

①

$$\chi^2 = \sum_i (y_i - (ax_i + b))^2$$

$$\chi^2 = \sum_i [y_i^2 - 2y_i(ax_i + b) + (ax_i + b)^2]$$

$$\chi^2 = \sum_i [y_i^2 - 2y_i ax_i - 2y_i b + a^2 x_i^2 + 2ax_i b + b^2]$$

$$\chi^2 = \sum_i [y_i^2 - 2a y_i x_i - 2b y_i + a^2 x_i^2 + 2ab x_i + b^2]$$

$$\chi^2 = \sum_i y_i^2 - 2a \sum_i y_i x_i - 2b \sum_i y_i + a^2 \sum_i x_i^2 + 2ab \sum_i x_i + \sum_i b^2$$

$$\frac{\partial \chi^2}{\partial a} = -2 \sum_i^N y_i x_i + 2a \sum_i^N x_i^2 + 2b \sum_i^N x_i = 0$$

$$\frac{\partial \chi^2}{\partial b} = -2 \sum_i^N y_i + 2a \sum_i^N x_i + 2 \sum_i^N b = 0$$

$$\rightarrow -2 \sum_i^N y_i x_i + 2a \sum_i^N x_i^2 + 2b \sum_i^N x_i = 0 \quad (1)$$

$$-2 \sum_i^N y_i + 2a \sum_i^N x_i + 2 \sum_i^N b = 0 \quad (2)$$

De ①

$$2b \sum_i^N x_i = 2 \sum_i^N y_i x_i - 2a \sum_i^N x_i^2$$

$$\rightarrow b = \frac{2 \sum_i y_i x_i - 2a \sum_i x_i^2}{2 \sum_i x_i} = \frac{\sum_i y_i x_i}{\sum_i x_i} - \frac{a \sum_i x_i^2}{\sum_i x_i}$$



$$-2 \sum_{i=1}^N y_i + 2a \sum_{i=1}^N x_i + 2Nb = 0$$

$$-2 \sum y_i + 2a \sum x_i + 2N \left[ \frac{\sum y_i x_i}{\sum x_i} - \frac{a \sum x_i^2}{\sum x_i} \right] = 0$$

$$-2 \sum y_i + 2a \sum x_i + 2N \frac{\sum y_i x_i}{\sum x_i} - 2Na \frac{\sum x_i^2}{\sum x_i} = 0$$

$$2a \sum x_i - 2Na \frac{\sum x_i^2}{\sum x_i} = 2 \sum y_i - 2N \frac{\sum y_i x_i}{\sum x_i}$$

$$2a \left[ \frac{\sum x_i - N \frac{\sum x_i^2}{\sum x_i}}{\sum x_i} \right] = 2 \sum y_i - 2N \frac{\sum y_i x_i}{\sum x_i}$$

$$a = \frac{\frac{2 \sum y_i}{\sum x_i} - \frac{2N \sum y_i x_i}{\sum x_i}}{\frac{2 \sum x_i}{\sum x_i} - \frac{2N \sum x_i^2}{\sum x_i}}$$

$$a = \frac{2 \sum y_i \sum x_i - 2N \sum y_i x_i}{\sum x_i} \cdot \frac{1}{2(\sum x_i)^2 - 2N \sum x_i^2}$$

$$a = \frac{2 \sum y_i \sum x_i - 2N \sum y_i x_i}{2(\sum x_i)^2 - 2N \sum x_i^2} = \frac{\sum y_i \sum x_i - N \sum y_i x_i}{(\sum x_i)^2 - N \sum x_i^2}$$

$$a = \frac{N \sum y_i x_i - \sum y_i \sum x_i}{N \sum x_i^2 - (\sum x_i)^2}$$



$$b = \frac{\sum y_i x_i}{\sum x_i} - a \frac{\sum x_i^2}{\sum x_i}$$

$$b = \frac{\sum y_i x_i}{\sum x_i} - \frac{\sum x_i^2}{\sum x_i} \left[ \frac{N \sum y_i x_i - \sum y_i \sum x_i}{N \sum x_i^2 - (\sum x_i)^2} \right]$$

$$b = \frac{\sum y_i x_i [N \sum x_i^2 - (\sum x_i)^2]}{\sum x_i [N \sum x_i^2 - (\sum x_i)^2]} - \frac{\sum x_i^2 [N \sum y_i x_i - \sum y_i \sum x_i]}{\sum x_i [N \sum x_i^2 - (\sum x_i)^2]}$$

$$b = \frac{N \sum y_i x_i \sum x_i^2 - N \sum y_i x_i (\sum x_i)^2 - N \sum x_i^2 \sum y_i x_i + \sum x_i^2 \sum y_i \sum x_i}{\sum x_i [N \sum x_i^2 - (\sum x_i)^2]}$$

$$b = \frac{N \sum x_i^2 \sum y_i \sum x_i - \sum y_i x_i (\sum x_i)^2}{\sum x_i [N \sum x_i^2 - (\sum x_i)^2]}$$

$$b = \frac{N \sum x_i^2 \sum y_i - \sum y_i x_i \sum x_i}{N \sum x_i^2 - (\sum x_i)^2}$$