Soft computing

Decision-making
multicriteria decision-making

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Overview

- *multicriteria classification*
- *multi criteria decision making, multi criteria analysis*
- *TOPSIS*
- *VIKOR*

Decision making

- ***Decision making*** *is regarded as the cognitive process resulting in the selection of a belief or a course of action among several alternatives.*
- *It is the process of identifying and choosing alternatives based on the values and* ***preferences*** *of the decision-Maker.*
- *An example of decision making is the purchase of the car when the buyer decides among limited number of possibilities (available models) based on his / her preferences (price, performance, color).*

Decision - making 2

- *During the decision making - the person applies different decision making processes.*
- *Decision making science deals with the description of decision making process.*
- *It can be used Decision Support Systems.*
  - *Decision Support Systems – DSS help its users with putting into practise of controlling and decision making activities.*
  - *A user can compare particular results of solutions with his / her ideas and according to that, the further process of the solution is influenced.*
  - *These systems provide users with solution offers and by questioning they give the direction of the further process.*
  - *These systems do not replace the decision makers. The results are not the decisions but just “set of variants”. They speed up and make the computation of consequences more precise. They quantify risks too.*
  - *part of knowledge-based systems*
    - *knowledge base and inference system (usually IF-THEN rules)*

Decision-making 3

- *Phases of decision making*
  - *reasoning*
    - *via thinking, reasoning - a user finds out which dependencies are valid in the task which should be decided.*
    - *a user is looking for connections / links and think about the consequences - so to find out objective facts - relevant knowledge.*
  - *comparing*
    - *here the user finds out subjective connections, how the alternatives are useful for “me”. Comparisons might be done as follows:*
      - *I know all the needed facts about the task (thus particularly conditioned by reasoning)*
      - *I know my own elementary preferences*
      - *I can compare these two “inputs” and make final preferences*
- *Based on final preferences, a user might make a decision.*

Decision-making 4 - some methods

- *Group techniques*
    - *Consensus decision-making tries to avoid "winners" and "losers"..*
    - *Voting-based methods:*
        - *Majority requires support from more than 50% of the members of the group.*
        - *Plurality, where the largest block in a group decides, even if it falls short of a majority.*
        - *Range voting lets each member score one or more of the available options (e.g. 1 to 10). The option with the highest average (or highest total score) is chosen. This method has experimentally been shown to produce the lowest Bayesian regret among common voting methods, even when voters are strategic.*
    - *other methods - Delphi method, Dotmocracy, Participative decision-making, Decision engineering - included machine learning techniques*
- *Individual techniques*
    - *lists of pros and cons (advantages and disadvantages)*
    - *toss a coin*
    - *priorities according to preferences*
    - *etc. including automatic decision making techniques*

Bayesian regret - parenthesis 1

- *Bayesian Regret (BR) (matematical definition)*
- *more about the voting systems https://rangevoting.org/BayRegDum.html*
  - *Bayesian "regret" (also called "loss") is just the maximum possible utility minus the Bayesian "expected utility." (Based on the application, the work with regret might be more comfortable then with utility. Regret and utility are common terminology in this area.*
  - *„"Social" utility is the sum of utility over all the members of some human population. For voting systems purposes, "Bayesian regret" is the expectation value of social regret. It depends on both the voting system, the number of candidates, the number of voters, and the probabilistic models of utilities, candidates, and voter behaviors..*
- *In theory, for a "perfect" election method, BR=0, and for all other election methods, BR>0. A scaling of BR is often used (multiply it by appropriate scale factor) that causes BR for "random winner" to be 1, which is sort of an estimate for "non-democracy."*

Bayesian regret - parenthesis 2

- *Simplified: The "Bayesian regret" of an election method E is the "expected avoidable human unhappiness" caused by using E.*
- *More precise procedure:*
- *1. Each voter has a personal "utility" value for the election of each candidate. (E.g., if Trump is elected, then voter Biden will acquire -55 extra lifetime happiness units.) In a computer simulation, the "voters" and "candidates" are artificial, and the utility numbers are generated by some randomized "utility generator" and assigned artificially to each candidate-voter pair.*
- *2. Now the voters vote, based both on their private utility values, and (if they are strategic voters) on their perception from "pre-election polls" (also generated artificially within the simulation, e.g. from a random subsample of "people") of how the other voters are going to act.*

