Rindler approximation to Kerr black hole

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Regular Article

Rindler approximation to Kerr-Newman black hole

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Abstract. We show that the Rindler approximation to the time-radial part of the Kerr and Kerr-Newman metrics near their external h_+ and internal h_- horizons **only** holds **outside** h_+ and **inside** h_-, so respectively inside and outside the external and internal ergospheres, regions where, in Boyer-Lindquist coordinates, both g_{tt} and g_{rr} are negative, but preserving the Lorentzian character of the metric, and $r > 0$ *i.e.* outside the region $r < 0$ where closed timelike curves exist. At each point, the choice of Rindler coordinates is not trivial, but depends on the polar angle θ. The approximation, as is known, automatically gives the absolute values of the surface gravities $\kappa_\pm$ as the corresponding proper accelerations, and therefore the Hawking temperatures $\tau_\pm$ at $h_\pm$.

1 Kerr-Newman black hole and Rindler space

It is well known that near the horizons of the Schwarzschild, Reissner-Nordstrom, Kerr and Kerr-Newman black holes, the time-radial part of the geometry can be approximated by the 2-dimensional Rindler space-time, with the proper acceleration representing the absolute value of the corresponding surface gravities. It is interesting to investigate, mainly in the case of two horizons, on which side of the outer (h_+) and inner (h_-) horizons the approximation holds. This is done here for the Kerr and Kerr-Newman cases, where it is shown that it occurs within the domains of outer communication, that is, outside the black hole and white hole regions *i.e.* outside h_+ and inside h_-. This allows to compute the Hawking temperature at h_+ and h_- as an Unruh effect. In the literature one often finds references only to the event horizon h_+ neglecting the analysis of the inner Cauchy horizon h_-. Our purpose is to add such a study.

The Kerr-Newman [1,2] metric in Boyer-Lindquist [3] coordinates is given by

$$\mathrm{d}s^2 = \frac{\Delta}{\Sigma}(\mathrm{d}t - a\sin^2\theta\mathrm{d}\varphi)^2 - \frac{\Sigma}{\Delta}\mathrm{d}r^2 - \Sigma\mathrm{d}\theta^2 - \frac{(r^2+a^2)^2}{\Sigma}\sin^2\theta\left(\mathrm{d}\varphi - \frac{a}{r^2+a^2}\mathrm{d}t\right)^2, \tag{1}$$

where $t, r \in (-\infty, +\infty)$, $\varphi \in [0, 2\pi)$, $\theta \in [0, \pi]$, $\Delta = r^2 + a^2 - 2Mr + Q^2$, $\Sigma = r^2 + a^2\cos^2\theta$, M is the gravitational mass, a the angular momentum/unit mass, and $Q^2 = q^2 + p^2$, with q the electric charge and p the magnetic (Dirac) charge. The Kerr-Newman metric reduces to the Kerr metric when $Q^2 = 0$. In the following we shall consider the case $M^2 - (a^2 + Q^2) > 0$.

Let us investigate this metric near the Killing horizons h_+ (*event* horizon) and h_- (*Cauchy* horizon), respectively at the roots of Δ:

$$r_\pm = M \pm \sqrt{M^2 - (a^2 + Q^2)}. \tag{2}$$

In the neighborhood of $r_\pm$, we define the radial coordinate ρ through

$$r := r_\pm + \frac{\alpha_\pm}{r_\pm}\rho^2 \tag{3}$$

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with $[\rho] = [L]^1$ and $\alpha_\pm$ functions to be determined later, with $[\alpha_\pm] = [L]^0$. We consider the different terms in (1) up to $O(\rho^2)$.

