Bolometer noise: nonequilibrium theory.

ARTICLE *in* APPLIED OPTICS · MARCH 1982

Impact Factor: 1.69 · DOI: 10.1364/AO.21.001125 · Source: PubMed

1 AUTHOR:

John C Mather
NASA

Bolometer noise: nonequilibrium theory

John C. Mather

New theoretical results for noise in cryogenic bolometers are derived. Johnson noise is reduced by as much as 60% by electrothermal feedback from the bias supply. Phonon noise in the thermal link is reduced by as much as 30% relative to the usual equilibrium formula. Photon noise in the Rayleigh-Jeans limit is computed with attention to the attenuation of the photon correlations in the light beam. Basic results on bolometer responsivity, time constant, and thermal properties are presented in a new and convenient form. Excess $1/f$ and contact shot noise are also discussed.

I. Introduction

The theory of bolometer operation has been treated by several authors.[1–7] The typical bolometer is a temperature-sensitive resistor, heated by incoming radiation and cooled by thermal conduction through its electrical lead wires. Figure 1 shows the electrical bias circuit, the thermal structure, a sample current–voltage plot, and the equivalent electrical circuit.

In this paper we first present the results of the small signal analysis of the bolometer as a transducer. These results are equivalent to the results of Jones[4] but are expressed in terms of directly observable parameters: the bolometer resistance $R = E/I$ at the operating point, the low frequency dynamic impedance $Z = dE/dI$, and the apparent time constant τ_e.

Subsequent sections present new analyses for noise in the bolometer in the presence of electrical bias power. We find that the major limiting noise sources have all been overestimated, so that the ultimate sensitivity of bolometers should be closer to the intrinsic photon noise limit. Johnson noise from the bolometer is reduced by electrothermal feedback by as much as 60%, since the bias current does work on the Johnson noise source. We also present a new analysis for the phonon noise in the thermal conductance from the bolometer to the heat sink. The result is a function of the temperature dependence of the thermal conductivity of the link material, but the detailed geometry of the link need not be known.

We also discuss photon noise in the Rayleigh-Jeans limit. We find that several previous authors have not accounted for the reduction in the Boson coherence factor suffered when a beam is attenuated or only partially absorbed by the detector. We conclude with a computation of the effect of amplifier and load resistor noise and brief discussion of excess bolometer noise.

The author is with NASA Goddard Space Flight Center, Laboratory for Extraterrestrial Physics, Greenbelt, Maryland 20771.

Received 4 September 1981.

We will use the following notation: E = potential across the bolometer; I = current through the bolometer; R = resistance = E/I; $Z_L(\omega)$ = dynamic load impedance; ω = angular frequency; time factor = $\exp(i\omega t)$; P = electrical power into bolometer = EI; Q = radiation power absorbed by bolometer element; W = total power = $P + Q$; T_c = ambient temperature; T = bolometer temperature; ΔT = temperature rise = $T - T_c$; Z = dynamic resistance = dE/dI at zero frequency; $Z(\omega)$ = complex dynamic impedance at angular frequency ω; G = dynamic thermal conductance = dP/dT at zero frequency; S = zero frequency voltage responsivity = dE/dQ; $S(\omega)$ = responsivity = dE/dQ at angular frequency ω; SD = spectral density operator; C = heat capacity at operating temperature; τ_e = effective time constant; $\tau = C/G$ = physical time constant; k_B = Boltzmann constant; R_N,L = parameters of the equivalent electrical circuit; T_L = temperature of load resistor; α = absorptivity of detector; ϵ = emissivity of thermal radiation source; f = transmissivity of optical system; $A\Omega$ = throughput of optical system; T_s = temperature of background radiation source.

II. Small Signal Analysis

We give the small signal parameters in terms of the measured time constant τ_e, the bolometer resistance R, and the impedance Z. These formulas will be used in the noise analysis that follows. Many bolometers have a maximum in voltage at a particular bias current at which point $Z = 0$. This will be used as a frequent example where the electrothermal feedback from the bias battery significantly changes the external properties of the bolometer.