Bayesian regret - parenthesis 3

- *3. The election system E elects some winning candidate W.*
- *4. The sum over all voters V of their utility for W, is the "achieved societal utility."*
- *5. The sum over all voters V of their utility for X, maximized over all candidates X, is the "optimum societal utility" which would have been achieved if the election system had magically chosen the societally best candidate.*
- *6. The difference between 5 and 4 is the "Bayesian Regret" of the election system E, at least in this experiment. It might be zero, but if E was bad or if this election was unlucky for E, then it will be positive because W and X will be different candidates.*
- *7. We now redo steps 1-6 a zillion times (i.e. running a zillion simulated elections) to find the average Bayesian regret of election system E.*

Decision-making 5

- *Essential part of decision making includes analysis of the terminal set of alternatives (output data) with usage of evaluation criteria (input data, features, attributes).*
- *Then it is **possible to order** these alternatives according to attractiveness for the decision maker, if all criteria are considered simultaneously.*
- *Or to f**ind out the best alternative** or to determine the relative total priority of each alternative (in cases that projects are in the competitions for financial sources), again supposed that all criteria are considered simultaneously.*
- ***multiple-criteria decision analysis - MCDA***
  - *different methods, which can give different results on the same data*
  - *still discussed if suitable… - soft computing*

MCDA - multi criteria decision analysis 1

- *example - possibility to go for holidays*
- *3 alternatives*
  - *Bali*
  - *Bibione*
  - *Karibik*
- *how to decide?*

MCDA - multi criteria decision analysis 2

- *each of alternatives will be evaluated by some criteria, e.g.*
- *travel expenses*
- *accommodation expenses*
- *fun, entertainment*
- *swimming quality*
- *hotel facilities, atd.*
- *selection of the criteria is based on the expert in that area same as in the classifiers*
    - *it is necessary to be careful with the criteria selection to achieve the best results*

MCDA - multi criteria decision analysis 3

- *it is necessary to take into consideration*
  - *information gain - library, internet, journals*
  - *reliability of the information*
  - *time to information retrieve*
  - *cost for information retrieve, etc.*

  - *information might be opposing or particular attributes (properties of the given task) for particular output classes (alternatives) might go against each other and it is necessary to decide somehow*

⇩

- *multi criteria problems - methods from multicriteria decision making (analysis) - MCDM (MCDA)*

MCDA 4 - methods

- *the best selection from the set of alternatives (outputs, classes) which each of it is evaluated by multiple criteria (inputs, features)*
- *techniques include:*
  - *SAW - Simple Additive Weighting*
  - ***TOPSIS** - Technique for Order Preference by Similarity to the Ideal Solution*
  - *ELECTRE - Elimination et Choice Translating Reality*
  - *AHP - Analytical Hiearchy Process*
  - *SMART - The Simple Multi Attribute Rating Technique*
  - *ANP - Analytic Network Process*
  - ***VIKOR***

MCDA 5 - terminology

- *alternatives - possibilities - output values (classes) - what should be decided as the best (in the example of holidays - Bali, Bibione, Karibik)*
- *Criteria, attributes - input variables (features) which have the infuence on the output alternatives (in the example with holidays - travel expenses, accommodation expenses, fun, entertainment, swimming quality, hotel facilities …)*

- *It is necessary to keep:*
  - *Completeness - all important attributes are included*
  - *Redundant attributes - to remove relatively not important or duplicates at the beginning*
  - *Functionality - each output value has to be decidable towards to attributes*

MCDA 6 - terminology

- *weights - importance of particular attributes (criteria)*
- *each attribute is assigned a weight in some range (0-10) or (0-100) by group of decision - makers (somebody who has the authority to make decisions)*
- *example*
  - *if a person wants to simplify computation, it is suitable to think about the benefits and costs during the design of the evaluation scale*
  - *if it is set so that everything is related to benefits, it is not necessary to think about benefits vs. costs during the process in some methods (TOPSIS)(low costs are benefits, water quality - higher value -benefit)*