i) In the last term of (1) we have, up to $O(\rho^2)$,

$$\begin{aligned}\mathrm{d}\varphi - \frac{a}{r^2+a^2}\mathrm{d}t &= \mathrm{d}\varphi - \frac{a\mathrm{d}t}{(r_\pm + \frac{\alpha_\pm}{r_\pm}\rho^2)^2 + a^2} = \mathrm{d}\varphi - \frac{a\mathrm{d}t}{r_\pm^2 + a^2 + 2\alpha_\pm\rho^2} = \mathrm{d}\varphi - \frac{a\mathrm{d}t}{r_\pm^2+a^2}\left(1 - \frac{2\alpha_\pm}{r_\pm^2+a^2}\rho^2\right)\\ &= \mathrm{d}\varphi - \omega_\pm \mathrm{d}t + \frac{2a\alpha_\pm\rho^2}{(r_\pm^2+a^2)^2}\mathrm{d}t = \mathrm{d}\tilde{\varphi}_\pm + \frac{2a\alpha_\pm\rho^2}{(r_\pm^2+a^2)^2}\mathrm{d}t, \end{aligned} \tag{4}$$

where

$$\omega_\pm = \omega(r_\pm,\theta) = \frac{a}{r_\pm^2+a^2} \tag{5}$$

is the dragging angular velocity of spacetime due to the rotation of the hole [4]

$$\omega(r,\theta) = \frac{a(2Mr - Q^2)}{(r^2+a^2)^2 - a^2\sin^2\theta\varDelta} \tag{6}$$

evaluated at the horizons $h_\pm$, and

$$\tilde{\varphi}_\pm = \varphi - \omega_\pm t \tag{7}$$

are co-rotating azimuthal angles. (To each horizon one can associate a co-rotating coordinate system, respectively $\{t, r, \theta, \tilde{\varphi}_\pm\}$ to $h_\pm$.) Since we are interested only in the time-radial part of the metric, we must consider $\tilde{\varphi}_\pm = \text{const}$. So, $\mathrm{d}\tilde{\varphi}_\pm = 0$ and therefore

$$\left(\mathrm{d}\varphi - \frac{a}{r^2+a^2}\mathrm{d}t\right)^2 = \frac{4a^2\alpha_\pm^2}{(r_\pm^2+a^2)^4}(\rho^2)^2\mathrm{d}t^2, \tag{8}$$

which can be neglected to $O(\rho^2)$.

ii) For the $\mathrm{d}r^2$ term: $\mathrm{d}r = \mathrm{d}(r_\pm + \frac{\alpha_\pm}{r_\pm}\rho^2) = \frac{2\alpha_\pm}{r_\pm}\rho\mathrm{d}\rho$, so $\mathrm{d}r^2 = \frac{4\alpha_\pm^2}{r_\pm^2}\rho^2\mathrm{d}\rho^2$. The surface gravities at $r_\pm$ are given by (see appendix A)

$$\kappa_+ = \frac{r_+ - r_-}{2(r_+^2+a^2)} = \frac{\sqrt{M^2-(a^2+Q^2)}}{2M(M+\sqrt{M^2-(a^2+Q^2)}) - Q^2} \tag{9a}$$

and

$$\kappa_- = -\frac{r_+ - r_-}{2(r_-^2+a^2)} = -\frac{\sqrt{M^2-(a^2+Q^2)}}{2M(M-\sqrt{M^2-(a^2+Q^2)}) - Q^2} \tag{9b}$$

with $\kappa_+ > 0$, $\kappa_- < 0$, and $|\kappa_-| > \kappa_+$ since $r_- < r_+$. From (9a), $r_+ - r_- = 2(r_+^2+a^2)\kappa_+$, which implies $2(r_+^2+a^2)\kappa_+ - r_+ = -r_-$, then $r - r_- = 2(r_+^2+a^2)\kappa_+ + (r - r_+) = 2(r_+^2+a^2)\kappa_+ + \frac{\alpha_+}{r_+}\rho^2$ and therefore

$$\varDelta = (r-r_+)(r-r_-) = \frac{\alpha_+}{r_+}\left(2(r_+^2+a^2)\kappa_+ + \frac{\alpha_+}{r_+}\rho^2\right) = \frac{2\alpha_+}{r_+}(r_+^2+a^2)\kappa_+\rho^2 + O(\rho^4); \tag{10a}$$

analogously, from (9b),

$$\varDelta = (r-r_+)(r-r_-) = \frac{\alpha_-}{r_-}\left(2(r_-^2+a^2)\kappa_- + \frac{\alpha_-}{r_-}\rho^2\right) = \frac{2\alpha_-}{r_-}(r_-^2+a^2)\kappa_-\rho^2 + O(\rho^4). \tag{10b}$$