The small signal voltage responsivity of the simple bolometer is given by

$$S(\omega) = \frac{1}{2I}\,\frac{(Z/R) - 1}{(Z/Z_L) + 1}\,\frac{1}{1 + j\omega\tau_e}\,. \qquad (1)$$

We note the limiting case for $Z = 0$, $\omega = 0$ is simply $S(0) = -1/2I$.

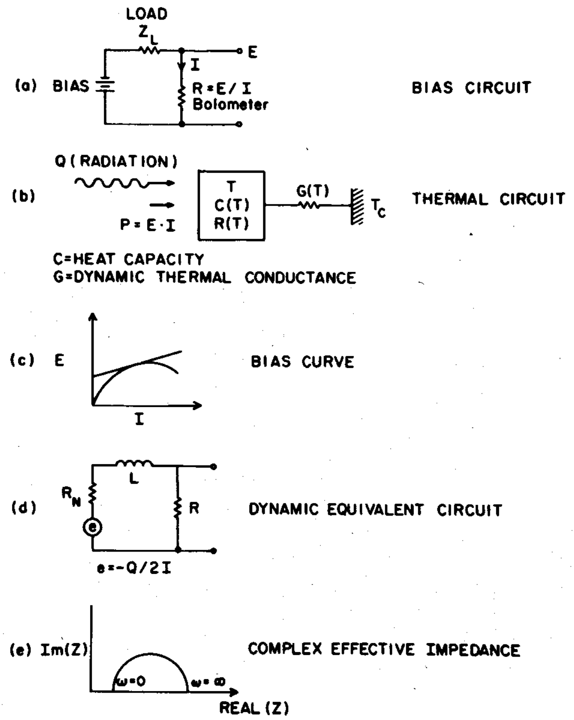

Fig. 1. Bolometer definitions: (a) bias circuit; (b) thermal design; (c) sample current–voltage plot; (d) equivalent small signal circuit; (e) complex impedance as a function of frequency.

The apparent time constant τ_e describes the response of the bolometer to modulated radiation. It also is the time constant for the decay of the transient part of the response to a change in the bias battery voltage. The physical time constant τ is related to the apparent time constant τ_e by

$$\tau = \frac{C}{G} = \tau_e \cdot \frac{2R}{Z+R} \frac{Z+Z_L}{R+Z_L}. \tag{2}$$

For $Z = 0$, $Z_L \to \infty$, we find that $\tau = 2\tau_e$, so that the bolometer appears to respond twice as fast as it would without electrothermal feedback.

The bolometer has an effective electrical impedance $Z(\omega)$ which is independent of the bias impedance. It is given by

$$Z(\omega) = Z \cdot \frac{1 + j\omega\tau \dfrac{Z+R}{2Z}}{1 + j\omega\tau \dfrac{Z+R}{2R}}. \tag{3}$$

This function has the values $Z(0) = Z$, $Z(\infty) = R$. At intermediate frequencies it traces a semicircle in the complex impedance plane, centered on $(Z + R)/2$, as shown by the equivalent form

$$Z(\omega) = \frac{Z+R}{2} + \frac{Z-R}{2} \frac{1 - j\omega\tau \dfrac{Z+R}{2R}}{1 + j\omega\tau \dfrac{Z+R}{2R}}, \tag{4}$$

where the second term has a constant modulus as a function of ω. This is shown in Fig. 1(e).

Both the bolometer responsivity and impedance are represented by the equivalent circuit given in Fig. 1(d). The effective voltage source is

$$e_g = -Q/2I, \tag{5}$$

while the circuit parameters are

$$R_N = RZ/(R - Z), \tag{6}$$

$$L = \frac{\tau}{2} R \frac{R+Z}{R-Z}. \tag{7}$$

We note that the effective inductance L is very large. If $\tau = 0.01$ sec, $R = 2 \times 10^6\ \Omega$, $Z = 0$, we find $L = 10^4$ H.

To determine the effective dynamic thermal conductance, the temperature coefficient of bolometer resistance must be measured separately by direct measurement. Then the thermal conductance is given by

$$G = I^2 \frac{dR}{dT} \frac{Z+R}{Z-R}. \tag{8}$$

The heat capacity can then be determined from Eq. (2).