Attribute (criterium)	Weight	Evaluation scale
cost for travelling	6	10 low - 1 high
costs for accommodation	8	10 low - 1 high
entertainment	2	10 very good - 1 no
water quality	4	10 very good - 1 bad

MCDA 6 - terminology

- *decision-makers - who has the right to decide (experts)*
- *they have attributes and evaluating scale at their disposal*
- *their decision is an average*
- *decision-table - is made for all possible output values (alternatives) - examples with Bali, Bibione, Karibik - artificial example*

- *Bali*

Attribute (criterium)	Decision - makers			Weight of the attribute
	Tom	Petr	Jan	
cost for travelling	6	2	4	(6 + 2 + 4)/3 = 4
costs for accommodation	8	8	5	(8 + 8 + 5)/3 = 7
entertainment	4	2	3	(4 + 2 + 3)/3 = 3
water quality	4	5	6	(4 + 5 + 6)/3 = 5

MCDA 8 - terminology

- *the same for other alternatives:*
- *Bibione*

Attribute (criterium)	Decision - makers			Weight of the attribute
	Tom	Petr	Jan	
cost for travelling	5	2	2	(5 + 2 + 2)/3 = 3
costs for accommodation	6	2	4	(6 + 2 + 4)/3 = 4
entertainment	9	9	3	(9 + 9 + 3)/3 = 7
water quality	2	1	3	(2 + 1 + 3)/3 = 2

- *Karibik*

Attribute (criterium)	Decision - makers			Weight of the attribute
	Tom	Petr	Jan	
cost for travelling	1	1	1	(1 + 1 + 1)/3 = 1
costs for accommodation	3	2	4	(3 + 2 + 4)/3 = 3
entertainment	9	9	9	(9 + 9 + 9)/3 = 9
water quality	10	10	7	(10 + 10 + 7)/3 = 9

MCDA 9 - decision matrix

- *total table of particular alternatives and criteria*

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	4	3	1
costs for accommodation	7	4	3
entertainment	3	7	9
water quality	5	2	9

- *Karibik*

Attribute (criterium)	Decision - makers			Weight of the attribute
	Tom	Petr	Jan	
cost for travelling	1	1	1	(1 + 1 + 1)/3 = 1
costs for accommodation	3	2	4	(3 + 2 + 4)/3 = 3
entertainment	9	9	9	(9 + 9 + 9)/3 = 9
water quality	10	10	7	(10 + 10 + 7)/3 = 9

TOPSIS 1

- *TOPSIS - Technique for Order Preference by Similarity to Ideal Solution*
- *2 artificial alternatives are assumed*
  - ***Ideal alternative** - the one with the best attributes (attributes with maximal benefit and minimal costs)*
  - ***Negative ideal alternative** - the one with worst attributes (minimal benefit and maximal costs)*
- *TOPSIS selects the alternative which is the closest to ideal alternative and the most distant from negative ideal alternative*

TOPSIS 2 - in a nutshell 1

- *step 1 - evaluation matrix -* ***m*** *alternatives,* ***n*** *criteria, item of the matrix* x_{ij}
- *step 2 - normalization of the evaluation matrix (decision-table)*

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}^2}, i = 1,2,...,m, j = 1,2,...n$$

- *step 3 - weighted normalized matrix* $t_{ij} = r_{ij} \cdot w_j$
  - *multiply the normalized evalutation matrix by weights* w_j

$$w_j = W_j / \sum_{j=1}^{n} W_j, j = 1, 2, \ldots, n$$

  - *where* W_j *is the value of the original attribute weight (average of decision makers)*
  - *so that* $\sum_{j=1}^{n} w_j = 1$
  - *(alternative, it is possible to use also the root square of the sum of weight squares)*

TOPSIS 2 - in nutshell 2

- *step 4 - ideal (the best, A_b) and the negative ideal (the worst, A_w) solution*
  - *while the benefit is maximized (positive influence) and costs are minimized (negative influence)*

$$A_w = \{\langle max(t_{ij} | i = 1,2,...,m) | j \in J_- \rangle, \langle min(t_{ij} | i = 1,2,...,m) | j \in J_+ \rangle\} \equiv \{t_{wj} | j = 1,2,...,n\}$$
$$A_b = \{\langle min(t_{ij} | i = 1,2,...,m) | j \in J_- \rangle, \langle max(t_{ij} | i = 1,2,...,m) | j \in J_+ \rangle\} \equiv \{t_{bj} | j = 1,2,...,n\}$$

