sx_2(s) = F(s)$$

For M2

$$11sx_2(s) = 8[sx_1(s) - sx_2(s)] - 5x_2(s)$$

Simplifying

$$11sx_2(s) + 8sx_2(s) + 5sx_2(s) = 8sx_2(s)$$

This becomes

$$(11s^2 + 8s + 5)x_2(s) = 8sx_1(s)$$

Solve for $x_1(s)$ and $x_1(s)$

From the second equation for $x_2(s)$

$$x_1(s) = \frac{(11s^2 + 8s + 5)}{8s}x_2(s)$$

Substitute into the first equation

$$(11s^2 + 8s + 5)\frac{(11s^2 + 8s + 5)}{8s}x_2(s) - 8sx_2(s) = F(s)$$

Multiply out the terms

$$\frac{(11s^2 + 8s + 5)(11s^2 + 8s + 5)}{8s}x_2(s) - 8sx_2(s) = F(s)$$

Simplify the expression

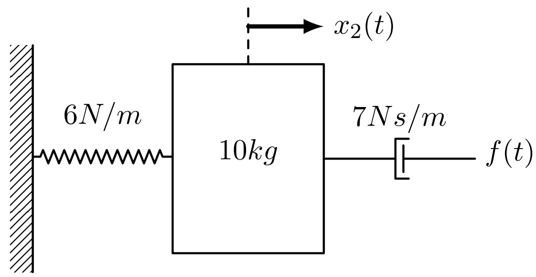
$$x_2(s)\left[\frac{(11s^2 + 8s + 5)(11s^2 + 8s + 5)}{8s} - 8s\right] = F(s)$$

Thus, the Transfer Function is

$$\boxed{G(s) = \frac{X_2(s)}{F(s)} = \frac{8s}{(11s^2 + 8s + 5)(11s^2 + 8s + 5) - 64s^2}}$$

Problem 28

Find the transfer function $\dfrac{X_2(s)}{F(s)}$



Given: As per diagram

Solution:

Formulate the Equation of Motion

$$m x_2(t) + c x_2(t) + k x_2(t) = f(t)$$

Substitute the Values

$$10 x_2(t) + 7 x_2(t) + 6 x_2(t) = f(t)$$

Take the Laplace Transform

$$10 s^2 x_2(s) + 7 s x_2(s) + 6 x_2(s) = f(s)$$

Solve for $\dfrac{X_2(s)}{F(s)}$

Rearrange the Equation

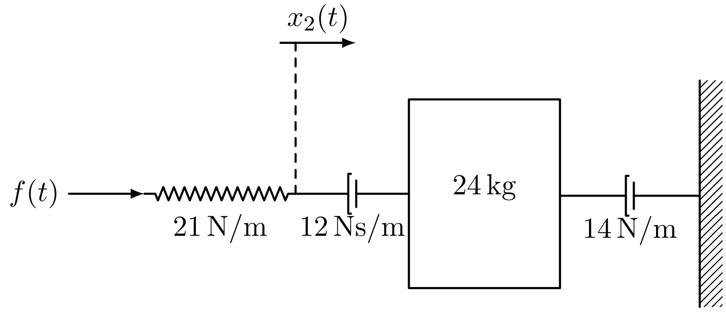
$$x_2(s)(10s^2 + 7s + 6) = F(s)$$

Transfer Function:

$$\boxed{G(s) = \frac{X_2(s)}{F(s)} = \frac{1}{10s^2 + 7s + 6}}$$

Problem 29

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Given: As per diagram

Solution:

Formulate the Equation of Motion

$$mx_2(t) = f(t) - c_1x_2(t) - kx_2(t) - c_2x_2(t)$$

Simplify the Equation

$$mx_2(t) + (c_1 + c_2)x_2(t) + kx_2(t) = f(t)$$

Substitute the Values

$$24x_2(t) + (12 + 4)x_2(t) + 21x_2(t) = f(t)$$

Simplify

$$24x_2(t) + 26x_2(t) + 21x_2(t) = f(t)$$

Take the Laplace Transform

$$24s^2x_2(s) + 26sx_2(s) + 21x_2(s) = f(s)$$

Solve for $\dfrac{X_2(s)}{F(s)}$

Rearrange the Equation

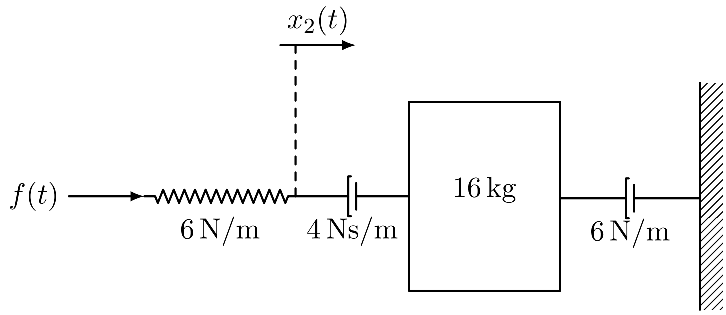
$$x_2(s)(24s^2 + 26s + 21) = F(s)$$

Transfer Function:

$$\boxed{G(s) = \frac{X_2(s)}{F(s)} = \frac{1}{24s^2 + 26s + 21}}$$

Problem 30

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Given: As per diagram

Solution:

Formulate the Equation of Motion

$$mx_2(t) = f(t) - c_1x_2(t) - kx_2(t) - c_2x_2(t)$$

Simplify the Equation

$$mx_2(t) + (c_1 + c_2)x_2(t) + kx_2(t) = f(t)$$

Substitute the Values

$$16x_2(t) + (4 + 6)x_2(t) + 6x_2(t) = f(t)$$

Simplify

$$16x_2(t) + 10x_2(t) + 6x_2(t) = f(t)$$

Take the Laplace Transform

$$16s^2x_2(s) + 10sx_2(s) + 6x_2(s) = f(s)$$

Solve for $\dfrac{X_2(s)}{F(s)}$

Rearrange the Equation

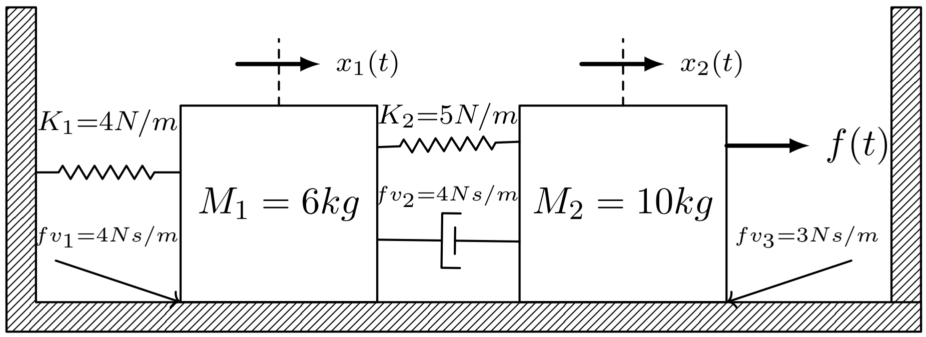
$$x_2(s)(16s^2 + 10s + 6) = F(s)$$

Transfer Function:

$$\boxed{G(s) = \frac{X_2(s)}{F(s)} = \frac{1}{16s^2 + 10s + 6}}$$

Problem 31

Find the transfer function $G(s) = \dfrac{X_1(s)}{F(s)}$



Given: As per diagram

Solution:

Derive the Equations of Motion:

Equation of Motion for M1:

$$M_1\ddot{x}_1(t) = -f_{v1}\dot{x}_1(t) - k_1x_1(t) + f_{v2}[\dot{x}_2(t) - \dot{x}_1(t)] + k_2[x_2(t) - x_1(t)]$$

Substitute the Values:

$$6\ddot{x}_1(t) = -4\dot{x}_1(t) - 4x_1(t) + 4[\dot{x}_2(t) - \dot{x}_1(t)] + 5[x_2(t) - x_1(t)]$$

Simplifying:

$$6\ddot{x}_1(t) = -(4+5)x_1(t) + 5x_2(t) - (4+4)\dot{x}_1(t) + 4\dot{x}_2(t)$$

$$6\ddot{x}_1(t) = -9x_1(t) + 5x_2(t) - 8\dot{x}_1(t) + 4\dot{x}_2(t)$$

Equation of Motion for M2:

$$M_2\ddot{x}_2(t) = k_2[x_1(t) - x_2(t)] + f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] - f_{v3}\dot{x}_2(t) + F(t)$$

Substituting the Given Values:

$$10\ddot{x}_2(t) = 5[x_1(t) - x_2(t)] + 4[\dot{x}_1(t) - \dot{x}_2(t)] - 3\dot{x}_2(t) + F(t)$$

Simplifying:

$$10\ddot{x}_2(t) = 5x_1(t) - 5x_2(t) + 4\dot{x}_1(t) - (4+3)\dot{x}_2(t) + F(t)$$

$$10\ddot{x}_2(t) = 5x_1(t) - 5x_2(t) + 4\dot{x}_1(t) - 7\dot{x}_2(t) + F(t)$$

Laplace Transform:

For M1:

$$6s^2X_1(s) = -9X_1(s) + 5X_2(s) - 8sX_1(s) + 4sX_2(s)$$

Simplifying:

$$(6s^2 + 8s + 9)X_1(s) = (5 + 4s)X_2(s)$$

For M2:

$$10s^2X_2(s) = 5X_1(s) - 5X_2(s) + 4sX_1(s) - 7sX_2(s) + F(s)$$

Simplifying:

$$(10s^2 + 7s + 5)X_2(s) = (5 + 4s)X_1(s) + F(s)$$

Solving for the Transfer Function:

Substitute $X_2(s)$ from the Second Equation into the First Equation:

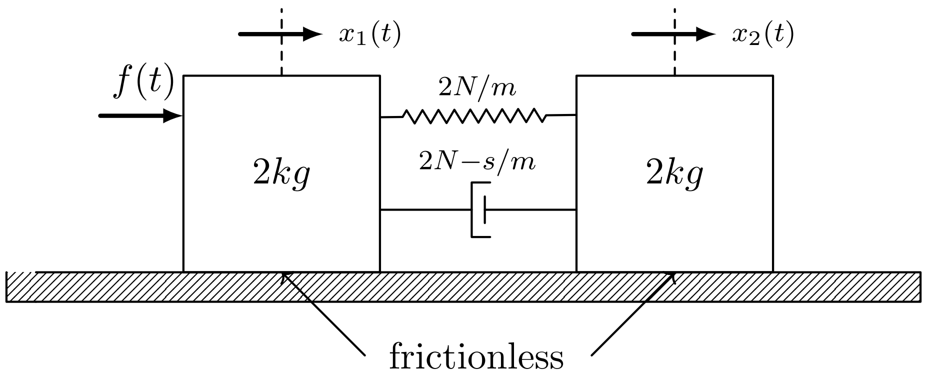
$$(6s^2 + 8s + 9)X_1(s) = (5 + 4s)\left[\frac{(5 + 4s)X_1(s) + F(s)}{10s^2 + 7s + 5}\right]$$

Final Transfer Function:

$$\boxed{\frac{X_1(s)}{F(s)} = \frac{5 + 4s}{(6s^2 + 8s + 9)(10s^2 + 7s + 5) - (5 + 4s)^2}}$$

Problem 32

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Given: As per diagram

Solution:

Equation for Mass 1:

Using Newton's second law:

$$M_1\ddot{x}_1(t) = f(t) - B[\dot{x}_1(t) - \dot{x}_2(t)] - K[x_1(t) - x_2(t)]$$

Taking the Laplace Transform (with zero initial conditions):

$$M_1 s^2 X_1(s) = F(s) - Bs[X_1(s) - X_2(s)] - K[X_1(s) - X_2(s)]$$

Substitute the Values:

$$2s^2 X_1(s) = F(s) - 2s[X_1(s) - X_2(s)] - 2[X_1(s) - X_2(s)]$$

$$2s^2 X_1(s) = F(s) - (2s+2)[X_1(s) - X_2(s)]$$

Equation for Mass 2:

Using Newton's second law:

$$M_2\ddot{x}_2(t) = B[\dot{x}_1(t) - \dot{x}_2(t)] - K[x_1(t) - x_2(t)]$$

Taking the Laplace Transform (with zero initial conditions):

$$M_2 s^2 X_2(s) = Bs[X_1(s) - X_2(s)] - K[X_1(s) - X_2(s)]$$

Substitute the Values:

$$2s^2X_2(s) = 2s[X_1(s) - X_2(s)] + 2[X_1(s) - X_2(s)]$$

$$2s^2X_2(s) = (2s+2)[X_1(s) - X_2(s)]$$

Solve the System of Equations:

$$X_1(s) - X_2(s) = \frac{2s^2X_2(s)}{2s+2}$$

$$X_1(s) = X_2(s) + \frac{s^2X_2(s)}{s+1}$$

$$X_1(s) = \frac{X_2(s)(s^2+s+1)}{s+1}$$

Substitute into the First Equation:

Substitute this Expression for $X_1(s)$ into the First Equation:

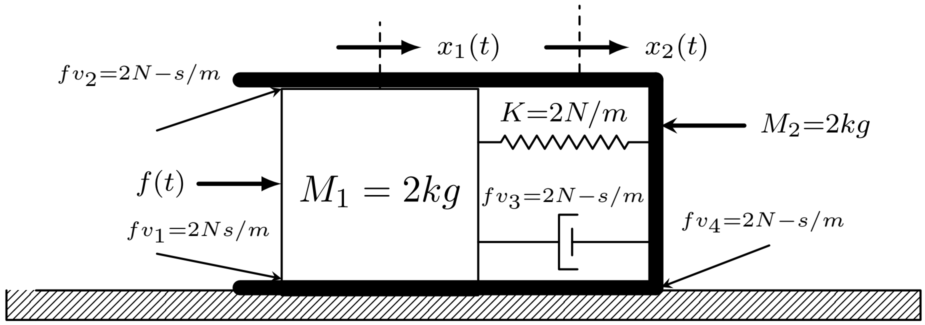
$$2s^2\frac{X_2(s)(s^2+s+1)}{s+1} = F(s) - (2s+2)\left[\frac{X_2(s)(s^2+s+1)}{s+1} - X_2(s)\right]$$

Final Transfer Function:

$$\boxed{G(s) = \frac{s+1}{2s^4+4s^3+4s^2+2s}}$$

Problem 33

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$ for the translational mechanical system shown in the figure



Given: As per diagram

Solution:

Equations of motion for each mass in the Laplace Domain:

For M_1:

$$M_1\ddot{x}_1(t) = F(t) - f_{v1}\dot{x}_1(t) - f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] - K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_1 s^2 X_1(s) = F(s) - f_{v1}sX_1(s) - f_{v2}[X_1(s) - X_2(s)] - K[X_1(s) - X_2(s)]$$

Substitute the values:

$$2s^2X_1(s) = F(s) - 2sX_1(s) - 2s[X_1(s) - X_2(s)] - 2[X_1(s) - X_2(s)]$$

Simplify the terms:

$$2s^2X_1(s) = F(s) - 4sX_1(s) + 2sX_2(s) - 2X_1(s) + 2X_2(s)$$

$$2s^2X_1(s) = F(s) - (2s+2)X_1(s) + (2s+2)X_2(s)$$

Rearrange to:

$$(2s^2 + 2s + 2)X_1(s) - (2s+2)X_2(s) = F(s)$$

For M_2:

$$M_2\ddot{x}_2(t) = -f_{v3}\dot{x}_2(t) + f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] + K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_2 s X_2(s) = -f_{v3} s X_2(s) + f_{v2}[X_1(s) - X_2(s)] + K[X_1(s) - X_2(s)]$$

Substitute the terms:

$$2s^2 X_2(s) = -2sX_2(s) + 2s[X_1(s) - sX_2(s)] + 2[X_1(s) - X_2(s)]$$

Simplify the terms:

$$2s^2 X_2(s) = -2sX_2(s) + 2sX_1(s) - 2sX_2(s) + 2X_1(s) - 2X_2(s)$$

$$2s^2 X_2(s) = 2sX_1(s) + 2X_1(s) - (2s+2)X_2(s)$$

Rearrange to:

$$(2s^2 + 2s + 2)X_2(s) = (2s+2)X_1(s)$$

Solving the System of Equations

Solve equation (2) for $X_1(s)$:

$$X_1(s) = \frac{(2s^2 + 2s + 2)X_2(s)}{2s+2}$$

Substitute this into equation (1):

$$(2s^2 + 2s + 2)\left[\frac{(2s^2 + 2s + 2)X_2(s)}{2s+2}\right] - (2s+2)X_2(s) = F(s)$$

Multiply out:

$$\frac{(2s^2 + 2s + 2)^2 X_2(s)}{2s+2} - (2s+2)X_2(s) = F(s)$$

Multiply both sides by $2s + 2$ to eliminate the denominator:

$$(2s^2 + 2s + 2)^2 X_2(s) - (2s+2)^2 X_2(s) = (2s+2)F(s)$$

Factor out $X_2(s)$ on the left-hand side:

$$[(2s^2 + 2s + 2)^2 - (2s+2)^2]X_2(s) = (2s+2)F(s)$$

Simplify the terms:

$$(2s^2 + 2s + 2)^2 = 4s^4 + 8s^3 + 12s^2 + 8s + 4$$

$$(2s+2)^2 = 4s^2 + 8s + 4$$

Subtract the two:

$$(4s^4 + 8s^3 + 12s^2 + 8s + 4) - (4s^2 + 8s + 4) = 4s^4 + 8s^3 + 8s^2$$

So:

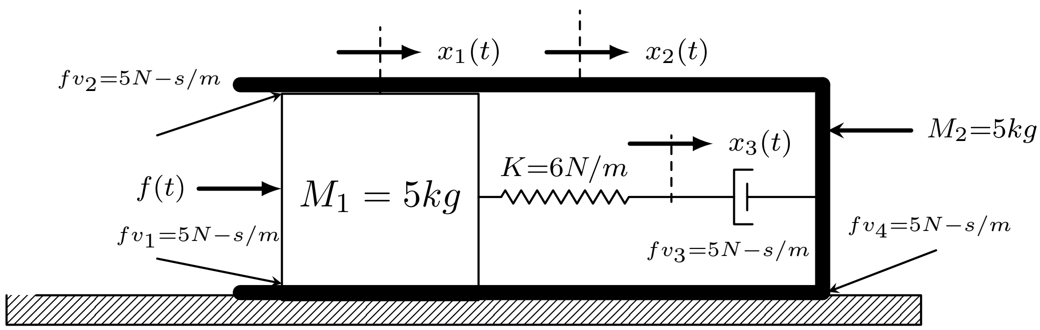
$$(4s^4 + 8s^3 + 8s^2)X_2(s) = (2s + 2)F(s)$$

Final Transfer Function:

$$\boxed{G(s) = \frac{2s + 2}{4s^4 + 8s^3 + 8s^2}}$$

Problem 34

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$ for the translational mechanical system shown in the figure



Given: As per diagram

Solution:

Define the Forces Acting on Each Mass:

For M_1:

$$M_1\ddot{x}_1(t) = F(t) - f_{v1}\dot{x}_1(t) - f_{v2}\dot{x}_2(t) - K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_1s^2X_1(s) = F(s) - f_{v1}sX_1(s) - f_{v2}sX_2(s) - K[X_1(s) - X_2(s)]$$

Substitute the values:

$$5s^2X_1(s) = F(s) - 5sX_1(s) - 5s[X_1(s) - X_2(s)] - 6[X_1(s) - X_2(s)]$$

Simplify the terms:

$$5s^2X_1(s) = F(s) - 10sX_1(s) - 6X_1(s) + 6X_2(s)$$

For M_2:

$$M_2\ddot{x}_2(t) = -f_{v3}\dot{x}_2(t) - f_{v4}\dot{x}_2(t) + K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_2sX_2(s) = -f_{v3}sX_2(s) - f_{v2}sX_2 + K[X_1(s) - X_2(s)]$$

Substitute the terms:

$$5s^2X_2(s) = -5sX_2(s) - 5sX_2(s) - 5sX_2 + 6[X_1(s) - X_2(s)]$$

Simplify the terms:

$$5s^2X_2(s) = 6X_1(s) - (6 + 15s)X_2(s)$$

Solve the Equations:

Solve equation (2) for $X_1(s)$:

$$X_1(s) = \frac{5s^2X_2(s) + (6 + 15s)X_2(s)}{6}$$

$$X_1(s) = \frac{X_2(s)(5s^2 + 15s + 6)}{6}$$

Substitute $X_1(s)$ into equation (1):

$$\frac{5s^2X_2(s)(5s^2 + 15s + 6)}{6} = F(s) - 10s\frac{X_2(s)(5s^2 + 15s + 6)}{6} - 6\frac{X_2(s)(5s^2 + 15s + 6)}{6} + 6X_2(s)$$

Simplifying and solving the algebra, the resulting transfer function $G(s) = \frac{X_2(s)}{F(s)}$ will involve terms of the form:

$$G(s) = \frac{6}{D(s)}$$

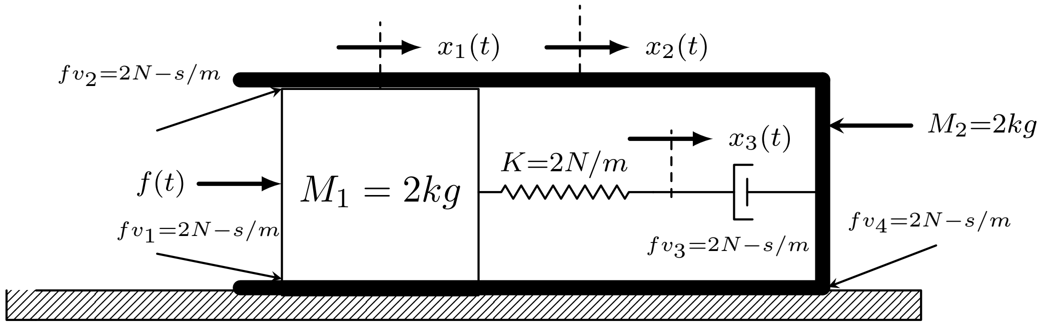
Where $D(s)$ is a polynomial depending on s.

Final Transfer Function:

$$\boxed{G(s) = \frac{6}{25s^4 + 125s^3 + 225s^2 + 195s + 72}}$$

Problem 35

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Given: As per diagram

Solution:

Define the Forces Acting on Each Mass:

For M_1:

$$M_1\ddot{x}_1(t) = F(t) - f_{v1}\dot{x}_1(t) - f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] - K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_1 s^2 X_1(s) = F(s) - f_{v1} s X_1(s) - f_{v2}[X_1(s) - X_2(s)] - K[X_1(s) - X_2(s)]$$

Substitute the values:

$$2s^2 X_1(s) = F(s) - 2sX_1(s) - 2s[X_1(s) - X_2(s)] - 2[X_1(s) - X_2(s)]$$

Simplify the terms:

$$2s^2 X_1(s) = F(s) - 4sX_1(s) + 2sX_2(s) - 2X_1(s) + 2X_2(s)$$

$$2s^2 X_1(s) = F(s) - (2s+2)X_1(s) + (2s+2)X_2(s)$$

For M_2:

$$M_2\ddot{x}_2(t) = -f_{v3}\dot{x}_2(t) + f_{v2}[\dot{x}_1(t) - \dot{x}_2(t)] + K[x_1(t) - x_2(t)]$$

In Laplace Domain:

$$M_2 s X_2(s) = -f_{v3} s X_2(s) + f_{v2}[X_1(s) - X_2(s)] + K[X_1(s) - X_2(s)]$$

Substitute the terms:

$$2s^2X_2(s) = -2sX_2(s) + 2s[X_1(s) - sX_2(s)] + 2[X_1(s) - X_2(s)]$$

Simplify the terms:

$$2s^2X_2(s) = -2sX_2(s) + 2sX_1(s) - 2sX_2(s) + 2X_1(s) - 2X_2(s)$$

$$2s^2X_2(s) = 2sX_1(s) + 2X_1(s) - (2s+2)X_2(s)$$

Solving the System of Equations

Solve equation (2) for $X_1(s)$:

$$2s^2X_2(s) + (2s+2)X_2(s) = (2s+2)X_1(s)$$

$$X_1(s) = \frac{2s^2X_2(s) + (2s+2)X_2(s)}{2s+2}$$

$$X_1(s) = \frac{X_2(s)(2s^2+2s+2)}{2s+2}$$

Substitute $X_1(s)$ into equation (1):

$$2s^2\frac{X_2(s)(2s^2+2s+2)}{2s+2} = F(s) - (2s+2)\frac{X_2(s)(2s^2+2s+2)}{2s+2} + (2s+2)X_2(s)$$

Cancel $(2s+2)$ from some terms:

$$2s^2(2s^2+2s+2)X_2(s) = F(s) - (2s^2+2s+2)X_2(s) + (2s+2)X_2(s)$$

Simplify Further:

$$(4s^4+4s^3+4s^2+2s^2+2s+2)X_2(s) = F(s)$$

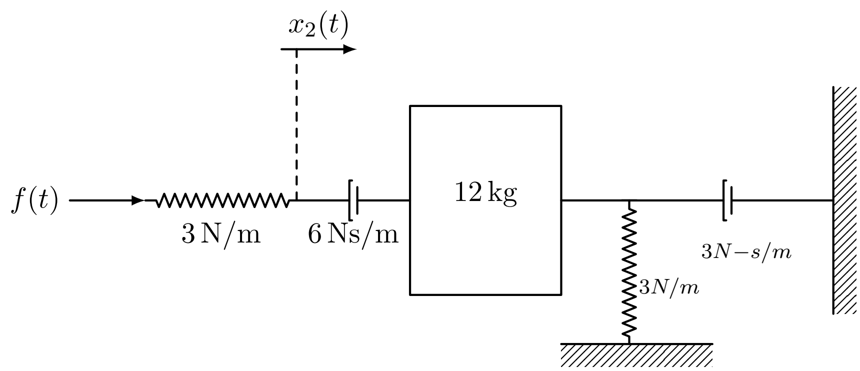
$$(4s^4+4s^3+6s^2+2s+2)X_2(s) = F(s)$$

Final Transfer Function:

$$\boxed{G(s) = \frac{1}{4s^4+4s^3+6s^2+2s+2}}$$

Problem 36

Find the transfer function $G(s) = \dfrac{X(s)}{F(s)}$ of the given mechanical system using translational mechanical system



Given: As per diagram

Solution:

Force Balance on M:

$$M\ddot{x}(t) + (b_1 + b_2)\dot{x}(t) + (k_1 + k_2)x(t) = f(t)$$

Laplace Transform:

$$Ms^2X(s) + (b_1 + b_2)sX(s) + (k_1 + k_2)X(s) = F(s)$$

Substitute the Given Values:

$$12s^2X(s) + (6 + 3)sX(s) + (3 + 3)X(s) = F(s)$$

Simplify:

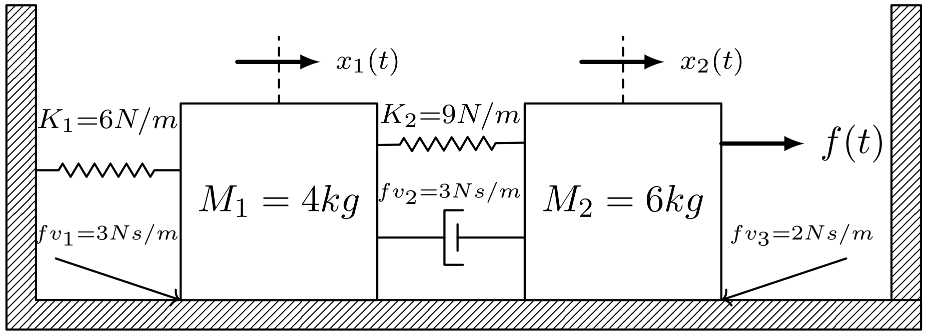
$$12s^2X(s) + 9sX(s) + 6X(s) = F(s)$$

Transfer function solving for $\dfrac{X(s)}{F(s)}$:

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{12s^2 + 9s + 6}}$$

Problem 37

Find the transfer function, $\dfrac{X_1(s)}{F(s)}$



Given: As per diagram

Solution:

Equation of Motion:

For M_1:

$$M_1\ddot{x}_1(t) = F_{v1}\dot{x}_1(t) - k_1x_1(t) + F_{v2}[\dot{x}_2(t) - \dot{x}_1(t)] + k_2[x_2(t) - x_1(t)]$$

Substitute :

$$4\ddot{x}_1(t) = -3\dot{x}_1(t) - 6x_1(t) + 3[\dot{x}_2(t) - \dot{x}_1(t)] + 9[x_2(t) - x_1(t)]$$

Simplify:

$$4\ddot{x}_1(t) = -6\dot{x}_1(t) - 15x_1(t) + 3\dot{x}_2(t) + 9x_2(t)$$

For M_2:

$$M_2\ddot{x}_2(t) = -F_{v2}[\dot{x}_2(t) - \dot{x}_1(t)] - k_2[x_2(t) - x_1(t)] - f_{v3}\dot{x}_2(t) + F(t)$$

Substitute :

$$6\ddot{x}_2(t) = -3[\dot{x}_2(t) - \dot{x}_1(t)] - 9[x_2(t) - x_1(t)] - 2\dot{x}_2(t) + F(t)$$

Simplify:

$$6\ddot{x}_2(t) = -5\dot{x}_2(t) + 3\dot{x}_1(t) - 9x_2(t) + 9x_1(t) + F(t)$$

Apply the Laplace Transform for M_1 and M_2

For M_1:

$$(4s^2 + 6s + 15)X_1(s) = (3s + 9)X_2(s)$$

For M_2

$$(6s^2 + 5s + 9)X_2(s) = (3s + 9)X_1(s) + F(s)$$

Solve for $G(s) = \dfrac{X_1(s)}{F(s)}$:

$$X_2(s) = \frac{(3s + 9)X_1(s) + F(s)}{6s^2 + 5s + 9}$$

Substitute $X_2(s)$ to $X_1(s)$

$$(4s^2 + 6s + 15)X_1(s) = \frac{(3s + 9)^2 X_1(s) + (3s + 9)F(s)}{6s^2 + 5s + 9}$$

$$(4s^2 + 6s + 15)(6s^2 + 5s + 9)X_1(s) = (9s^2 + 54s + 81)X_1(s) + (3s + 9)F(s)$$

Simplify:

$$(24s^4 + 44s^3 + 111s^2 + 135s + 135)X_1(s) = (9s^2 + 54s + 81)X_1(s) + (3s + 9)F(s)$$

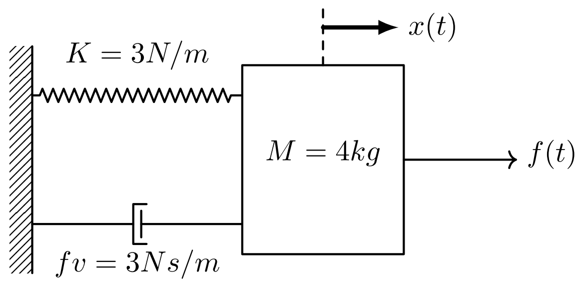
$$(24s^4 + 44s^3 + 102s^2 + 81s + 54)X_1(s) = (3s + 9)F(s)$$

Thus $G(s) = \dfrac{X_1(s)}{F(s)}$

$$\boxed{G(s) = \frac{3s + 9}{24s^4 + 44s^3 + 102s^2 + 81s + 54}}$$

Problem 38

Find the transfer function, $\frac{X(s)}{F(s)}$



Given: As per diagram

Solution:

Equation of Motion:

$$M\frac{d^2x(t)}{dt^2} + fv\frac{dx(t)}{dt} + Kx(t) = f(t)$$

Substitute the Given:

$$4\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 3x(t) = f(t)$$

Laplace Transform:

(Consider that the Initial Conditionis 0)

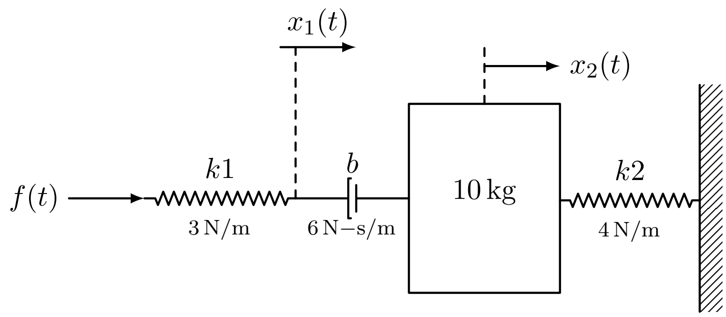
$$4s^2X(s) + 3sX(s) + 3X(s) = F(s)$$

Transfer function $\frac{X(s)}{F(s)}$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{4s^2 + 3s + 3}}$$

Problem 39

Find the transfer function $\dfrac{X_2(s)}{F(s)}$



Given: As per diagram

Solution:

Equation for $X_1(t)$:

$$F(t) - k_1 x_1(t) - b[\dot{x}_1(t) - \dot{x}_2(t)] = 0$$

Apply the Laplace Transform:

$$F(s) - k_1 X_1(s) - b[sX_1(s) - sX_2(s)] = 0$$

Simplify:

$$F(s) = (k_1 + bs)X_1(s) - bsX_2(s)$$

Equation for $X_2(t)$:

$$b[\dot{x}_1(t) - \dot{x}_2(t)] + k_2 x_2(t) = M\ddot{x}_2(t)$$

Apply Laplace Transform:

$$b[sX_1(s) - sX_2(s)] + k_2 X_2(s) = Ms^2 X_2(s)$$

Simplify:

$$bsX_1(s) - (bs + k_2)X_2(s) = Ms^2 X_2(s)$$

Solving for $\dfrac{X_2(s)}{F(s)}$:

$X_1(s)$:

$$X_1(s) = \frac{F(s) + bsX_2(s)}{k_1 + bs}$$

Substitute $X_1(s)$ to 2^{nd} Equation:

$$b\left(s\frac{F(s) + bsX_2(s)}{k_1 + bs}\right) - (bs + k_2)X_2(s) = Ms^2X_2(s)$$

Simplify:

$$\frac{bsF(s)}{k_1 + bs} + \frac{b^2s^2X_2(s)}{k_1 + bs} - (bs + k_2)X_2(s) = Ms^2X_2(s)$$

Multiply Both Sides By $(k_1 + bs)$:

$$bsF(s) + b^2s^2X_2(s) - (bs + k_2)(k_1 + bs)X_2(s) = Ms^2(k_1 + bs)X_2(s)$$

Collecting Terms:

$$bsF(s) = [Ms^2(k_1 + bs) + (bs + k_2)(k_1 + bs) - b^2s^2]X_2(s)$$

Hence:

$$\frac{X_2(s)}{F(s)} = \frac{bs}{Ms^2(k_1 + bs) + (bs + k_2)(k_1 + bs) - b^2s^2}$$

Substitute the Given Values:

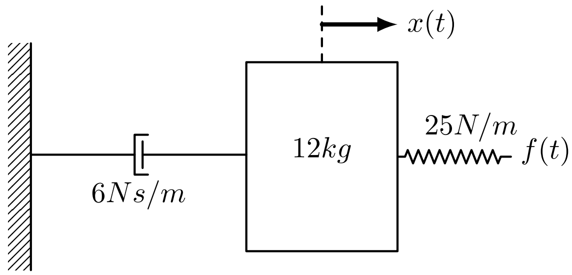
$$\frac{X_2(s)}{F(s)} = \frac{6s}{10s^2(3 + 6s) + (6s + 4)(3 + 6s) - 36s^2}$$

Or

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{6s}{60s^3 + 30s^2 + (36s^2 + 42s + 12) - 36s^2}}$$

Problem 40

Find the transfer function, $G(s) = \dfrac{X_1(s)}{F(s)}$ for the translational mechanical system shown in Figure



Given: As per diagram

Solution:

Equation of Motion:

$$M\ddot{x}(t) = F(t) - F_s - F_d$$

Substitute the Given Values:

$$12\ddot{x}(t) = F(t) - 25x(t) - 6\dot{x}(t)$$

Apply the Laplace Transform (zero initial conditions):

$$12s^2X(s) = F(s) - 25X(s) - 6sX(s)$$

Rearrange:

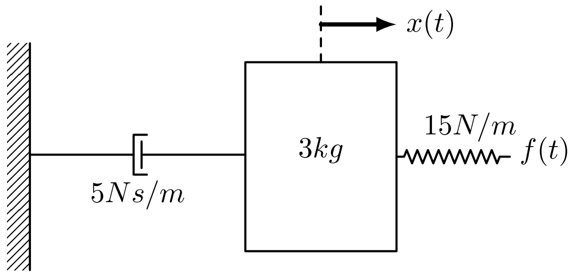
$$X(s)(12s^2 + 6s + 25) = F(s)$$

Final Transfer Function:

$$\boxed{G(s) = \frac{1}{12s^2 + 6s + 25}}$$

Problem 41

Find the transfer function, $G(s) = \dfrac{X_1(s)}{F(s)}$ for the translational mechanical system shown in Figure



Given: As per diagram

Solution:

Equation of Motion:

$$M\ddot{x}(t) = F(t) - F_s - F_d$$

Substitute the Given Values:

$$3\ddot{x}(t) = F(t) - 15x(t) - 5\dot{x}(t)$$

Apply the Laplace Transform (zero initial conditions):

$$3s^2X(s) = F(s) - 15X(s) - 5sX(s)$$

Rearrange:

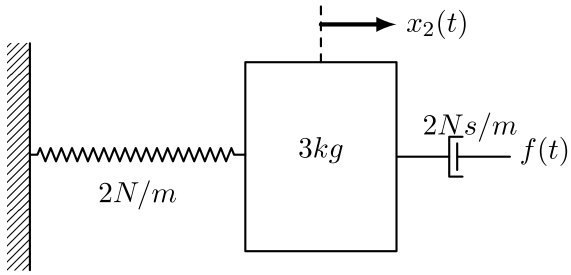
$$X(s)(3s^2 + 5s + 15) = F(s)$$

Final Transfer Function:

$$\boxed{G(s) = \frac{1}{3s^2 + 5s + 15}}$$

Problem 42

For the translational mechanical system shown below, find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Given: As per diagram

Solution:

Equation of Motion:

$$M\ddot{x}_2(t) = F(t) - F_s - F_d$$

Apply Laplace Transform:

$$3s^2X_2(s) = F(s) - 2X_2(s) - 2sX_2(s)$$

Simplify:

$$3s^2X_2(s) + 2sX_2(s) + 2X_2(s) = F(s)$$

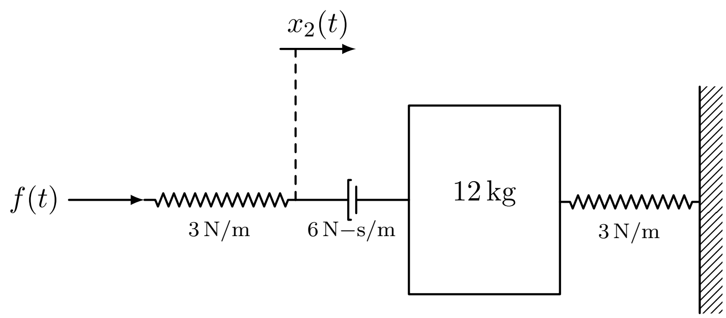
$$X_2(s)(3s^2 + 2s + 2) = F(s)$$

Thus $G(s) = \dfrac{X_2(s)}{F(s)}$

$$\boxed{G(s) = \frac{1}{3s^2 + 2s + 2}}$$

Problem 43

Find the transfer function $G(s) = \dfrac{X_2(s)}{F(s)}$



Given: As per diagram

Solution:

For Mass M:

$$M\ddot{x}_2(t) = F(t) - F_{s1} - F_d - F_{s2}$$

$$12\ddot{x}_2(t) = F(t) - 3[x_2(t) - x_1(t)] - 6\dot{x}_2(t) - 3x_2(t)$$

Simplify:

$$12\ddot{x}_2(t) + 6\dot{x}_2(t) + 6x_2(t) - 3x_1(t) = F(t)$$

Taking Laplace Transform:

$$12s^2X_2(s) + 6sX_2(s) + 6X_2(s) - 3X_1(s) = F(s)$$

Solve for $X_2(s)$:

$$X_2(s)(12s^2 + 6s + 6) - 3X_1(s) = F(s)$$

Relate $X_1(s)$ to $X_2(s)$:

$$3x_1(t) + 6\dot{x}_1(t) = F(t)$$

Laplace Transform:

$$3X_1(s) + 6sX_1(s) = F(s)$$

$$X_1(s)(3 + 6s) = F(s)$$

$$X_1(s) = \frac{F(s)}{3 + 6s}$$

Substitute $X_1(s)$ Back:

$$X_2(s)(12s^2 + 6s + 6) - 3\left(\frac{F(s)}{3 + 6s}\right) = F(s)$$

Solve for Transfer Function:

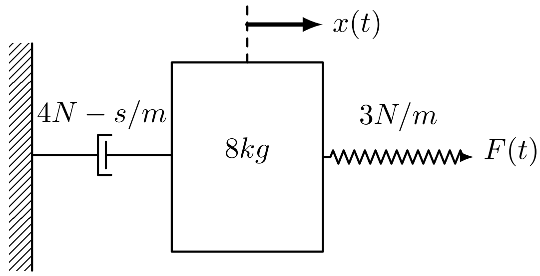
$$X_2(s) = \frac{F(s) + \left(\frac{3F(s)}{3+6s}\right)}{12s^2 + 6s + 6}$$

Thus $G(s) = \dfrac{X_2(s)}{F(s)}$

$$\boxed{G(s) = \frac{6s + 4}{(3 + 6s)(12s^2 + 6s + 6)}}$$

Problem 44

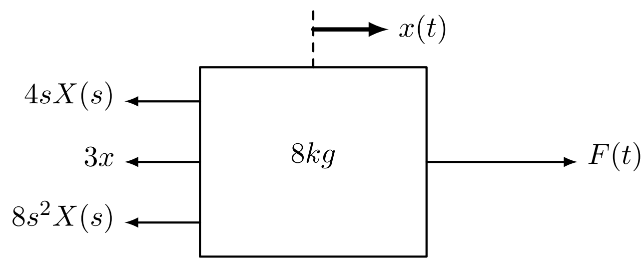
Determine the transfer function $\frac{X(s)}{F(s)}$ of the system below



Given: As per diagram

Solution:

Free Body Diagram:



Force Balance:

$$F(t) - 3x(t) - 4\dot{x}(t) - 8\ddot{x}(t) = 0$$

Laplace Transform:

$$F(s) - 3X(s) - 4sX(s) - 8s^2X(s) = 0$$

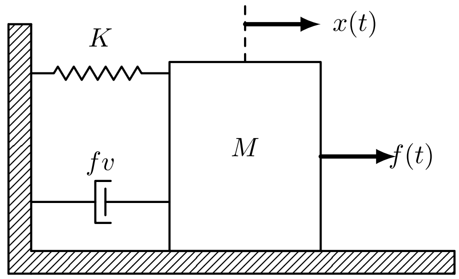
$$F(s) = (8s^2 + 4s + 3)X(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{8s^2 + 4s + 3}}$$

Problem 45

1. Determine the transfer function $\dfrac{X(s)}{F(s)}$ of the system below

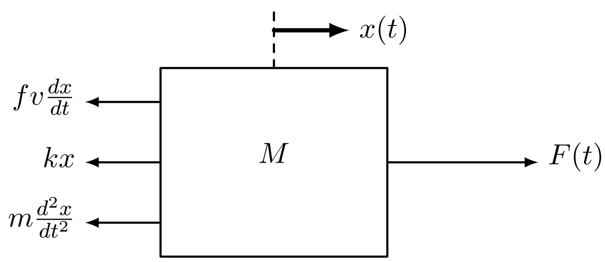
$K = 4Nm; fv = 5Nm/s; M = 2kg$



Given: As per diagram

Solution:

FBD of M



$$\sum F_x = 0 \quad \rightarrow \quad \oplus$$

$$f(t) - m\tfrac{d^2x}{dt^2} - fv\tfrac{dx}{dt} - kx = 0$$

$$F(s) - 2s^2X(s) - 5sX(s) - 4X(s) = 0$$

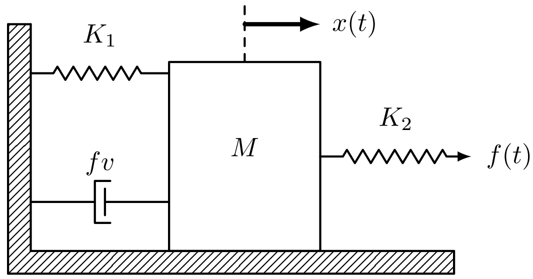
$$F(s) = (2s^2 + 5s + 4)X(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{2s^2 + 5s + 4}}$$

Problem 46

1. Determine the transfer function $\dfrac{X(s)}{F(s)}$ of the system below

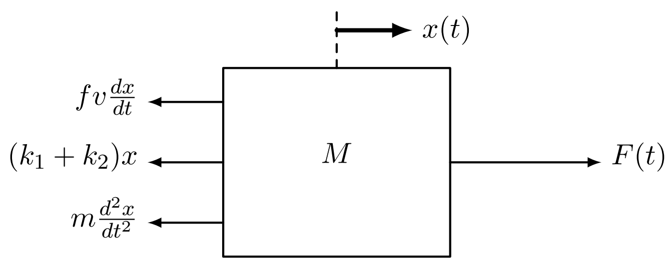
$K_1 = 4Nm; K_2 = 3Nm; fv = 2Nm/s; M = 1kg$



Given: As per diagram

Solution:

FBD of M



$$\sum F_x = 0 \quad \rightarrow \quad \oplus$$

$$f(t) - m\frac{d^2x}{dt^2} - fv\frac{dx}{dt} - (k_1 + k_2)x = 0$$

$$F(s) - s^2X(s) - 2sX(s) - (4+3)X(s) = 0$$

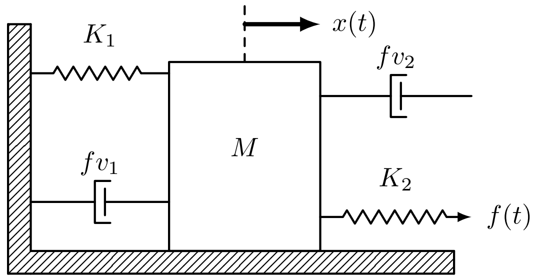
$$F(s) = (s^2 + 5s + 7)X(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{s^2 + 5s + 7}}$$

Problem 47

1. Determine the transfer function $\dfrac{X(s)}{F(s)}$ of the system below

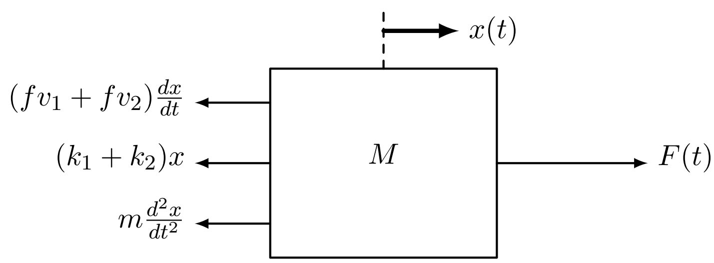
$K_1 = 2Nm; K_2 = 6Nm; fv_1 = 4Nm/s; fv_2 = 2Nm/s; M = 5kg$



Given: As per diagram

Solution:

FBD of M



$$\sum F_x = 0 \quad \rightarrow \quad \oplus$$

$$f(t) - m\frac{d^2x}{dt^2} - (fv_1 + fv_2)\frac{dx}{dt} - (k_1 + k_2)x = 0$$

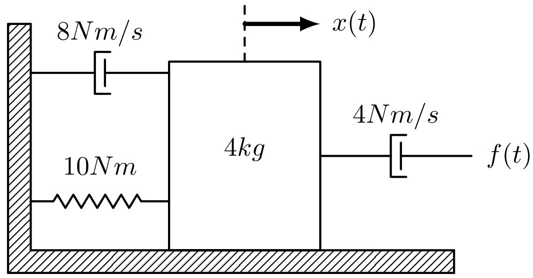
$$F(s) - 5s^2X(s) - (4+2)sX(s) - (2+6)X(s) = 0$$

$$F(s) = (5s^2 + 6s + 8)X(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{5s^2 + 6s + 8}}$$

Problem 48

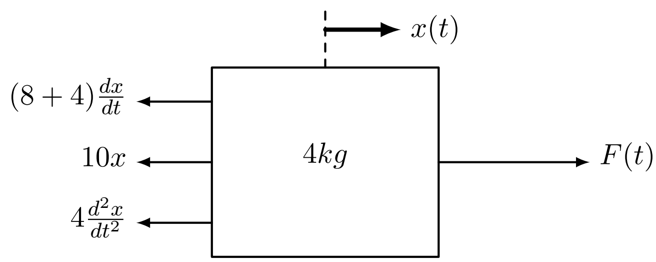
Determine the transfer function $\dfrac{X(s)}{F(s)}$ of the system below



Given: As per diagram

Solution:

FBD @ $4kg$ mass



$$\sum F_x = 0 \quad \rightarrow \quad \oplus$$

$$f(t) - 4\tfrac{d^2x}{dt^2} - 12\tfrac{dx}{dt} - 10x = 0$$

$$F(s) - 4s^2X(s) - 12sX(s) - 10X(s) = 0$$

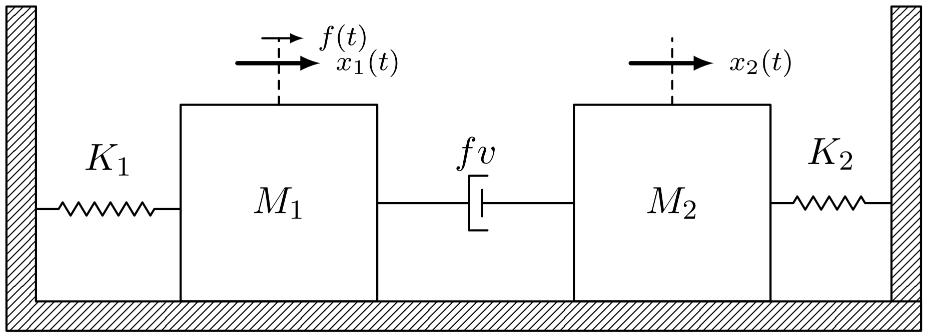
$$F(s) = (4s^2 + 12s + 10)X(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{4s^2 + 12s + 10}}$$

Problem 49

Determine the transfer function $\dfrac{X_2(s)}{F(s)}$ of the system below

$K_1 = 3Nm; K_2 = 6Nm; fv = 6Nm/s; M_1 = M_2 = 3kg$



Given: As per diagram

Solution:

FBD of M_1

$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$f(t) + 6\frac{dx_2}{dt} - 3\frac{d^2x_1}{dt^2} - 6\frac{dx_1}{dt} - 3x_1 = 0$

$F(s) + 6sX_2(s) - 3s^2X_1(s) - 6sX_1(s) - 3X_1(s) = 0$

$F(s) = (3s^2 + 6s + 3)X_1(s) - 6sX_2(s) \quad \text{(eq. 1)}$

FBD of M_2

$\sum F_x = 0 \quad \rightarrow \quad \oplus$

$6\frac{dx_1}{dt} - 3\frac{d^2x_2}{dt^2} - 6\frac{dx_2}{dt} - 6x_2 = 0$

$6sX_1(s) - 3s^2X_2(s) - 6sX_2(s) - 6X_2(s) = 0$

$6sX_1(s) = (3s^2 + 6s + 6)X_2(s)$

$$X_1(s) = \frac{3s^2 + 6s + 6}{6s}X_2(s) \quad \text{(eq. 2)}$$

Substitute eq. 2 into eq. 1:

$$F(s) = (3s^2 + 6s + 3)\left(\frac{3s^2 + 6s + 6}{6s}X_2(s)\right) - 6sX_2(s)$$

$$F(s) = X_2(s)\left(\frac{(3s^2 + 6s + 3)(3s^2 + 6s + 6)}{6s} - 6s\right)$$

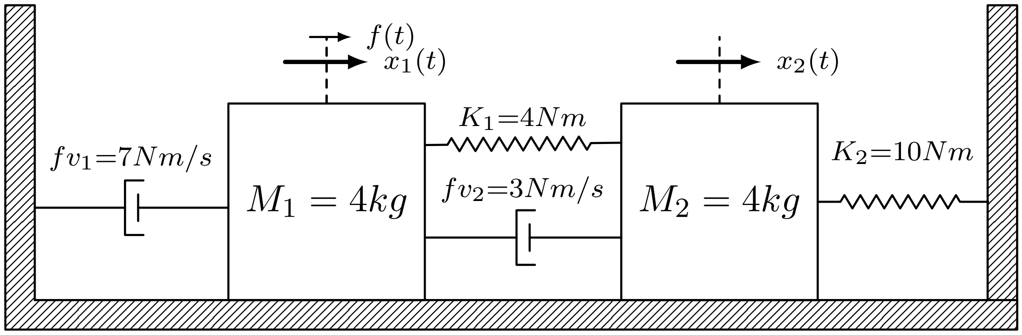
$$F(s) = X_2(s)\left(\frac{9s^4 + 36s^3 + 63s^2 + 54s + 18 - 36s^2}{6s}\right)$$

$$\frac{F(s)}{X_2(s)} = \frac{9s^4 + 36s^3 + 27s^2 + 54s + 18}{6s}$$

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{6s}{9s^4 + 36s^3 + 27s^2 + 54s + 18}}$$

Problem 50

Determine the transfer function $\dfrac{X_2(s)}{F(s)}$ of the system below



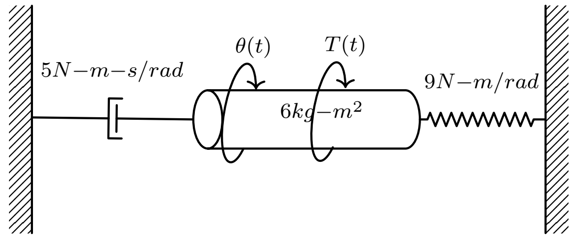
Given: As per diagram

Solution:

$$
\begin{gathered}
\text{FBD of } M_1 \\
\sum F_x = 0 \quad \rightarrow \quad \oplus \\
f(t) + 3\tfrac{dx_2}{dt} + 4x_2 - 4\tfrac{d^2x_1}{dt^2} - 10\tfrac{dx_1}{dt} - 4x_1 = 0 \\
F(s) + 3sX_2(s) + 4X_2(s) - 4s^2X_1(s) - 10sX_1(s) - 4X_1(s) = 0 \\
F(s) = (4s^2 + 10s + 4)X_1(s) - (3s + 4)X_2(s) \rightarrow \text{eq.1} \\
\text{FBD of } M_2 \\
\sum F_x = 0 \quad \rightarrow \quad \oplus \\
3\tfrac{dx_1}{dt} + 4x_1 - 4\tfrac{d^2x_2}{dt^2} - 3\tfrac{dx_2}{dt} - 14x_2 = 0 \\
3sX_1(s) + 4X_1(s) - 4s^2X_2(s) - 3sX_2(s) - 14X_2(s) = 0 \\
(3s + 4)X_1(s) = (4s^2 + 3s + 14)X_2(s) \\
X_1(s) = \frac{4s^2 + 3s + 14}{3s + 4}X_2(s) \rightarrow \text{eq.2} \\
\text{Substitute eq.2 to eq.1} \\
F(s) = (4s^2 + 10s + 4)\left(\frac{4s^2 + 3s + 14}{3s + 4}X_2(s)\right) - (3s + 4)X_2(s) \\
F(s) = X_2(s)\left(\frac{16s^4 + 52s^3 + 93s^2 + 138s + 40}{3s + 4}\right) \\
\frac{F(s)}{X_2(s)} = \frac{16s^4 + 52s^3 + 93s^2 + 138s + 40}{3s + 4} \\
\boxed{\frac{X_2(s)}{F(s)} = \frac{3s + 4}{16s^4 + 52s^3 + 93s^2 + 138s + 40}}
\end{gathered}
$$

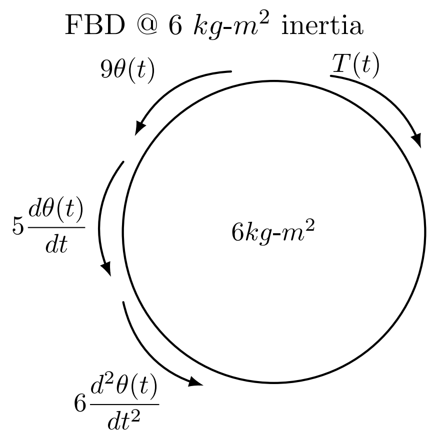
Problem 51

1. Determine the transfer function $\dfrac{\theta(s)}{T(s)}$ of the system below



Given: As per diagram

Solution:



$$\sum T = 0 \quad \oplus$$

$$T(t) - 6\frac{d^2\theta(t)}{dt^2} - 5\frac{d\theta(t)}{dt} - 9\theta(t) = 0$$

$$T(s) - 6s^2\theta(s) - 5s\theta(s) - 9\theta(s) = 0$$

$$(6s^2 + 5s + 9)\theta(s) = T(s)$$

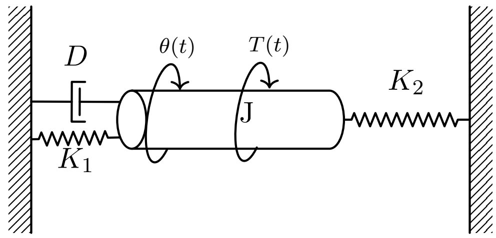
$$\frac{T(s)}{\theta(s)} = (6s^2 + 5s + 9)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{1}{6s^2 + 5s + 9}}$$

Problem 52

1. Determine the transfer function $\dfrac{\theta(s)}{T(s)}$ of the system below

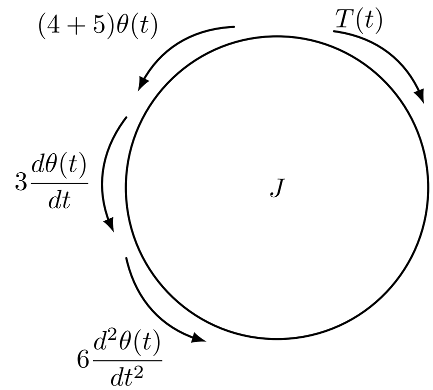
$D = 3N - m - s/rad; K_1 = 4N - m/rad; K_2 = 5N - m/rad; J = 6kg - m^2$



Given: As per diagram

Solution:

FBD of J inertia



$$\sum T = 0 \qquad \oplus$$

$$T(t) - 6\frac{d^2\theta(t)}{dt^2} - 3\frac{d\theta(t)}{dt} - (4+5)\theta(t) = 0$$

$$T(s) - 6s^2\theta(s) - 3s\theta(s) - 9\theta(s) = 0$$

$$(6s^2 + 3s + 9)\theta(s) = T(s)$$

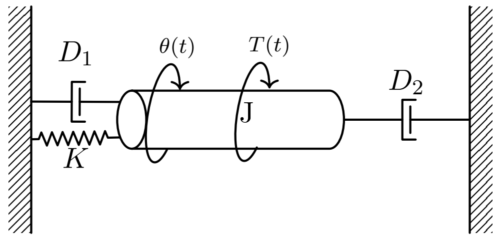
$$\frac{T(s)}{\theta(s)} = (6s^2 + 3s + 9)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{1}{6s^2 + 3s + 9}}$$

Problem 53

1. Determine the transfer function $\dfrac{\theta(s)}{T(s)}$ of the system below

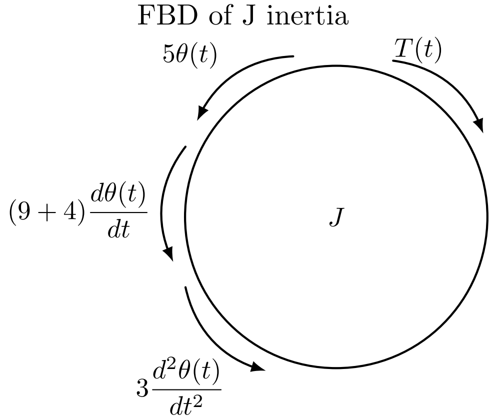
$D_1 = 9N - m - s/rad; D_2 = 4N - m - s/rad; K = 5N - m/rad; J = 3kg - m^2$



Given: As per diagram

Solution:

FBD of J inertia



$$\sum T = 0 \quad \oplus$$

$$T(t) - 3\frac{d^2\theta(t)}{dt^2} - (9+4)\frac{d\theta(t)}{dt} - 5\theta(t) = 0$$

$$T(s) - 3s^2\theta(s) - 13s\theta(s) - 5\theta(s) = 0$$

$$(3s^2 + 13s + 5)\theta(s) = T(s)$$

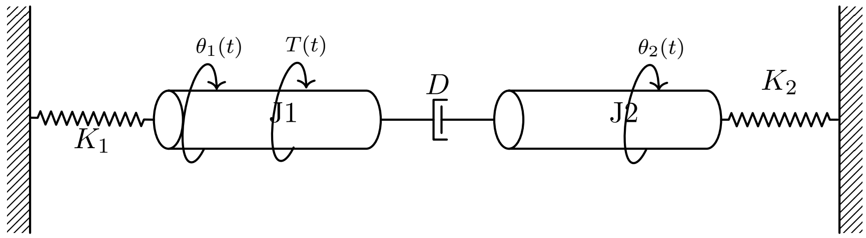
$$\frac{T(s)}{\theta(s)} = (3s^2 + 13s + 5)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{1}{3s^2 + 13s + 5}}$$

Problem 54

1. Determine the transfer function $\dfrac{\theta_1(s)}{T(s)}$ of the system below

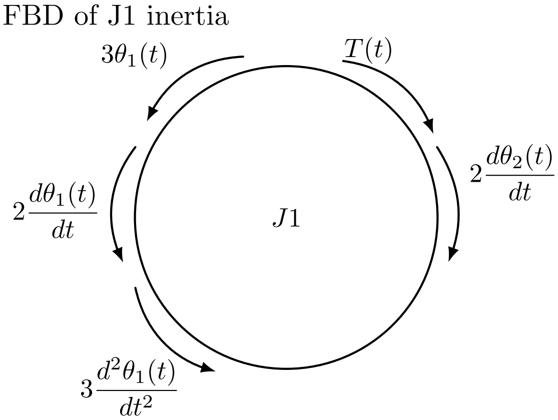
$D = 2N - m - s/rad; K_1 = 3N - m/rad; K_2 = 6N - m/rad; J1 = J2 = 5kg - m^2$



Given: As per diagram

Solution:

FBD of J1 inertia



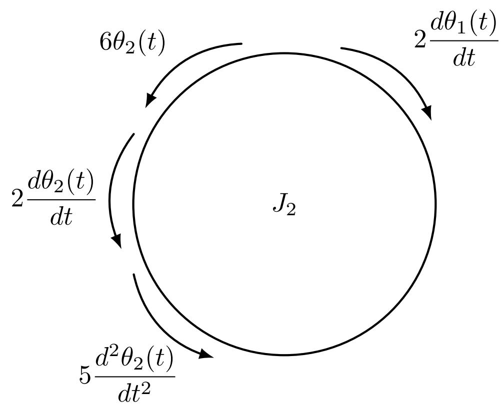
$\sum T = 0 \quad \oplus$

$$T(t) + 2\frac{d\theta_2(t)}{dt} - 5\frac{d^2\theta_1(t)}{dt^2} - 2\frac{d\theta_1(t)}{dt} - 3\theta_1(t) = 0$$

$$T(s) + 2s\theta_2(s) - 5s^2\theta_1(s) - 2s\theta_1(s) - 3\theta_1(s) = 0$$

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J_2 inertia:



$\sum T = 0 \quad \circlearrowright +$

$$2\frac{d\theta_1(t)}{dt} - 5\frac{d^2\theta_2(t)}{dt^2} - 2\frac{d\theta_2(t)}{dt} - 6\theta_2(t) = 0$$

$$2s\theta_1(s) - 5s^2\theta_2(s) - 2s\theta_2(s) - 6\theta_2(s) = 0$$

$$2s\theta_1(s) = (5s^2 + 2s + 6)\theta_2(s)$$

$$\theta_2(s) = \frac{2s}{5s^2 + 2s + 6}\theta_1(s) \quad \text{(eq. 2)}$$

Substitute eq. 2 into the torque equation:

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\theta_2(s) = T(s)$$

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\left(\frac{2s}{5s^2 + 2s + 6}\theta_1(s)\right) = T(s)$$

$$\Rightarrow \theta_1(s)\left((5s^2 + 2s + 3) - \frac{4s^2}{5s^2 + 2s + 6}\right) = T(s)$$

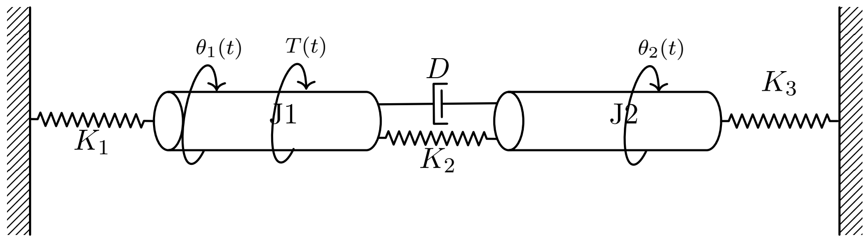
$$\Rightarrow \frac{T(s)}{\theta_1(s)} = \frac{25s^4 + 20s^3 + 30s^2 + 12s + 9}{5s^2 + 2s + 6}$$

$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{5s^2 + 2s + 6}{25s^4 + 20s^3 + 30s^2 + 12s + 9}}$$

Problem 55

1. Determine the transfer function $\dfrac{\theta_1(s)}{T(s)}$ of the system below

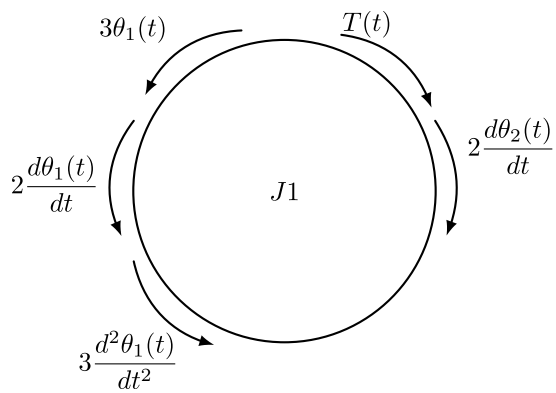
$D = 8N - m - s/rad; K_1 = 5N - m/rad; K_2 = K_3 = 7N - m/rad; J1 = J2 = 7kg - m^2$



Given: As per diagram

Solution:

FBD of J1 inertia



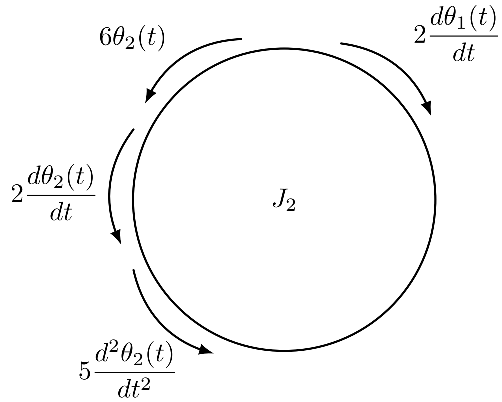
$\sum T = 0 \quad \oplus$

$$T(t) + 2\frac{d\theta_2(t)}{dt} - 5\frac{d^2\theta_1(t)}{dt^2} - 2\frac{d\theta_1(t)}{dt} - 3\theta_1(t) = 0$$

$$T(s) + 2s\theta_2(s) - 5s^2\theta_1(s) - 2s\theta_1(s) - 3\theta_1(s) = 0$$

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J_2 inertia:



$$\sum T = 0 \quad \circlearrowright +$$

$$2\frac{d\theta_1(t)}{dt} - 5\frac{d^2\theta_2(t)}{dt^2} - 2\frac{d\theta_2(t)}{dt} - 6\theta_2(t) = 0$$

$$2s\theta_1(s) - 5s^2\theta_2(s) - 2s\theta_2(s) - 6\theta_2(s) = 0$$

$$2s\theta_1(s) = (5s^2 + 2s + 6)\theta_2(s)$$

$$\theta_2(s) = \frac{2s}{5s^2 + 2s + 6}\theta_1(s) \quad \text{(eq. 2)}$$

Substitute eq. 2 into the torque equation:

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\theta_2(s) = T(s)$$

$$(5s^2 + 2s + 3)\theta_1(s) - 2s\left(\frac{2s}{5s^2 + 2s + 6}\theta_1(s)\right) = T(s)$$

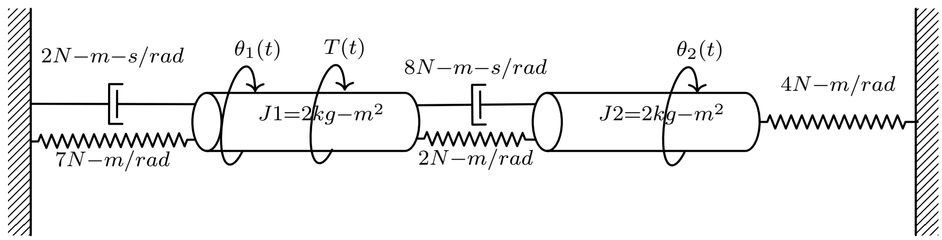
$$\Rightarrow \theta_1(s)\left((5s^2 + 2s + 3) - \frac{4s^2}{5s^2 + 2s + 6}\right) = T(s)$$

$$\Rightarrow \frac{T(s)}{\theta_1(s)} = \frac{25s^4 + 20s^3 + 30s^2 + 12s + 9}{5s^2 + 2s + 6}$$

$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{5s^2 + 2s + 6}{25s^4 + 20s^3 + 30s^2 + 12s + 9}}$$

Problem 56

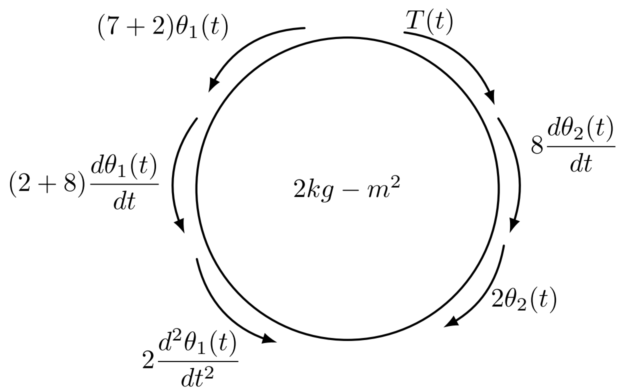
1. Determine the transfer function $\dfrac{\theta_2(s)}{T(s)}$ of the system below



Given: As per diagram

Solution:

FBD of J1 inertia



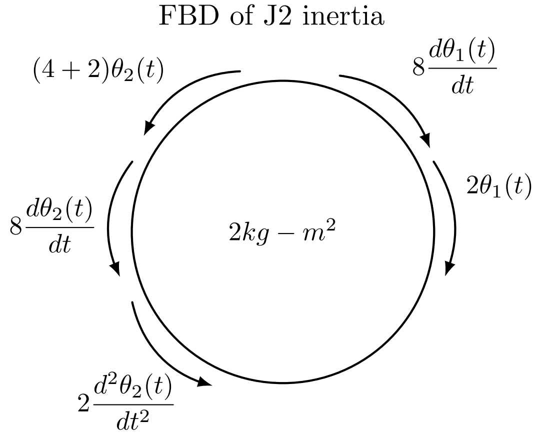
$$\sum T = 0 \quad \oplus$$

$$T(t)+8\frac{d\theta_2(t)}{dt}+2\theta_2(t)-2\frac{d^2\theta_1(t)}{dt^2}-(2+8)\frac{d\theta_1(t)}{dt}-(7+2)\theta_1(t)=0$$

$$T(s)+8s\theta_2(s)+2\theta_2(s)-2s^2\theta_1(s)-10s\theta_1(s)-9\theta_1(s)=0$$

$$(2s^2+10s+9)\theta_1(s)-(8s+2)\theta_2(s)=T(s)\rightarrow eq.1$$

FBD of J2 inertia



$$\sum T = 0 \quad \oplus$$

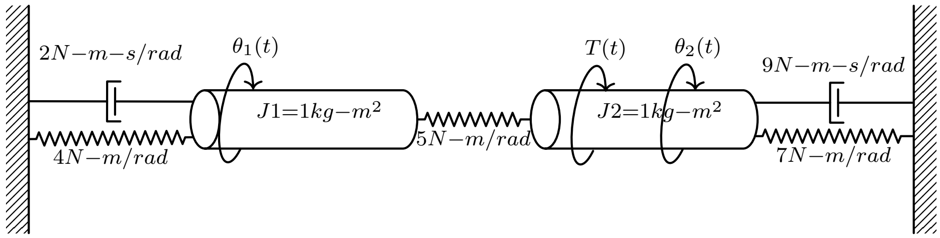
$$8\frac{d\theta_1(t)}{dt}+2\theta_1(t)-2\frac{d^2\theta_2(t)}{dt^2}-8\frac{d\theta_2(t)}{dt}-(4+2)\theta_2(t)=0$$

$$8s\theta_1(s)+2\theta_1(s)-2s^2\theta_2(s)-8s\theta_2(s)-6\theta_2(s)=0$$

$$(8s+2)\theta_1(s) = (2s^2+8s+6)\theta_2(s)$$
$$\theta_1(s) = \frac{2s^2+8s+6}{8s+2}\theta_2(s) \rightarrow eq.2$$
Substitute eq.2 to eq.1
$$T(s) = (2s^2+10s+9)\theta_1(s) - (8s+2)\theta_2(s)$$
$$T(s) = (2s^2+10s+9)\left(\frac{2s^2+8s+6}{8s+2}\theta_2(s)\right) - (8s+2)\theta_2(s)$$
$$T(s) = \left(\frac{(2s^2+10s+9)(2s^2+8s+6)}{8s+2}\theta_2(s)\right) - (8s+2)\theta_2(s)$$
$$T(s) = \left(\frac{4s^4+36s^3+110s^2+132s+54}{8s+2} - (8s+2)\right)\theta_2(s)$$
$$T(s) = \left(\frac{4s^4+36s^3+110s^2+132s+54-64s^2-32s-4}{8s+2}\right)\theta_2(s)$$
$$T(s) = \left(\frac{4s^4+36s^3+46s^2+100s+50}{8s+2}\right)\theta_2(s)$$
$$\frac{T(s)}{\theta_2(s)} = \left(\frac{4s^4+36s^3+46s^2+100s+50}{8s+2}\right)$$
$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{8s+2}{4s^4+36s^3+46s^2+100s+50}}$$

Problem 57

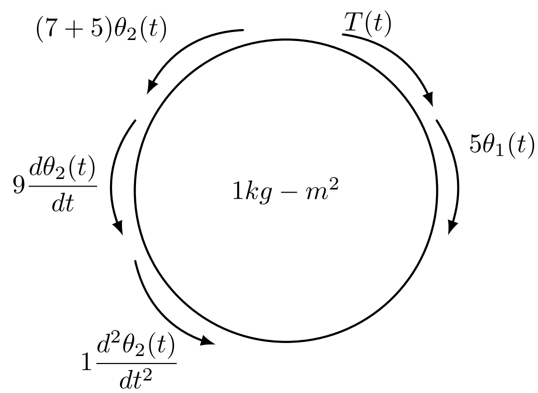
1. Determine the transfer function $\dfrac{\theta_2(s)}{T(s)}$ of the system below



Given: As per diagram

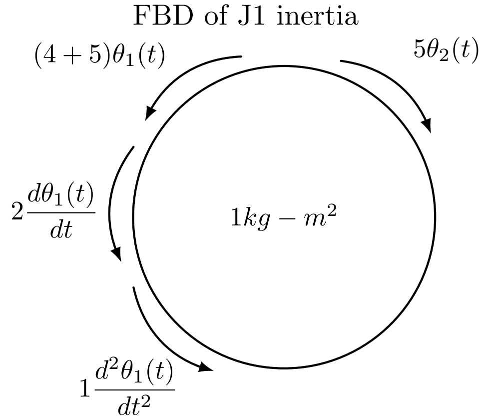
Solution:

FBD of J2 inertia



$$\sum T = 0 \qquad \oplus$$
$$T(t) + 5\theta_1(t) - 1\frac{d^2\theta_2(t)}{dt^2} - 9\frac{d\theta_2(t)}{dt} - (7+5)\theta_2(t) = 0$$
$$T(s) + 5\theta_1(s) - s^2\theta_2(s) - 9s\theta_2(s) - 12\theta_2(s) = 0$$
$$-5\theta_1(s) + (s^2+9s+12)\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J1 inertia



$$\sum T = 0 \qquad \oplus$$
$$5\theta_2(t) - 1\frac{d^2\theta_1(t)}{dt^2} - 2\frac{d\theta_1(t)}{dt} - (4+5)\theta_1(t) = 0$$
$$5\theta_1(s) - s^2\theta_1(s) - 2s\theta_1(s) - 9\theta_1(s) = 0$$
$$5\theta_1(s) = (s^2+2s+9)\theta_2(s)$$

$$\theta_1(s) = \frac{s^2+2s+9}{5}\theta_2(s) \to eq.2$$

Substitute eq.2 to eq.1

$$T(s) = -5\theta_1(s) + (s^2+9s+12)\theta_2(s)$$

$$T(s) = -5\left(\frac{s^2+2s+9}{5}\theta_2(s)\right) + (s^2+9s+12)\theta_2(s)$$

$$T(s) = \left(\frac{(-5)(s^2+2s+9)}{5}\theta_2(s)\right) + (s^2+9s+12)\theta_2(s)$$

$$T(s) = \left(\frac{-5s^2-10s-45}{5} + (s^2+9s+12)\right)\theta_2(s)$$

$$T(s) = \left(\frac{-5s^2-10s-45+5s^2+45s+60}{5}\right)\theta_2(s)$$

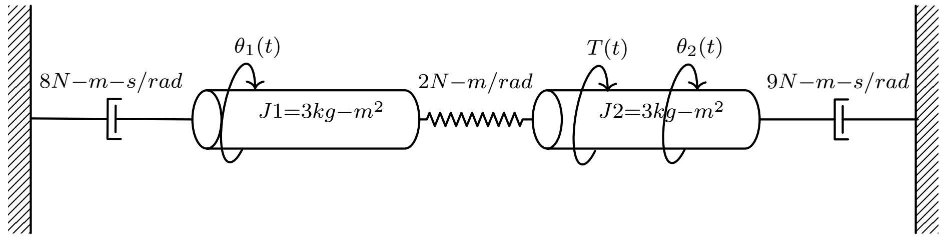
$$T(s) = \left(\frac{35s+15}{5}\right)\theta_2(s)$$

$$\frac{T(s)}{\theta_2(s)} = \frac{35s+15}{5}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{5}{35s+15}}$$

Problem 58

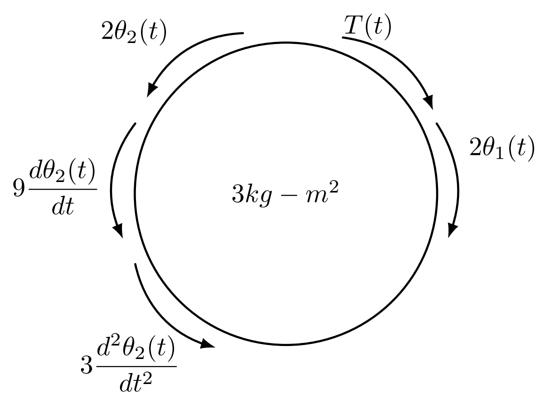
Determine the transfer function $\dfrac{\theta_2(s)}{T(s)}$ of the system below



Given: As per diagram

Solution:

FBD of J2 Inertia

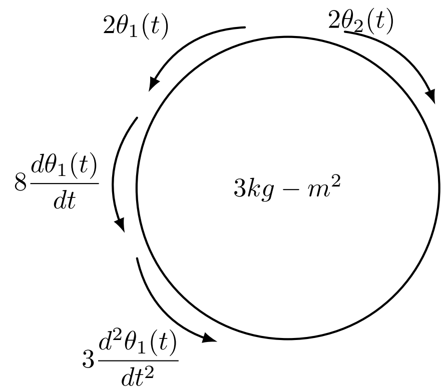


Equation of Motion for J2

$$\sum T = 0 \quad \circlearrowleft +$$
$$T(t) + 2\theta_1(t) - 3\frac{d^2\theta_2(t)}{dt^2} - 9\frac{d\theta_2(t)}{dt} - 2\theta_2(t) = 0$$
Laplace Transform: $T(s) + 2\theta_1(s) - 3s^2\theta_2(s) - 9s\theta_2(s) - 2\theta_2(s) = 0$

Rearranged: $-2\theta_1(s) + (3s^2 + 9s + 2)\theta_2(s) = T(s)$ **(Equation 1)**

FBD of J1 Inertia



Equation of Motion for J1

$$\sum T = 0 \quad \circlearrowright +$$

$$2\theta_2(t) - 3\frac{d^2\theta_1(t)}{dt^2} - 8\frac{d\theta_1(t)}{dt} - 2\theta_1(t) = 0$$

$$\text{Laplace Transform: } 2\theta_2(s) - 3s^2\theta_1(s) - 8s\theta_1(s) - 2\theta_1(s) = 0$$

$$\text{Rearranged: } (3s^2 + 8s + 2)\theta_1(s) = 2\theta_2(s)$$

$$\theta_1(s) = \frac{2}{3s^2 + 8s + 2}\theta_2(s) \quad \textbf{(Equation 2)}$$

Solving the System

Substitute Equation 2 into Equation 1:

$$T(s) = -2\left(\frac{2}{3s^2 + 8s + 2}\theta_2(s)\right) + (3s^2 + 9s + 2)\theta_2(s)$$

$$T(s) = \left(\frac{-4}{3s^2 + 8s + 2} + 3s^2 + 9s + 2\right)\theta_2(s)$$

$$T(s) = \left(\frac{-4 + (3s^2 + 9s + 2)(3s^2 + 8s + 2)}{3s^2 + 8s + 2}\right)\theta_2(s)$$

After expansion and simplification:

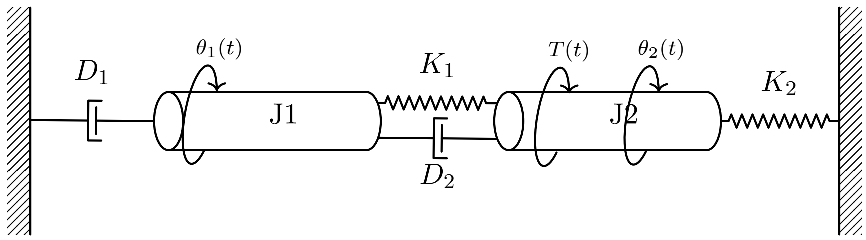
$$T(s) = \left(\frac{9s^4 + 51s^3 + 88s^2 + 42s}{3s^2 + 8s + 2}\right)\theta_2(s)$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{3s^2 + 8s + 2}{9s^4 + 51s^3 + 88s^2 + 42s}}$$

Problem 59

1. Determine the transfer function $\dfrac{\theta_2(s)}{T(s)}$ of the system below

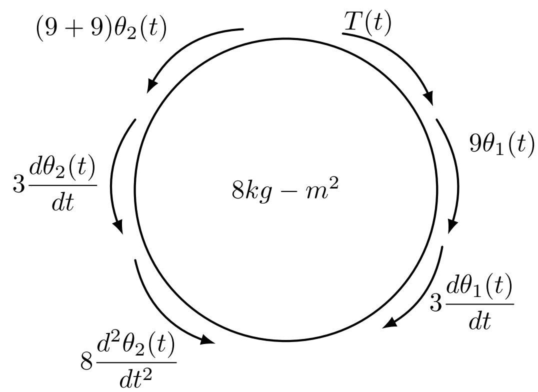
$D_1 = 5N - m - s/rad; D_2 = 3N - m - s/rad; K_1 = K_2 = 9N - m/rad; J1 = J2 = 8kg - m^2$



Given: As per diagram

Solution:

FBD of J2 inertia



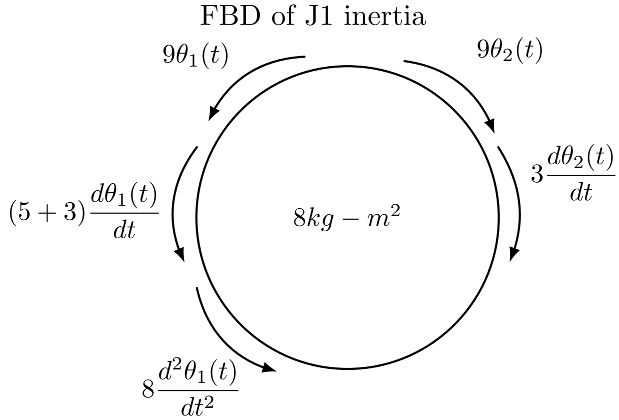
$$\sum T = 0 \quad \oplus$$

$$T(t) + 9\theta_1(t) + 3\frac{d\theta_1(t)}{dt} - 8\frac{d^2\theta_2(t)}{dt^2} - 3\frac{d\theta_2(t)}{dt} - (9+9)\theta_2(t) = 0$$

$$T(s) + 9\theta_1(s) + 3s\theta_1(s) - 8s^2\theta_2(s) - 3s\theta_2(s) - 18\theta_2(s) = 0$$

$$-(3s+9)\theta_1(s) + (8s^2 + 3s + 18)\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J1 inertia



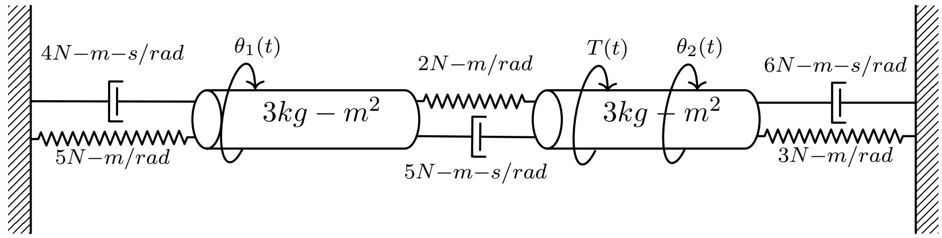
$$\sum T = 0 \quad \oplus$$

$$3\frac{d\theta_2(t)}{dt} + 9\theta_2(t) - 8\frac{d^2\theta_1(t)}{dt^2} - (5+3)\frac{d\theta_1(t)}{dt} - 9\theta_1(t) = 0$$

$$3s\theta_2(s) + 9\theta_2(s) - 8s^2\theta_1(s) - 8s\theta_1(s) - 9\theta_1(s) = 0$$
$$(8s^2 + 8s + 9)\theta_1(s) = (3s + 9)\theta_2(s)$$
$$\theta_1(s) = \frac{3s + 9}{8s^2 + 8s + 9}\theta_2(s) \rightarrow eq.2$$
Substitute eq.2 to eq.1
$$T(s) = -(3s + 9)\theta_1(s) + (8s^2 + 3s + 18)\theta_2(s)$$
$$T(s) = -(3s + 9)\left(\frac{3s + 9}{8s^2 + 8s + 9}\theta_2(s)\right) + (8s^2 + 3s + 18)\theta_2(s)$$
$$T(s) = \left(\frac{-9s^2 - 54s - 81}{8s^2 + 8s + 9}\theta_2(s)\right) + (8s^2 + 3s + 18)\theta_2(s)$$
$$T(s) = \left(\frac{-9s^2 - 54s - 81}{8s^2 + 8s + 9} + (8s^2 + 3s + 18)\right)\theta_2(s)$$
$$T(s) = \left(\frac{-9s^2 - 54s - 81 + 64s^4 + 88s^3 + 240s^2 + 171s + 162}{8s^2 + 8s + 9}\right)\theta_2(s)$$
$$T(s) = \left(\frac{64s^4 + 88s^3 + 231s^2 + 117s + 81}{8s^2 + 8s + 9}\right)\theta_2(s)$$
$$\frac{T(s)}{\theta_2(s)} = \frac{64s^4 + 88s^3 + 231s^2 + 117s + 81}{8s^2 + 8s + 9}$$
$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{8s^2 + 8s + 9}{64s^4 + 88s^3 + 231s^2 + 117s + 81}}$$

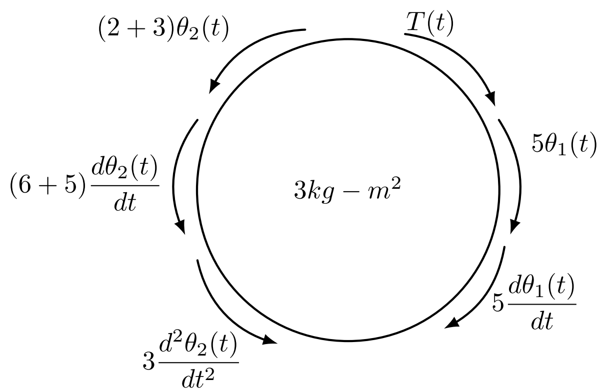
Problem 60

1. Determine the transfer function $\dfrac{\theta_1(s)}{T(s)}$ of the system below



Solution:

FBD of J2 inertia



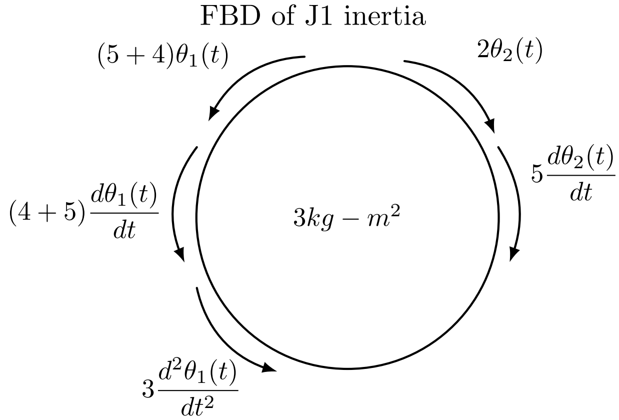
$$\sum T = 0 \quad \oplus$$

$$T(t) + 5\theta_1(t) + 5\frac{d\theta_1(t)}{dt} - 3\frac{d^2\theta_2(t)}{dt^2} - (6+5)\frac{d\theta_2(t)}{dt} - (2+3)\theta_2(t) = 0$$

$$T(s) + 5\theta_1(s) + 5s\theta_1(s) - 3s^2\theta_2(s) - 11s\theta_2(s) - 5\theta_2(s) = 0$$

$$-(5s+5)\theta_1(s) + (3s^2+11s+5)\theta_2(s) = T(s) \rightarrow eq.1$$

FBD of J1 inertia



$$\sum T = 0 \quad \oplus$$

$$5\frac{d\theta_2(t)}{dt} + 2\theta_2(t) - 3\frac{d^2\theta_1(t)}{dt^2} - (4+5)\frac{d\theta_1(t)}{dt} - (5+4)\theta_1(t) = 0$$

$$5s\theta_2(s) + 2\theta_2(s) - 3s^2\theta_1(s) - 9s\theta_1(s) - 9\theta_1(s) = 0$$

$$(3s^2+9s+9)\theta_1(s) = (5s+2)\theta_2(s)$$

$$\theta_2(s) = \frac{3s^2+9s+9}{5s+2}\theta_1(s) \rightarrow eq.2$$

Substitute eq.2 to eq.1

$$T(s) = -(5s+5)\theta_1(s) + (3s^2+11s+5)\theta_2(s)$$

$$T(s) = -(5s+5)\theta_1(s) + (3s^2+11s+5)\left(\frac{3s^2+9s+9}{5s+2}\theta_1(s)\right)$$

$$T(s) = -(5s+5)\theta_1(s) + \left(\frac{9s^4+54s^3+129s^2+144s+45}{5s+2}\theta_1(s)\right)$$

$$T(s) = \theta_1(s)\left(\frac{9s^4+54s^3+129s^2+144s+45}{5s+2} - (5s+5)\right)$$

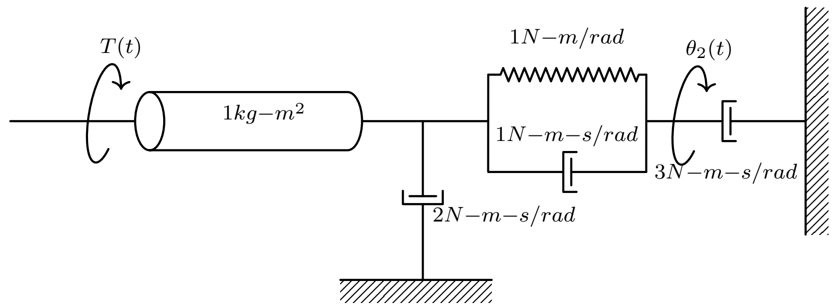
$$T(s) = \theta_1(s)\left(\frac{9s^4+54s^3+129s^2+144s+45-25s^2-35s-10}{5s+2}\right)$$

$$T(s) = \theta_1(s)\left(\frac{9s^4+54s^3+104s^2+109s+35}{5s+2}\right)$$

$$\frac{T(s)}{\theta_1(s)} = \frac{9s^4+54s^3+104s^2+109s+35}{5s+2}$$

$$\boxed{\frac{\theta_1(s)}{T(s)} = \frac{5s+2}{9s^4+54s^3+104s^2+109s+35}}$$

Problem 61
Find the transfer function $\theta_2(s)T(s)$



Given: As per diagram

Solution:

Equation of Motion

For θ_1

$$J\frac{d^2\theta_1(t)}{dt^2} + (b_1 + b_2)\frac{d\theta_1(t)}{dt} + k[\theta_1(t) - \theta_2(t)] = T(t)$$

