Truss Analysis Calculations

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1. Truss Geometry

The overall configuration of the 2-dimensional truss is shown in Figure 1. The specific node and member configurations are also summarized in Table 1 and Table 2 below.

The total span of the truss is 3.24 m and overall height of the truss is 3.106 m.

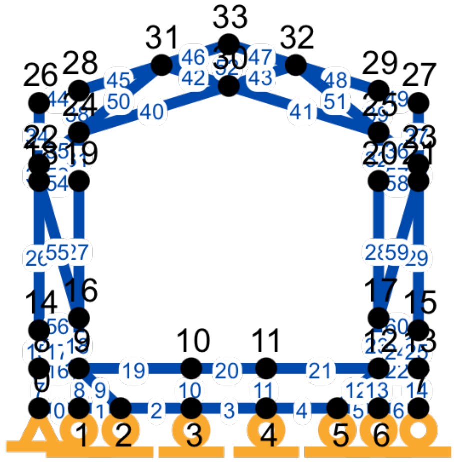

Figure 1: Truss global configuration

Node ID	X-Position (m)	Y-Position (m)	Fixity (if not free)
0	0	0	pin
1	0.34	0	roller
2	0.696	0	roller
3	1.3	0	roller
4	1.94	0	roller
5	2.55	0	roller
6	2.9	0	roller
7	3.24	0	roller
8	0	0.34	--
9	0.34	0.34	--
10	1.3	0.34	--
11	1.94	0.34	--
12	2.9	0.34	--
13	3.24	0.34	--
14	0	0.665	--
15	3.24	0.665	--
16	0.34	0.77	--

17	2.9	0.77	--
18	0	1.94	--
19	0.34	1.94	--
20	2.9	1.94	--
21	3.24	1.94	--
22	0	2.08	--
23	3.24	2.08	--
24	0.34	2.35	--
25	2.9	2.35	--
26	0	2.6	--
27	3.24	2.6	--
28	0.34	2.71	--
29	2.9	2.71	--
30	1.62	2.75	--
31	1.05	2.93	--
32	2.19	2.93	--
33	1.62	3.11	--

Table 1: Structure node geometry

Member ID	Start -> End Node	Length (m)
0	0 → 1	0.34
1	1 → 2	0.356
2	2 → 3	0.604
3	3 → 4	0.64
4	4 → 5	0.607
5	5 → 6	0.353
6	6 → 7	0.34
7	0 → 8	0.34
8	1 → 9	0.34
9	2 → 9	0.4923
10	3 → 10	0.34
11	4 → 11	0.34
12	5 → 12	0.4901
13	6 → 12	0.34
14	7 → 13	0.34
15	8 → 14	0.325
16	8 → 9	0.34

17	9 → 14	0.4703
18	9 → 16	0.43
19	9 → 10	0.96
20	10 → 11	0.64
21	11 → 12	0.96
22	12 → 13	0.34
23	12 → 17	0.43
24	12 → 15	0.4703
25	13 → 15	0.325
26	14 → 18	1.275
27	16 → 19	1.17
28	17 → 20	1.17
29	15 → 21	1.275
30	18 → 22	0.139
31	19 → 24	0.415
32	20 → 25	0.415
33	21 → 23	0.139
34	22 → 26	0.526
35	22 → 24	0.4379
36	23 → 25	0.4379
37	23 → 27	0.526
38	24 → 28	0.355
39	25 → 29	0.355
40	24 → 30	1.34
41	25 → 30	1.34
42	30 → 31	0.6
43	30 → 32	0.6
44	26 → 28	0.3558
45	28 → 31	0.7401
46	31 → 33	0.5997
47	33 → 32	0.5997
48	32 → 29	0.7401
49	29 → 27	0.3558
50	24 → 31	0.9107
51	25 → 32	0.9107

52	33 → 30	0.355
53	22 → 19	0.3673
54	19 → 18	0.34
55	18 → 16	1.218
56	16 → 14	0.3558
57	23 → 20	0.3673
58	20 → 21	0.34
59	21 → 17	1.218
60	17 → 15	0.3558

Table 2: Structure member geometry

2. Applied Loading to Nodes

The loads applied to this truss structure are represented in Figure 2 and summarized in detail below in Table 3. Note that if a node is omitted from Table 3, no loads have been applied to it.