III. Noise

The noise in a bolometer is produced by a variety of uncorrelated sources analyzed in the following sections. We summarize the results here. The electrical noise equivalent power (NEP) is given by

$$\mathrm{NEP}^2 = \mathrm{NEP}^2_{\mathrm{Johnson}} + \mathrm{NEP}^2_{\mathrm{thermal}} + \mathrm{NEP}^2_{\mathrm{photon}} + \mathrm{NEP}^2_{\mathrm{load}} + \mathrm{NEP}^2_{\mathrm{amplifier}} + \mathrm{NEP}^2_{\mathrm{excess}}, \tag{9}$$

where

$$\mathrm{NEP}^2_{\mathrm{Johnson}} = 4k_BTP \left|\frac{Z+R}{Z-R}\right|^2 (1 + \omega^2\tau^2), \tag{10}$$

$$\mathrm{NEP}^2_{\mathrm{thermal}} = 4k_BGT^2 \frac{\displaystyle\int_{T_c}^{T} \left[\frac{tk(t)}{Tk(T)}\right]^2 dt}{\displaystyle\int_{T_c}^{T} \left[\frac{k(t)}{k(T)}\right] dt}, \tag{11}$$

$$\mathrm{NEP}^2_{\mathrm{photon}} = 4 \frac{A\Omega}{c^2} \frac{(k_BT_s)^5}{h^3} \int \frac{x^4 dx}{e^x - 1} \left(1 + \frac{\alpha\epsilon f}{e^x - 1}\right) (\alpha\epsilon f), \tag{12a}$$

where

$$x = h\nu/k_BT_s, \tag{12b}$$

$$\mathrm{NEP}^2_{\mathrm{load}} = \frac{4k_BT_L}{Z_L} \left|\frac{2ZI}{\dfrac{Z}{R} - 1}\right|^2 \left[1 + \omega^2\tau^2 \left(\frac{Z+R}{2Z}\right)^2\right], \tag{13}$$

$$\mathrm{NEP}^2_{\mathrm{amp}} = i_A^2 \left|\frac{2ZI}{\dfrac{Z}{R} - 1}\right|^2 \left[1 + \omega^2\tau^2 \left(\frac{Z+R}{2Z}\right)^2\right] + \frac{e_A^2}{|S^2(\omega)|}. \tag{14}$$

IV. Johnson Noise Analysis

Most authors have computed bolometer Johnson noise from the resistance R, assuming the bolometer to be a simple resistor at temperature T. Other authors have assumed that the Johnson noise arises from the real part of $Z(\omega)$, which clearly cannot be true since Z

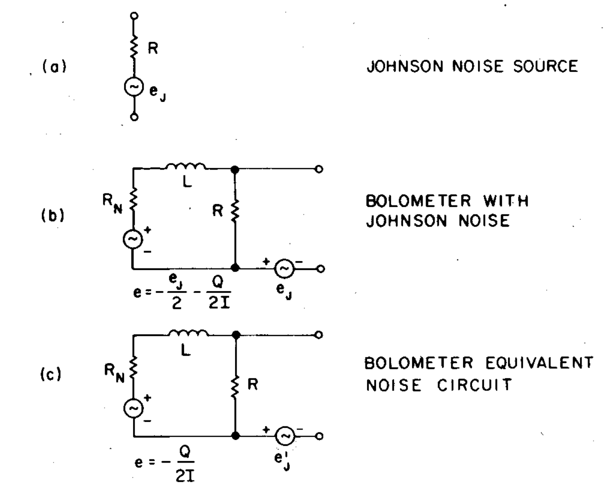

Fig. 2. (a) Johnson noise of a resistor or bolometer with thermal feedback neglected; (b) equivalent circuit including feedback; (c) Thevenin equivalent of (b).

can be zero or negative. We give here the argument for a new formula including electrothermal feedback from the bias source.