On the other hand,

$$\varSigma = r^2 + a^2\cos^2\theta = \left(r_\pm + \frac{\alpha_\pm}{r_\pm}\rho^2\right)^2 + a^2\cos^2\theta = r_\pm^2 + a^2\cos^2\theta + 2\alpha_\pm\rho^2 + O(\rho^4) := \varSigma_\pm + 2\alpha_\pm\rho^2 + O(\rho^4) \tag{11}$$

with

$$\varSigma_\pm = r_\pm^2 + a^2\cos^2\theta. \tag{12}$$

Then, up to $O(\rho^2)$ terms,

$$-\frac{\varSigma}{\varDelta}\mathrm{d}r^2 = -\frac{4\alpha_\pm\varSigma_\pm}{r_\pm(r_\pm - r_\mp)}\mathrm{d}\rho^2. \tag{13}$$

iii) For the third term in (1), at fixed θ,

$$-\Sigma d\theta^2 = 0. \tag{14}$$

iv) For the first term in (1), from (10a) and (10b), $\Delta = \frac{\alpha_\pm}{r_\pm}(r_\pm - r_\mp)\rho^2$, and then $\frac{\Delta}{\Sigma} = \frac{\alpha_\pm(r_\pm - r_\mp)}{r_\pm \Sigma_\pm}\rho^2$. Also, again for constant values of $\tilde{\varphi}_\pm$,

$$\mathrm{d}t - a\sin^2\theta \mathrm{d}\varphi = \mathrm{d}t - a\sin^2\theta(\mathrm{d}\tilde{\varphi} + \omega_\pm \mathrm{d}t) = \mathrm{d}t - a\sin^2\theta\omega_\pm \mathrm{d}t = \mathrm{d}t\left(1 - \frac{a^2\sin^2\theta}{r_\pm^2 + a^2}\right) = \mathrm{d}t\left(\frac{\Sigma_\pm}{r_\pm^2 + a^2}\right).$$

Then,

$$\frac{\Delta}{\Sigma}(\mathrm{d}t - a\sin^2\theta \mathrm{d}\varphi)^2 = \Sigma_\pm \frac{\alpha_\pm(r_\pm - r_\mp)}{r_\pm(r_\pm^2 + a^2)^2}\rho^2 \mathrm{d}t^2 = \frac{4\Sigma_\pm\alpha_\pm}{r_\pm(r_\pm - r_\mp)}(\kappa_\pm\rho)^2\mathrm{d}t^2. \tag{15}$$

So, from (8), (13), (14) and (15),

$$\mathrm{d}s^2|_{\substack{\theta,\, \tilde{\varphi}_\pm = \mathrm{const} \\ r \sim r_\pm,}} = \frac{4\Sigma_\pm\alpha_\pm}{r_\pm(r_\pm - r_\mp)}((|\kappa_\pm|\rho)^2\mathrm{d}t^2 - \mathrm{d}\rho^2), \tag{16}$$

which is conformal to the *Rindler metric with proper acceleration* $|\kappa_\pm| > 0$ [5,6]. Choosing

$$\alpha_\pm := \tilde{\alpha}_\pm = \frac{r_\pm(r_\pm - r_\mp)}{4\Sigma_\pm} = \begin{cases} \frac{r_+(r_+ - r_-)}{4(r_+^2 + a^2\cos^2\theta)} > 0 & \text{at } h_+, \\ -\frac{r_-(r_+ - r_-)}{4(r_-^2 + a^2\cos^2\theta)} < 0 & \text{at } h_-, \end{cases} \tag{17}$$

one obtains

$$\mathrm{d}s^2|_{\substack{\theta,\, \tilde{\varphi}_\pm = \mathrm{const} \\ r \sim r_\pm, \\ \alpha_\pm = \tilde{\alpha}_\pm,}} = \mathrm{d}s^2_{Rindler} = (|\kappa_\pm|\rho)^2\mathrm{d}t^2 - \mathrm{d}\rho^2, \tag{18}$$