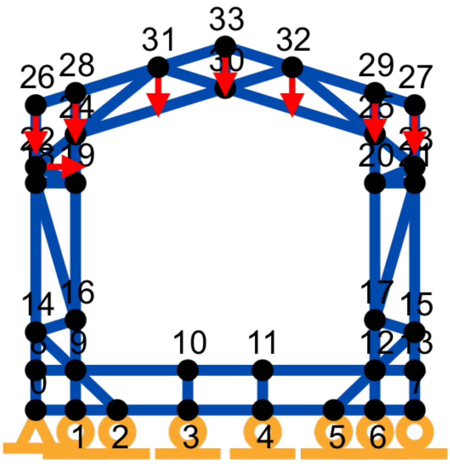

Figure 2: Graphical representation of loads applied to the structure (arrow length not to scale)

Node ID	Fx (kN)	Fy (kN)
22	2.5	0
26	0	-3
27	0	-3
28	0	-3
29	0	-3
31	0	-3
32	0	-3
33	0	-3

Table 3: Applied loading to nodes

3. Truss Analysis Using the Direct Stiffness Method

With the truss geometry and loading defined above, the member forces and deflections are calculated using the direct stiffness method. It is assumed that all members behave elastically and have sufficient strength at connections to transfer the required load to the member.

3.1 Member Stiffness Matrix

First, each member stiffness matrix is composed in the global coordinate system. For truss analysis, it is assumed that both ends of the member are rotationally unconstrained so that each member will only be loaded axially. The member stiffness matrix in the global coordinate system will be a 4x4 matrix for a 2-dimensional truss. Each member will be defined as follows:

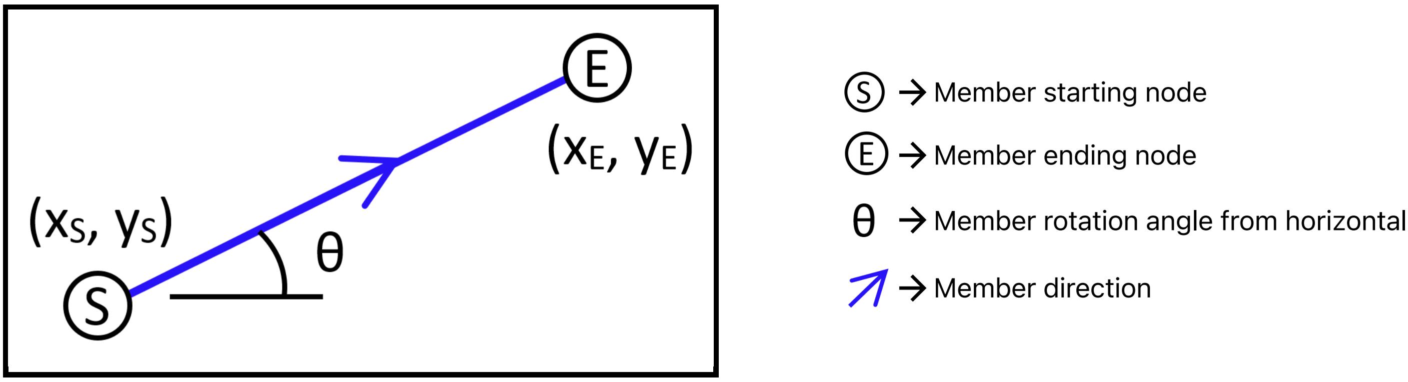

Figure 3: General member geometry definition

Having member properties:

L → Member length

A → Member cross-sectional area

E → Member material modulus of elasticity

In this analaysis, A and E have been set to the following values:

Member ID	Cross-sectional Area (mm^2)	Elastic Modulus (MPa)
0	2400	8480
1	2400	8480
2	2400	8480
3	2400	8480
4	2400	8480
5	2400	8480
6	2400	8480
7	2400	8480
8	2400	8480
9	2400	8480
10	2400	8480
11	2400	8480
12	2400	8480
13	2400	8480

14	2400	8480
15	2400	8480
16	2400	8480
17	2400	8480
18	2400	8480
19	2400	8480
20	2400	8480
21	2400	8480
22	2400	8480
23	2400	8480
24	2400	8480
25	2400	8480
26	2400	8480
27	2400	8480
28	2400	8480
29	2400	8480
30	2400	8480
31	2400	8480
32	2400	8480
33	2400	8480
34	2400	8480
35	2400	8480
36	2400	8480
37	2400	8480
38	2400	8480
39	2400	8480
40	2400	8480
41	2400	8480
42	2400	8480
43	2400	8480
44	2400	8480
45	2400	8480
46	2400	8480
47	2400	8480
48	2400	8480
49	2400	8480

50	2400	8480
51	2400	8480
52	2400	8480
53	2400	8480
54	2400	8480
55	2400	8480
56	2400	8480
57	2400	8480
58	2400	8480
59	2400	8480
60	2400	8480

For simplicity in this general example, the following constants are calculated:

$$c = cos\theta$$
$$s = sin\theta$$

And a stiffness matrix is assembled for each member using the following equation:

$$k_i = \frac{AE}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

For example, the stiffness matrix for member 0 is:

$$k_0 = \frac{2400\ mm^2 * 8480\ MPa}{0.340m}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3.2 Global Structure Stiffness Matrix

All of the member stiffness matrices will be combined to form the global structure stiffness matrix, K, by grouping each nodal degree of freedom and summing the attached member stiffness matrix elements. For this 2-dimensional truss with 34 nodes, the global stiffness matrix will be 68x68.