The bolometer is now considered to be a resistor in series with a fluctuating noise voltage generator e_J, as shown in Fig. 2(a). If the value of G (the thermal conductance) were very large, the full equivalent circuit for noise would be the same regardless of dc bias current. Electrical work done on the Johnson noise source would be dissipated in the bolometer but would not affect the bolometer temperature or resistance. This noise has a mean value of zero and so produces no average heat flow to the bolometer.

When the value of G is now reduced to its normal value, two new effects appear: the effective bolometer impedance becomes $Z(\omega)$ instead of R, and the bias current dissipates a power in the Johnson noise source, which may actually be detected by the bolometer. For the usual bolometer with a negative temperature coefficient, this second effect opposes the original noise voltage, since an increase in e_J increases the power dissipated in the bolometer, increases its temperature, decreases its resistance, and decreases the output voltage. The equivalent circuit is now given by Fig. 2(b). We will use the shorthand that

$$e_J(\omega) = [SD(e_J)|_\omega]^{1/2}, \tag{15}$$

where the units are $V/Hz^{1/2}$.

By Thevenin's theorem, Fig. 2(b) may be converted to Fig. 2(c), with the new noise source $e'_J(\omega)$ given by

$$e'_J(\omega) = e_J(\omega)\,\frac{j\omega\tau + 1}{j\omega\tau + \dfrac{2R}{R+Z}}\,. \tag{16}$$

We see that for $\omega = 0, Z = 0, e'_J = e_J/2$. Even greater reductions are possible for $Z < 0$, which can occur for bolometers with very large negative temperature coefficients. When the bolometer is inserted in the bias circuit, the load resistor modifies the noise further, giving an actual noise output voltage of

$$e''_J(\omega) = e_J(\omega)\,\frac{Z_L}{R+Z_L}\,\frac{\omega + \dfrac{1}{j\tau}}{\omega + \dfrac{1}{j\tau_e}}\,. \tag{17}$$

The noise voltage e_J has of course the spectral density

$$SD(e_J) = 4k_BTR. \tag{18}$$

It is interesting to compute the NEP implied by this noise source alone. It is given by

$$\begin{aligned}\mathrm{NEP} &= e''_J(\omega)/S(\omega)\\ &= \sqrt{4k_BTP}\,\frac{|Z+R|}{|Z-R|}\,|1+j\omega\tau|,\end{aligned} \tag{19}$$

which is independent of the load resistance. This important result shows that changing the electrothermal feedback effect by varying Z_L has no effect on the NEP, as one would expect. The NEP also depends on the real physical time constant C/G rather than the effective time constant τ_e.

V. Phonon Noise

Spontaneous random energy flows into and out of the bolometer occur at all times. On the microscopic scale this is produced by the random propagation of energy carriers, be they electrons, phonons, photons, or quasi-particles. On a microscopic scale it is readily shown from thermodynamics that this power flow (call it P_n) has the spectral density

$$SD(P_n) = 4k_BT^2G, \tag{20}$$

where G is the thermal conductance between bolometer and heat sink. This result applies strictly to an isothermal system. We again define the shorthand notation that

$$P_n(\omega) = [SD(P_n)|_\omega]^{1/2}. \tag{21}$$

In this section we will generalize the result to a bolometer with a temperature gradient in the thermal link. The result will be independent of the geometrical properties of the link, providing that the mean free path of the energy carriers is short compared with the length (defined as X) of the link. This is the opposite limit from that considered by Boyle and Rogers.[5] Essentially the result is a weighted mean temperature to use in Eq. (20), with the weighting function depending on the thermal conductivity of the link as a function of temperature.

The approach is to imagine the link divided into small elements in series. Each element is nearly isothermal and produces a random power flow given by applying Eq. (20) to the small element. Most of the power flow is immediately conducted back again through the element, but a small amount reaches the bolometer. We compute this amount and then add the contributions from the various elements.