with the change of coordinates in (3) given by

$$r = r_\pm + \frac{\tilde{\alpha}_\pm(\theta)}{r_\pm}\rho^2 = \begin{cases} r_+ + \frac{r_+ - r_-}{4\Sigma_+(\theta)}\rho^2 > r_+ \text{ near } h_+, \\ r_- - \frac{r_+ - r_-}{4\Sigma_-(\theta)}\rho^2 < r_- \text{ near } h_-. \end{cases} \tag{19}$$

Thus, the choice of the local Rindler coordinate system (t, ρ) depends on θ. In particular, at the equator,

$$\tilde{\alpha}_\pm\left(\frac{\pi}{2}\right) = \frac{r_\pm - r_\mp}{4r_\pm} = \begin{cases} \frac{r_+ - r_-}{4r_+} > 0 & \text{at } h_+, \\ -\frac{r_+ - r_-}{4r_-} < 0 & \text{at } h_-. \end{cases} \tag{20}$$

The points at $r_+(\rho)$ near r_+ are outside h_+ but inside S_+, the external ergosphere, while the points at $r_-(\rho)$ near r_- are inside h_- but outside S_-, the internal ergosphere, and therefore outside the region with $-\infty < r < 0$ where closed timelike curves exist. In the first two regions the metric coefficients g_{tt} and g_{rr} are negative; the metric, however, is Lorentzian. In fig. 1, the shadowed regions in the elementary cell of the Penrose-Carter diagram of the Kerr-Newman spacetime, indicate the zones of validity of the Rindler approximation. The total geometry in these zones is the product of the flat Rindler space and the 2-sphere.

Finally, the Hawking temperatures at the horizons can be read as an Unruh effect [7] due to the uniformly accelerated motions with accelerations $|\kappa_\pm|$, and are given by

$$\tau_\pm = \frac{|\kappa_\pm|}{2\pi}. \tag{21}$$

2 Comment on the approximation

From (3) and (17), it can be easily seen that $O(\rho^4) \ll O(\rho^2)$ means that