This operation yields the following structural stiffness matrix for the above defined truss:

59900	0	-59900	0	0	0	0	0	0	0	0	0	0	0	0	(
0	59900	0	0	0	0	0	0	0	0	0	0	0	0	0	(
-59900	0	117000	0	-57200	0	0	0	0	0	0	0	0	0	0	(
0	0	0	59900	0	0	0	0	0	0	0	0	0	0	0	(
0	0	-57200	0	112000	-20600	-33700	0	0	0	0	0	0	0	0	(
0	0	0	0	-20600	19700	0	0	0	0	0	0	0	0	0	(
0	0	0	0	-33700	0	65500	0	-31800	0	0	0	0	0	0	(
0	0	0	0	0	0	0	59900	0	0	0	0	0	0	0	(
0	0	0	0	0	0	-31800	0	65300	0	-33500	0	0	0	0	(
0	0	0	0	0	0	0	0	0	59900	0	0	0	0	0	(
0	0	0	0	0	0	0	0	-33500	0	113000	20700	-57700	0	0	(

0	0	0	0	0	0	0	0	0	0	20700	20000	0	0	0	(
0	0	0	0	0	0	0	0	0	0	-57700	0	118000	0	-59900	(
0	0	0	0	0	0	0	0	0	0	0	0	0	59900	0	(
0	0	0	0	0	0	0	0	0	0	0	0	-59900	0	59900	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	599
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	-59900	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	-21600	20600	0	0	0	0	0	0	0	0	0	(
0	0	0	-59900	20600	-19700	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	-59900	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	-59900	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	-21500	-20700	0	0	0	(
0	0	0	0	0	0	0	0	0	0	-20700	-20000	0	-59900	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-59
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	(

Structure Stiffness Matrix, K

3.3 Reduced Structure Stiffness Matrix

With the reactions at the structure supports being unknown, the structure stiffness matrix is reduced by removing the rows and columns which correspond to the node support directions, resulting in the reduced structure stiffness matrix, K_R :

117000	-57200	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-57200	112000	-33700	0	0	0	0	0	0	-21600	20600	0	0	0	0	0
0	-33700	65500	-31800	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-31800	65300	-33500	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-33500	113000	-57700	0	0	0	0	0	0	0	0	0	-2150
0	0	0	0	-57700	118000	-59900	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-59900	59900	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	59900	0	-59900	0	0	0	0	0	0
0	0	0	0	0	0	0	0	122000	0	0	0	0	0	0	0
0	-21600	0	0	0	0	0	-59900	0	125000	-42300	-21200	0	0	0	0
0	20600	0	0	0	0	0	0	0	-42300	148000	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-21200	0	53000	0	-31800	0	0
0	0	0	0	0	0	0	0	0	0	0	0	59900	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-31800	0	53000	0	-2120
0	0	0	0	0	0	0	0	0	0	0	0	0	0	59900	0
0	0	0	0	-21500	0	0	0	0	0	0	0	0	-21200	0	12500
0	0	0	0	-20700	0	0	0	0	0	0	0	0	0	0	4240
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5990
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-22600	21600	0	0	0	0	0
0	0	0	0	0	0	0	0	-62600	21600	-20700	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2260
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2160
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-47300	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced Structure Stiffness Matrix

3.4 Reduced structure force matrix

Given the loads applied to the structure, as described in Table 3, the global force matrix, Q, is assembled to match the dimensional size of the reduced structure stiffness matrix. Each node degree of freedom for the structure will match between the structure force and structure stiffness matrices. Since the reactions at the constrained nodes are unknown until the analysis is completed, the node support direction forces are removed from the global structure force matrix to create the reduced structure load matrix, Q_R :

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & -3 & 0 & -3 & 0 & -3 & 0 \end{bmatrix}$$

Structural Load Matrix (reduced)

3.5 Analysis for global displacements

The unknown nodal displacements are calculated by inverting the reduced stiffness matrix and multiplying it with the reduced structure force matrix: $K_R^{-1} \cdot Q_R$

Then, the known support displacements of 0 are added to compose the global stiffness matrix, D.

