LATEX

Allgemein

Grundlagen

Mengen:

 $\mathbb{N} = \{1, 2, 3, \dots, \}$ Natürliche Zahlen Natürliche Zahlen (+0) $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$ Ganze Zahlen $\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$ $\mathbb{Q} = \{\frac{\mathbb{Z}}{\mathbb{N}}\}$ Rationale Zahlen $\mathbb{R} = \{\dots, \sqrt{3}, \pi, \dots\}$ Reelle Zahlen

Trigometrie

Rechtwinkliges Dreieck:



 $\sin(\alpha) = \frac{a}{2}$

$$\cos(\alpha) = \frac{b}{c}$$

 $tan(\alpha) = \frac{a}{b}$

 $tan(\alpha) = \frac{sin(\alpha)}{cos(\alpha)}$ $\sin(\alpha) = \tan(\alpha) \cdot \cos(\alpha)$

$$\cos(\alpha) = \frac{\sin(\alpha)}{\tan(\alpha)}$$

Allgemeines Dreieck:



Sinussatz:

 $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$

Kosinussatz:

 $a^2 = b^2 + c^2 - 2bc\cos(\alpha)$ $b^2 = a^2 + c^2 - 2ac\cos(\beta)$ $c^2 = a^2 + b^2 - 2ab \cdot \cos(\gamma)$

Umrechnung Gradmass/Bogenmass:

 $x_{Bogenmass} = \frac{\alpha}{180^{\circ}} \cdot \pi$ $x_{Gradmass} = \frac{\alpha}{\pi} \cdot 180^{\circ}$

Binomische Formeln

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b) \cdot (a-b) = a^2 - b^2$$

Brüche

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c} = \frac{ad}{bc}$$

Summenzeichen

$$\sum_{i=0}^{n} a = a_i + a_{i+1} + \ldots + a_n$$

Gaussche Summenformel:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Für k^2 :

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Rechenregeln:

 $\sum_{k=s}^{n} (c \cdot a_k) = c \cdot \sum_{k=s}^{n} a_k$

$$\sum_{k=s}^{n} (a_k + b_k) = \sum_{k=s}^{n} a_k + \sum_{k=s}^{n} b_k$$

$$\sum_{k=s}^{n} (a_k \cdot b_k) = \sum_{k=s}^{n} a_k \cdot \sum_{k=s}^{n} b_k$$

$$\sum_{k=s}^{n} a_k + \sum_{k=n+1}^{m} a_k = \sum_{k=s}^{m} a_k$$

$$\sum_{k,l=s}^{n} a_{kl} = \sum_{k=s}^{n} \sum_{l=s}^{n} a_{kl}$$

Indextransformation:

$$\sum_{k=s}^{n} a_k = \sum_{k=s-k_0}^{n-k_0} a_k + k_0 \to$$

Produktzeichen

$$\prod^{n} a = a_i \cdot a_{i+1} \cdot \ldots \cdot a_n$$

Logarithmen $a^y = x \rightarrow \log_a x = y$

$$\log_a(u \cdot v) = \log_a u + \log_a v \qquad \log_{10} x = \log x$$

$$\log_a \frac{u}{v} = \log_a u - \log_a v \qquad \qquad \log_e x = \ln x$$

$$\log_a u^x = x \cdot \log_a u$$

$$\log_a(x) = \frac{\log(x)}{\log(a)} = \frac{\ln(x)}{\ln(a)} = \frac{\log_c(x)}{\log_c(a)}$$
 Basiswechsel

$$\log_a(a^x) = x \qquad \qquad \log_a(1) = 0$$

$$a^n = x \rightarrow n = \frac{\log(x)}{\log(a)}$$

$$ln(x) = y \qquad (|e^{\cdots}) \to \quad x = e^y$$

Potenzen $a^n = a \cdot a \cdot \ldots \cdot a$

$$\cdot \, a^n = a^{m+n} \qquad \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$a^n \cdot b^n = (a \cdot b)^n \qquad \qquad \frac{a^n}{b^n} = (\frac{a}{b})^n$$

$$(a^n)^m = a^{m \cdot n} = (a^m)^n$$

$$a^0 = 1 \qquad \qquad a^{-n} = \frac{1}{a^n}$$

Wurzel

Quadratwurzel: $\sqrt{a} = b \rightarrow b^2 = a$

$$\sqrt{a} \cdot \sqrt{a} = a$$

$$\frac{\sqrt{a}}{\sqrt{a}} = a$$

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{a} = a$$

$$\sqrt{a^2} = |a| \to a \in \mathbb{R}$$

Wurzel Tricks:

$$\frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$$

$$2\sqrt{x} \cdot \sqrt{1 + \frac{1}{x}} = 2\sqrt{x \cdot (1 + \frac{1}{x})} = 2\sqrt{x + 1}$$