$$|\rho| \ll 2\sqrt{\frac{r_\pm\Sigma_\pm}{r_+ - r_-}}. \tag{22}$$

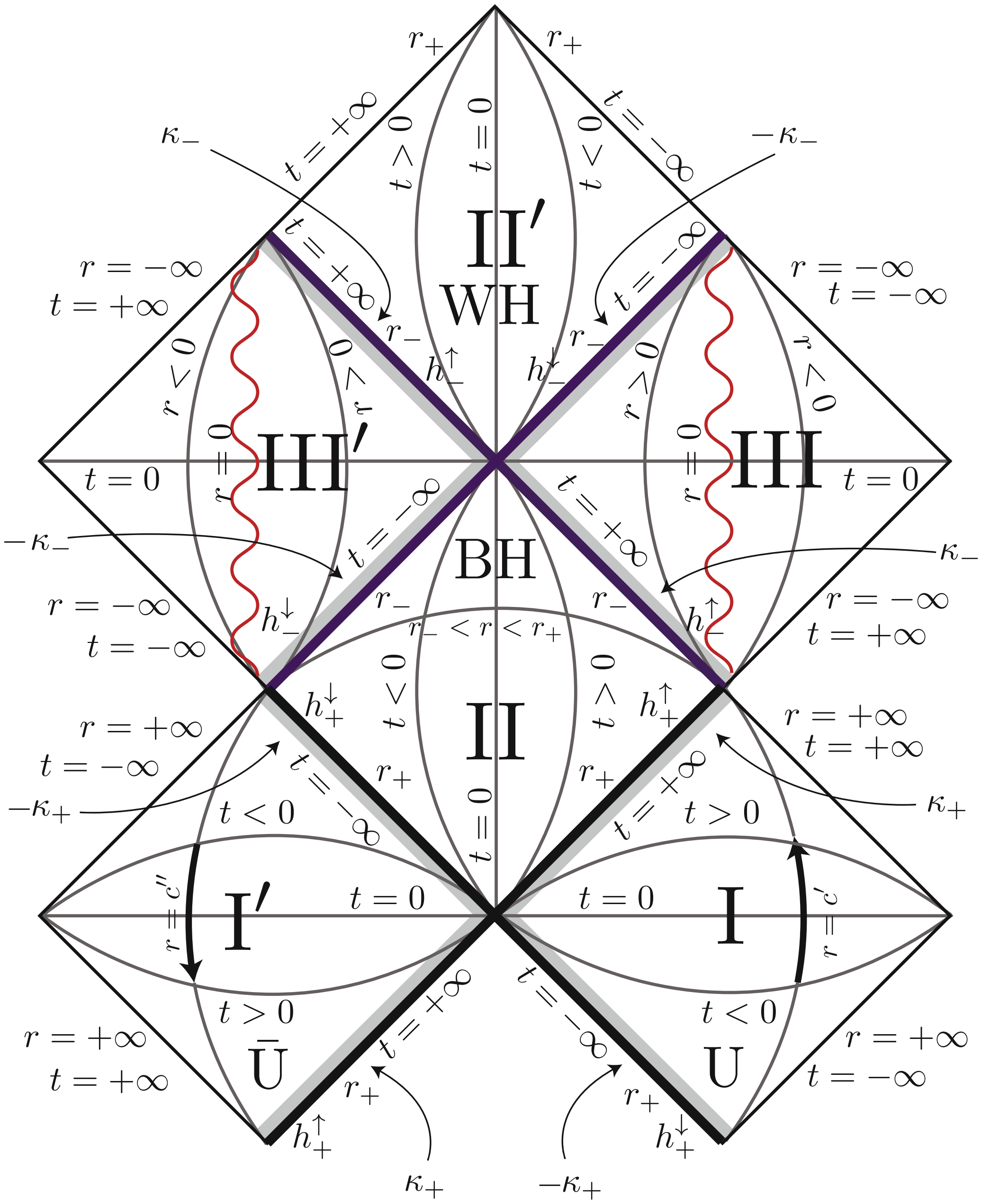

Fig. 1. In the shadowed regions Rindler aproximation holds; $\pm\kappa_\pm, \mp\kappa_\pm$: surface gravities; $h_\pm^{\uparrow\downarrow}$: future (↑) and past (↓) event (+) and Cauchy (-) horizons; U: universe, $\bar{U}$: anti-universe; BH: black hole; WH: white hole.

In particular, for $\theta = \frac{\pi}{2}$,

$$\left|\rho\left(\frac{\pi}{2}\right)\right| \ll 2\sqrt{\frac{r_\pm^3}{r_+ - r_-}} = \begin{cases} \frac{2r_+}{\sqrt{1-\frac{r_-}{r_+}}} & \text{at } r_+, \\ \frac{2r_-}{\sqrt{\frac{r_+}{r_-}-1}} & \text{at } r_-. \end{cases} \tag{23}$$

For example, for $\frac{r_-}{r_+} = \frac{1}{2}$ *i.e.* for $M^2 = \frac{9}{8}(a^2 + Q^2)$,

$$\left|\rho\left(\frac{\pi}{2}\right)\right| \ll \begin{cases} 2\sqrt{2}r_+ & \text{at } r_+, \\ 2r_- & \text{at } r_-. \end{cases} \tag{24}$$

Appendix A.

In this appendix we present a detailed derivation of the surface gravities $\kappa_\pm$ at the horizons $h_\pm$.

