Physics 212

Exam 1 Study Guide

1 Coulomb's Law

- Describes the force from one charged particle onto another charged particle

$$\hat{F_{1,2}} = k\frac{q_1 q_2}{r^2}\hat{r}$$

- $\hat{F_{1,2}}$ = Force from point q_1 onto q_2, (Newtons)

- k = $\frac{1}{4\pi\epsilon_0}$ = $9.0 * 10^9 \frac{Nm^2}{C^2}$

- q_1, q_2= Charge of q_1 and $q_2$respectively, (Coulombs)

- r=Distance between the two charges, (Meters)

- $\hat{r}$=Unit vector pointing in the direction of $F_{1,2}$

Question 1.1: Coulomb's law example

Consider two particles, one carrying a charge of +1.5 nC and the other a charge of -2.0 nC, separated by a distance of 1.5 cm. Find the electric force which the positive charge exerts on the negative charge. n = 10^{-9}

Since we're attempting to find the electric force between two charges and are given our charges and distance, we can plug in our formula. Additionally, we're finding a value, so no need for vector hats or the unit vector r.

$$F_{1,2} = k\frac{q_1 q_2}{r^2}\hat{r}$$
$$F_{1,2} = 9.0 * 10^9 \frac{Nm^2}{C^2}\frac{1.5 * 10^{-9}C * -2.0 * 10^{-9}C}{0.015m}$$
$$F_{1,2} = -1.8 * 10^{-6}N$$

1.1 Superposition

Since force is a vector, it can be added to other vectors to create a net force. This translates to Coulomb's law. If multiple charges create forces on a single charge, then the net force on that single charge is the sum all the forces acting upon it. $\sum_a^b F_a + F_{a+1} + F_{a+2} + \ldots + F_b$

Question 1.2: Superposition

Suppose we had two balls, b_1 and b_2, of mass M and with charges of +Q. b_1 is a distance d meters from b_2 and is currently directly above b_2. Calculate the $\hat{F_y}$ on b_1.

The two forces which play a part in affecting the balls are gravity and the electric force. Also, since there is no horizontal force, our net force is equal to F_y

$$\hat{F_y} = \hat{F_g} + \hat{F_{ef}}$$
$$\hat{F_g} = -Mg\hat{j}$$
$$\hat{F_{ef}} = k\frac{Q^2}{d^2}\hat{j}$$
$$\hat{F_y} = (k\frac{Q^2}{d^2} - Mg)\hat{j}$$

Question 1.3: Not fun

If we placed a particle of mass M and of +Q a distance D away from a fixed charge of +Q, calculate the time it would take for the particle to reach a speed of v.

2 Electric Fields

- Charges create electric fields
- Electric fields emitted from a positive charge point outward, ones from a negative charge point inward
- Two charges, one negative and one positive, are referred to as a dipole
- To envision an electric field, imagine if a positive point charge was placed
- On a graph, electric field line density determines relative magnitude, ie. more lines in an area means the magnitude of the electric field at that

area is larger than the magnitude of the electric field of an area of equal size with fewer lines.

$$E = \frac{F_e}{q_2} = \frac{kq_1}{r^2}$$

- Essentially, the electric field is equal to Coulomb's force but without the second charge
- E = Strength of the Electric field in N/C

Question 2.1: Electric Fields

If we placed a charge of 5 C in an electric field with 900 N/C, what is the force exerted onto the charge?

$$\begin{aligned}
E &= \frac{F_e}{q_2} \\
F_e &= Eq_2 \\
F_e &= 900\frac{N}{C} * 5C \\
F_e &= 4500N
\end{aligned}$$

2.1 Superposition

- lol electric fields add up

Question 2.2: Electric Fields

Determine the Electric field a distance r away of an infinitely long line of linear charge density λ C/m.

$$E_y = \int k\frac{\lambda dx}{h^2}\sin\theta \tag{1}$$

$$x = r\cot\theta \tag{2}$$

$$dx = \frac{r}{\csc^2\theta}d\theta \tag{3}$$

$$h = r\sin\theta \tag{4}$$

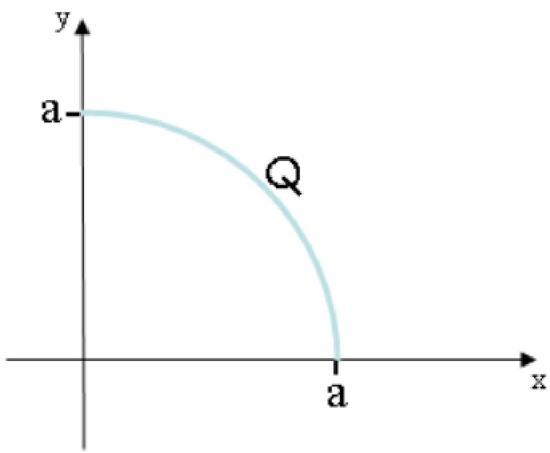
$$E_y = \int k\frac{\lambda\left(\frac{r}{\csc^2\theta}d\theta\right)}{r^2\sin^2\theta}\sin\theta \tag{5}$$