We assume that the thermal link is narrow, so that temperature t, thermal conductivity k, and cross-sectional area A are all functions of a single spatial coor-

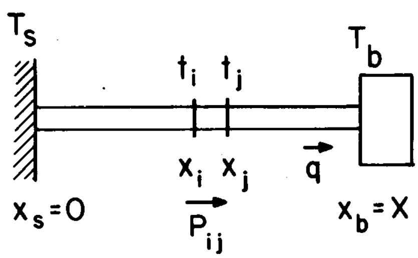

Fig. 3. Elements of thermal noise analysis. Terms are defined in text.

dinate x, defined as the distance from the heat sink. The bolometer temperature is T, and the heat sink temperature is T_c. We further assume for simplicity that the link is of homogeneous material so that k is a function of t only. Under steady state conditions, the power flow through the link is $W(T,T_c)$. The equation for the temperature gradient in the link is

$$W(T,T_c) = k(t)A(x)\frac{dt}{dx}, \tag{22}$$

which is immediately integrable to

$$W(T,T_c) = \int_{T_c}^{T} k(t)dt \div \int_0^X [A(x)]^{-1}dx, \tag{23}$$

and then differentiable to give

$$G = dW/dT = k(T) \div \int_0^X [A(x)]^{-1}dx. \tag{24}$$

This definition of the thermal conductance G is the same one used in the small signal analysis. Equation (24) shows that the variation of G with T is the same as the variation of k, justifying the arguments made by Low[6] in determining the scaling laws for bolometer properties as a function of temperature.

We now compute the noise power q into the bolometer produced by random fluctuations in the interval from x_i to x_j, with $0 < x_i < x_j < X$, as shown in Fig. 3. Subscript s refers to the heat sink, i and j to the two ends of the element under consideration, and b to the bolometer chip. Each part of the link is described by its own heat flow function W having the dimension of power. For example, $W_{is}(t_i,t_s) = -W_{si}(t_i,t_s)$. We define $G_{is} = \partial W_{is}/\partial t_i, G_{si} = \partial W_{si}/\partial t_s$, etc. as thermal conductances and $R_{is} = G_{is}^{-1}$ etc. as thermal resistances. We hold t_s and t_b fixed and introduce a random noise power source p_{ij} across the link G_{ij}. Using $\Delta t_i,\Delta t_j$ as the fluctuating parts of t_i,t_j we obtain the nodal equations

$$q = G_{jb}\Delta t_j, \tag{25}$$

$$p_{ij} = (G_{jb} + G_{ji})\Delta t_j - G_{ij}\Delta t_i, \tag{26}$$

$$-p_{ij} = (G_{is} + G_{ij})\Delta t_i - G_{ji}\Delta t_j. \tag{27}$$

The solution to these equations is

$$q = p_{ij} \cdot [(G_{jb} + G_{ji})(R_{jb}) + G_{ij}R_{is}]^{-1}. \tag{28}$$

As $x_j - x_i \to 0$, $G_{ij} \to \infty$, and therefore

$$q \to p_{ij}R_{ij}/(R_{is} + R_{ib}). \tag{29}$$

From Eq. (24),

$$\begin{aligned} R_{is} + R_{ib} &= \frac{1}{k(t_i)}\left[\int_0^{x_i} \frac{1}{A(x)}dx + \int_{x_i}^{x_s} \frac{1}{A(x)}dx\right] \\ &= \frac{1}{k(t_i)}\int_0^x \frac{dx}{A(x)} = \frac{k(t_b)}{k(t_i)}R_{bs}. \end{aligned} \tag{30}$$

Similarly,

$$R_{ij} = \frac{(x_j - x_i)}{k(t_i)A(x_i)}. \tag{31}$$

Now the spectral density of p_{ij} is $SD(p_{ij}) = 4k_BT_i^2G_{ij}$ from the classical equilibrium fluctuation formula. The spectral density of q is given by

$$SD(q) = \sum_{\text{elements}} \left(\frac{R_{ij}}{R_{is} + R_{ib}}\right)^2 4k_Bt_i^2G_{ij}. \tag{32}$$

This sum may be transformed with the aid of Eqs. (22)–(24) to

$$SD(q) = 4k_BGT^2 \frac{\int_{T_c}^{T}\left[\frac{tk(t)}{Tk(T)}\right]^2 dt}{\int_{T_c}^{T}\frac{k(t)}{k(T)}dt}. \tag{33}$$