In the retarded Kerr coordinates (u, r, θ, χ) (Eddington-Finkelstein type), with $u, r \in (-\infty, +\infty)$, $\theta \in [0, \pi]$, and $\chi \in [0, 2\pi)$, the Kerr-Newman metric is given by

$$g_{\mu\nu} = \begin{pmatrix} g_{uu} & g_{ur} & g_{u\theta} & g_{u\chi} \\ \cdot & g_{rr} & g_{r\theta} & g_{r\chi} \\ \cdot & \cdot & g_{\theta\theta} & g_{\theta\chi} \\ \cdot & \cdot & \cdot & g_{\chi\chi} \end{pmatrix} = \begin{pmatrix} 1 - \frac{2Mr-Q^2}{\Sigma} & 1 & 0 & a\sin^2\theta \frac{2Mr-Q^2}{\Sigma} \\ \cdot & 0 & 0 & -a\sin^2\theta \\ \cdot & \cdot & -\Sigma & 0 \\ \cdot & \cdot & \cdot & -\sin^2\theta \frac{A}{\Sigma} \end{pmatrix} \tag{A.1}$$

with inverse

$$g^{\mu\nu} = \begin{pmatrix} g^{uu} & g^{ur} & g^{u\theta} & g^{u\chi} \\ \cdot & g^{rr} & g^{r\theta} & g^{r\chi} \\ \cdot & \cdot & g^{\theta\theta} & g^{\theta\chi} \\ \cdot & \cdot & \cdot & g^{\chi\chi} \end{pmatrix} = \begin{pmatrix} \frac{-a^2\sin^2\theta}{\Sigma} & \frac{r^2+a^2}{\Sigma} & 0 & -\frac{a}{\Sigma} \\ \cdot & -\frac{\Sigma-2Mr+Q^2+a^2\sin^2\theta}{\Sigma} & 0 & \frac{a}{\Sigma} \\ \cdot & \cdot & -\frac{1}{\Sigma} & 0 \\ \cdot & \cdot & \cdot & -\frac{1}{\Sigma\sin^2\theta} \end{pmatrix}. \tag{A.2}$$

In (A.1),

$$A = \Sigma(r^2+a^2) + a^2\sin^2\theta \frac{2Mr-Q^2}{\Sigma} = (r^2+a^2)^2 - a^2\sin^2\theta\Delta. \tag{A.3}$$

Since $\partial_u g_{\mu\nu} = 0$ and $\partial_\chi g_{\mu\nu} = 0$, ∂_u and ∂_χ are Killing vector fields.

The horizons $h_\pm$ are given by the equations

$$\mathcal{K}^\pm(r) = 0 \tag{A.4}$$

where $\mathcal{K}^\pm$ are the hypersurfaces defined by

$$\mathcal{K}^\pm(r) = r - r_\pm. \tag{A.5}$$

Their normal vector fields are

$$l_\pm = \tilde{f}_\pm g^{\mu\nu}(\partial_\nu \mathcal{K}^\pm)\partial_\mu = l^u_\pm \partial_u + l^r_\pm \partial_r + l^\chi_\pm \partial_\chi, \tag{A.6}$$

where

$$l^u_\pm = \tilde{f}_\pm \frac{r^2+a^2}{\Sigma}, \quad l^r_\pm = -\frac{\tilde{f}_\pm \Delta}{\Sigma}, \quad l^\chi_\pm = \tilde{f}\frac{a}{\Sigma}, \tag{A.7}$$

and $\tilde{f}_\pm$ are arbitrary non vanishing functions.

$\mathcal{K}^\pm$ are null surfaces: in fact, an easy calculation leads to

$$l^2_\pm|_{r_\pm} = (g_{\mu\nu} l^\mu_\pm l^\nu_\pm)|_{r_\pm} \sim \Delta(r_\pm) = 0 \tag{A.8}$$

with

$$l_\pm|_{r_\pm} = \tilde{f}_\pm \frac{r^2_\pm + a^2}{\Sigma_\pm} \xi_\pm|_{r_\pm} := (f_\pm)^{-1}\xi_\pm|_{r_\pm}, \tag{A.9}$$

where $f_\pm = \frac{\Sigma_\pm}{r^2_\pm + a^2}(\tilde{f}_\pm)^{-1}$ and

$$\xi_\pm|_{r_\pm} = \partial_u|_{r_\pm} + \omega_\pm \partial_\chi|_{r_\pm}, \quad \xi^2_\pm|_{r_\pm} = 0 \tag{A.10}$$

i.e.

$$\xi^u_\pm|_{r_\pm} = 1, \quad \xi^r_\pm|_{r_\pm} = 0, \quad \xi^\chi_\pm|_{r_\pm} = \omega_\pm. \tag{A.11}$$

Being a linear combination of Killing vectors, $\xi_\pm|_{r_\pm}$ are also Killing vectors. Then, h_+ and h_- are Killing horizons.

