Allgemein

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LaTeX

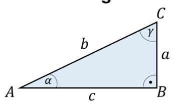
Grundlagen

Mengen:

$\mathbb{N} = \{1, 2, 3, \ldots, \}$ Natürliche Zahlen
$\mathbb{N}_0 = \{0, 1, 2, \ldots\}$ Natürliche Zahlen (+0)
$\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$ Ganze Zahlen
$\mathbb{Q} = \{\frac{\mathbb{Z}}{\mathbb{N}}\}$ Rationale Zahlen
$\mathbb{R} = \{\ldots, \sqrt{3}, \pi, \ldots\}$ Reelle Zahlen

Trigometrie

Rechtwinkliges Dreieck:

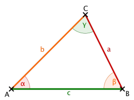


$\sin(\alpha) = \frac{a}{c}$ $\quad \cos(\alpha) = \frac{b}{c}$ $\quad \tan(\alpha) = \frac{a}{b}$

$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$ $\quad \cos(\alpha) = \frac{\sin(\alpha)}{\tan(\alpha)}$

$\sin(\alpha) = \tan(\alpha) \cdot \cos(\alpha)$

Allgemeines Dreieck:



Sinussatz:
$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$

Kosinussatz:
$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$
$b^2 = a^2 + c^2 - 2ac\cos(\beta)$
$c^2 = a^2 + b^2 - 2ab \cdot \cos(\gamma)$

Umrechnung Gradmass/Bogenmass:

$x_{Bogenmass} = \frac{\alpha}{180°} \cdot \pi$ $\quad x_{Gradmass} = \frac{\alpha}{\pi} \cdot 180°$

Binomische Formeln

$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$
$(a + b) \cdot (a - b) = a^2 - b^2$

Brüche

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c} = \frac{ad}{bc}$$

Summenzeichen

$$\sum_{i}^{n} a = a_i + a_{i+1} + \ldots + a_n$$

Gaussche Summenformel:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Für k^2:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Rechenregeln:

$\sum_{k=s}^{n} (c \cdot a_k) = c \cdot \sum_{k=s}^{n} a_k$

$\sum_{k=s}^{n} (a_k + b_k) = \sum_{k=s}^{n} a_k + \sum_{k=s}^{n} b_k$

$\sum_{k=s}^{n} (a_k \cdot b_k) = \sum_{k=s}^{n} a_k \cdot \sum_{k=s}^{n} b_k$

$\sum_{k=s}^{n} a_k + \sum_{k=n+1}^{m} a_k = \sum_{k=s}^{m} a_k$

$\sum_{k,l=s}^{n} a_{kl} = \sum_{k=s}^{n} \sum_{l=s}^{n} a_{kl}$

Indextransformation:
$\sum_{k=s}^{n} a_k = \sum_{k=s-k_0}^{n-k_0} a_k + k_0 \rightarrow$

Produktzeichen

$$\prod^{n} a = a_i \cdot a_{i+1} \cdot \ldots \cdot a_n$$

Logarithmen $a^y = x \rightarrow \log_a x = y$

$\log_a(u \cdot v) = \log_a u + \log_a v$ $\quad \log_{10} x = \log x$

$\log_a \frac{u}{v} = \log_a u - \log_a v$ $\quad \log_e x = \ln x$

$\log_a u^x = x \cdot \log_a u$

$\log_a(x) = \frac{\log(x)}{\log(a)} = \frac{\ln(x)}{\ln(a)} = \frac{\log_c(x)}{\log_c(a)}$ Basiswechsel

$\log_a(a^x) = x$ $\quad \log_a(1) = 0$

$$a^n = x \quad \rightarrow \quad n = \frac{\log(x)}{\log(a)}$$

$$\ln(x) = y \quad (|e^{\cdots}) \rightarrow \quad x = e^y$$

Potenzen $a^n = a \cdot a \cdot \ldots \cdot a$

$a^m \cdot a^n = a^{m+n}$ $\quad \frac{a^m}{a^n} = a^{m-n}$

$a^n \cdot b^n = (a \cdot b)^n$ $\quad \frac{a^n}{b^n} = (\frac{a}{b})^n$

$(a^n)^m = a^{m \cdot n} = (a^m)^n$

$a^0 = 1$ $\quad a^{-n} = \frac{1}{a^n}$

Tricks:
$x^{1+a} = y \cdot (a + 2) \quad \rightarrow \quad x = (y \cdot (a + 2))^{\frac{1}{1+a}}$