Laplace Domain:

$$Js^2\theta_1(s) + (b_1 + b_2)s\theta_1(s) + k[\theta_1(s) - \theta_2(s)] = T(s)$$

Substitute the Given Values:

$$1s^2\theta_1(s) + (1 + 2)s\theta_1(s) + 1[\theta_1(s) - \theta_2(s)] = T(s)$$

$$s^2\theta_1(s) + 3s\theta_1(s) + \theta_1(s) - \theta_2(s) = T(s)$$

$$(s^2 + 3s + 1)\theta_1(s) - \theta_2(s) = T(s)$$

$$(s^2 + 3s + 1)\theta_1(s) = T(s) + \theta_2(s)$$

Equation of Motion

For θ_2

$$(b_2 + b_3)\frac{d\theta_2(t)}{dt} + k[\theta_2(t) - \theta_1(t)] = 0$$

Laplace Domain:

$$(b_2 + b_3)s\theta_2(s) + k[\theta_2(s) - \theta_1(s)] = 0$$

Substitute the Given Values:

$$(2+3)s\theta_2(s) + 1[\theta_2(s) - \theta_1(s)] = 0$$

$$5s\theta_2(s) + \theta_2(s) - \theta_1(s) = 0$$

$$(5s+1)\theta_2(s) = \theta_1(s)$$

$$\theta_1(s) = (5s+1)\theta_2(s)$$

Substitute θ_1 Into First Equation:

$$(s^2+3s+1)(5s+1)\theta_2(s) = T(s) + \theta_2(s)$$

$$(5s+1)(s^2+3s+1)\theta_2(s) = T(s) + \theta_2(s)$$

$$(5s^3+16s^2+8s+1)\theta_2(s) = T(s) + \theta_2(s)$$

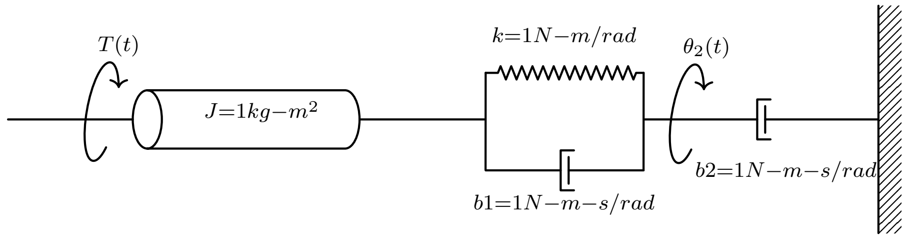
$$(5s^3+16s^2+8s)\theta_2(s) = T(s)$$

Solve for $G(s) = \dfrac{\theta_2(s)}{T(s)}$:

$$\boxed{G(s) = \frac{\theta_2(s)}{T(s)} = \frac{1}{5s^3+16s^2+8s}}$$

Problem 62

Find the transfer function $\dfrac{\theta_2(s)}{T(s)}$



Given: As per diagram

Solution:

Force Balance on M

First Mass (Moment of Inertia J):

$$J\frac{d^2\theta_1(t)}{dt^2} + b_1\frac{d\theta_1(t)}{dt} + k[\theta_1(t) - \theta_2(t)] = T(t)$$

Second Mass:

$$J\frac{d^2\theta_2(t)}{dt^2} + b_2\frac{d\theta_2(t)}{dt} + k[\theta_2(t) - \theta_1(t)] = T(t)$$

Laplace Transform Where Initial Condition is 0:

First Inertia:

$$Js^2\theta_1(s) + b_1 s\theta_1(s) + k[\theta_1(s) - \theta_2(s)] = T(s)$$

Second Inertia:

$$Js^2\theta_2(s) + b_2 s\theta_2(s) + k[\theta_2(s) - \theta_1(s)] = 0$$

Solving for $\theta_2(s)$

From 2^{nd} Equation:

$$(Js^2 + b_2 s + k)\theta_2(s) = k\theta_1(s)$$

$$\theta_2(s) = \frac{k\theta_1(s)}{Js^2 + b_2 s + k}$$

Substitute it to 1^{st} Equation:

$$Js^2\theta_1(s) + b_1 s\theta_1(s) + k[\theta_1(s) - \frac{k\theta_1(s)}{Js^2 + b_2 s + k}] = T(s)$$

Simplify:

Factor out $\theta_1(s)$:

$$\theta_1(s)\left(Js^2 + b_1 s + k - \frac{k^2}{Js^2 + b_2 s + k}\right) = T(s)$$

$$\theta_1(s)\left[\frac{(Js^2 + b_2 s + k)(Js^2 + b_1 s + k) - k^2}{Js^2 + b_2 s + k}\right] = T(s)$$

$$\theta_1(s) = \frac{T(s)(Js^2 + b_2 s + k)}{(Js^2 + b_2 s + k)(Js^2 + b_1 s + k) - k^2}$$

Substitute θ_2 in Equation:

$$\theta_2(s) = \frac{(k)[T(s)](Js^2 + b_2 s + k)}{(Js^2 + b_2 s + k)[(Js^2 + b_1 s + k)(Js^2 + b_2 s + k)] - k^2}$$

Simplify:

$$\frac{\theta_2(s)}{T(s)} = \frac{k(Js^2 + b_2 s + k)}{(Js^2 + b_1 s + k)(Js^2 + b_2 s + k) - k^2}$$

Substitute the Given Values:

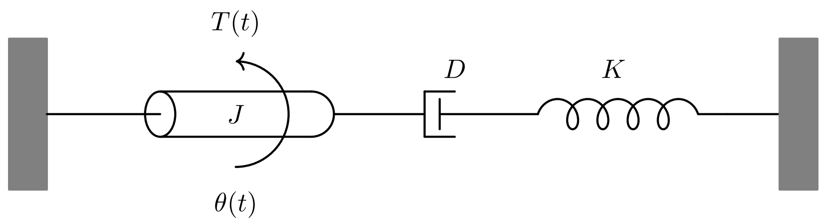
$$\frac{\theta_2(s)}{T(s)} = \frac{s^2 + s + 1}{(s^2 + s + 1)^2 - 1}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{s^2 + s + 1}{s^4 + 2s^3 + s^2}}$$

Problem 63
Find the transfer function

$$G(s) = \frac{\Theta(s)}{T(s)}$$

for the system below:



Given: As per diagram

Solution:

Solution:
From the diagram, this is a single-degree-of-freedom rotational mechanical system with moment of inertia J, damper D, and torsional spring K.

The equation of motion in the Laplace domain is:

$$(Js^2 + Ds + K)\Theta(s) = T(s)$$

Solving for the transfer function:

$$G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2 + Ds + K}$$

Substitute the given values:

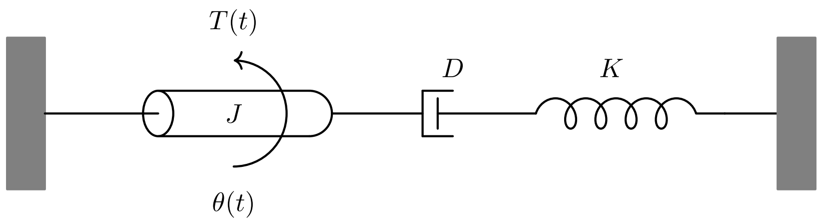
$$G(s) = \frac{1}{2s^2 + 0.5s + 3}$$

Answer:

$$\boxed{G(s) = \frac{1}{2s^2 + 0.5s + 3}}$$

Problem 64
Find the transfer function

$$G(s) = \frac{\Theta(s)}{T(s)}$$



Given: As per diagram

Solution:

Single-degree-of-freedom rotational mechanical system The equation of motion in the Laplace domain is:

$$(Js^2 + Ds + K)\Theta(s) = T(s)$$

Solving for the transfer function:

$$G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2 + Ds + K}$$

Substitute the given values:

$$G(s) = \frac{1}{4s^2 + 2s + 5}$$

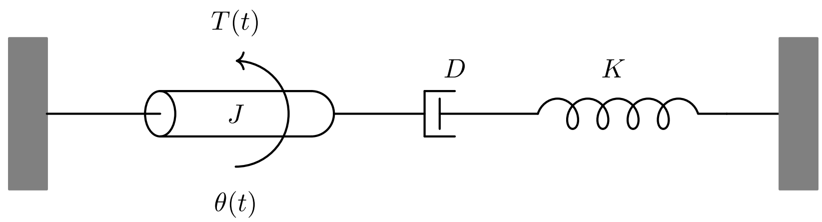
Answer:

$$\boxed{G(s) = \frac{1}{4s^2 + 2s + 5}}$$

Problem 65 Find the transfer function

$$G(s) = \frac{\Theta(s)}{T(s)}$$

for the system below:



Given: As per diagram

Solution:

From the diagram, this is a single-degree-of-freedom rotational mechanical system with moment of inertia J, damper D, and torsional spring K.

The equation of motion in the Laplace domain is:

$$(Js^2 + Ds + K)\Theta(s) = T(s)$$

Solving for the transfer function:

$$G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2 + Ds + K}$$

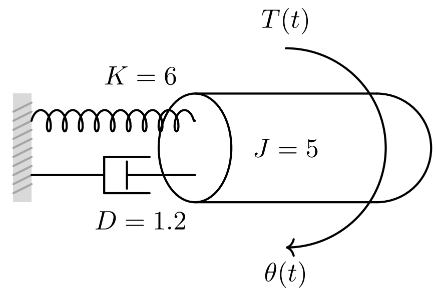
Substitute the given values:

$$G(s) = \frac{1}{6s^2 + s + 4}$$

Answer:

$$\boxed{G(s) = \frac{1}{6s^2 + s + 4}}$$

Problem 66
Find the transfer function and identify the damping ratio.



Given: As per diagram

Solution:

The standard form of the transfer function is:

$$G(s) = \frac{1}{Js^2 + Ds + K}$$

Substituting the values:

$$G(s) = \frac{1}{5s^2 + 1.2s + 6}$$

The damping ratio is:

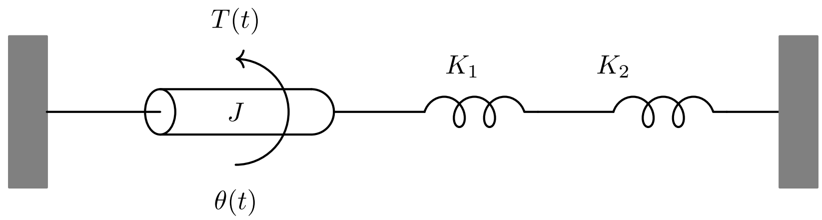
$$\zeta = \frac{D}{2\sqrt{JK}} = \frac{1.2}{2\sqrt{5 \cdot 6}} = \frac{1.2}{2\sqrt{30}} \approx 0.110$$

Answer:

$$\boxed{G(s) = \frac{1}{5s^2 + 1.2s + 6}}, \quad \boxed{\zeta \approx 0.110}$$

Problem 67

Determine the transfer function $G(s)$ for:



with $J = 1$, $K_1 = 3$, $K_2 = 2$.

Given: As per diagram

Solution:

The two torsional springs K_1 and K_2 are connected in series to the mass moment of inertia J. Since they are both acting in parallel on the same shaft, their equivalent stiffness is simply:

$$K = K_1 + K_2 = 3 + 2 = 5$$

The standard transfer function for such a rotational system is:

$$G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2 + K}$$

Substituting the values:

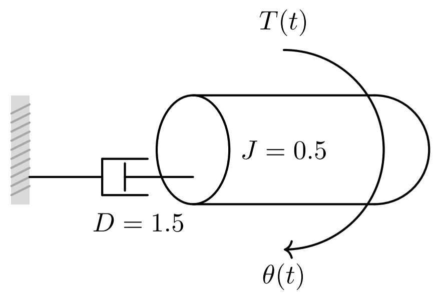
$$G(s) = \frac{1}{s^2 + 5}$$

Answer:

$$\boxed{G(s) = \frac{1}{s^2 + 5}}$$

Problem 68

Compute the time constant for a rotational system with $J = 0.5$ and $D = 1.5$ (no spring).



Given: $J = 0.5$ and $D = 1.5$

Solution:

The time constant for a first-order rotational system is:

$$\tau = \frac{J}{D}$$

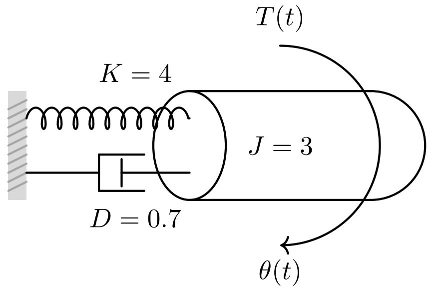
Substituting the values:

$$\tau = \frac{0.5}{1.5} = \frac{1}{3}$$

Answer:

$$\boxed{\tau = \frac{1}{3} \text{ s}}$$

Problem 69 Find the transfer function and identify the damping ratio.



Given: As per diagram

Solution:

The standard form of the transfer function is:

$$G(s) = \frac{1}{Js^2 + Ds + K}$$

Substituting the values:

$$G(s) = \frac{1}{3s^2 + 0.7s + 4}$$

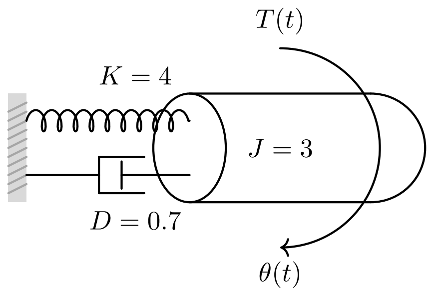
The damping ratio is:

$$\zeta = \frac{D}{2\sqrt{JK}} = \frac{0.7}{2\sqrt{3 \cdot 4}} = \frac{0.7}{2\sqrt{12}} \approx 0.101$$

Answer:

$$\boxed{G(s) = \frac{1}{3s^2 + 0.7s + 4}}, \quad \boxed{\zeta \approx 0.101}$$

Problem 70
Determine θ_{ss} for a unit step torque input.



Given: As per diagram

Solution:

For a step input, steady-state displacement is:

$$\theta_{ss} = \lim_{s \to 0} G(s) = \frac{1}{K}$$

From Problem 7, $K = 4$:

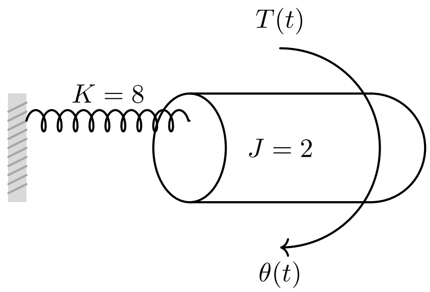
$$\theta_{ss} = \frac{1}{4} = 0.25$$

Answer:

$$\boxed{\theta_{ss} = 0.25 \text{ rad}}$$

Problem 71

For a system with $J = 2$, $K = 8$, find ω_n.



Given: As per diagram

Solution:

The natural frequency in radians per second is given by:

$$\omega_n = \sqrt{\frac{K}{J}}$$

Substituting the known values:

$$\omega_n = \sqrt{\frac{8}{2}} = \sqrt{4}$$

$$\omega_n = 2\,\text{rad/s}$$

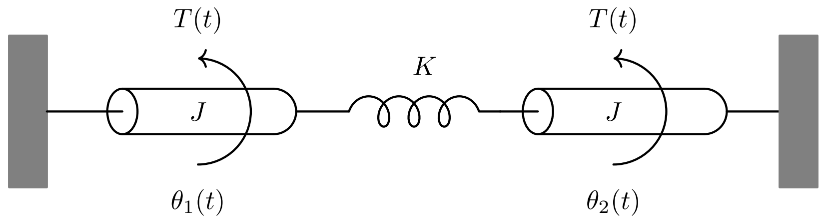
So, the system will oscillate naturally at 2 radians per second when displaced from its equilibrium position and released, assuming no damping is present.

Answer:

$$\boxed{\omega_n = 2\,\text{rad/s}}$$

Problem 72

A shaft connects two disks as shown:



with $J_1 = 1$, $J_2 = 2$, $K = 5$. Find $G(s) = \frac{\Theta_2(s)}{T(s)}$.

Given: As per diagram

Solution:

The total equivalent moment of inertia is:

$$J = J_1 + J_2 = 1 + 2 = 3$$

The transfer function is:

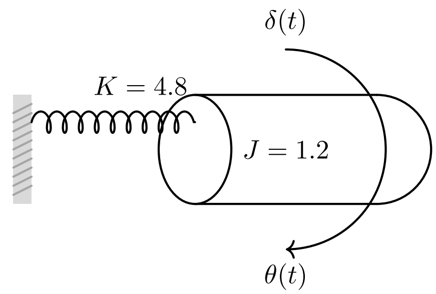
$$G(s) = \frac{1}{Js^2 + K} = \frac{1}{3s^2 + 5}$$

Answer:

$$\boxed{G(s) = \frac{1}{3s^2 + 5}}$$

Problem 73

A rotational system has $J = 1.2$ and $K = 4.8$. If subjected to an impulse torque, find the maximum angular velocity.



Given: As per diagram

Solution:

An impulse torque causes an instantaneous change in angular velocity:

$$\omega_{\text{max}} = \frac{1}{J}$$

Substitute the given value:

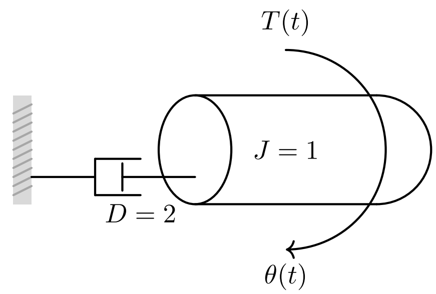
$$\omega_{\text{max}} = \frac{1}{1.2} \approx 0.833$$

Answer:

$$\boxed{\omega_{\text{max}} \approx 0.833 \text{ rad/s}}$$

Problem 74

Determine the settling time (5%) for a system with $J = 1$, $D = 2$.



Given: As per diagram

Solution:

Time constant is:

$$\tau = \frac{J}{D} = \frac{1}{2} = 0.5$$

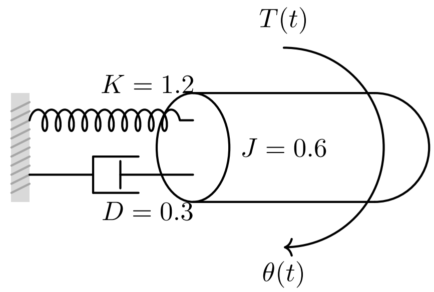
The 5% settling time is:

$$t_s = 4\tau = 4 \cdot 0.5 = 2$$

Answer:

$$\boxed{t_s = 2 \text{ s}}$$

Problem 75
Find the transfer function.



Given: As per diagram

Solution:

The transfer function is:

$$G(s) = \frac{1}{Js^2 + Ds + K}$$

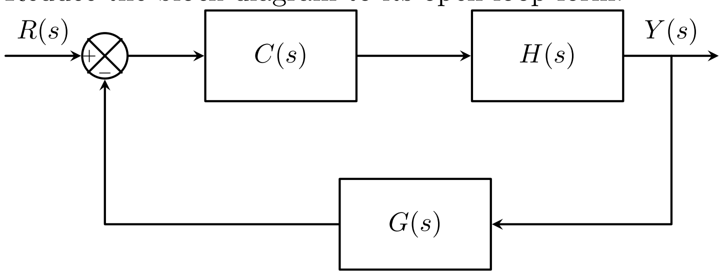
Substitute the given values:

$$G(s) = \frac{1}{0.6s^2 + 0.3s + 1.2}$$

Answer:

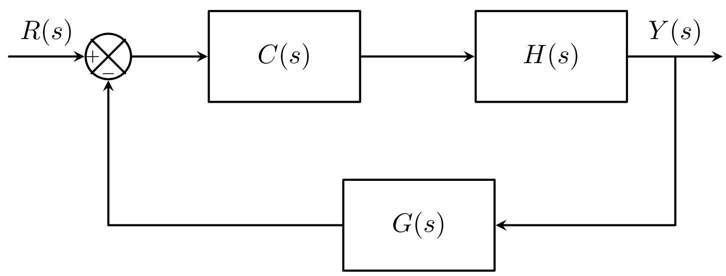
$$\boxed{G(s) = \frac{1}{0.6s^2 + 0.3s + 1.2}}$$

Problem 76
Reduce the block diagram to its open-loop form:

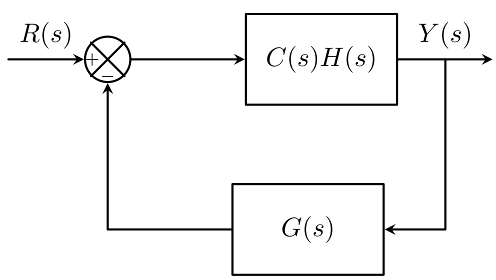


Given: As per diagram

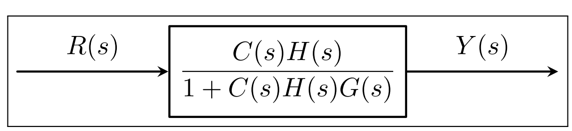
Solution:



Used Cascaded rule of $C(s)$ and $H(s)$

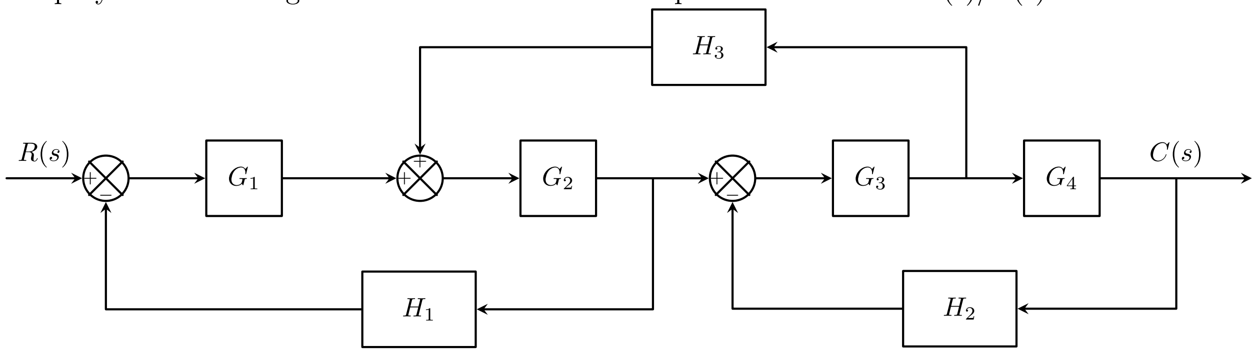


Used Feedback rule of $C(s)H(s)$ and $G(s)$



Problem 77

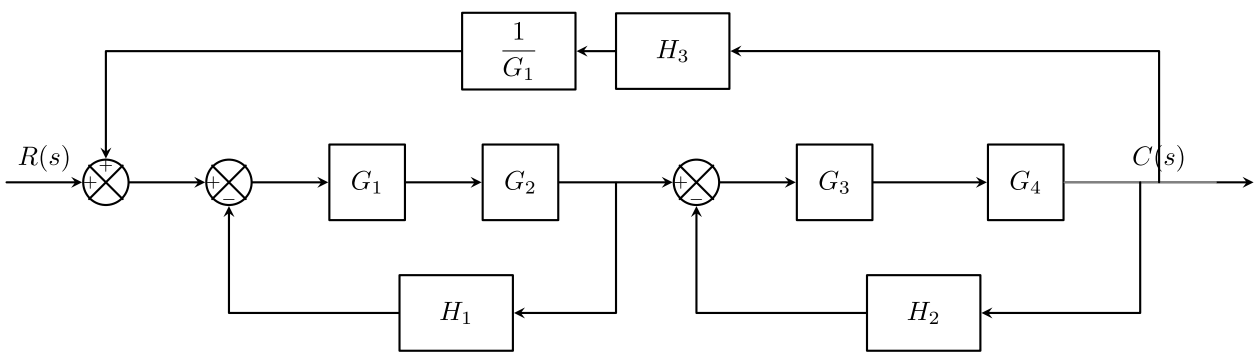
Simplify the block diagram then obtain the close-loop transfer function C(s)/R(s).



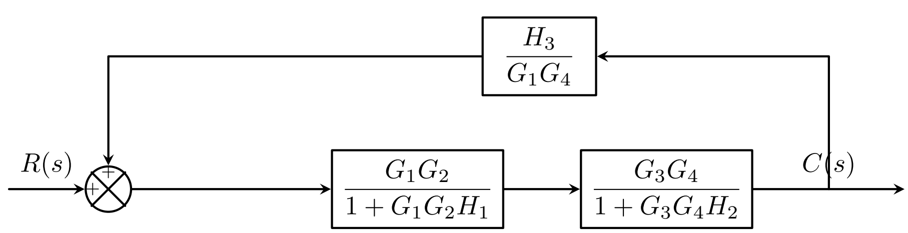
Given: As per diagram

Solution:

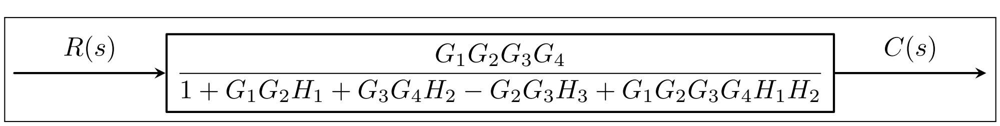
First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4 and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point.



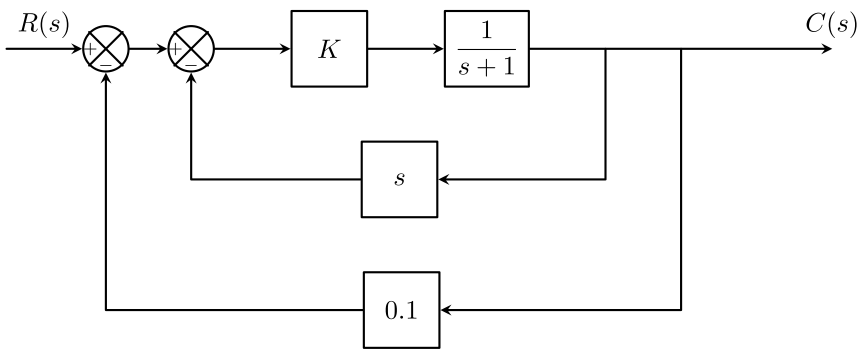
By simplifying each loop, the block diagram can be modified as



Further simplification results in



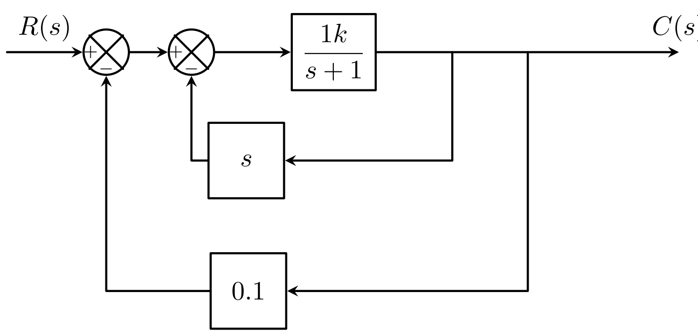
Problem 78
Reduce the given block diagram to canonical form.



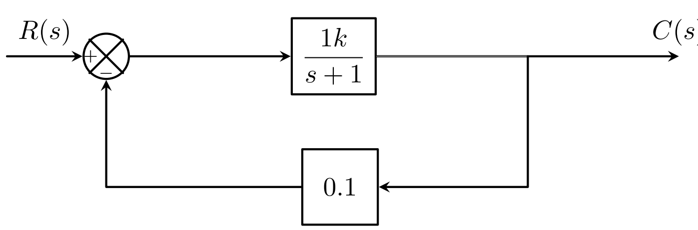
Given: As per diagram

Solution:

Used Cascaded rule of K and $\frac{1}{s+1}$

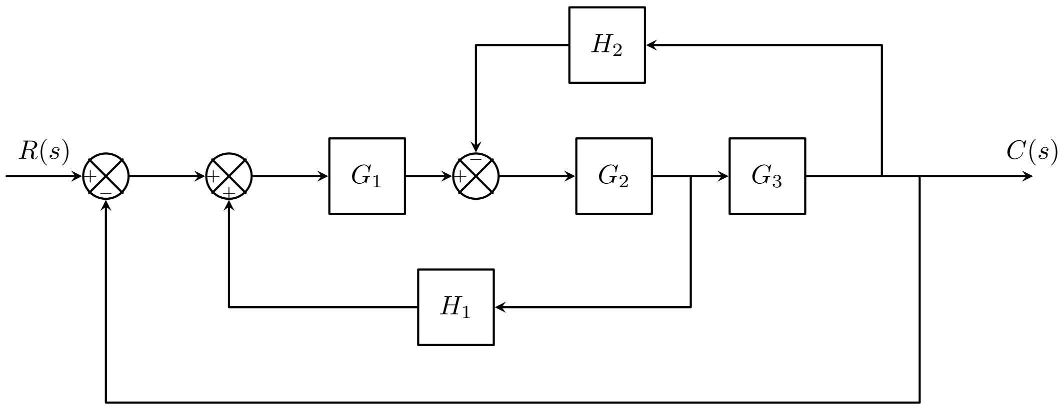


Used Feedback rule of $\frac{1k}{s+1}$ and s



$$\boxed{\frac{C(s)}{R(s)} = \frac{K}{s^2 + (1+K)s + (0.1K+1)}}$$

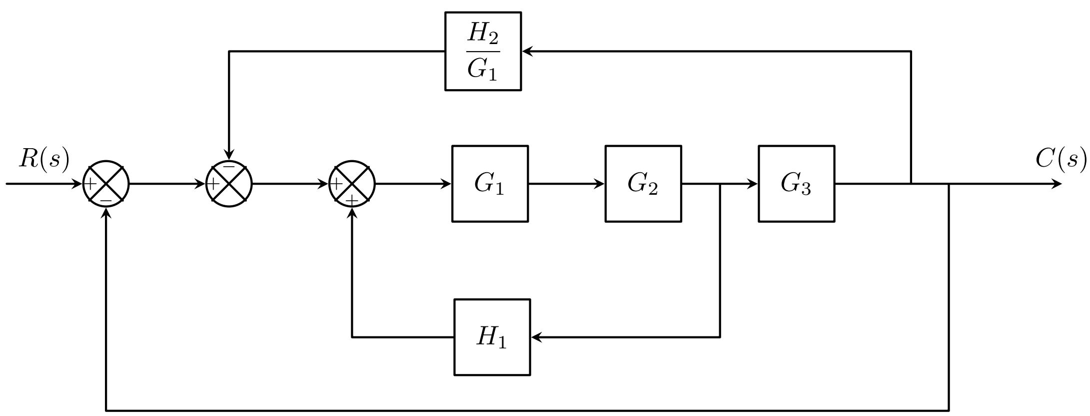
Problem 79
Simplify the block diagram.