$J_+ = \{j = 1,2,...,n | j\}$ criteria with positive influence (benefits)

$J_- = \{j = 1,2,...,n | j\}$ criteria with negative influence (costs)

  - *it is necessary to pay attention - the best is to reverse the costs scale so that to express benefits - to have all scales in the same sense*
    - *then it is enough to sort out just the half of the expression*

$$A_w = \{\langle min(t_{ij} | i = 1,2,...,m) | j \in J_+ \rangle\} \equiv \{t_{wj} | j = 1,2,...,n\}$$
$$A_b = \{\langle max(t_{ij} | i = 1,2,...,m) | j \in J_+ \rangle\} \equiv \{t_{bj} | j = 1,2,...,n\}$$

TOPSIS 2 - in a nutshell 3

- *step 5 - the distance from the best and worst solution, where d_{iw} (S') - is the worst and d_{ib} (S*) is the best conditions of the attributes to the class (alternative)*

$$d_{iw} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{wj})^2}, i = 1, 2, \ldots, m,$$

$$d_{ib} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{bj})^2}, i = 1, 2, \ldots, m$$

- *step 6 - to compute the similarity towards to the worst solution s_{iw}*

$$s_{iw} = d_{iw} / (d_{iw} + d_{ib}), 0 \leq s_{iw} \leq 1, i = 1, 2, \ldots, m \;.$$

  - *s_{iw} = 1, if the attribute has the best conditions (values)*
  - *s_{iw} = 0, if the attribute has the worst conditions (values)*

- *step 7 - to order the alternatives according to the value S_{iw}, from the worst (the smallest value) to the best (the highest value)*

TOPSIS 4 - algorithm + steps in the example

- *Step 1 - evaluation (decision) matrix*

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	4	3	1
costs for accommodation	7	4	3
entertainment	3	7	9
water quality	5	2	9

decision matrix

TOPSIS 4 - algorithm + steps in the example

- *Step 2 - normalization of the decision matrix*
  - *transforms different dimensions of attributes to nondimensional space to be able to compare them across the criteria*
  - *each column of the decision matrix is divided by the root square of the sum of weight squares of particular attribute*

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$$

Attribute (criterium)	Alternatives			square root of the sum…
	Bali	Bibione	Karibik	
cost for travelling	4	3	1	$\sqrt{4^2+3^2+1^2} = 5.099$
costs for accommodation	7	4	3	$\sqrt{7^2+4^2+3^2} = 8.602$
entertainment	3	7	9	$\sqrt{3^2+7^2+9^2} = 11.79$
water quality	5	2	9	$\sqrt{5^2+2^2+9^2} = 10.488$

rozhodovací matice

TOPSIS 5 - algorithm + steps in the example

- *Step 2 - normalization of the decision matrix*
  - *transforms different dimensions of attributes to nondimensional space to be able to compare them across the criteria*
  - *each column of the decision matrix is divided by the root square of the sum of weight squares of particular attribute*

Attribute (criterium)	Alternatives			$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}^2}$
	Bali	Bibione	Karibik	
cost for travelling	4	3	1	$\sqrt{4^2+3^2+1^2} = 5.099$
costs for accommodation	7	4	3	$\sqrt{7^2+4^2+3^2} = 8.602$ $\sqrt{3^2+7^2+9^2} = 11.79$
entertainment	3	7	9	$\sqrt{5^2+2^2+9^2} = 10.488$
water quality	5		9	

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	4 / 5.099	3 / 5.099	1 / 5.099
costs for accommodation			
entertainment			
water quality			

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	0.784	0.588	0.196
costs for accommodation			
entertainment			
water quality			

TOPSIS 5 - algorithm + steps in the example

- *Step 2 - normalization of the decision matrix*
  - *transforms different dimensions of attributes to nondimensional space to be able to compare them across the criteria*
  - *each column of the decision matrix is divided by the root square of the sum of weight squares of particular attribute*