Wurzel

Quadratwurzel: $\sqrt{a} = b \rightarrow b^2 = a$

$\sqrt{a} \cdot \sqrt{a} = a$ $\quad \frac{\sqrt{a}}{\sqrt{a}} = 1$

$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ $\quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$\sqrt{a^2} = |a| \rightarrow a \in \mathbb{R}$

Wurzel Tricks:

$\frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$

$2\sqrt{x} \cdot \sqrt{1 + \frac{1}{x}} = 2\sqrt{x \cdot (1 + \frac{1}{x})} = 2\sqrt{x + 1}$

Algem. Wurzel: $\sqrt[n]{a} = b \rightarrow b^n = a$

$\sqrt[n]{a} \cdot \sqrt[m]{a} = a^{\frac{1}{n}} \cdot a^{\frac{1}{m}} = a^{\frac{m+n}{m \cdot n}}$

$\sqrt[n]{a} : \sqrt[m]{a} = a^{\frac{1}{n}} : a^{\frac{1}{m}} = a^{\frac{m-n}{m \cdot n}}$

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$ $\quad \sqrt[n]{a^m} = a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ $\quad \sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[m \cdot n]{a}$

$\sqrt[0]{a} \rightarrow$ nicht definiert $\quad \sqrt[2n]{-1} \rightarrow$ nicht definiert

$\sqrt[2n+1]{-1} = 1$

Exponentialfunktion

$f(x) = a \cdot b^x$
$a \quad \rightarrow$ Anfangswert
$b \quad \rightarrow$ Wachstumsfaktor
$\rightarrow \quad b > 1$ Wachstum
$\rightarrow \quad b < 1$ Abnahme

Prozentuale Zu- oder Abnahme (p%)
$b = 1 \pm \frac{p}{100}$

Volumen und Flächen

Quadrat:
$A = a^2$ $\quad U = 4 \cdot a$ $\quad d = \sqrt{2} \cdot a$

Rechteck:
$A = a \cdot b$ $\quad U = 2a + 2b$ $\quad d = \sqrt{a^2 + b^2}$

Dreieck:
$A = \frac{1}{2} \cdot a \cdot h$ $\quad U = a + b + c$

Kreis:
$A = \pi \cdot r^2$ $\quad U = 2\pi \cdot r$
$A = \frac{d^2 \cdot \pi}{4}$

Zylinder:
$V = \pi \cdot r^2 \cdot h$ $\quad A = 2 \cdot \pi(r^2 + r \cdot h)$

Pyramide:
$G \rightarrow$ Grundfläche (Quadrat, Rechteck, Dreieck)
$V = \frac{1}{3} \cdot G \cdot h$ $\quad A = G + S_1, S_2, \ldots$

Kugel:
$V = \frac{4}{3} \cdot \pi \cdot r^3$ $\quad A = 4 \cdot \pi \cdot r^2$

Nullstellen von Gleichungen

Grad 1

$$x_0 = -\frac{b}{a}$$

Grad 2

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Weitere Methoden:

- Polynomdivision
- Faktorisierung
- Binomisierung

Tricks mit $\lim$

$$\lim_{n\to\infty} (1 + \frac{2}{5n})^{-\frac{2n}{5}} = \lim_{n\to\infty} (1 + \frac{1}{\frac{5n}{2}})^{-\frac{2n}{5}} = \lim_{n\to\infty} (1 + \frac{1}{\frac{5n}{2}})^{\frac{5n}{2} \cdot \frac{-4}{25}} = e^{-\frac{4}{25}}$$

$$\lim_{n\to\infty} (\sqrt{n^2+n} - \sqrt{n^2-2n}) = \lim_{n\to\infty} \left(\sqrt{n^2+n} - \sqrt{n^2-2n}\right) \cdot \frac{\sqrt{n^2+n} + \sqrt{n^2-2n}}{\sqrt{n^2+n} + \sqrt{n^2-2n}}$$
$$= \lim_{n\to\infty} \frac{n^2+n-(n^2-2n)}{\sqrt{n^2+n} + \sqrt{n^2-2n}} = \lim_{n\to\infty} \frac{3n}{\sqrt{n^2+n} + \sqrt{n^2-2n}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$
$$= \lim_{n\to\infty} \frac{3}{\sqrt{\frac{n^2+n}{n^2}} + \sqrt{\frac{n^2-2n}{n^2}}} = \lim_{n\to\infty} \frac{3}{\sqrt{1+\frac{1}{n}} + \sqrt{1-\frac{2}{n}}} = \frac{3}{\sqrt{1} + \sqrt{1}} = \frac{3}{2}$$