The resulting displacement at each node along with known support displacements are given below:

Node ID	Δx (m)	Δy (m)
0	0	0
1	0.0000418	0
2	0.0000855	0
3	0.000102	0
4	0.000119	0
5	0.000136	0
6	0.000136	0
7	0.000136	0
8	0.000025	0.000013
9	0.000025	-0.000157
10	0.0000705	0
11	0.000101	0
12	0.000146	0.0000162
13	0.000146	-0.000201

14	0.000243	0.0000255
15	0.000604	-0.000392
16	0.000386	-0.000376
17	0.00076	0.0000174
18	0.00242	-0.00000328
19	0.00244	-0.000759
20	0.00554	-0.000307
21	0.00551	-0.00102
22	0.00276	-0.0000295
23	0.00588	-0.00106
24	0.0034	-0.000887
25	0.00644	-0.000435
26	0.00389	-0.000107
27	0.00536	-0.00113
28	0.00415	-0.000939
29	0.00556	-0.000487
30	0.00485	-0.00405
31	0.00495	-0.00353
32	0.00468	-0.00333
33	0.00478	-0.004

Table 4: Structure node displacements derived from global stiffness matrix

3.6 Calculate member axial demands

Using the relative displacements of each member's start and end nodes along with a transformed stiffness matrix, the axial demand on a member, q, is calculated as follows:

$$q_i = \frac{AE}{L} \begin{bmatrix} -c & -s & c & s \end{bmatrix} \begin{bmatrix} \Delta_{Sx} \\ \Delta_{Sy} \\ \Delta_{Ex} \\ \Delta_{Ey} \end{bmatrix}$$

Where Δ_{Sx} is the displacement of the starting node in the x-direction for member i.

The member axial demands for the truss described above are displayed in Figure 4 and summarized in detail in Table 5 along with the member's length. Tensile axial loads are represented as negative forces, and compression axial demands are represented as positive forces.

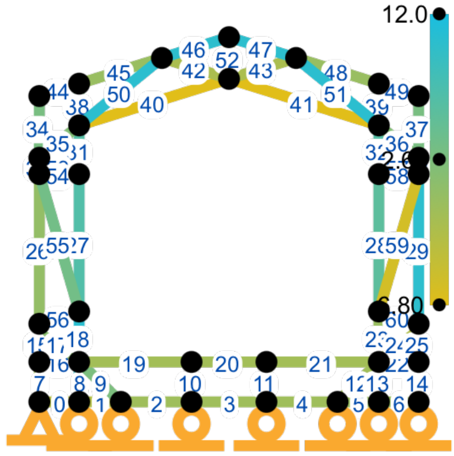

Figure 4: Structure member loading (kN)

Member ID	Length (m)	Axial Demand (kN)
0	0.34	-2.5
1	0.356	-2.5
2	0.604	-0.556
3	0.64	-0.556
4	0.607	-0.556
5	0.353	0
6	0.34	0
7	0.34	-0.7804
8	0.34	9.424
9	0.4923	2.688
10	0.34	0
11	0.34	0
12	0.4901	-0.772
13	0.34	-0.9708
14	0.34	12.01
15	0.325	-0.7804
16	0.34	0
17	0.4703	1.356
18	0.43	10.34
19	0.96	-0.9636
20	0.64	-0.9636
21	0.96	-0.9636
22	0.34	0
23	0.43	-0.05373

24	0.4703	-2.102
25	0.325	12.01
26	1.275	0.4594
27	1.17	6.667
28	1.17	5.645
29	1.275	10.08
30	0.139	3.833
31	0.415	6.266
32	0.415	6.266
33	0.139	4.855
34	0.526	3
35	0.4379	1.957
36	0.4379	1.957
37	0.526	3
38	0.355	3
39	0.355	3
40	1.34	-6.804
41	1.34	-6.804
42	0.6	-1.747
43	0.6	-1.747
44	0.3558	0
45	0.7401	0
46	0.5997	10.14
47	0.5997	10.14
48	0.7401	0
49	0.3558	0
50	0.9107	10.33
51	0.9107	10.33
52	0.355	-2.985
53	0.3673	1.059
54	0.34	-0.9803
55	1.218	3.513
56	0.3558	-1.026
57	0.3673	-1.642
58	0.34	1.52
59	1.218	-5.446

60	0.3558	1.591

Table 5: Structure member demand summary (+Compression/-Tension)

3.7 Calculate support reactions

First, the unknown values in the global force matrix, Q, is assembled by multiplying the global stiffness matrix by the global displacement matrix: Q = K · D

The total force at the truss supports are then found by removing all of the free degrees of freedom (reduced structural load matrix, Q_R) from the force matrix so that only the supported degrees of freedom remain.

Finally, to calculate the supports, any loads applied to the supports are subtracted out of this reduced force matrix yielding the following support reactions:

Node ID	Rx (kN)	Ry (kN)
0	-2.5	-0.7804
1	0	9.424
2	0	1.857
3	0	0
4	0	0
5	0	-0.5356
6	0	-0.9708
7	0	12.01

Table 6: Structure support reaction summary

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