

Segundo de los estimadores de Harn-Won.

$$\xi_H^{(2)}(r) = \frac{DD(r) RR(r)}{[DR(r)]^2}$$

con $n = \bar{n}(1+\delta)$

Primero calculamos

$$DD(r) = \int n_1 w_1 dV_1 \int n_2 w_2 dV_2 = \int \bar{n}_1 \bar{n}_2 w_1 w_2 dV_1 dV_2 = \int \bar{n}_1 (1+\delta_1) \bar{n}_2 (1+\delta_2) w_1 w_2 dV_1 dV_2 \\ = \bar{n}^2 \iint (1+\delta_1 + \delta_2 + \delta_1 \delta_2) w_1 w_2 dV_1 dV_2$$

$$\text{Como } \Psi = \frac{\langle\langle \delta(r) W(r) W(r) \rangle\rangle}{\langle\langle W(r) W(r) \rangle\rangle} \therefore \Psi \langle\langle W(r) W(r) \rangle\rangle = \langle\langle \delta(r) W(r) W(r) \rangle\rangle$$

$$\xi^{(2)}(r) = \frac{\langle\langle W(r) W(r) \delta(r) \delta(r) \rangle\rangle}{\langle\langle W(r) W(r) \rangle\rangle} \therefore \xi^{(2)}(r) = \langle\langle W(r) W(r) \delta(r) \delta(r) \rangle\rangle = \langle\langle \delta(r) \delta(r) W(r) W(r) \rangle\rangle$$

Entonces $DD(r)$:

$$DD(r) = \bar{n}^2 \left[\langle\langle w_1 w_2 \rangle\rangle + \Psi_1 \langle\langle w_1 w_2 \rangle\rangle + \Psi_2 \langle\langle w_1 w_2 \rangle\rangle + \xi^{(2)} \langle\langle w_1 w_2 \rangle\rangle \right] = \bar{n}^2 \langle\langle w_1 w_2 \rangle\rangle (1 + \Psi_1 + \Psi_2 + \xi^{(2)})$$

Ahora calculamos

$$RR(r) = \int \bar{n}_1 w_1 dV_1 \int \bar{n}_2 w_2 dV_2 = \bar{n}^2 \int w_1 w_2 dV_1 dV_2 = \bar{n}^2 \langle\langle w_1 w_2 \rangle\rangle$$

Finalmente

$$DR = \int n_1 w_1 dV_1 \int n_2 w_2 dV_2 = \iint \bar{n}_1 \bar{n}_2 (1+\delta_1) w_1 w_2 dV_1 dV_2 = \bar{n}^2 \left[\iint w_1 w_2 dV_1 dV_2 + \iint \delta_1 w_1 w_2 dV_1 dV_2 \right] \\ = \bar{n}^2 [\langle\langle w_1 w_2 \rangle\rangle + \Psi_1 \langle\langle w_1 w_2 \rangle\rangle] = \bar{n}^2 (1 + \Psi_1) \langle\langle w_1 w_2 \rangle\rangle$$

$$\Rightarrow \xi_H^{(2)}(r) = \frac{\bar{n}^2 \langle\langle w_1 w_2 \rangle\rangle (1 + \Psi_1 + \Psi_2 + \xi^{(2)}) - \bar{n}^2 \langle\langle w_1 w_2 \rangle\rangle}{\bar{n}^4 (1 + \Psi_1)^2 \langle\langle w_1 w_2 \rangle\rangle^2} = \frac{1 + \Psi_1 + \Psi_2 + \xi^{(2)}}{(1 + \Psi_1)^2}$$

Segundo de la estimación Lundy - Szalay

$$\xi_{LZ}^{(2)}(r) = 2 + \frac{1}{N_{est}} \frac{DD(r)}{RR(r)} - \frac{2}{N_{est}} \frac{DR(r)}{RR(r)}$$

Observamos que ya tenemos todos los términos, a excepción de N_{est} :

$$N_{est} = \frac{D(r)}{R(r)} = \frac{\int n w dv}{\int \bar{n} w dv} = \frac{\int \bar{n} (1+\delta) w dv}{\int \bar{n} w dv} = \frac{\langle w \rangle + \bar{\delta} \langle w \rangle}{\langle w \rangle} = 1 + \bar{\delta}$$

$$N_{est} = 1 + \bar{\delta}$$

$$\xi_{LZ}^{(2)}(r) = 2 + \frac{1}{(1+\bar{\delta})^2} \frac{\pi^2 \langle \langle w_1 w_2 \rangle \rangle (1 + \psi_1 + \psi_2 + \xi^{(2)})}{\pi^2 \langle \langle w_1 w_2 \rangle \rangle} - \frac{2}{(1+\bar{\delta})} \frac{\pi^2 (1 + \psi_1) \langle \langle w_1 w_2 \rangle \rangle}{\pi^2 \langle \langle w_1 w_2 \rangle \rangle}$$

$$= 2 + \frac{1 + \psi_1 + \psi_2 + \xi^{(2)}}{(1+\bar{\delta})^2} - 2 \frac{1 + \psi_1}{1 + \bar{\delta}}$$