Attribute (criterium)	Alternatives			$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}^2}$
	Bali	Bibione	Karibik	
cost for travelling	4	3	1	$\sqrt{4^2+3^2+1^2} = 5.099$
costs for accommodation	7	4	3	$\sqrt{7^2+4^2+3^2} = 8.602$ $\sqrt{3^2+7^2+9^2} = 11.79$
entertainment	3	7	9	$\sqrt{5^2+2^2+9^2} = 10.488$
water quality	5		9	

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	4 / 5.099	3 / 5.099	1 / 5.099
costs for accommodation	7 / 8.602	4 / 8.602	3 / 8.602
entertainment			
water quality			

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	0.784	0.588	0.196
costs for accommodation	0.814	0.465	0.349
entertainment			
water quality			

TOPSIS 5 - algorithm + steps in the example

- *Step 2 - normalization of the decision matrix*
  - *transforms different dimensions of attributes to nondimensional space to be able to compare them across the criteria*
  - *each column of the decision matrix is divided by the root square of the sum of weight squares of particular attribute*

Attribute (criterium)	Alternatives			$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}^2}$
	Bali	Bibione	Karibik	
cost for travelling	4	3	1	$\sqrt{4^2+3^2+1^2} = 5.099$
costs for accommodation	7	4	3	$\sqrt{7^2+4^2+3^2} = 8.602$ $\sqrt{3^2+7^2+9^2} = 11.79$
entertainment	3	7	9	$\sqrt{5^2+2^2+9^2} = 10.488$
water quality	5		9	

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	4 / 5.099	3 / 5.099	1 / 5.099
costs for accommodation	7 / 8.602	4 / 8.602	3 / 8.602
entertainment	3 / 11.79	7 / 11.79	9 / 11.79
water quality	5 / 10.488	2 / 10.488	9 / 10.488

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	0.784	0.588	0.196
costs for accommodation	0.814	0.465	0.349
entertainment	0.254	0.594	0.763
water quality	0.477	0.191	0.858

TOPSIS 6 - algorithm + steps in the example

- *Step 3 - weighted normalized matrix*

$$t_{ij} = r_{ij} \cdot w_j$$

- *weighted matrix*

$$w_j = W_j / \sum_{j=1}^{n} W_j, j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} w_j = 1$$

Attribute (criterium)	Alternatives			sum of weights
	Bali	Bibione	Karibik	
cost for travelling	4	3	1	(4+3+1)= 8
costs for accommodation	7	4	3	(7+4+3)= 14
entertainment	3	7	9	(3+7+9)= 19
water quality	5		9	(5+2+9)= 16

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	4 / 8	3 / 8	1 / 8
costs for accommodation	7 / 14	4 / 14	3 / 14
entertainment			
water quality			

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	0,5	0,375	0,125
costs for accommodation	0,5	0,286	0,214
entertainment			
water quality			

TOPSIS 7 - algorithm + steps in the example

- *Step 3 - weighted normalized matrix*
  - *weighted matrix*

$$w_j = W_j / \sum_{j=1}^{n} W_j, j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} w_j = 1$$

Attribute (criterium)	Alternatives			sum of weights
	Bali	Bibione	Karibik	
cost for travelling	4	3	1	(4+3+1)= 8
costs for accommodation	7	4	3	(7+4+3)= **14**
entertainment	3	7	9	(3+7+9)= 19
water quality	5		9	(5+2+9)= 16

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	4 / 8	3 / 8	1 / 8
costs for accommodation	7 / **14**	4 / **14**	3 / **14**
entertainment	3 / 19	7 / 19	9 / 19
water quality	5 / 16	2 / 16	9 / 16

- ***weighted matrix***

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	0,5	0,375	0,125
costs for accommodation	0,5	0,286	0,214
entertainment	0,158	0,368	0,474
water quality	0,313	0,125	0,562

TOPSIS 8 - algorithm + steps in the example

- *Step 3 - second part - to create weighted normalized decision matrix*
  - *each attribute in normalized matrix is necessary to multiply by the weight*

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}^2}$$

$$w_j = W_j / \sum_{j=1}^{n} W_j, j = 1, 2, \ldots, n$$

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	0.784	0.588	0.196
costs for accommodation	0.814	0.465	0.349
entertainment	0.254	0.594	0.763
water quality	0.477	0.191	0.858