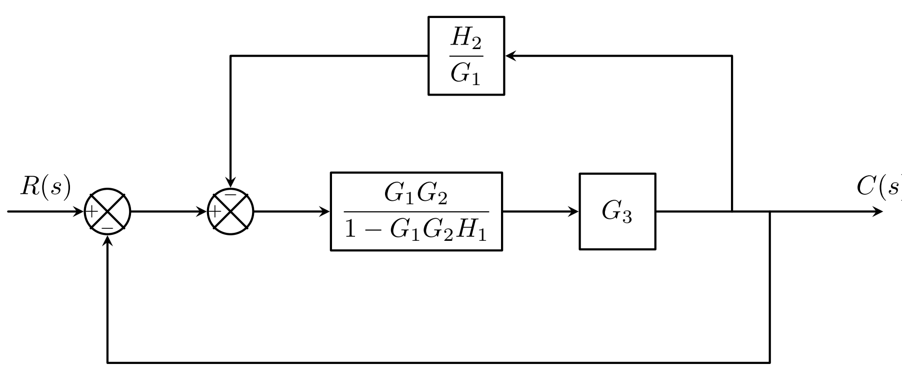
Given: As per diagram

Solution:

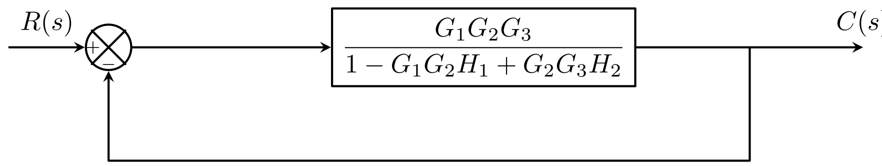
By moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1,we obtain figure



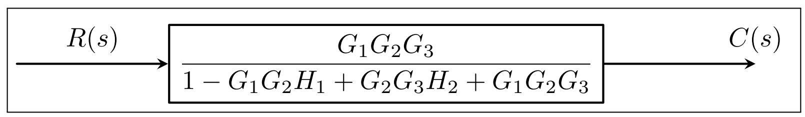
Eliminating the positive feedback loop, we have



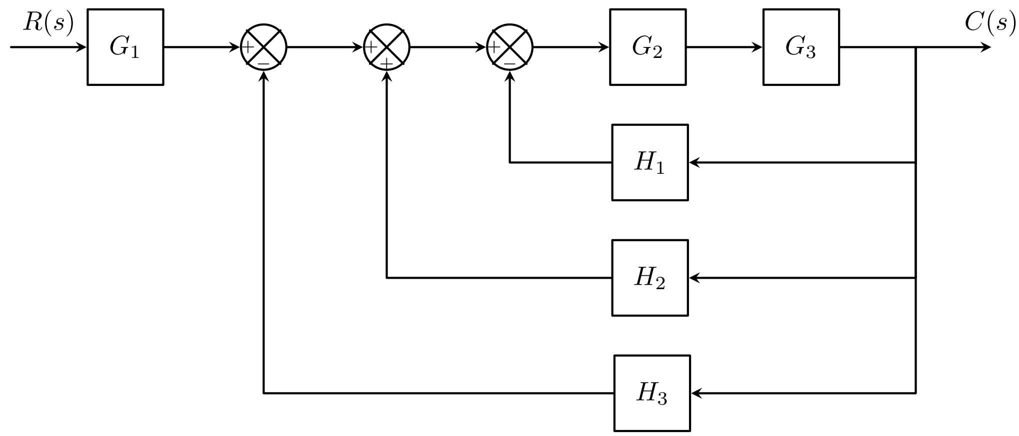
The elimination of the loop containing $\dfrac{H_2}{G_1}$ gives



Finally,eliminating the feedback loop results in



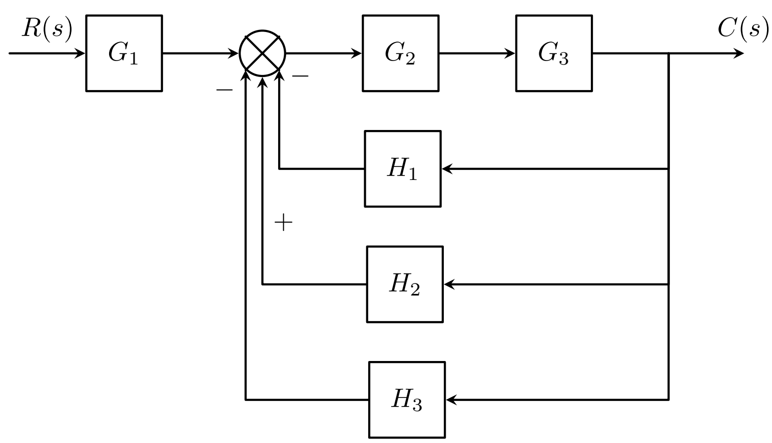
Problem 80
Reduce the Block Diagram.



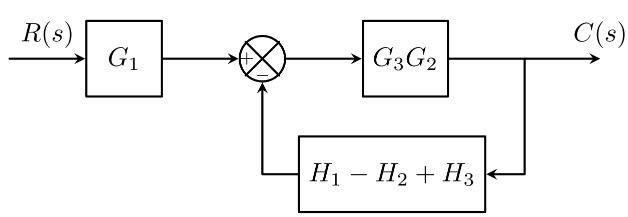
Given: As per diagram

Solution:

Look at three summing junctions.



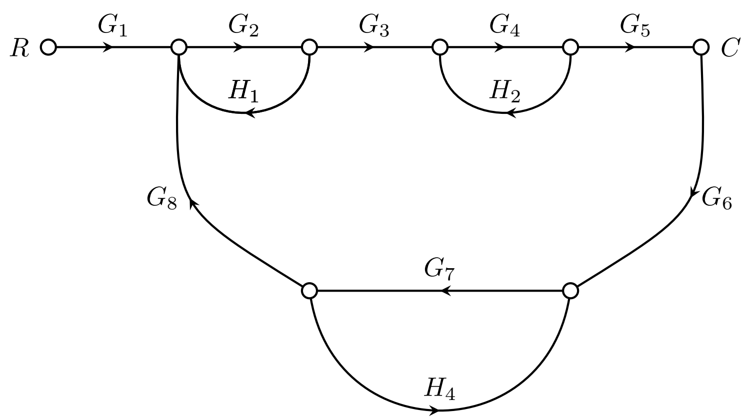
The the three feedback functions, H_1, H_2 and H_3 are connected in parallel



The transfer function shown in figure is:

$$R(s) \longrightarrow \boxed{\dfrac{G_3 G_2 G_1}{1 + G_3 G_2 H_1 - H_2 + H_3}} \longrightarrow C(s)$$

Problem 81
Find the transfer function of the signal flow graph below



Given: As per diagram

Solution:

Forward Path:

$$P_1 = G_1G_2G_3G_4G_5$$

Mason's Gain Formula:

$$\frac{C}{R} = \frac{\sum T_k\Delta_k}{\Delta}$$

Loop Gains:

$$L_1 = G_2H_1$$
$$L_2 = G_4H_2$$
$$L_3 = G_7H_4$$
$$L_4 = G_2G_3G_4G_5G_6G_7G_8$$

Determinant (Δ):

$$\begin{aligned}\Delta = 1 &- (G_2H_1 + G_4H_2 + G_7H_4 + G_2G_3G_4G_5G_6G_7G_8)\\ &+ (G_2G_4H_1H_2 + G_2G_7H_1H_4 + G_4G_7H_2H_4)\\ &- G_2G_4G_7H_1H_2H_4\end{aligned}$$

Non-Touching Loop Combinations:

2 Non-Touching: $(G_2H_1)(G_4H_2), \quad (G_2H_1)(G_7H_4), \quad (G_4H_2)(G_7H_4)$
3 Non-Touching: $(G_2H_1)(G_4H_2)(G_7H_4)$
4 Non-Touching: N/A

Co-factor (Δ_1) for Path P_1:

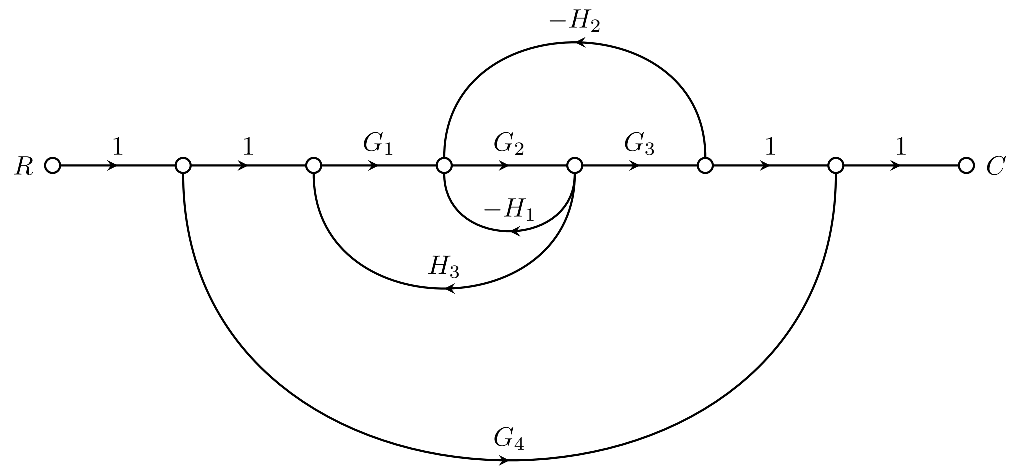
$$\Delta_1 = 1 - G_7H_4$$

Transfer Function:

$$\frac{C}{R} = \frac{G_1G_2G_3G_4G_5(1 - G_7H_4)}{\Delta}$$

$$\boxed{\begin{aligned}\frac{C}{R} = \frac{G_1G_2G_3G_4G_5 - G_1G_2G_3G_4G_5G_7H_4}{1 - G_2H_1 - G_4H_2 - G_7H_4 - G_2G_3G_4G_5G_6G_7G_8} \\ + G_2G_4H_1H_2 + G_2G_7H_1H_4 + G_4G_7H_2H_4 \\ - G_2G_4G_7H_1H_2H_4\end{aligned}}$$

Problem 82
For the system shown,obtain the closed loop transfer function



Given: As per diagram

Solution:

Forward Paths:

$$P_1 = G_1G_2G_3$$
$$P_2 = G_4$$

Mason's Rule:

$$\frac{C}{R} = \frac{\sum T_k\Delta_k}{\Delta}$$

Loop Gains:

$$L_1 = -G_2H_1$$
$$L_2 = G_1G_2H_1$$
$$L_3 = -G_2G_3H_2$$

Determinant:

$$\Delta = 1 - (L_1 + L_2 + L_3) = 1 + G_2H_1 - G_1G_2H_1 + G_2G_3H_2$$

Cofactors:

$$\Delta_1 = 1, \text{ for } P_1$$
$$\Delta_2 = 1, \text{ for } P_2$$

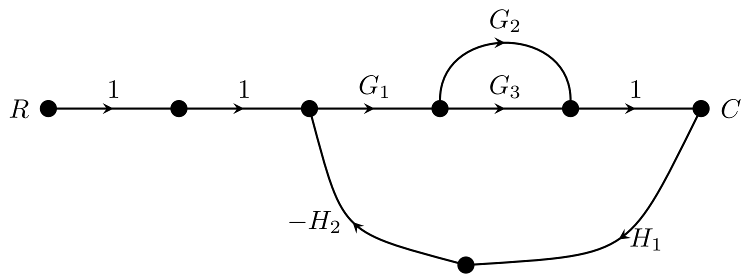
2 Non-touching Loops:

N/A

Final Transfer Function:

$$\boxed{\frac{C}{R} = \frac{G_1G_2G_3 + G_4}{1 + G_2H_1 - G_1G_2H_1 + G_2G_3H_2}}$$

Problem 83
Find the transfer function of the signal flow graph below



Given: As per diagram

Solution:

Forward Paths:

$$P_1 = G_1 G_2$$
$$P_2 = G_1 G_3$$

Mason's Rule:

$$\frac{C}{R} = \frac{\sum T_k \Delta_k}{\Delta}$$

Loop Gains:

$$L_1 = -G_1 G_3 H_1 H_2$$
$$L_2 = -G_1 G_2 H_1 H_2$$

Determinant:

$$\Delta = 1 - (L_1 + L_2) = 1 + G_1 G_3 H_1 H_2 + G_1 G_2 H_1 H_2$$

Cofactors:

$\Delta_1 = 1$, for P_1
$\Delta_2 = 1$, for P_2

2 Non-touching Loops:

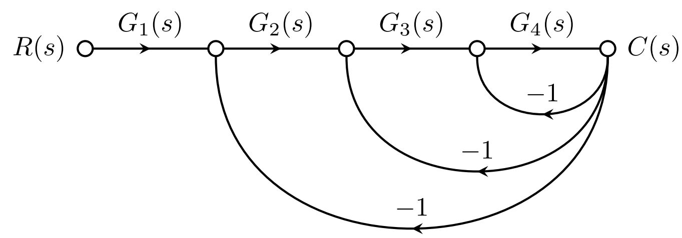
N/A

Final Transfer Function:

$$\boxed{\frac{C}{R} = \frac{G_1 G_2 + G_1 G_3}{1 + G_1 G_3 H_1 H_2 + G_1 G_2 H_1 H_2}}$$

Problem 84

Find the transfer function of the signal flow graph below



Given: As per diagram

Solution:

Forward Path:

$$P_1 = G_1G_2G_3G_4$$

Mason's Rule:

$$\frac{C}{R} = \frac{\sum T_k\Delta_k}{\Delta}$$

Loop Gains:

$$L_1 = -G_2G_3G_4$$
$$L_2 = -G_3G_4$$
$$L_3 = -G_4$$

Determinant:

$$\Delta = 1 - (L_1 + L_2 + L_3) = 1 + G_2G_3G_4 + G_3G_4 + G_4$$

Cofactors:

$$\Delta_1 = 1$$

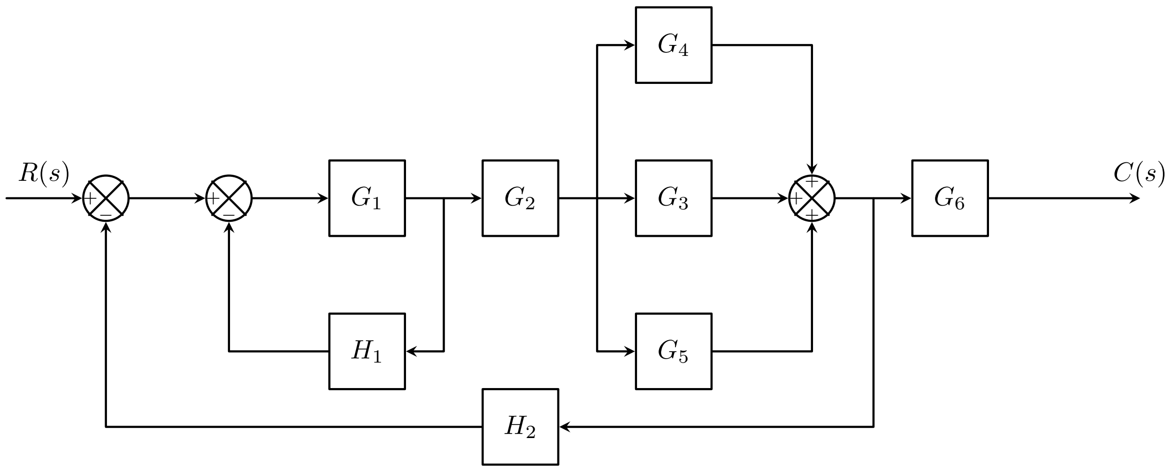
2 Non-touching Loops:

N/A

Final Transfer Function:

$$\boxed{\frac{C}{R} = \frac{G_1G_2G_3G_4}{1 + G_2G_3G_4 + G_3G_4 + G_4}}$$

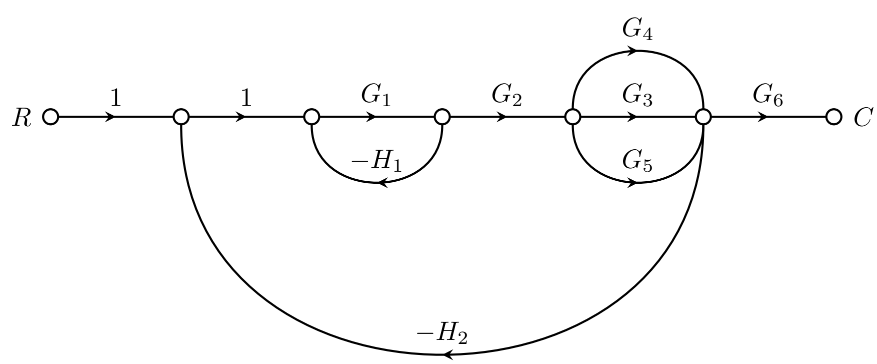
Problem 85
Find the transfer function of the system shown by a block diagram



Given: As per diagram

Solution:

Converting into SFG



Solution:

Forward Paths:

$P_1 = G_1G_2G_3G_6$

$P_2 = G_1G_2G_4G_6$

$P_3 = G_1G_2G_5G_6$

Mason's Rule:

$$\frac{C}{R} = \frac{\sum T_k\Delta_k}{\Delta}$$

Loop Gains:

$L_1 = -G_1H_1$

$L_2 = -G_1G_2G_3H_2$

$L_3 = -G_1G_2G_4H_2$

$L_4 = -G_1G_2G_5H_2$

Determinant:

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) = 1 + G_1H_1 + G_1G_2G_3H_2 + G_1G_2G_4H_2 + G_1G_2G_5H_2$$

Cofactors:

$\Delta_1 = 1 \quad \Delta_2 = 1 \quad \Delta_3 = 1$

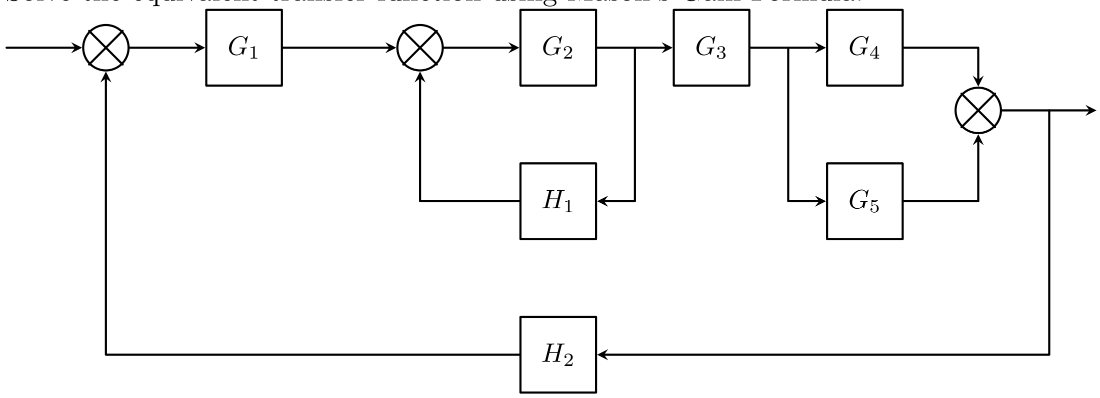
2 Non-touching Loops:

N/A

Final Transfer Function:

$$\boxed{\frac{C}{R} = \frac{G_1G_2G_3G_6 + G_1G_2G_4G_6 + G_1G_2G_5G_6}{1 + G_1H_1 + G_1G_2G_3H_2 + G_1G_2G_4H_2 + G_1G_2G_5H_2}}$$

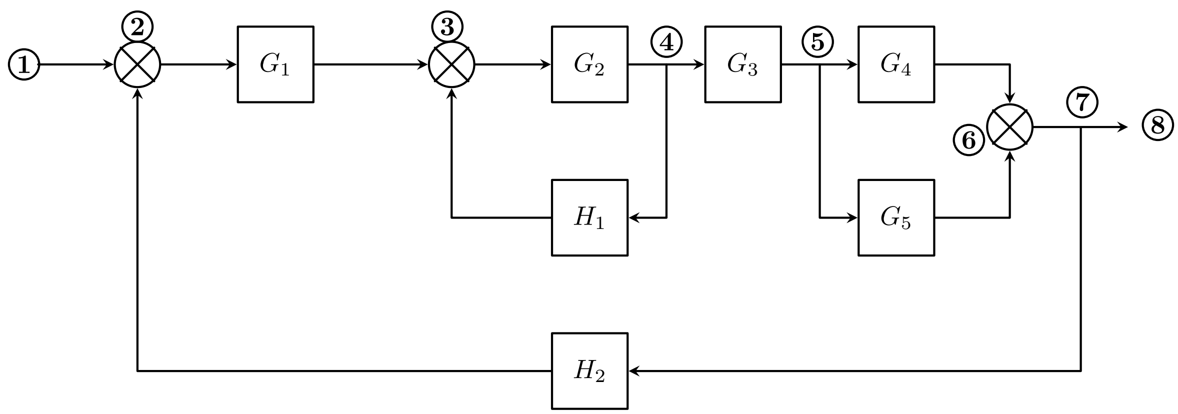
Problem 86
Solve the equivalent transfer function using Mason's Gain Formula:



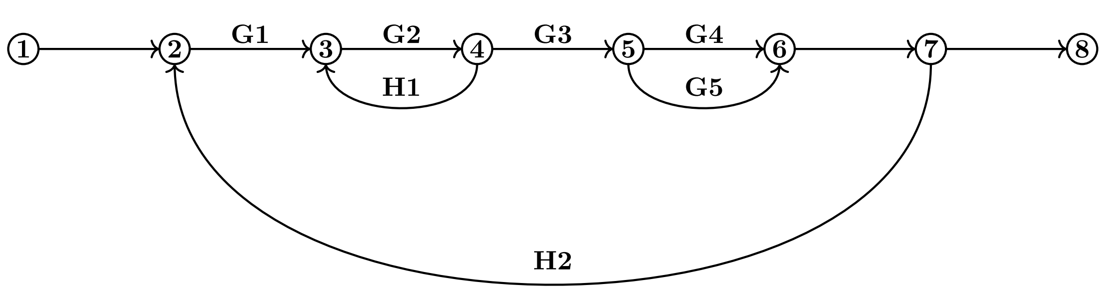
Given: As per diagram

Solution:

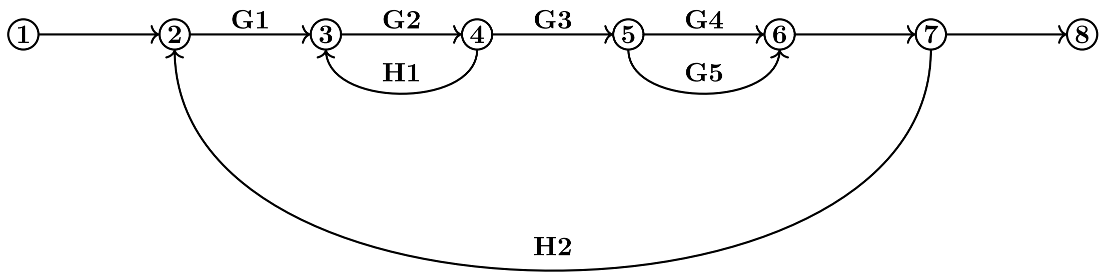
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify forward path and all loops



$$F_1 = G_2G_3G_4$$

$$\Delta_1 = 0$$

$$F_2 = G_2G_3G_5$$

$$\Delta_2 = 0$$
$$L_1 = G_2H_1$$

$$L_2 = G_1G_2G_3G_4H_2$$

$$L_3 = G_1G_2G_3G_5H_2$$

Step 4. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

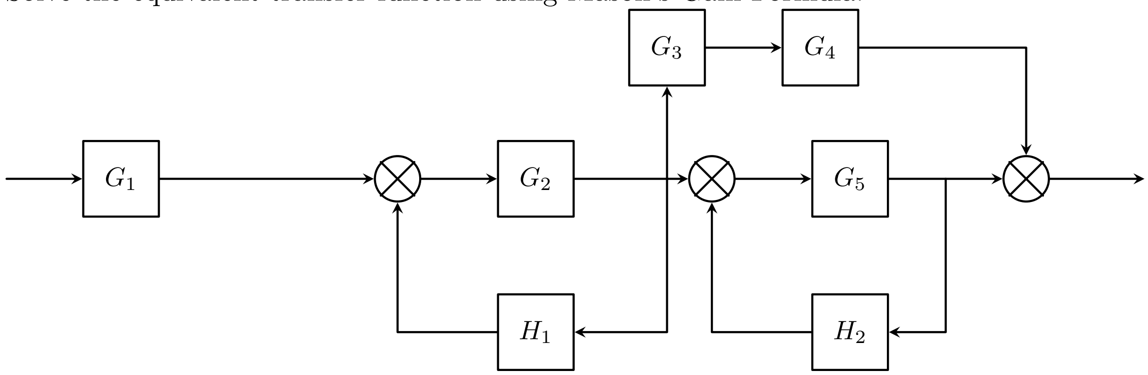
$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_3G_4(1-0) + G_1G_2G_3G_5(1-0)}{1 + (G_2H_1 + G_1G_2G_3G_4H_2 + G_1G_2G_3G_5H_2) + (0) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_3G_4 + G_1G_2G_3G_5}{1 + G_2H_1 + G_1G_2G_3G_4H_2 + G_1G_2G_3G_5H_2}}$$

Problem 87

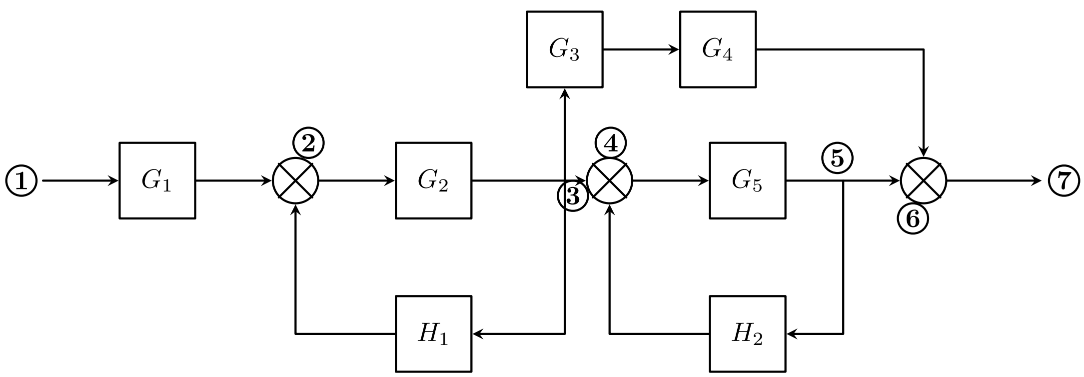
Solve the equivalent transfer function using Mason's Gain Formula:



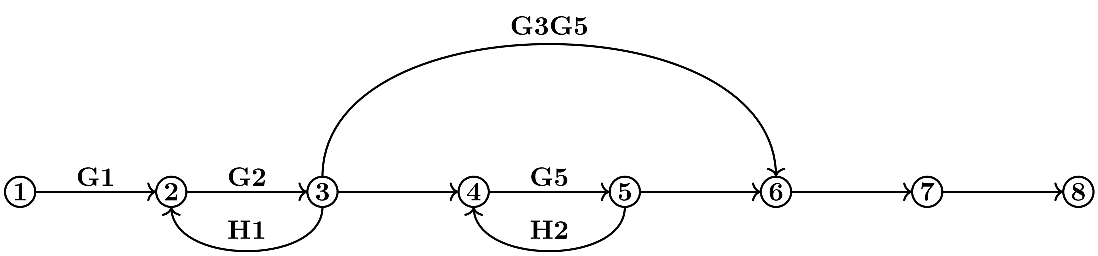
Given: As per diagram

Solution:

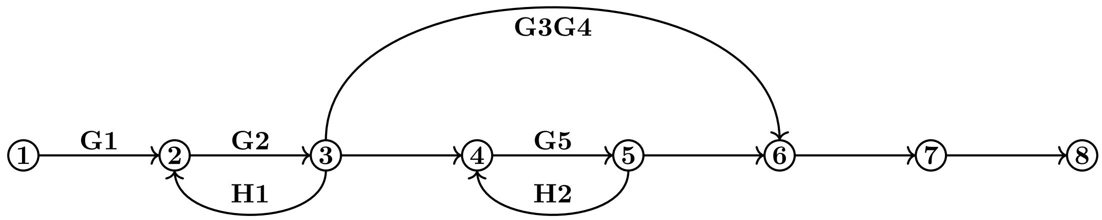
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



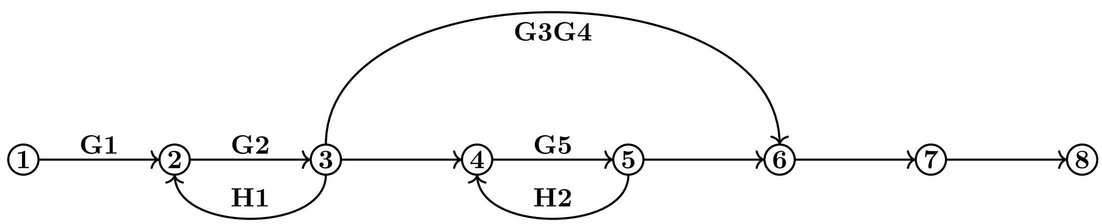
$$F_1 = G_1G_2G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_2G_3G_4$$

$$\Delta_2 = G_5H_2$$

Step 4. Identify all loops



$$L_1 = G_2H_1$$

$$L_2 = G_5H_1$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

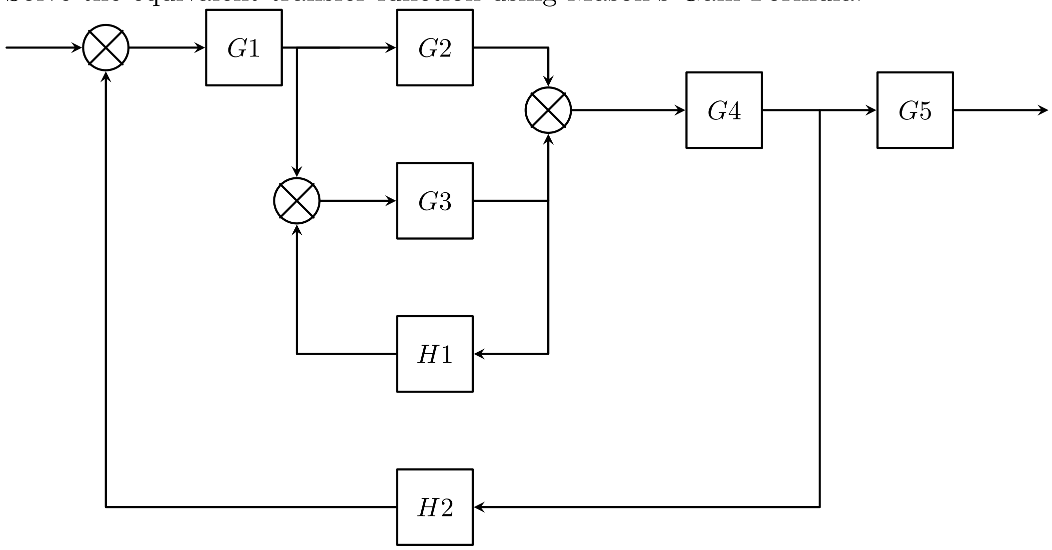
$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_5(1+0) + G_1G_2G_3G_4(1 + G_5H_2)}{1 + (G_2H_1 + G_5H_2) + (G_2G_5H_1H_2) + (0) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_5 + G_1G_2G_3G_4 + G_1G_2G_3G_4G_5H_2}{1 + G_2H_1 + G_5H_2 + G_2G_5H_1H_2}}$$

