Starting from the Bernoulli equation:

$$\frac{1}{2}\rho v^2 + \rho g z + P = const \tag{1}$$

Where ρ is the density, v is the velocity at any point inside the liquid, $g = 9.81$, and z is the distance above the bottom of the tank at this point. Now we take this equation at the bottom of the tank, and set it equal to the same equation of the liquid at the top of the tank. We can get rid of P because it doesn't relate to the height of the liquid. All the ρ's have been cancelled.

$$\frac{1}{2}v_a^2 + g z_a = \frac{1}{2}v_b^2 + g z_b \tag{2}$$

Rearranging this gives:

$$\frac{v_a^2 + v_b^2}{2} + g\left(z_a - z_b\right) = 0 \tag{3}$$

$$v_a^2 + v_b^2 = 2g\left(z_b - z_a\right) \tag{4}$$

$$v_a^2 + v_b^2 = 2gH\left(t\right) \tag{5}$$

Next, we need to relate the volume of water lost from the top, to the water lost from the bottom. Where s_a is the cross sectional length of the tank, and s_b is the cross sectional length of the hole at the bottom of the tank:

$$s_a v_a \Delta t = s_b v_b \Delta t \tag{6}$$

And because $\alpha = \frac{s_a}{s_b}$

$$\frac{v_b}{v_a} = \alpha \tag{7}$$

So by rearranged Equation 5, we can get:

$$2gH\left(t\right) = \left(\left(\frac{v_b}{v_a}\right)^2 - 1\right) v_a^2 \tag{8}$$

So substituting in Equation 7, we get:

$$2gH\left(t\right) = \left(\alpha^2 - 1\right) v_a^2 \tag{9}$$

Rearranging this gives:

$$v_a^2 = \frac{2gH\left(t\right)}{\alpha^2 - 1} \tag{10}$$

$$v_a = \sqrt{\frac{2gH\left(t\right)}{\alpha^2 - 1}} \tag{11}$$

Now, v_a is actually dependant on time, since it's always the height of the water at the top of the tank. But since in all our equations so far, the positive direction has been up, but now the positive direction will be down, $\frac{dH(t)}{dt} = -v_a$ So:

$$\frac{dH(t)}{dt} = -\sqrt{\frac{2gH\left(t\right)}{\alpha^2 - 1}} \tag{12}$$