*

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	0,5	0,375	0,125
costs for accommodation	0,5	0,286	0,214
entertainment	0,158	0,368	0,474
water quality	0,313	0,125	0,562

$$t_{ij} = r_{ij} \cdot w_j$$

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	0.392	0.221	0.025
costs for accommodation	0.407	0.133	0.075
entertainment	0.04	0.219	0.362
water quality	0.149	0.024	0.483

TOPSIS 9 - algorithm + steps in the example

- *Step 4 - ideal solution (max.) and negative ideal solution (min.)*
- *at the beginning there was a **evaluation scale** in the sense of **benefits**, i.e. it is necessary to sort out only maximum and minimum*
  - ***it is not necessary to sort** out the ideal solution for benefits (max) vs. ideal solution for costs (min) and viceversa negative ideal solution for benefits (min) and negative ideal solution for costs (max)*

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	0.392	0.221	0.025
costs for accommodation	0.407	0.133	0.075
entertainment	0.04	0.219	0.362
water quality	0.149	0.024	0.483

- *ideal solution - {0.392, 0.407, 0.362, 0.483}*
- *negative ideal solution - {0.025, 0.075, 0.4, 0.024}*

TOPSIS 10 - algorithm + steps in the example

- *Step 5 - to compute the separation from the ideal solution d_{ib} (S*)*

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	$(0.392 - 0.392)^2 = 0.0$	$(0.221 - 0.392)^2 = 0.029$	$(0.026 - 0.392)^2 = 0.135$
costs for accommodation	$(0.407 - 0.407)^2 = 0.0$	$(0.133 - 0.407)^2 = 0.075$	$(0.075 - 0.407)^2 = 0.11$
entertainment	$(0.04 - 0.362)^2 = 0.103$	$(0.219 - 0.362)^2 = 0.02$	$(0.362 - 0.362)^2 = 0.0$
water quality	$(0.149 - 0.483)^2 = 0.111$	$(0.024 - 0.483)^2 = 0.211$	$(0.483 - 0.483)^2 = 0.0$
$d_{ib} = \sqrt{\sum(\ )^2}$	0.463	0.579	0.496

$$d_{ib} = \sqrt{\sum_{j=1}^{n} (t_{ij} - t_{bj})^2}, i = 1, 2, \ldots, m$$

TOPSIS 11 - algorithm + steps in the example

- *Step 6 - to compute separation from the negative ideal solution d_{iw} (S')*

Attribute (criterium)	Alternatives		
	Bali	Bibione	Karibik
cost for travelling	$(0.392 - 0.026)^2 = 0.135$	$(0.221 - 0.026)^2 = 0.038$	$(0.026 - 0.026)^2 = 0.0$
costs for accommodation	$(0.407 - 0.075)^2 = 0.11$	$(0.133 - 0.075)^2 = 0.003$	$(0.075 - 0.075)^2 = 0.0$
entertainment	$(0.04 - 0.04)^2 = 0.0$	$(0.219 - 0.04)^2 = 0.032$	$(0.362 - 0.04)^2 = 0.103$
water quality	$(0.149 - 0.024)^2 = 0.016$	$(0.024 - 0.024)^2 = 0.0$	$(0.483 - 0.024)^2 = 0.211$
$d_{iw} = \sqrt{\sum(\ )^2}$	0.529	0.285	0.561

$$d_{iw} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{wj})^2}, i = 1, 2, \ldots, m,$$

TOPSIS 12 - algorithm + steps in the example

- *Step 7 - to compute the relative closeness to ideal solution*

$$s_{iw} = d_{iw}/(d_{iw} + d_{ib}), 0 \leq s_{iw} \leq 1, i = 1, 2, \ldots, m \; .$$