Problem 88

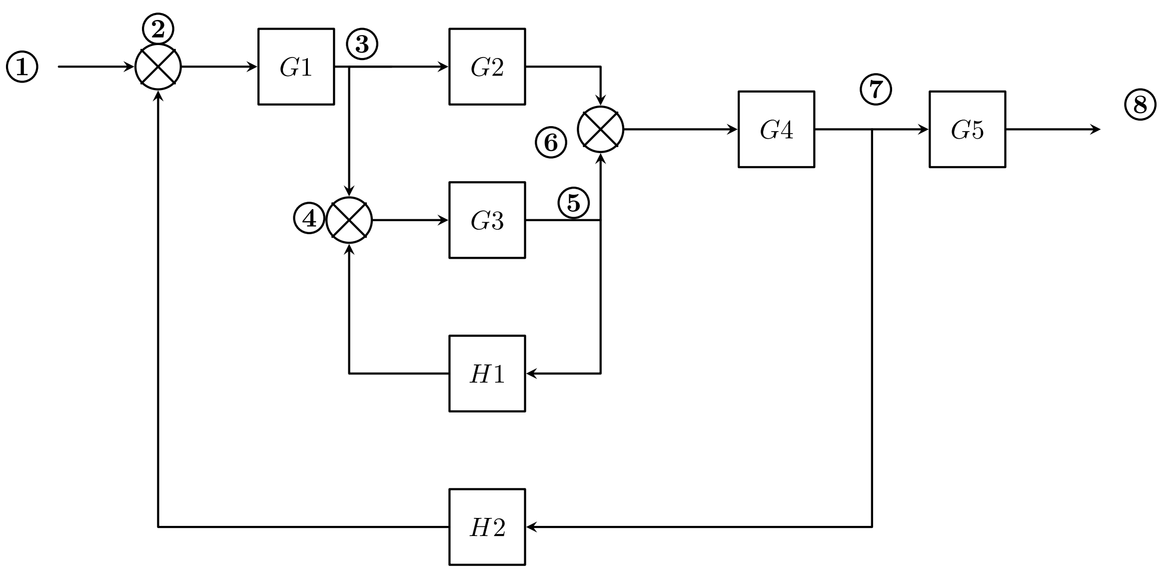
Solve the equivalent transfer function using Mason's Gain Formula:



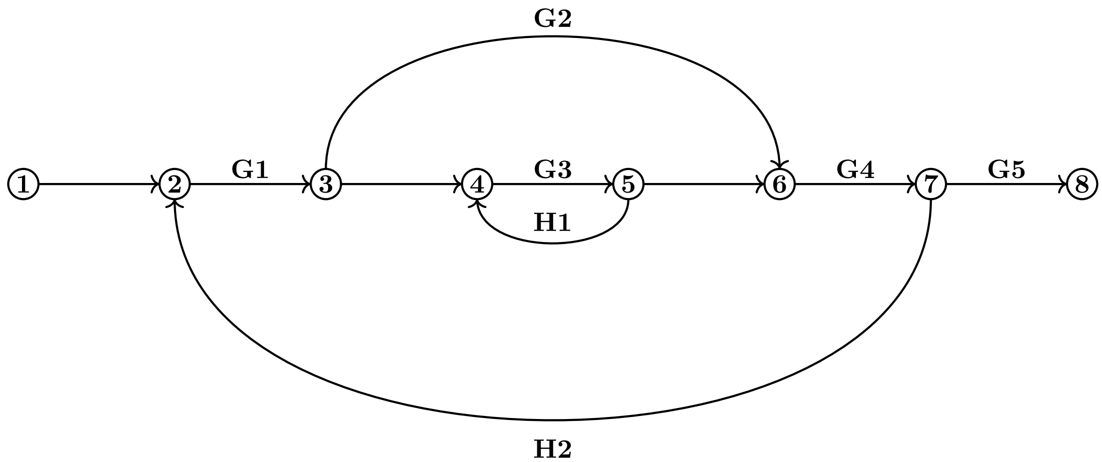
Given: As per diagram

Solution:

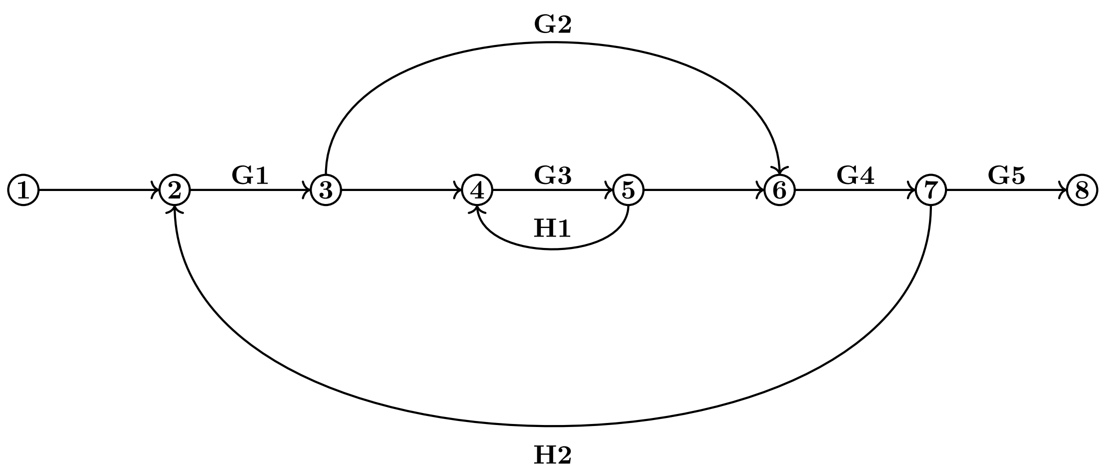
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



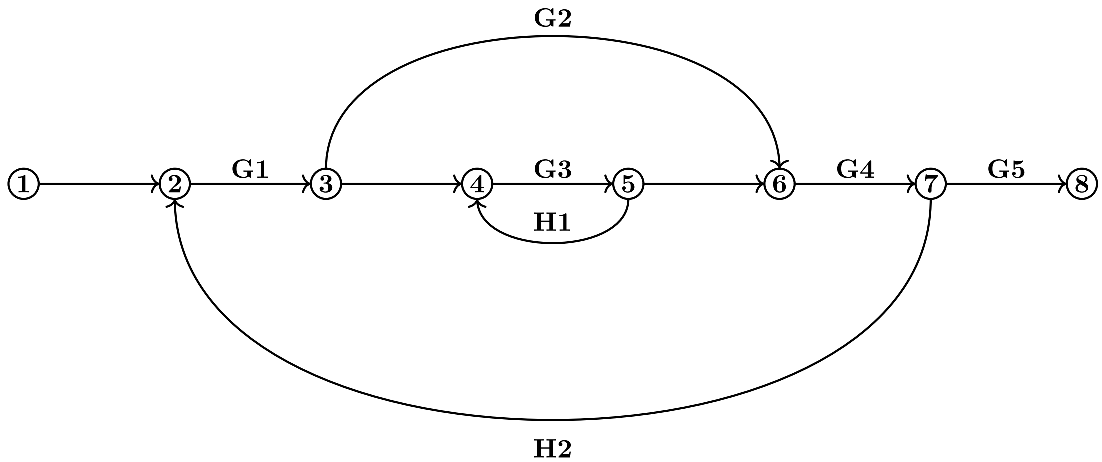
$$F_1 = G_1G_3G_4G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_2G_4G_5$$

$$\Delta_2 = G_3H_1$$

Step 4. Identify all loops



$$L_1 = G_3H_1$$

$$L_2 = G_1G_3G_4H_2$$

$$L_3 = G_1G_2G_4H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

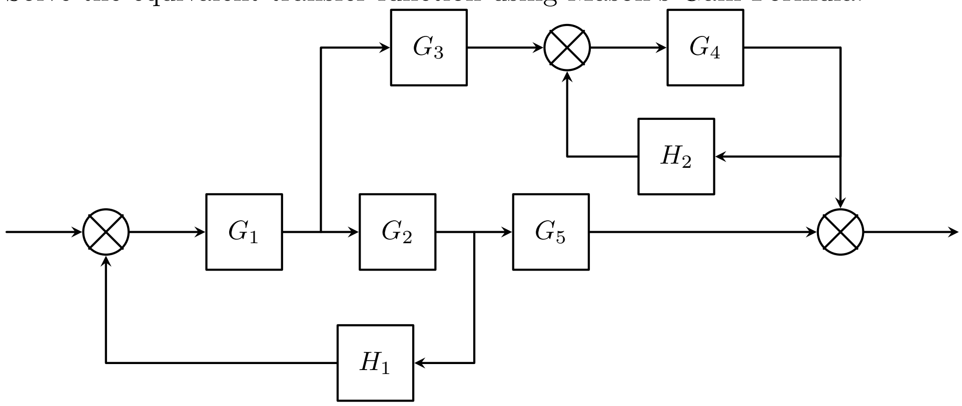
$(*)$ means not touching each other

$$TF = \frac{G_1G_3G_4G_5(1+0) + G_1G_2G_4G_5(1+G_3H_1)}{1 + (G_3H_1 + G_1G_3G_4H_2 + G_1G_2G_4H_2) + (G_3H_1 \cdot G_1G_2G_4H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_3G_4G_5 + G_1G_2G_4G_5 + G_1G_2G_3G_4G_5H_1}{1 + G_3H_1 + G_1G_3G_4H_2 + G_1G_2G_4H_2 + G_1G_2G_3G_4H_1H_2}}$$

Problem 89

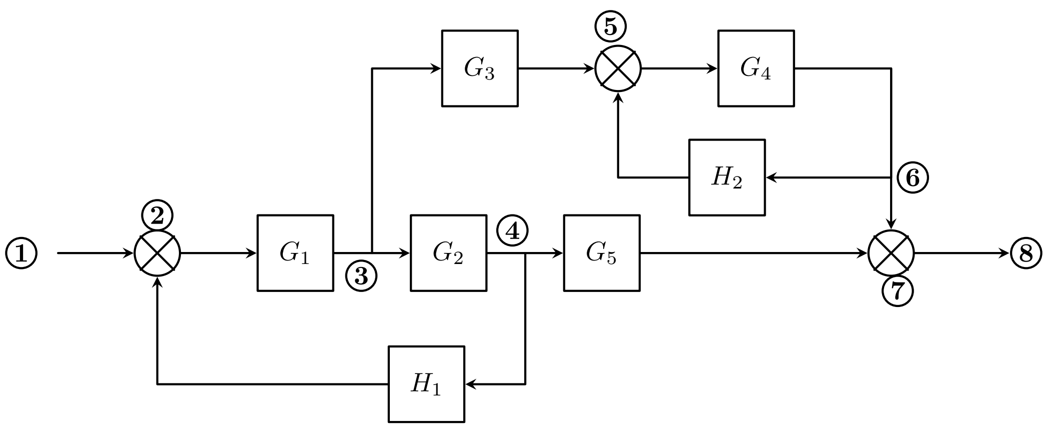
Solve the equivalent transfer function using Mason's Gain Formula:



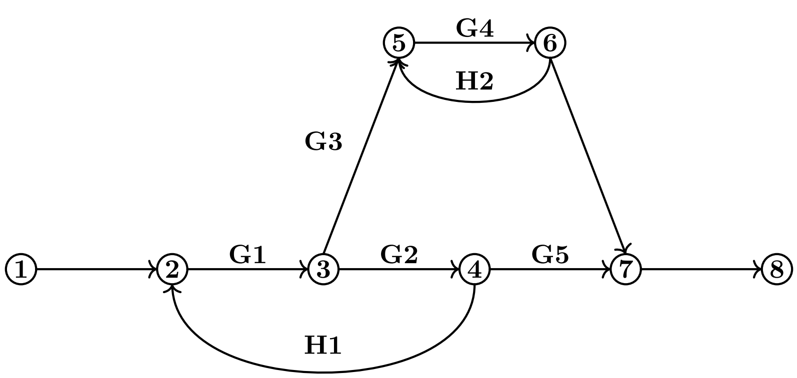
Given: As per diagram

Solution:

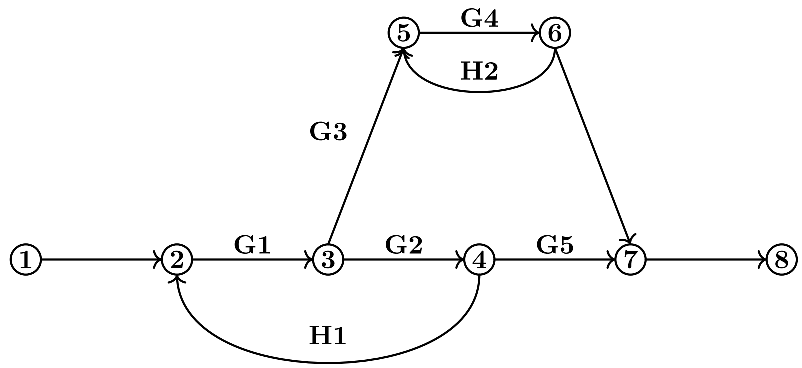
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



$$F_1 = G_1 G_2 G_5$$

$$\Delta_1 = G_4 H_2$$

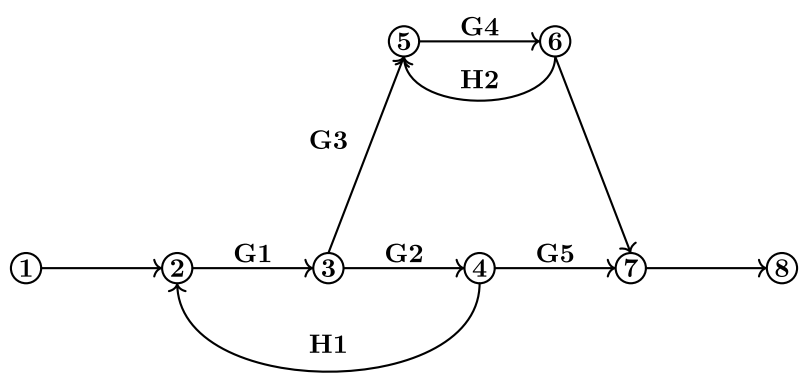
$$F_2 = G_1 G_3 G_4$$

$$\Delta_2 = 0$$

$$L_1 = G_1 G_2 H_1$$

$$L_2 = G_4 H_2$$

Step 4. Identify all loops



Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

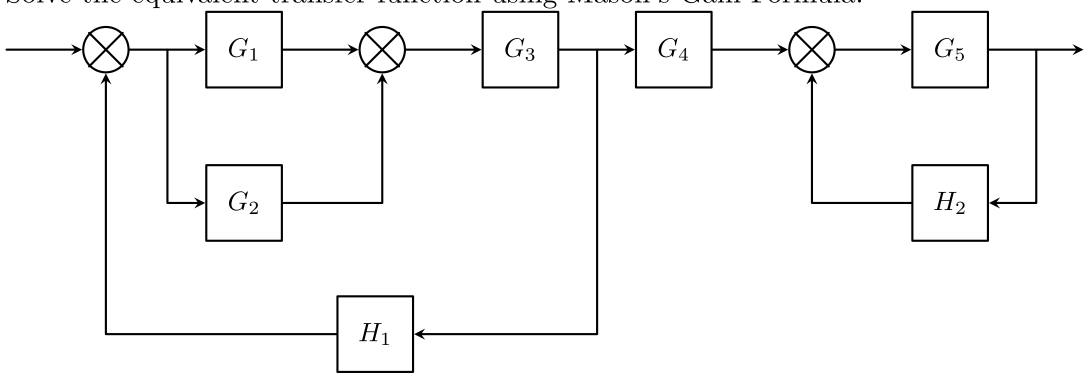
$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_5(1+G_4H_2) + G_1G_3G_4(1+0)}{1+(G_1G_2H_1+G_4H_2)+(G_1G_2H_1 \cdot G_4H_2)+(0)+...}$$

$$\boxed{TF = \frac{G_1G_2G_5 + G_1G_3G_4 + G_1G_2G_4G_5H_2}{1+G_1G_2H_1+G_4H_2+G_1G_2G_4H_1H_2}}$$

Problem 90

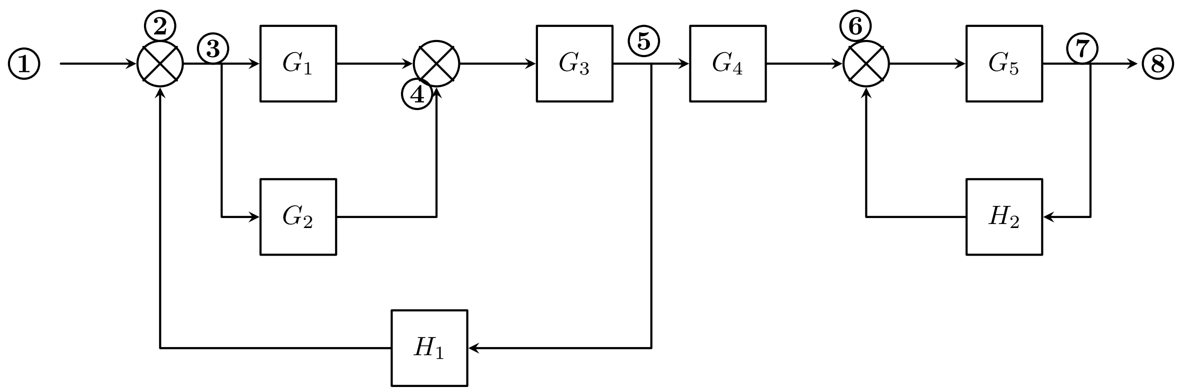
Solve the equivalent transfer function using Mason's Gain Formula:



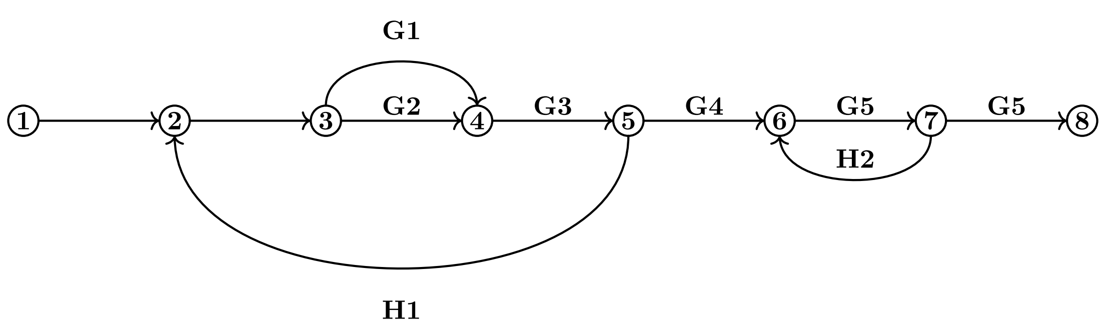
Given: As per diagram

Solution:

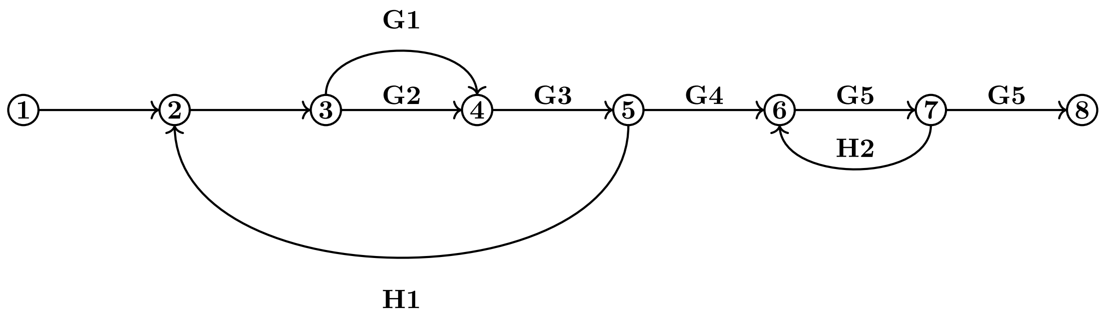
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



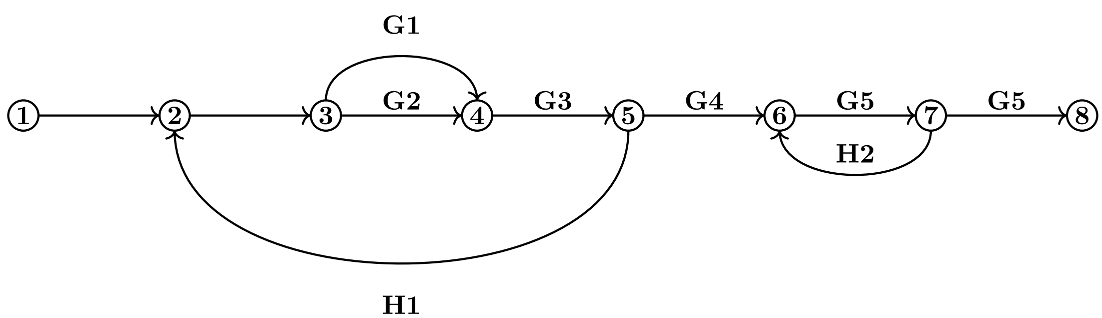
$$F_1 = G_1G_3G_4G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_2G_3G_4G_5$$

$$\Delta_2 = 0$$

Step 4. Identify all loops



$$L_1 = G_2G_3H_1$$

$$L_2 = G_1G_3H_1$$

$$L_3 = G_5H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

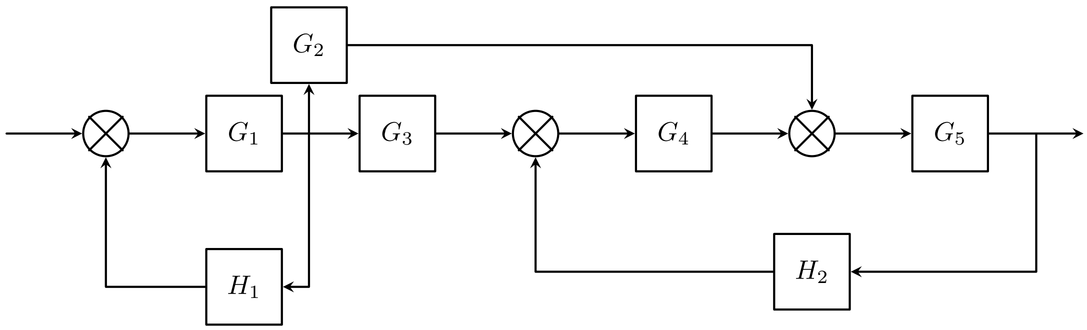
$(*)$ means not touching each other

$$TF = \frac{G_1G_3G_4G_5(1+0) + G_2G_3G_4G_5(1+0)}{1 + (G_2G_3H_1 + G_1G_3H_1 + G_5H_2) + (G_2G_3H_1 \cdot G_5H_2 + G_1G_3H_1 \cdot G_5H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_3G_4G_5 + G_2G_3G_4G_5}{1 + G_1G_3H_1 + G_2G_3H_1 + G_5H_2 + G_1G_3G_5H_1H_2 + G_2G_3G_5H_1H_2}}$$

Problem 91

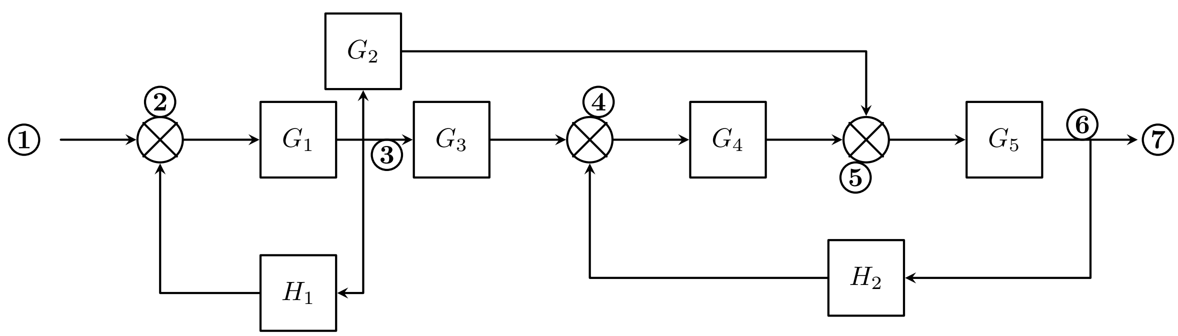
Solve the equivalent transfer function using Mason's Gain Formula:



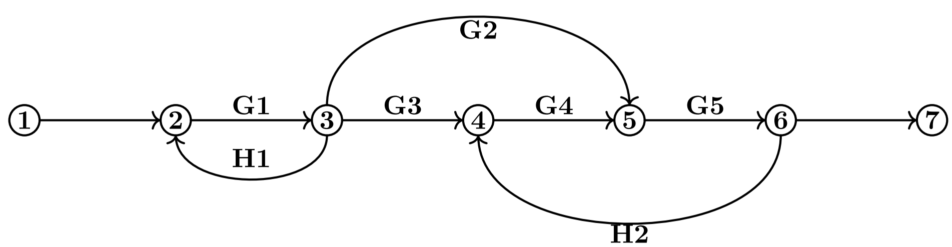
Given: As per diagram

Solution:

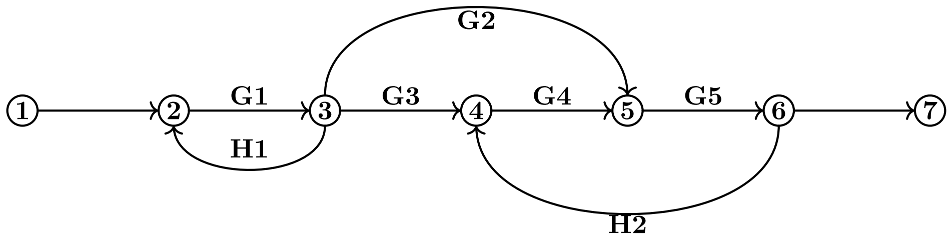
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



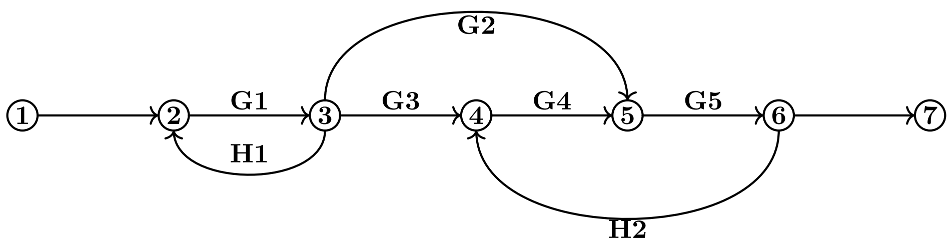
$$F_1 = G_1G_2G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_3G_4G_5$$

$$\Delta_2 = 0$$

Step 4. Identify all loops



$$L_1 = G_1H_1$$

$$L_2 = G_4G_5H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

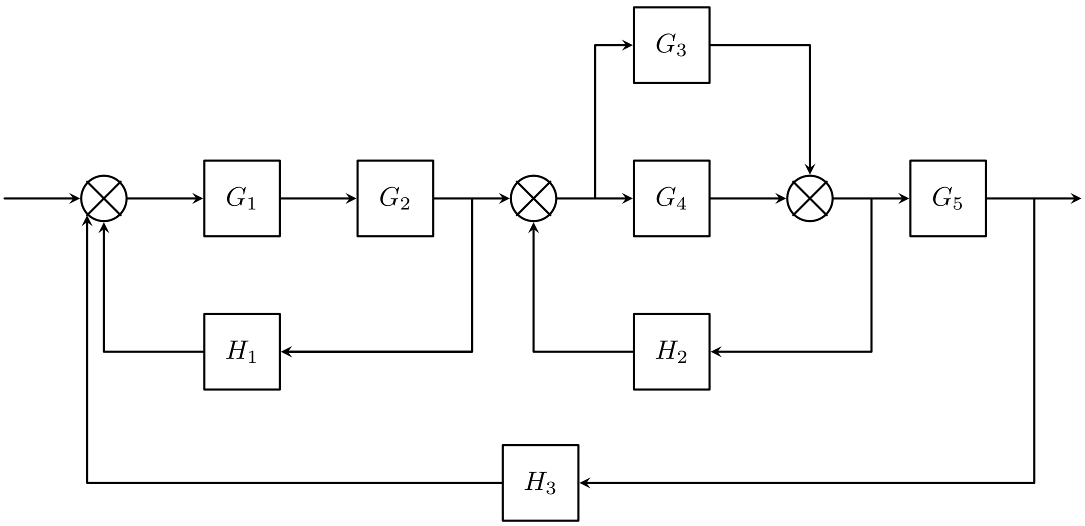
$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_5(1+0) + G_1G_3G_4G_5(1+0)}{1 + (G_1H_1 + G_4G_5H_2) + (G_1H_1 \cdot G_4G_5H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_5 + G_1G_3G_4G_5}{1 + G_1H_1 + G_4G_5H_2 + G_1G_4G_5H_1H_2}}$$

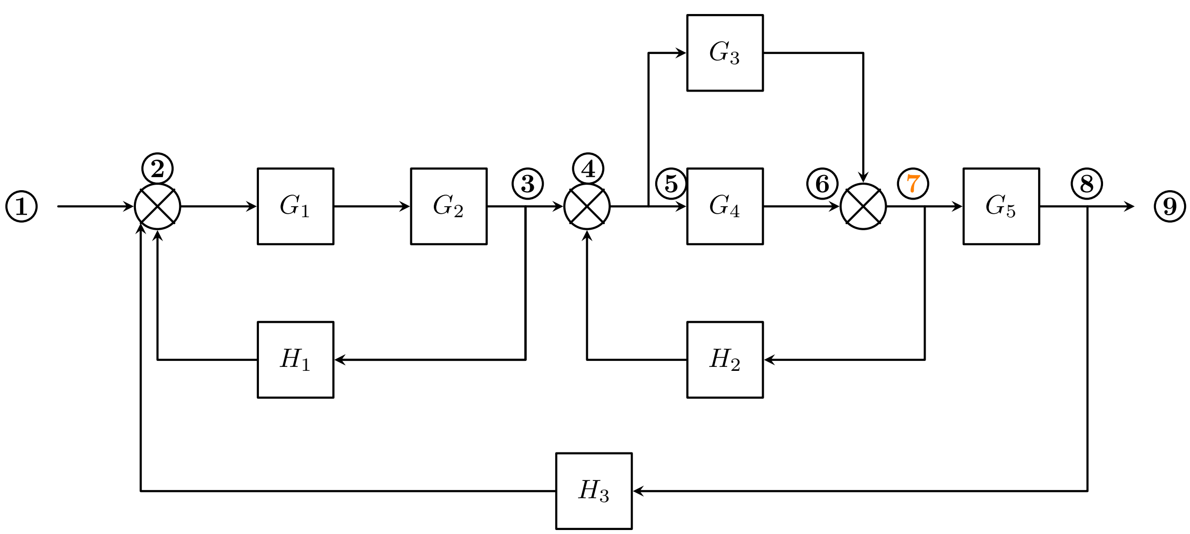
Problem 92
Solve the equivalent transfer function using Mason's Gain Formula:



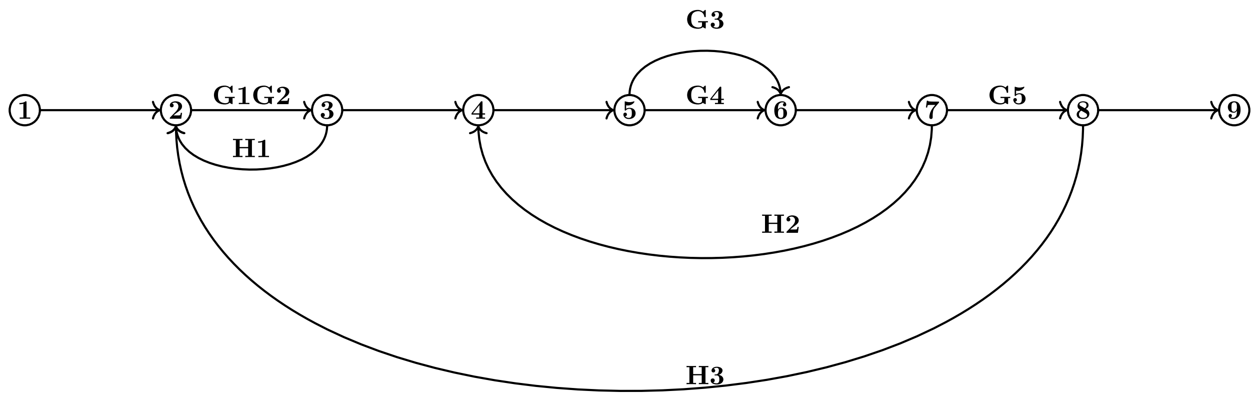
Given: As per diagram

Solution:

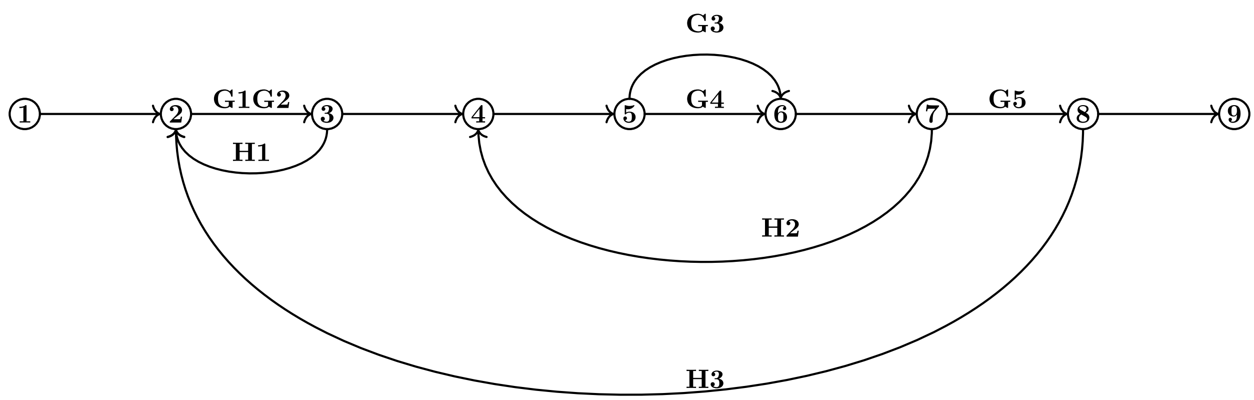
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



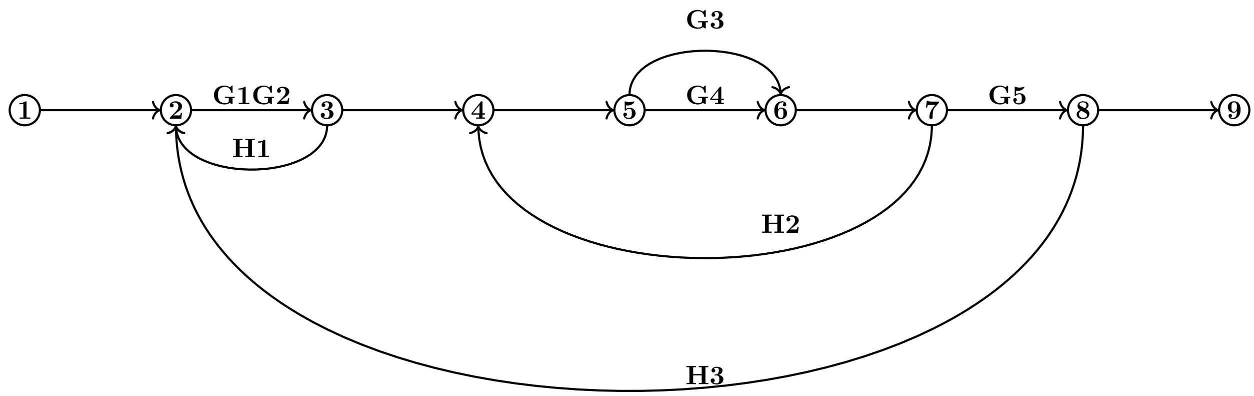
$$F_1 = G_1G_2G_3G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_2G_4G_5$$

$$\Delta_2 = 0$$

Step 4. Identify all loops



$$L_1 = G_1G_2H_1$$

$$L_2 = G_3H_2$$

$$L_3 = G_4H_2$$

$$L_4 = G_1G_2G_3G_5H_3$$

$$L_5 = G_1G_2G_4G_5H_3$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

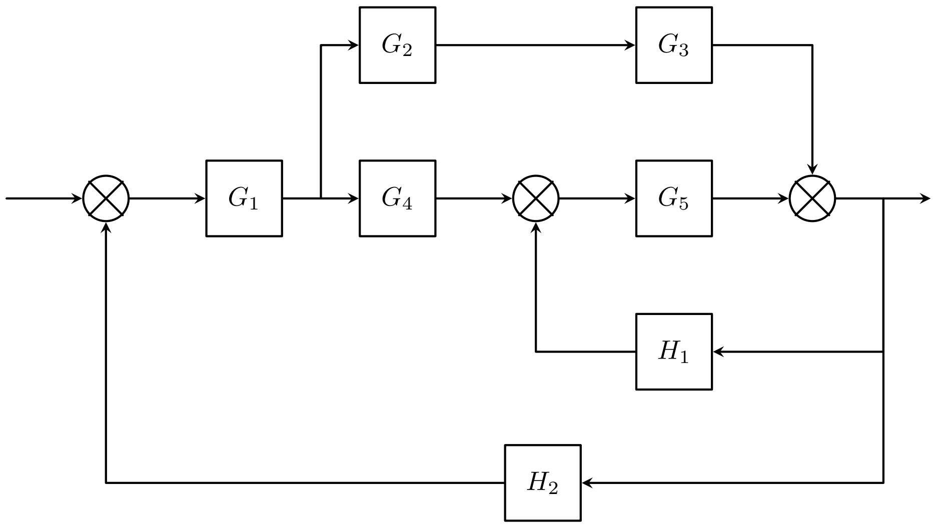
$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_3(1+0) + G_1G_2G_4G_5(1+0)}{1 + (G_1G_2H_1 + G_3H_2 + G_4H_2 + G_1G_2G_3G_5H_3 + G_1G_2G_4G_5H_3) + (G_1G_2H_1 \cdot G_3H_2 + G_1G_2H_1 \cdot G_4H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_3G_5 + G_1G_2G_4G_5}{1 + G_1G_2H_1 + G_3H_2 + G_4H_2 + G_1G_2G_3G_5H_3 + G_1G_2G_4G_5H_3 + G_1G_2G_3H_1H_2 + G_1G_2G_4H_1H_2}}$$

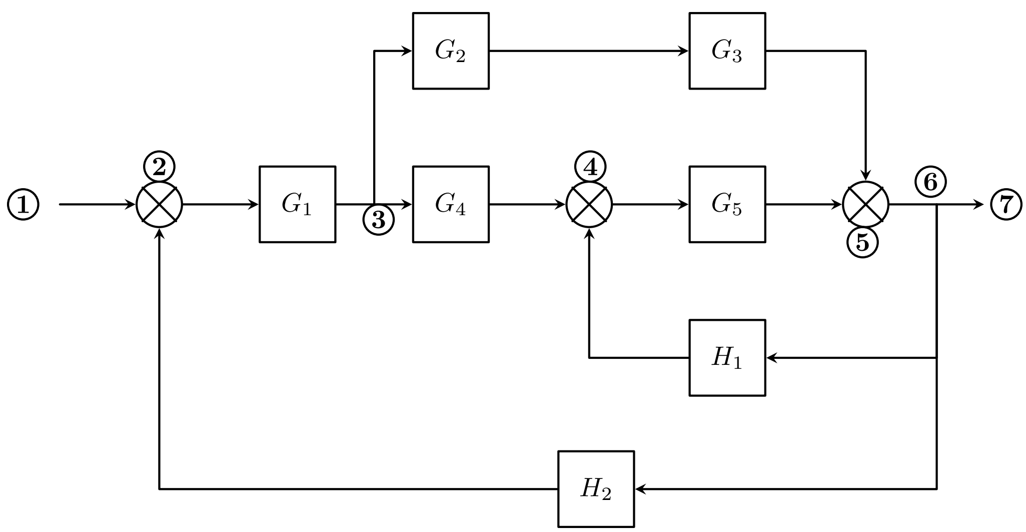
Problem 93
Solve the equivalent transfer function using Mason's Gain Formula:



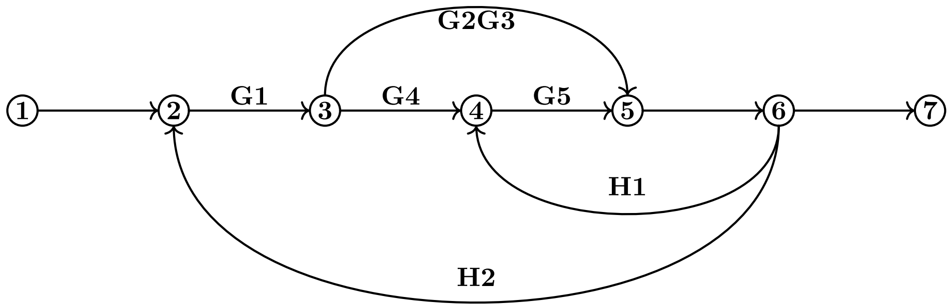
Given: As per diagram

Solution:

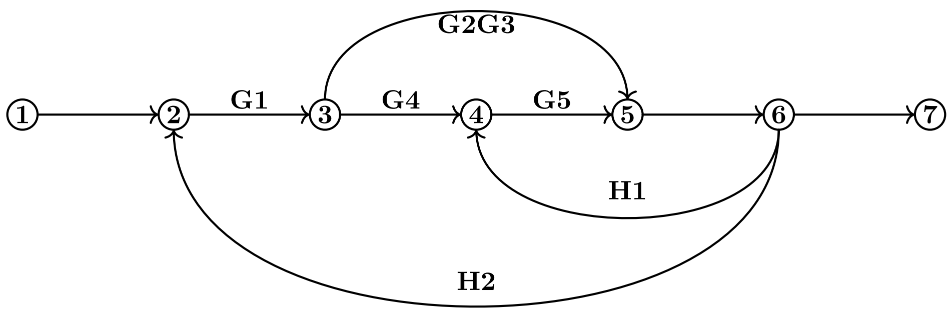
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



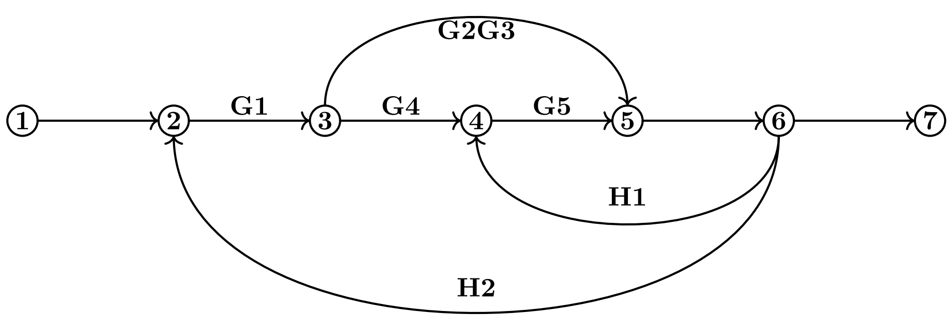
$$F_1 = G_1G_2G_3$$

$$\Delta_1 = 0$$

$$F_2 = G_1G_4G_5$$

$$\Delta_2 = 0$$

Step 4. Identify all loops



$$L_1 = G_5H_1$$

$$L_2 = G_1G_4G_5H_2$$

$$L_3 = G_1G_2G_3H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

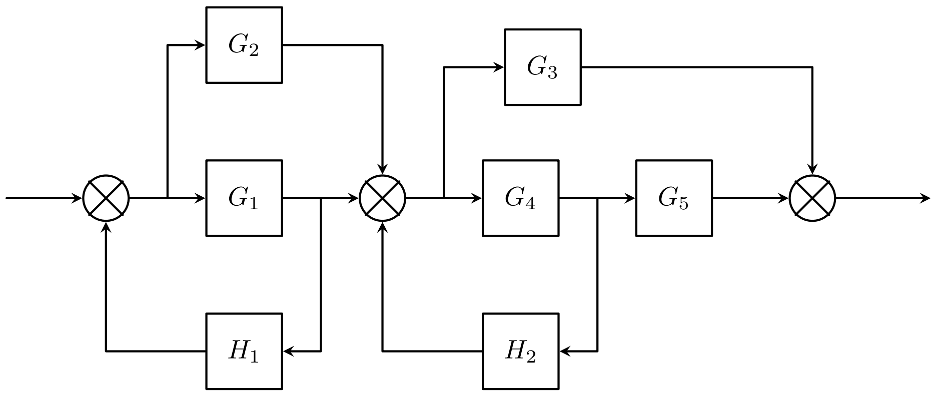
$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_3(1+0) + G_1G_4G_5(1+0)}{1 + (G_5H_1 + G_1G_4G_5H_2 + G_1G_2G_3H_2) + (0) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_3 + G_1G_4G_5}{1 + G_5H_1 + G_1G_4G_5H_2 + G_1G_2G_3H_2}}$$

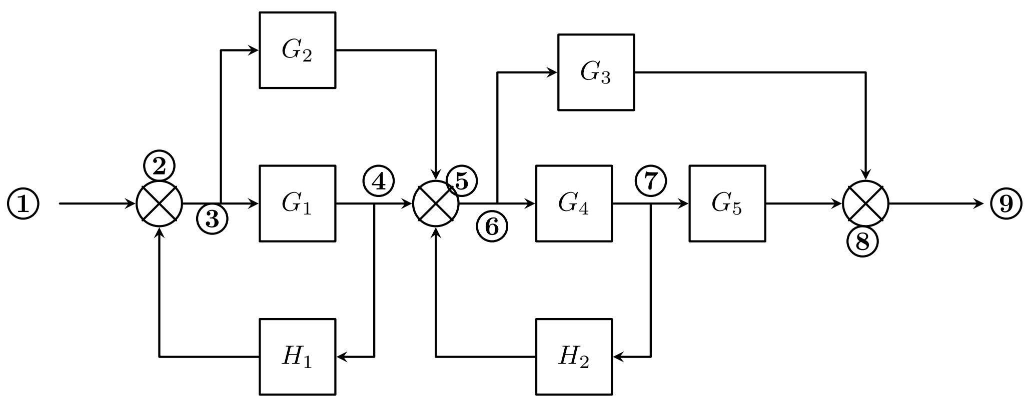
Problem 94
Solve the equivalent transfer function using Mason's Gain Formula:



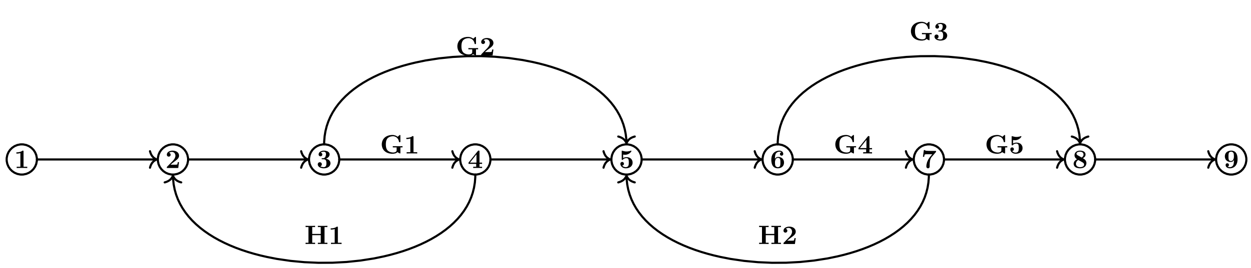
Given: As per diagram

Solution:

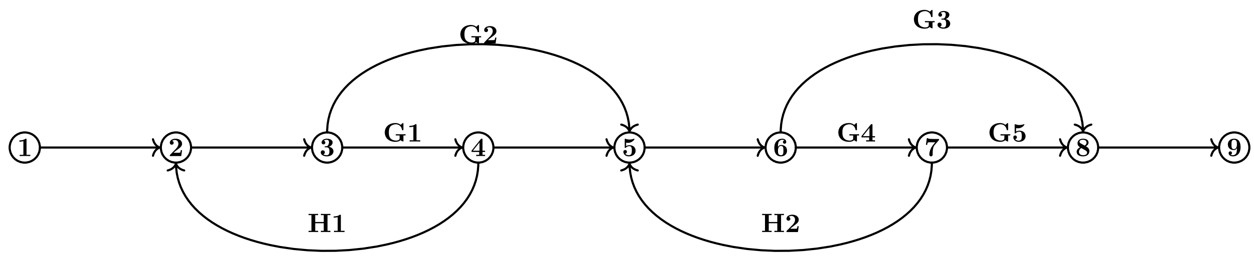
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them if it exist



$$F_1 = G_1 G_4 G_5$$

$$\Delta_1 = 0$$

$$F_2 = G_2 G_4 G_5$$

$$\Delta_2 = 0$$

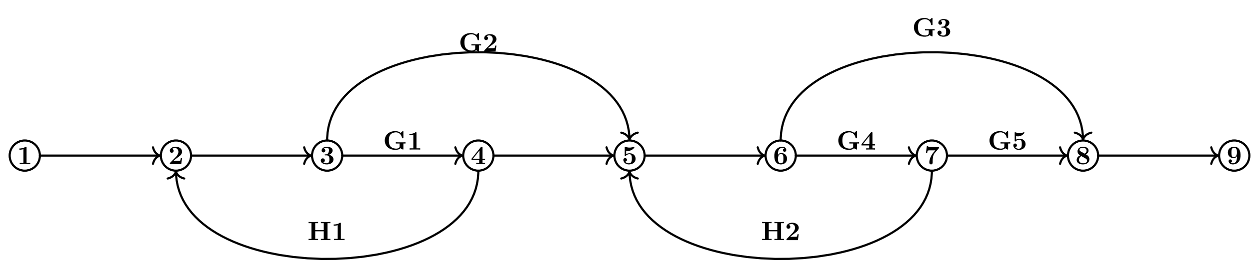
$$F_3 = G_1 G_3$$

$$\Delta_3 = 0$$

$$F_4 = G_2 G_3$$

$$\Delta_4 = 0$$

Step 4. Identify all loops



$$L_1 = G_1 H_1$$

$$L_2 = G_4 H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

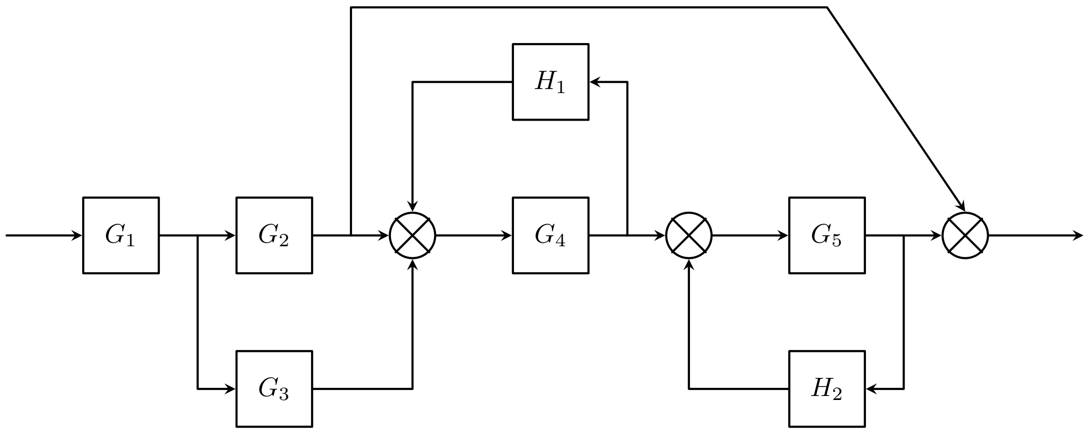
$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops\ *) + (set\ of\ 3\ *) + (set\ of\ 4*) + ...}$$

$(*)$ means not touching each other

$$TF = \frac{G_1G_4G_5(1+0) + G_2G_4G_5(1+0) + G_1G_3(1+0) + G_2G_3(1+0)}{1 + (G_1H_1 + G_4H_2) + (G_1H_1 \cdot G_4H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_4G_5 + G_2G_4G_5 + G_1G_3 + G_2G_3}{1 + G_1H_1 + G_4H_2 + G_1G_4H_1H_2}}$$

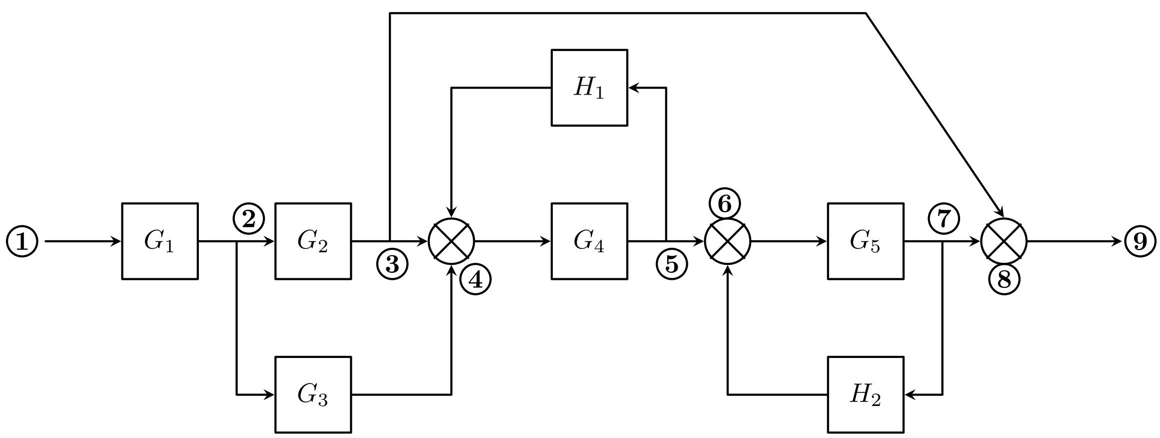
Problem 95
Solve the equivalent transfer function using Mason's Gain Formula:



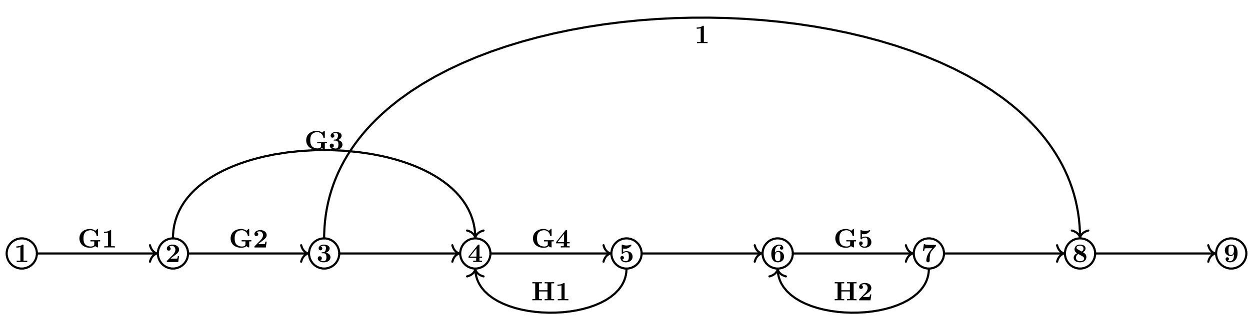
Given: As per diagram

Solution:

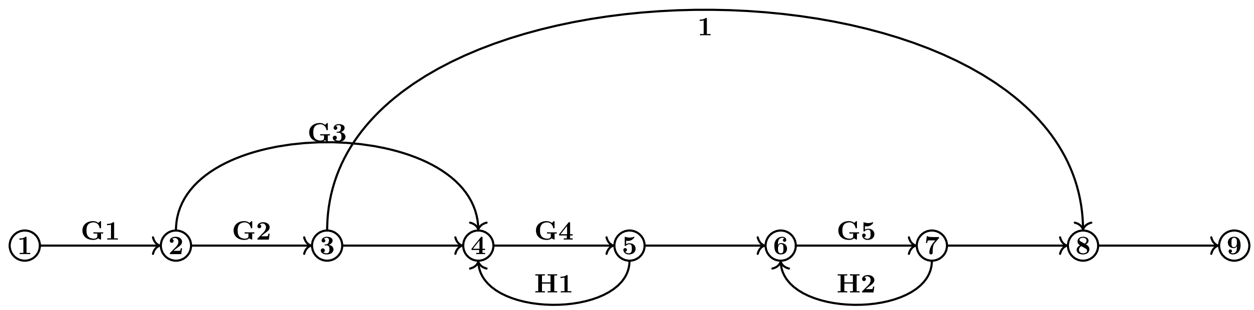
Step 1. Identify the longest forward path, assign all terminals, summing points, and nodes as separate points.



Step 2. Generate the signal flow graph.



Step 3. Identify all possible forward paths, including the loops that does not touch them (if there exist)



$$F_1 = G_1G_2G_4G_5$$

$$\Delta_1 = 0$$

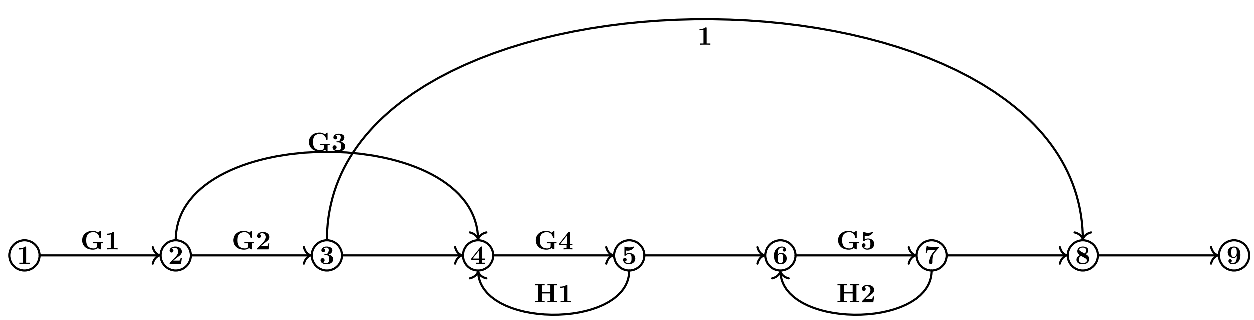
$$F_2 = G_1G_2$$

$$\Delta_2 = G_4H_1 \ \& \ G_5H_2$$

$$F_3 = G_1G_3G_4G_5$$

$$\Delta_3 = 0$$

Step 4. Identify all loops



$$L_1 = G_4H_1$$

$$L_2 = G_5H_2$$

Step 5. Use the Mason's Gain Formula (positive loop technique)

$$TF = \frac{\Sigma(forward\ path)(1 + loop\ not\ touching\ the\ forward\ path)}{1 + (loops) + (pair\ of\ loops *) + (set\ of\ 3 *) + (set\ of\ 4*) + ...}$$

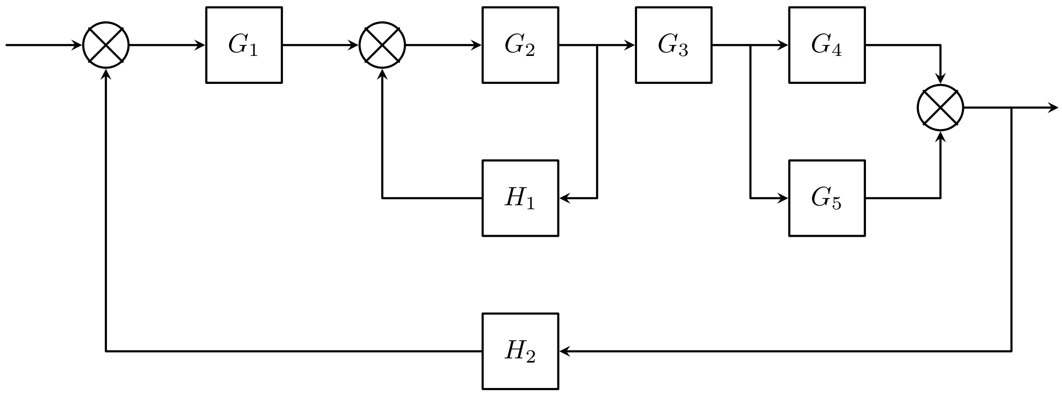
$(*)$ means not touching each other

$$TF = \frac{G_1G_2G_4G_5(1+0) + G_1G_2(1 + G_4H_1 + G_5H_2 + G_4G_5H_1H_2) + G_1G_3G_4G_5}{1 + (G_4H_1 + G_5H_2) + (G_4H_1 \cdot G_5H_2) + (0) + ...}$$

$$\boxed{TF = \frac{G_1G_2G_4G_5 + G_1G_2 + G_1G_2G_4H_1 + G_1G_2G_5H_2 + G_1G_2G_4G_5H_1H_2 + G_1G_3G_4G_5}{1 + G_4H_1 + G_5H_2 + G_4G_5H_1H_2}}$$

Problem 96

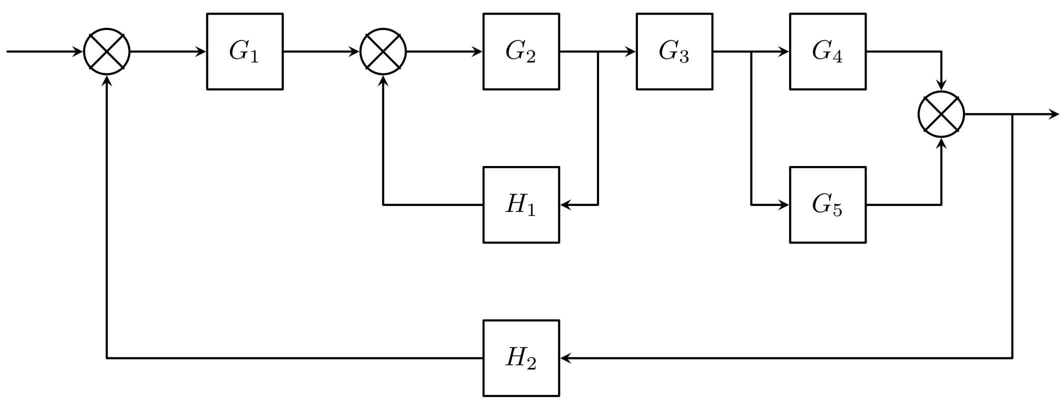
Simplify the following Block Diagram



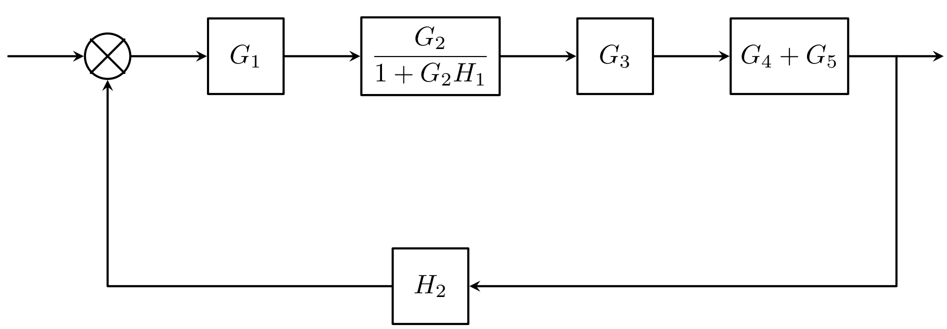
Given: As per diagram

Solution:

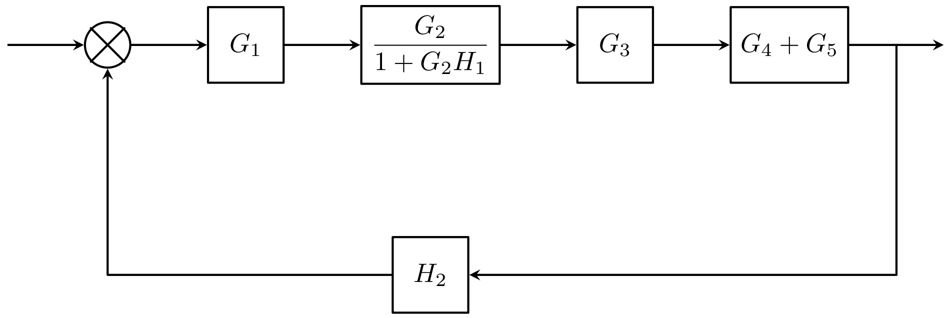
Step 1



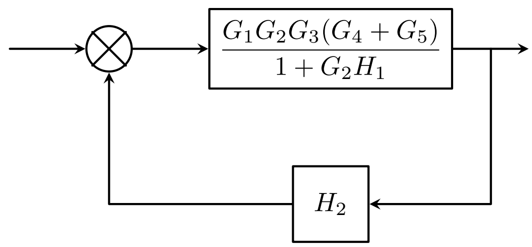
Simplify



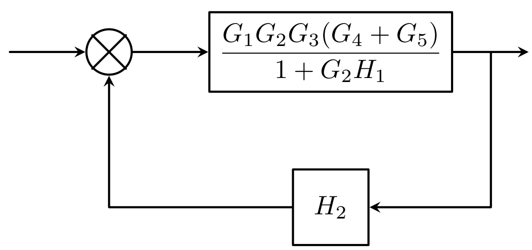
Step 2



Simplify



Step 3

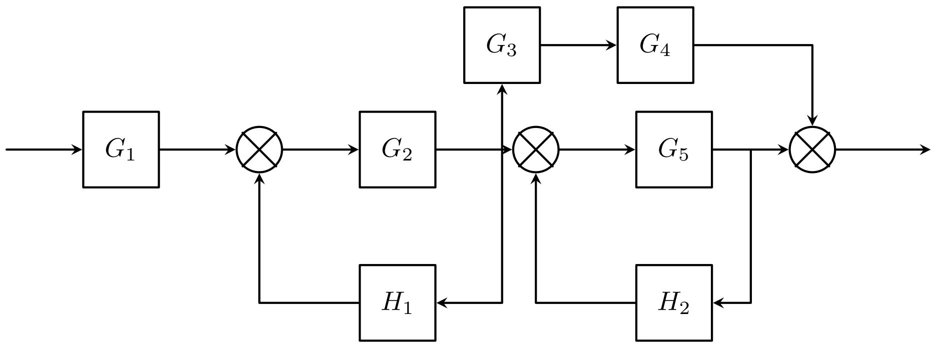


Simplify (Final Transfer Function)

$$TF = \frac{\left[\dfrac{G_1G_2G_3(G_4+G_5)}{1+G_2H_1}\right]}{1+\left[\dfrac{G_1G_2G_3(G_4+G_5)}{1+G_2H_1}\right]H_2}$$

$$\boxed{TF = \frac{G_1G_2G_3(G_4+G_5)}{1+G_2H_1+G_1G_2G_3(G_4+G_5)+H_2}}$$

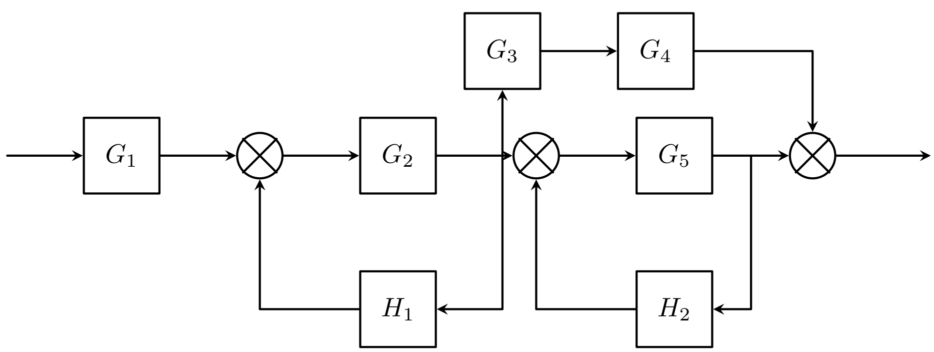
Problem 97
Simplify the following Block Diagram



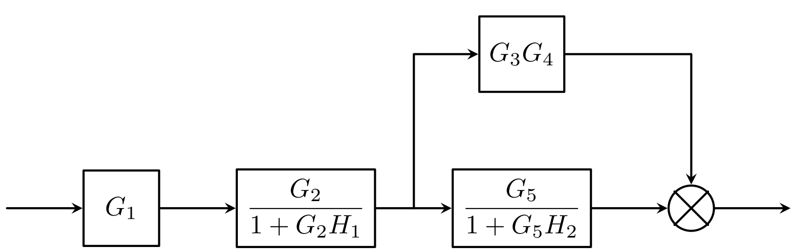
Given: As per diagram

Solution:

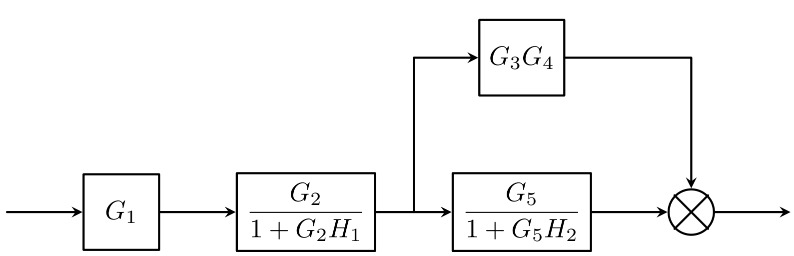
Step 1



Simplify



Step 2



Simplify

$$\longrightarrow \boxed{G_1} \longrightarrow \boxed{\frac{G_2}{1+G_2H_1}} \longrightarrow \boxed{G_3G_4 + \frac{G_5}{1+G_5H_2}} \longrightarrow$$

Step 3

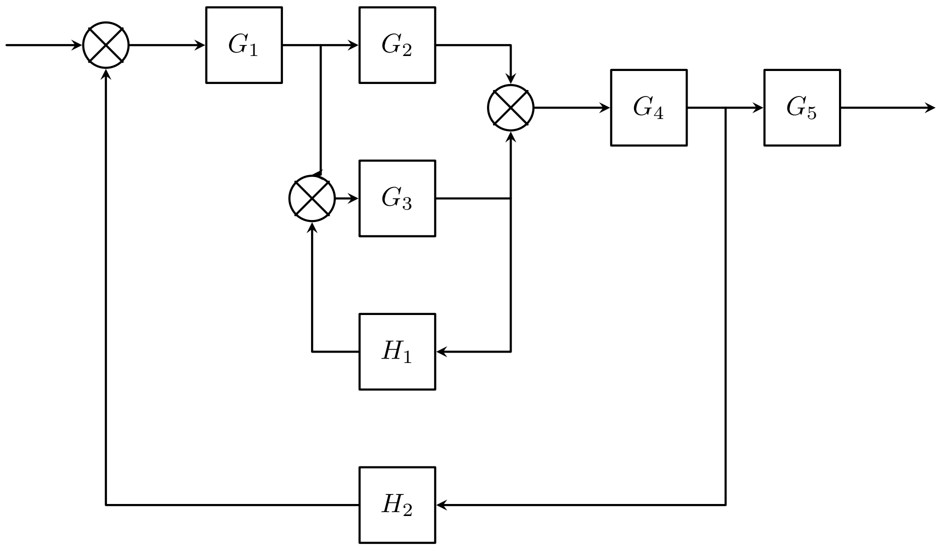
$$\longrightarrow \boxed{G_1} \longrightarrow \boxed{\frac{G_2}{1+G_2H_1}} \longrightarrow \boxed{G_3G_4 + \frac{G_5}{1+G_5H_2}} \longrightarrow$$

Simplify (Final Transfer Function)

$$TF = (G_1)\left(\frac{G_2}{1+G_2H_1}\right)\left(G_3G_4 + \frac{G_5}{1+G_5H_2}\right)$$

$$\boxed{TF = \frac{G_1G_2G_5 + G_1G_2G_3G_4 + G_1G_2G_3G_4G_5H_2}{1 + G_2H_1 + G_5H_2 + G_2G_5H_1H_2}}$$

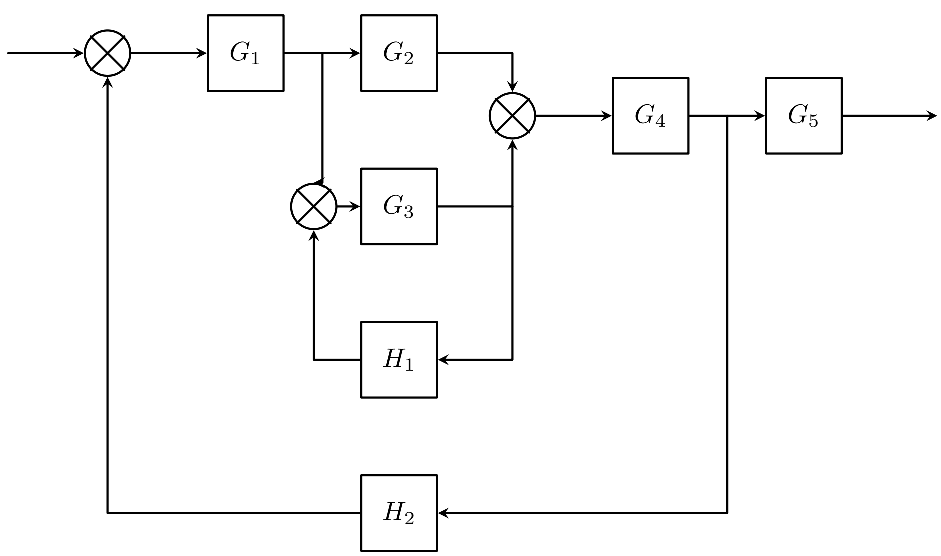
Problem 98
Simplify the following Block Diagram using Block Diagram Algebra



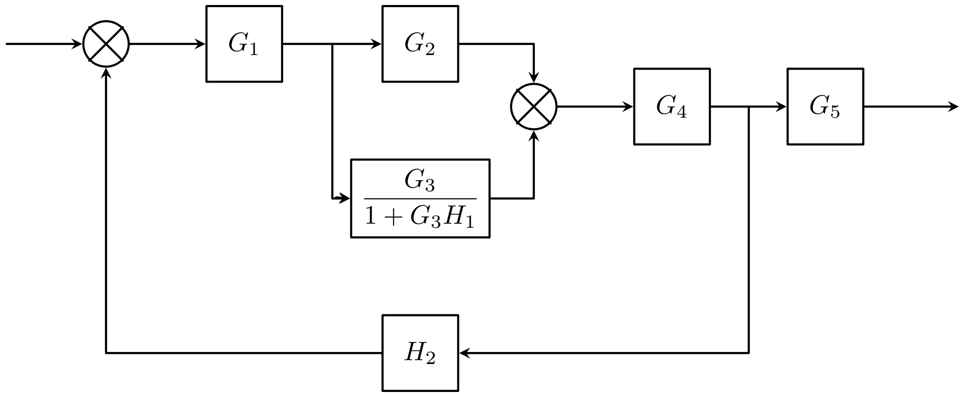
Given: As per diagram

Solution:

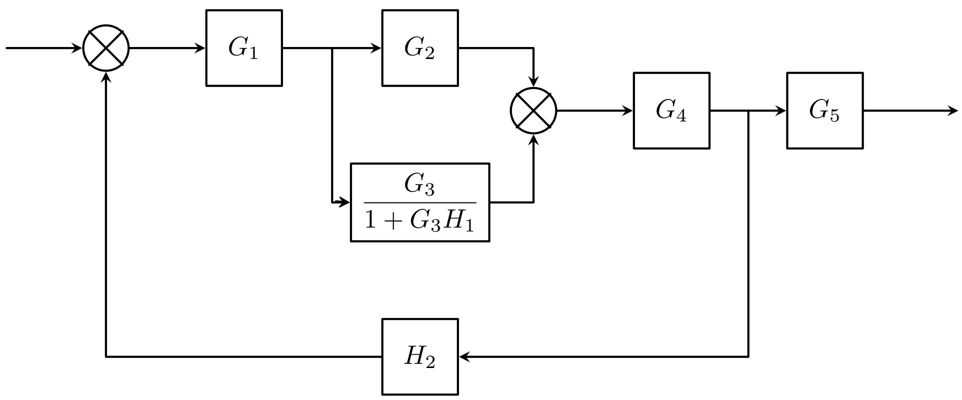
Step 1



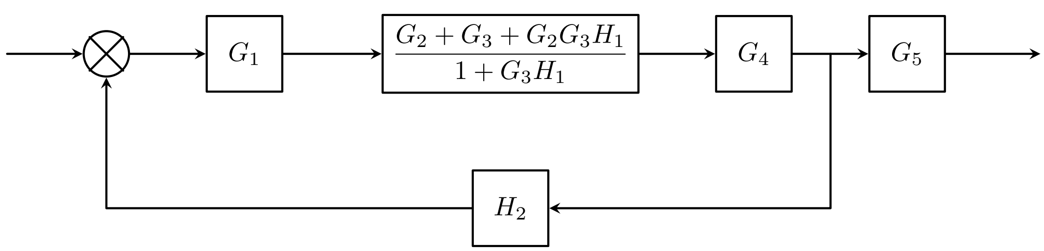
Simplify



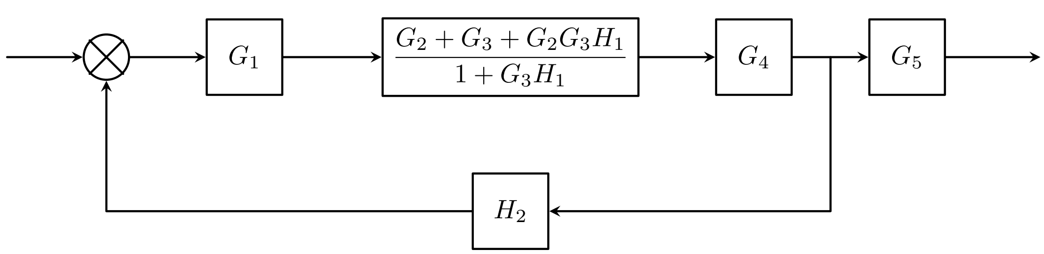
Step 2



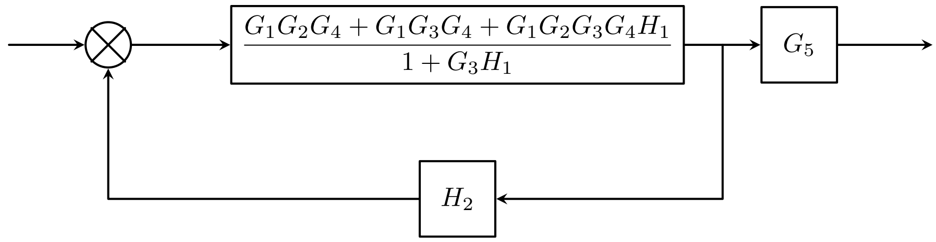
Simplify



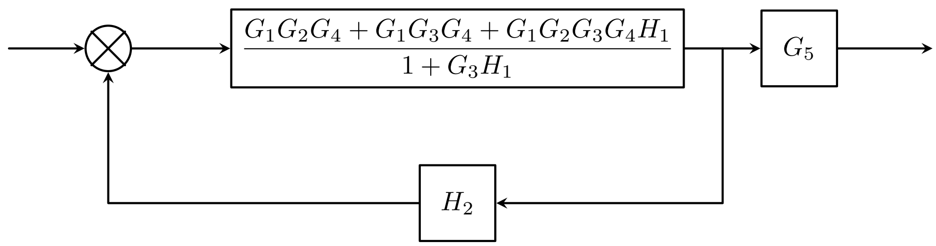
Step 3



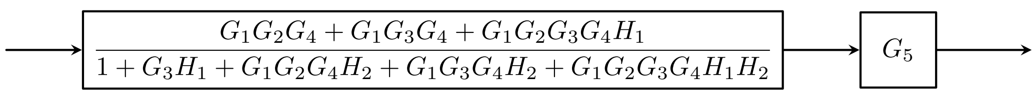
Simplify



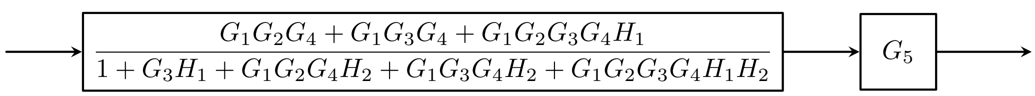
Step 4



Simplify



Step 5

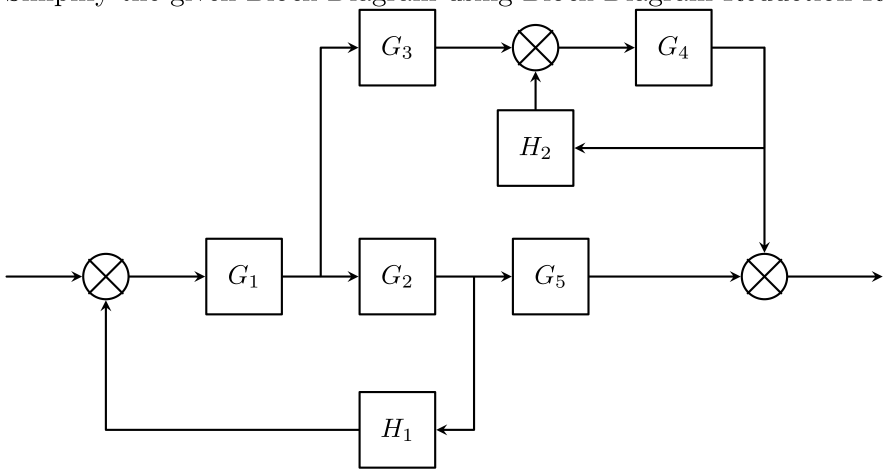


Simplify (Final Transfer Function)

$$TF = \left(\frac{G_1G_2G_4 + G_1G_3G_4 + G_1G_2G_3G_4H_1}{1 + G_3H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2 + G_1G_2G_3G_4H_1H_2} \right) (G_5)$$

$$\boxed{TF = \frac{G_1G_2G_4G_5 + G_1G_3G_4G_5 + G_1G_2G_3G_4G_5H_1}{1 + G_3H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2 + G_1G_2G_3G_4H_1H_2}}$$

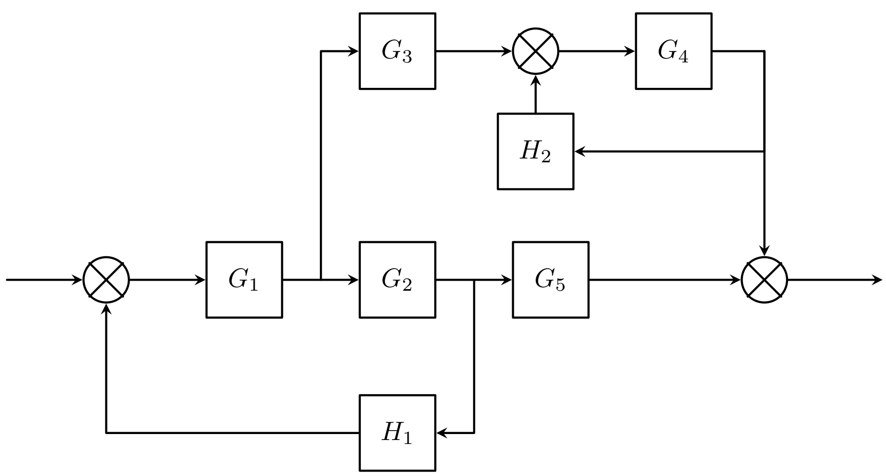
Problem 99
Simplify the given Block Diagram using Block Diagram Reduction Rules.



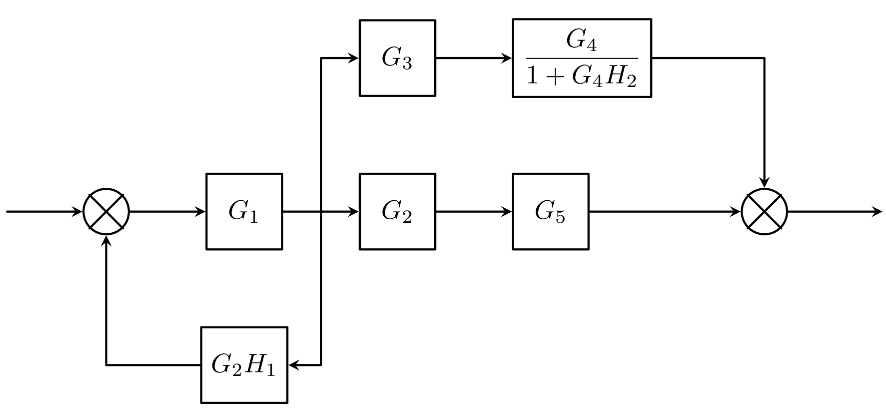
Given: As per diagram

Solution:

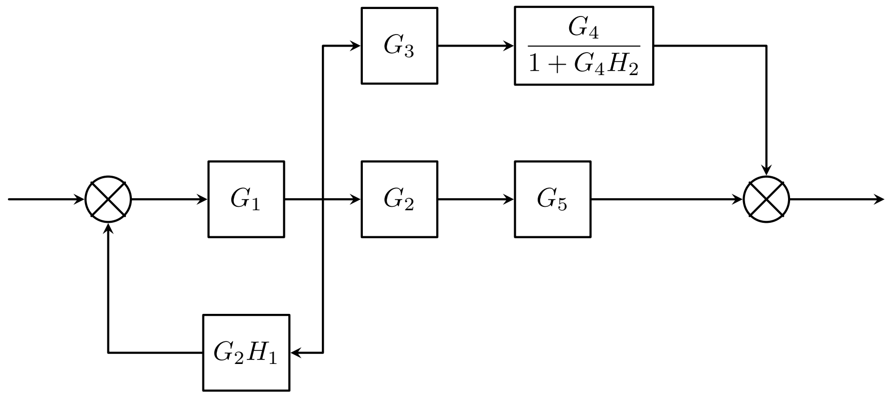
Step 1



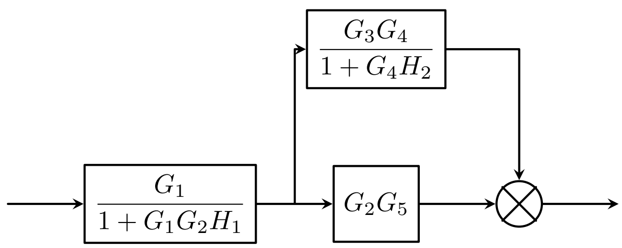
Simplify



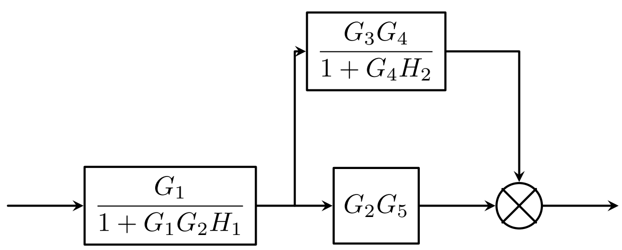
Step 2



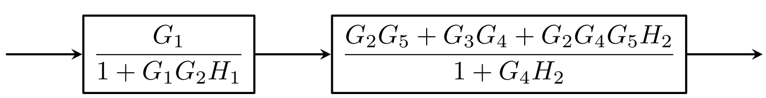
Simplify



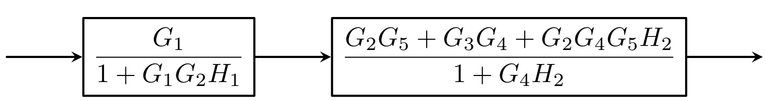
Step 3



Simplify



Step 4



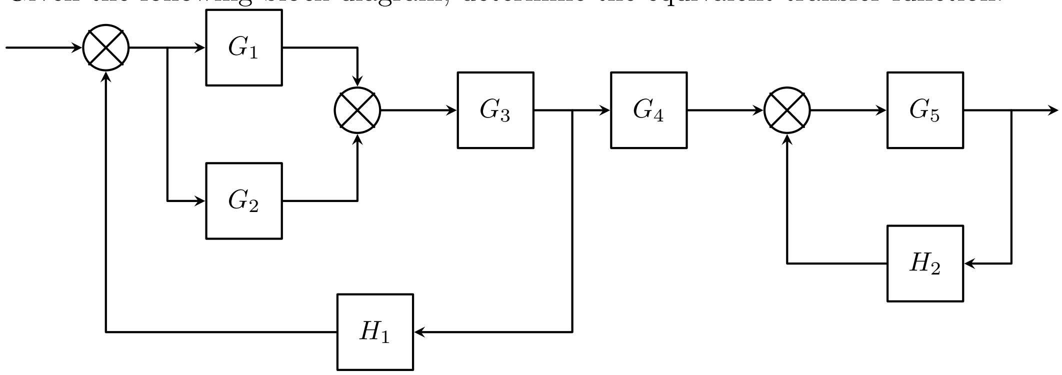
Simplify (Final Transfer Function)

$$G(s) = \left(\frac{G_1}{1+G_1G_2H_1}\right)\left(\frac{G_2G_5+G_3G_4+G_2G_4G_5H_2}{1+G_4H_2}\right)$$

$$\boxed{G(s) = \frac{G_1G_2G_5+G_1G_3G_4+G_1G_2G_4G_5H_2}{1+G_1G_2H_1+G_4H_2+G_1G_2G_4H_1H_2}}$$

Problem 100

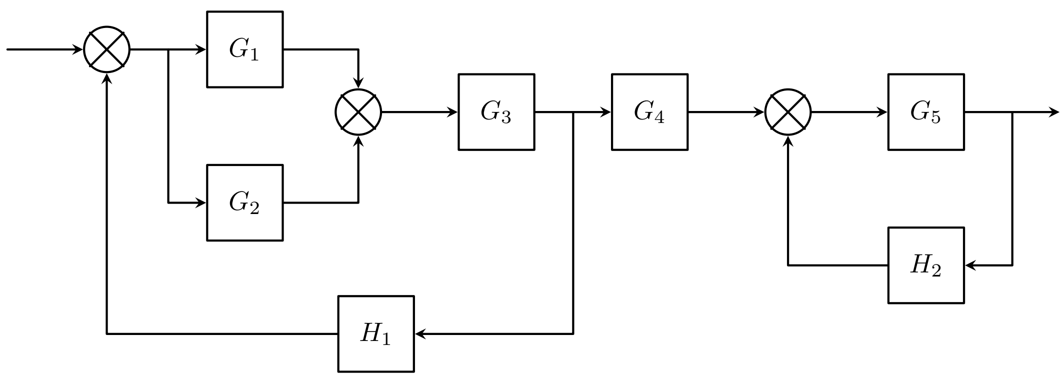
Given the following block diagram, determine the equivalent transfer function.



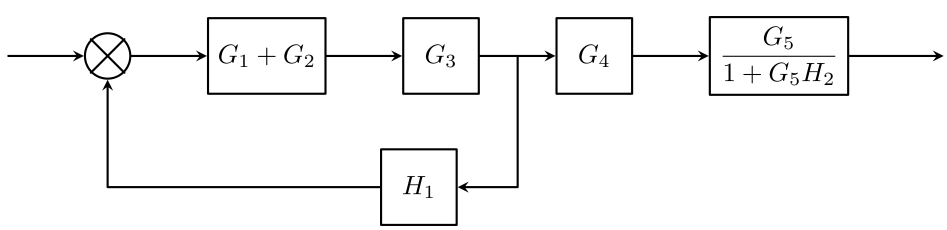
Given: As per diagram

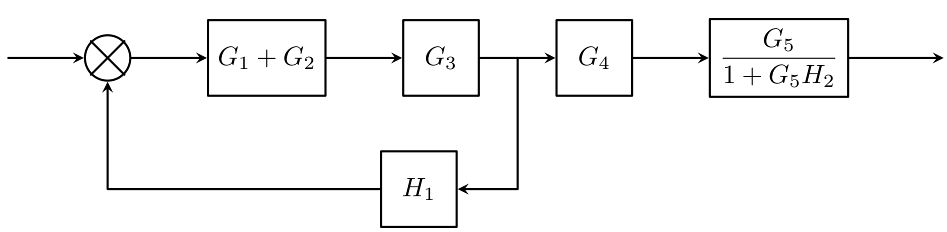
Solution:

Find parallel and series

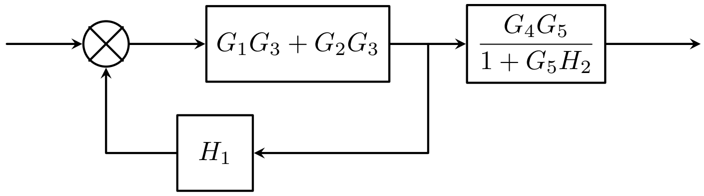


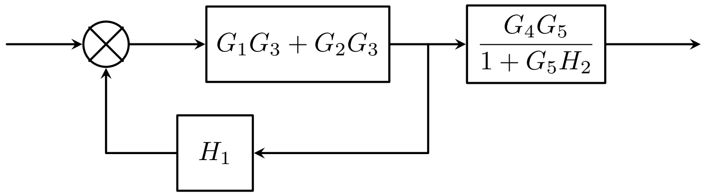
Simplify



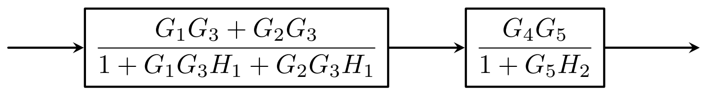


Simplify

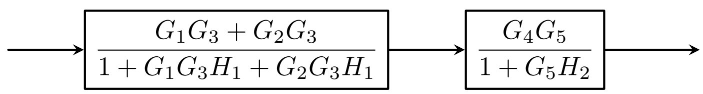




Simplify



Then



$$G(s) = \left(\frac{G_1G_3 + G_2G_3}{1 + G_1G_3H_1 + G_2G_3H_1}\right)\left(\frac{G_4G_5}{1+G_5H_2}\right)$$

$$\boxed{G(s) = \frac{G_1G_3G_4G_5 + G_2G_3G_4G_5}{1 + G_1G_3H_1 + G_2G_3H_1 + G_5H_2 + G_1G_3G_5H_1H_2 + G_2G_3G_5H_1H_2}}$$