For typical materials at low temperature, $k(t) \propto t^{\beta}$. For normal metals, $\beta = 1$, while for crystalline dielectrics or superconductors, $\beta = 3$. For conductivities of this form, integrals in Eq. (33) may be done explicitly. The result may be expanded to lowest orders in $\Delta = 1 - (T_c/T)$, giving

$$SD(q) = 4k_BGT^2\left[1 - \left(\frac{\beta}{2} + 1\right)\Delta + \frac{(\beta + 2)(3\beta + 2)}{12}\Delta^2 \ldots\right]. \tag{34}$$

For a typical case, $\beta = 3$, $\Delta = 0.2$, this formula gives the value in brackets as 0.683, while the true value is 0.652. In this case, the effective mean value of T is close to T_c rather than T. We remind the reader that G is defined as the derivative of W with respect to T.

VI. Photon Noise

Photon noise on bolometers arises from the random fluctuations in the rate of absorption of radiation quanta. We discuss this briefly because many authors ignore the degradation of photon coherence by beam attenuation and by partial absorption.[1–3,5–7] We cite van Vliet[8] and Hauser[9] for a better answer; van Vliet's Eq. (68) implies that the spectral density of fluctuations in absorbed power is

$$SD(Q) = 2\int d\nu Q_{\nu}h\nu[1 + \eta(\nu)], \tag{35}$$

where Q is the radiant power absorbed, Q_{ν} is the same quantity per unit optical bandwidth, and η is the effective photon mode occupation number. The value of η is given by

$$\eta(\nu) = \alpha\epsilon f/(e^x - 1), \tag{36}$$

where α is the detector absorptivity for the particular detective process producing a measurable result, ϵ is the emissivity of the source, f is the transmissivity of the optical system connecting them, α, ϵ, and f are evaluated at ν, and $x = h\nu/k_BT_s$, where T_s is the temperature of the source. Equations (12) are also derived by substituting into Eqs. (35) and (36) the following expression for Q_{ν}:

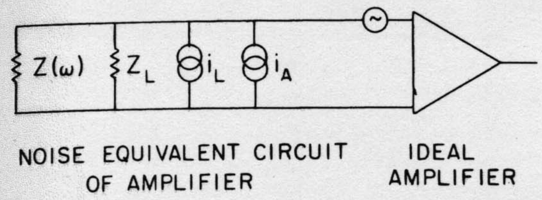

Fig. 4. Equivalent noise circuit of bolometer and amplifier. Terms are defined in text.

$$Q_\nu = 2(\alpha\epsilon f)A\Omega \frac{h\nu^3}{c^2}\frac{1}{e^x - 1}. \tag{37}$$

For far IR detectors operating in a high radiation background from a room temperature optical system, the difference between $\alpha\epsilon f$ and 1 may be very important. Note that the NEP in Eq. (12) is the electrical NEP; the optical detector NEP is Eq. (12) divided by α.

For completeness, similar terms should be included for all elements of the telescope and detector system, including one term for the self-emission of the detector. For cryogenic detectors this term is extremely small.

VII. Load Resistor and Amplifier Noise

Amplifier and load resistor noise are typically represented by the equivalent circuit in Fig. 4 with noise voltage and noise current generators as shown. We find that the total noise voltage from these sources at the amplifier input is given by

$$e_n^2(\omega) = \left(\frac{4k_BT_L}{Z_L} + i_A^2(\omega)\right)\left(\frac{ZZ_L}{Z + Z_L}\right)^2 \left|\frac{1 + j\omega\tau \dfrac{Z + R}{2Z}}{1 + j\omega\tau_e}\right|^2 + e_A^2(\omega). \tag{38}$$

The term from the amplifier current noise and load resistor noise Z is frequency dependent because of the frequency dependence of $Z(\omega)$. Equations (13) and (14) are derived by dividing Eq. (38) by $|S(\omega)|^2$.