$$E_y = k\frac{\lambda}{r}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin\theta d\theta \tag{6}$$

$$E_y = k\frac{2\lambda}{r} \tag{7}$$

Question 2.3: Electric Fields

Something about arc radius a, total charge of Q, and finding the E_x



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- ## 3 Gauss's Law

  - Gauss's law enables us to determine an electric field around an object. In order to do so, we create a surface called a Gaussian surface, around/surrounding the object.

  - Gauss's law essentially is referencing the flux, or the dot product of the electric field through an area made by the Gaussian surface, $EA\cos\theta$
  - It's important to sum the flux through every single part of the Gaussian surface, pieces which are perpendicular to the Gaussian surface can be discarded because $\cos\frac{\pi}{2} = 0$
  - A Gaussian surface is typically a 3D shape, usually a cylinder, rectangular prism, or sphere.
  - For the sake of simplicity, we only uses Gauss's law when the electric field is even across a Gaussian surface.
  - To

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

- Basically the first half of this equation is just the flux
- $\vec{E}$ = Electric Field and it's direction, C/N
- $d\vec{A}$ yea bro its the surface area
- Q_{enc} = Total charge enclosed within the surface, C
- $\epsilon_0 = 8.854 * 10^{-12} \frac{C^2}{Nm^2}$
- In words it just means the flux generated from the electric field and surface area is equal to the charge enclosed by the surface, divided by a constant

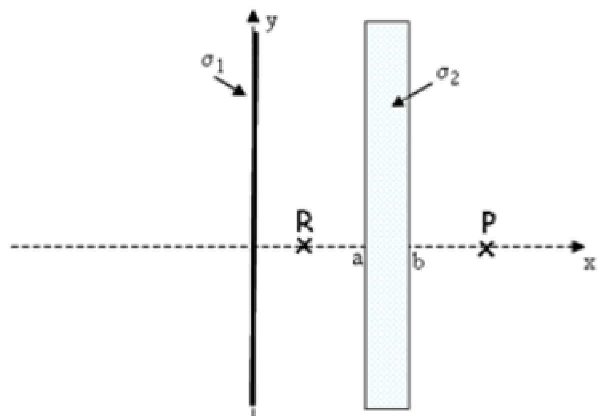
Question 3.1: Gauss's Law

Determine the Electric field a distance r away of an infinitely long line of linear charge density λ C/m.

$$\int \vec{E} d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$
$$EA = \frac{Q_{enc}}{\epsilon_0}$$
$$E = \frac{Q_{enc}}{\epsilon_0 A}$$
$$E = \frac{\lambda l}{2\pi r l}$$
$$E = \frac{\lambda}{2\pi r} = 2k\frac{\lambda}{r}$$

Question 3.2: Gauss's Law

An infinite sheet of charge, oriented perpendicular to the xaxis, passes through x = 0. It has a surface charge density $\sigma_1 = -3.3\mu\frac{C}{m^2}$. A thick, infinite conducting slab, also oriented perpendicular to the x-axis occupiees the region between $a = 2.5cm$ and $b = 4.4cm$. The conducting slab has a net charge per unit area of $\sigma_2 = 70\mu\frac{C}{m^2}$. What is σ_b, the charge per unit area on the surface of the slab located at x = 4.4 cm?



In order to solve this problem, we'll establish a system of equations.
Our first equation will be the charge of the slab over an area.

$$\sigma_a + \sigma_b = \sigma_2$$

The sum of the charge density of both sides of the slab, is equal to the net charge of the conductor.
Our second equation utilizes the fact that the net electric field inside of a conductor, in this case the slab, is zero.
To take advantage of this fact, we use the superposition formula for electric fields

$$\begin{aligned} \Sigma_b^a E &= E_a + E_{a+1} + \ldots + E_b \\ E_{insideslab} &= E_{\sigma_1} + E_{\sigma_a} - E_{\sigma b} \\ 0 &= E_{\sigma_1} + E_{\sigma_a} - E_{\sigma b} \end{aligned}$$

The sign of E_{σ_b} is negative in the equation above, because this side lies to the right of the conductor. If it generated a positive electric field in the slab, the electric field in the slab would be pointing left and therefore needs to negative.
We now have our 2 equations.