	Alternatives		
	Bali	Bibione	Karibik
d_{ib}	0.463	0.579	0.496
d_{iw}	0.525	0.285	0.561
$d_{ib} + d_{iw}$	0.463 + 0.525 = 0.988	0.579 + 0.285 = 0.864	0.496 + 0.561 = 1.057
$d_{iw} / (d_{ib} + d_{iw})$	0.525 / 0.988 = $\boxed{0.5309}$	0.285 / 0.864 = 0,329	0.561 / 1.057 = 0.5308

max

- *min < max*

- ***the worst*** *Bibione (0,329) < Karibik (0,5308)<* $\boxed{\textit{Bali (0,5309)}}$ ***the best***
- *according to the TOPSIS, the variant Bali was selected (Karibik is very close, almost next to each other)*

VIKOR 1

- *for conflict and incomparable data (different units)*
- *compromise is acceptable for conflict solution*
- *basic ideas 1979 - Serafim Opricovic, name VIKOR 1990*
    - *serbian VIseKriterijumska Optimizacija I Kompromisno Resenje (multicriteria optimization and compromise solution)*

VIKOR 2

- *some parts of the algorithm are similar to TOPSIS*
- *the best solution over all criteria (multi criteria function) - benefit*
- *the worst solution over all criteria (multi criteria function) - costs / price*
- *to compute values S_i and R_i, where weights w are relative weights of particular attributes (given by experts)*
  - *index i means number of alternatives (classes)*
  - *n - number of criteria (attributes) - index j*

$$f_j^* = \max_i f_{ij} \qquad f_j^- = \min_i f_{ij}$$

for benefits $$S_i = \sum_{j=1}^{n} w_j (f_j^* - f_{ij})/(f_j^* - f_j^-)$$

for costs, to change

$(f_j^* - f_{ij})$ for $(f_{ij} - f_j^-)$

$$R_i = \max_j w_j (f_j^* - f_{ij})/(f_j^* - f_j^-)$$

VIKOR 3

- *to compute Q_i* $Q_i = \nu((S_i - S^*)/(S^- - S^*) + (1-\nu)((R_i - R^*)/(R^- - R^*)$
- *or alternatively* $Q_i = \nu((S_i - S^-)/(S^* - S^-) + (1-\nu)((R_i - R^-)/(R^* - R^-)$
- *where* $S^* = \min_i S_i, S^- = \max_i S_i$ $R^* = \min_i R_i, R^- = \max_i R_i$
  - ν *the weight of the strategy - maximum of group utility*
  - $1-\nu$ *the weight of the individual regret*
  - *compromise between strategies* $\nu = 0.5$

VIKOR 4

- *order output alternatives according to Q, S, and R from minimal (best) to maximal value*
- *the results are the 3 ranked evaluated lists*
- *compromise - alternative A(1), which is the best according to Q (minimum) at 2 conditions*
  - *acceptable advantage Q(A(2)) - Q(A(1)) ≥ DQ, where A(2) is the second best alternative according to Q and DQ=1/(1-J) and J is the total number of alternatives*
  - *acceptable stability in decision making A(1) must be the best also according to S and R, then the solution is stable for v = 0.5 (maximal group utility), or v is around 0.5 (in the case of consensus) and or v < 0.5 (veto)*
  - *if any of these two conditions are not valid, then*

VIKOR 5

- *compromise set of solutions is designed and consisted of*
  - *A(1) and A(2), if the condition 2 is not valid*
  - *A(1), A(2)…A(M), if condition 1 is not valid, where A(M) is determined as Q(A(M)) - Q(A(1)) < DQ for maximal M (position of these alternatives are mutual in closenest)*
- *this solution can be accepted because maximal utility from majority of (min S) and minimal individual regret (min R)*
- *measures S and R are integrated in Q for compromise solution*

- *VIKOR is effective, mainly when decision - makers are not able to express the preferences at the beginning of the process*
- *resulting compromise solution can be further discussed and can include weighting of the decision-makers preferences*

VIKOR 6 - variants

- *comprehensive VIKOR (2011)*
- *fuzzy VIKOR (2011)*
- *regret theory-based VIKOR (2000)*
- *modified VIKOR (2011)*
- *interval VIKOR (2009)*

- *briefly, they differ by the computation of S, R, Q*

Fuzzy VIKOR

- *criteria and weights might be fuzzy sets*
- *usually triangle fuzzy numbers are used*
- *fuzzy VIKOR is then used for aggregation of the quality which represents the distance from the ideal solution*
- *fuzzy operations are used for evaluation and ranking of fuzzy numbers*