VIII. Bolometer Excess Noise

Most bolometers have excess electrical noise, especially at low frequencies. We will discuss two types, flicker ("$1/f$") noise and contact shot noise. Both are modified by thermal feedback in the same way as the Johnson noise discussed above.

Van der Ziel[10] describes McWhorter's theory of flicker noise. In this theory flicker noise is due to trapping centers on the surface of a semiconducting filament, actually located within the first 2–4 nm of the oxide layer. The resistance of the element is modulated by the slow trapping and release of charge carriers, giving a voltage noise which is proportional to the bias voltage, with a $1/f^{1/2}$ dependence on frequency. This model of a fluctuating resistance permits estimation of the NEP scaling law as the thermal conductance G is varied. The result is that the NEP is proportional to $\sqrt{G}$ as usual with the provision that the modulation frequency scales as G. A reduced G value allows a smaller bias voltage which reduces the flicker noise to exactly compensate for the increase caused by operating at a lower frequency.

Another common noise source appears to be contact shot noise. Some germanium bolometers built in our laboratory have an electrical nonlinearity which is not due to the bias power heating the bolometer. It is observable in three ways: (1) in Jones's ac bridge method[4] for determining responsivity, spurious high readings are obtained at high ac modulation frequencies, especially at low dc bias. (2) A plot of resistance R vs bias power P does not show a smooth nearly straight line approach to a constant as P is reduced. (3) The responsivity deduced from Eq. (1) has a spurious high value at low bias current and does not vary with bias current in the same way as the optically determined responsivity.

This noise source may be modeled as a barrier having a dynamic impedance Z_{bar} with the full shot noise voltage $e_n = Z_{\text{bar}}(2qI)^{1/2}$ V/Hz$^{1/2}$. If Z_{bar} is more than a few percent of the bolometer impedance, the shot noise may exceed the Johnson noise. For one bolometer tested, approximate values were $Z_{\text{bar}} = 1$ MΩ, $R = 20$ MΩ, $I = 30$ nA, $T = 2$ K. With these values, the shot noise is 98 nV/Hz$^{1/2}$, while the Johnson noise is 47 nV/Hz$^{1/2}$. This shot noise becomes even more important at lower bias current, since Z_{bar} seems to scale as I^{-1} up to ~50 MΩ for this particular bolometer.

We cannot distinguish shot noise from Johnson noise by their spectral dependence, since both are white noise and both are modified by thermal feedback in the same way.

IX. Summary and Conclusions

We have given new estimates for noise in bolometers. Previous comparisons of measured bolometer performance to the ideal limit may need to be revised, and there may be further room for improvement before fundamental limits are achieved. Johnson, photon, and phonon noise sources are all reduced relative to commonly used estimates due to the nonequilibrium state of the bolometer and to the inclusion of the photon coherence factors.

I greatly appreciate the comments and suggestions made by Paul L. Richards, who knew that bolometer noise theory was incomplete, and by Rainer Weiss, Michael Hauser, and Harvey Moseley. Hauser also independently derived Eqs. (12).

References

1. F. R. Arams, Ed., *Infrared-to-Millimeter Wavelength Detectors* (Artech House, Dedham, Mass., 1973).
2. E. H. Putley, Phys. Status Solidi **6**, 571 (1964).
3. F. J. Low and A. R. Hoffman, Appl. Opt. **2**, 649 (1963).
4. R. C. Jones, J. Opt. Soc. Am. **43**, 1 (1953).
5. W. S. Boyle and K. F. Rogers, Jr., J. Opt. Soc. Am. **49**, 66 (1959).
6. F. J. Low, J. Opt. Soc. Am. **51**, 1300 (1961).
7. S. Zwerdling, R. A. Smith, and J. P. Theriault, Infrared Phys. **8**, 271 (1968).
8. K. M. van Vliet, Appl. Opt. **6**, 1145 (1967).
9. M. G. Hauser, "Calculation of Expected Infrared Signals and Background-Induced Noise Limitations," available from the author, NASA/GSFC, Greenbelt, Md. 20771.
10. A. Van der Ziel, *Noise* (Prentice-Hall, Englewood Cliffs, N.J., 1970), p. 109.