$$\begin{cases} 0 & = E_{\sigma_1} + E_{\sigma_a} - E_{\sigma b} \\ \sigma_a + \sigma_b & = \sigma_2 \end{cases}$$

Using Gauss's law, we know the electric field from an infinite sheet is equal to $\frac{\sigma}{2\epsilon_0}$. Substituting this in for our E fields in our first equation we get

$$\begin{cases} 0 & = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_a}{2\epsilon_0} - \frac{\sigma b}{2\epsilon_0} \\ \sigma_a + \sigma_b & = \sigma_2 \end{cases}$$

Next for our first equation, we can multiply both sides by $2\epsilon_0$ to remove the denominators, and we can move σ_1 to the opposite side because it's a given value.

$$\begin{cases} \sigma_a - \sigma_b = -\sigma_1 \\ \sigma_a + \sigma_b = \sigma_2 \end{cases}$$

To solve for σ_a and σ_b, add or subtract the two equations respectively.

$$\begin{aligned} \sigma_a - \sigma_b + \sigma_a + \sigma_b &= -\sigma_1 + \sigma_2 \\ 2\sigma_a &= -\sigma_1 + \sigma_2 \\ \sigma_a &= \frac{-\sigma_1 + \sigma_2}{2} \end{aligned}$$

$$\begin{aligned} \sigma_a - \sigma_b - \sigma_a - \sigma_b &= -\sigma_1 - \sigma_2 \\ -2\sigma_b &= -\sigma_1 - \sigma_2 \\ \sigma_b &= \frac{\sigma_1 + \sigma_2}{2} \end{aligned}$$

4 Electric Potential Energy

- As Khan Academy put it, "Electric potential energy is the energy needed to move a charge against/away from an electric field."
- An oppositely charged field from a particle, requires lots of energy to separate them.

$$\Delta U_E = -W_E = -qEr\cos\theta$$

- ΔU_E = The change in the electrical potential energy of a charge
- W_E = Work done by an electrical force, Nm
- q = Charge, coulombs
- E = Strength of Electric Field
- θ = angle between line from initial pos and final pos, to the direction of the electric field

$$W_{A\to B} = \int_{r_A}^{r_B} \vec{F}_E \cdot d\vec{r}$$

- Above is the work equation. It's equal to the equation -qERcos θ

$$U_r = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

  - Above is the eqatuion for the potential energy from one charge onto another a distance r away from each other

  - Electric potential superpositions too, in o

5 Electric Potential

  - Just like how electric fields are imagined by placing a positive charge, with electric potential, we ask ourselves how much the potential energy of an imaginary charge changes when we move it from one spot to another

  - $$U_E = qV$$

  - U_E pretty standard potential energy
  - q is charge
  - V is electric potential or the electric potential energy per unit charge, J/C or Volts
  - also correlates to $\Delta V = \frac{\Delta U_E}{q}$

$$V = k \sum_i \frac{q_i}{r_i}$$

  - Essentially it's a repeat of superposition, must sum all electric potentials in an area
  - Equipotential surfaces, surfaces of constant potential

6 Conductors and Capacitance

  - Conductor, a material that permits the flow of electrons, electrons can freely move
  - Between the two surfaces of a conductor, the electric field is equal to 0
  - No electric field exists inside of a conductor
  - Charge typically is around edges
  - Insulator, electrons cannot move freely
  - Insulators have their charge spread out evenly

– Semiconductors, a bit of both

$$C = \frac{Q}{\Delta V}$$

– C = Capacitance of the Capacitor expressed in farads
– Q = Charges of plates
– V = Potential Energy

7 Capacitors

$$C = k\frac{\epsilon_0 A}{d}$$

– Here k represents the dielectric constant, usually it's just a vaccum constant
– ϵ_0 = it's j the constant from earlier
– A = area of one parallel plate
– d = Distance between the parallel plates
– The above is the Parallel plate capacitor equation
– Basically capacitors work because one side has a lot of negative and the other has a lot of positive charge
– Capacitors store charge, p sure it resists change bu tneed to check
– When current intially begins flowing through a capacitor, it is limited because capacitor begins storing charge
– When the battery in a circuit is disconnected, the capcitor begins to discharge in the equation
– For capacitors in parallel, net capacitance is the sum of the capacitors in parallel
– $\sum_a^b C_{net} = C_a + C_{a+1} + \ldots + C_b$
– Capacitors in series circuit must be added like a resistor in parallel
– $\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$$U_c = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2$$

– U = Energy stored in a capacitor
– Q = Charge on the plate of the positive capacitor, (they're both equal but just use positive because potentia energy)
– C = Capacitance V = Electric Potential/Voltage