For later use, from (A.7) and the definition (A.10),

$$\xi^u_\pm = 1, \quad \xi^r_\pm = -\frac{\Delta}{r^2+a^2}, \quad \xi^\chi_\pm = \frac{a}{r^2+a^2}. \tag{A.12}$$

If $t^\mu_\pm$ are tangent to $\mathcal{K}_\pm$, then $t_\pm \cdot l_\pm|_{r_\pm} = 0$, and since $l^2_\pm|_{r_\pm} = 0$, then $l_\pm|_{r_\pm}$ are also tangent to $\mathcal{K}_\pm$. So, $l^\mu_\pm|_{r_\pm} = \frac{dx^\mu}{d\lambda}$ for some curve $x^\mu(\lambda) \subset \mathcal{K}_\pm$. If λ is an affine parameter, then [8]

$$l \cdot D l^\mu_\pm|_{r_\pm} = 0 \tag{A.13}$$

and the Killing vectors obey the equation

$$\xi_\pm \cdot D\xi_\pm|_{r_\pm} = \kappa_\pm \xi_\pm|_{r_\pm} \tag{A.14}$$

where $\kappa_\pm = \xi_\pm \cdot \partial(ln|f_\pm|)$. ($D$ and ∂ are respectively covariant and ordinary derivatives.) The l.h.s. of (A.13) can be written

$$\xi_\pm \cdot D\xi_{\pm\mu} = -\frac{1}{2}\xi^2_{\pm,\nu} \tag{A.15}$$

which from

$$-\frac{1}{2}\xi^2_{\pm,\nu}|_{r_\pm} = \kappa_\pm(\xi_\pm)_\nu|_{r_\pm}, \tag{A.16}$$

allows the determination of $\kappa_\pm$.

From (A.1) and (A.11) one obtains

$$(\xi_\pm)_u|_{r_\pm} = (\xi_\pm)_\theta|_{r_\pm} = (\xi_\pm)_\chi|_{r_\pm} = 0, \quad (\xi_\pm)_r|_{r_\pm} = 1 - a\omega_\pm \sin^2\theta. \tag{A.17}$$

So, from (A.15),

$$-\frac{1}{2}\xi^2_{\pm,r}|_{r_\pm} = \kappa_\pm(\xi_\pm)_r|_{r_\pm} = (1 - a\omega_\pm \sin^2\theta)\kappa_\pm = \frac{\Sigma}{r^2_\pm + a^2}\kappa_\pm. \tag{A.18}$$

Using (A.12),

$$\partial_r\xi^2_\pm|_{r_\pm} = \partial_r(\xi_\pm)_u|_{r_\pm} - \frac{1}{r^2_\pm + a^2}\partial_r\Delta|_{r_\pm}(\xi_\pm)_r|_{r_\pm} + \frac{a}{r^2_\pm + a^2}\partial_r(\xi_\pm)_\chi|_{r_\pm}, \quad \partial_r\Delta|_{r_\pm} = 2(r_\pm - M) = r_\pm - r_\mp, \tag{A.19}$$

and from

$$(\xi_\pm)_u = g_{u\mu}\xi^\mu_\pm, \text{ and } (\xi_\pm)_\chi = g_{\chi\mu}\xi^\mu_\pm, \tag{A.20}$$

a long but straightforward calculation gives

$$\partial_r(\xi_\pm)_u|_{r_\pm} = 0, \text{ and } \partial_r(\xi_\pm)_\chi|_{r_\pm} = 0. \tag{A.21}$$

Putting these results together in (A.18) one finally obtains

$$\kappa_\pm = \frac{r_\pm - r_\mp}{2(r^2_\pm + a^2)}. \tag{A.22}$$

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