Algem. Wurzel:

$$\sqrt[n]{a}=b\to b^n=a$$

$$\sqrt[n]{a} \cdot \sqrt[m]{a} = a^{\frac{1}{n}} \cdot a^{\frac{1}{m}} = a^{\frac{m+n}{m+n}}$$

$$\sqrt[n]{a}$$
: $\sqrt[m]{a} = a^{\frac{1}{n}}$: $a^{\frac{1}{m}} = a^{\frac{m-n}{m+n}}$

$$\sqrt[n]{a} : \sqrt[m]{a} = a^{\frac{n}{n}} : a^{\frac{m}{m}} = a^{\frac{m}{m-1}}$$

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$
 $\sqrt[n]{a^m} = a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m]{\sqrt[m]{a}} = \sqrt[m-n]{a}$$

$$\sqrt[0]{a}
ightarrow$$
 nicht definiert

$$\sqrt[2n]{-1} o ext{nicht definiert}$$

$$\sqrt[2n+1]{-1} = 1$$

Exponentialfunktion

$$f(x) = a \cdot b^x$$

 $a \rightarrow Anfangswert$ → Wachstumsfaktor b > 1 Wachstum b < 1 Abnahme

Prozentuale Zu- oder Abnahme (p%)

 $b = 1 \pm \frac{p}{100}$

Volumen und Flächen

Quadrat: $A = a^2$

$$U=4\cdot a \qquad \qquad d=\sqrt{2}\cdot a$$

Rechteck:

$$A=a\cdot b \hspace{1cm} U=2a+2b \hspace{1cm} d=\sqrt{a^2+b^2}$$

Dreieck:

$$A = \frac{1}{2} \cdot a \cdot h \qquad \qquad U = a + b + c$$

Kreis:

$$A = \pi \cdot r^2$$

$$A = \frac{d^2 \cdot \pi}{4}$$

$$U = 2\pi \cdot r$$

Zvlinder:

$$V = \pi \cdot r^2 \cdot h$$

$$A = 2 \cdot \pi (r^2 + r \cdot h)$$

Pyramide:

$$G o$$
 Grundfläche (Quadrat, Rechteck, Dreieck) $V = \frac{1}{3} \cdot G \cdot h$ $A = G + S_1, S_2, \ldots$

Kugel:
$$V = \frac{4}{3} \cdot \pi \cdot r^3 \qquad \qquad A = 4 \cdot \pi \cdot r^2$$

Nullstellen von Gleichungen

Grad 1

$$x_0 = -\frac{b}{a}$$

Grad 2

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Weitere Methoden:

- Polynomdivision
- Faktorisierung
- Binomisierung

Tricks mit lim

$$\begin{split} &\lim_{n\to\infty} (1+\frac{2}{5n})^{-\frac{2n}{5}} = \lim_{n\to\infty} (1+\frac{1}{\frac{5n}{2}})^{-\frac{2n}{5}} = \\ &\lim_{n\to\infty} (1+\frac{1}{\frac{5n}{2}})^{\frac{5n}{2}\cdot\frac{-4}{25}} = e^{-\frac{4}{25}} \end{split}$$

$$\begin{split} & \lim_{n \to \infty} (\sqrt{n^2 + n} - \sqrt{n^2 - 2n}) = \lim_{n \to \infty} (\sqrt{n^2 + n} - \sqrt{n^2 - 2n}) \cdot \frac{\sqrt{n^2 + n} + \sqrt{n^2 - 2n}}{\sqrt{n^2 + n} + \sqrt{n^2 - 2n}} \\ & = \lim_{n \to \infty} \frac{n^2 + n - (n^2 - 2n)}{\sqrt{n^2 + n} + \sqrt{n^2 - 2n}} = \lim_{n \to \infty} \frac{3n}{\sqrt{n^2 + n} + \sqrt{n^2 - 2n}} = \lim_{n \to \infty} \frac{1}{\sqrt{n^2 + n} + \sqrt{n^2 - 2n}} \\ & = \lim_{n \to \infty} \frac{3}{\sqrt{n^2 + n} + \sqrt{n^2 - 2n}} = \lim_{n \to \infty} \frac{3}{\sqrt{1 + 1} + \sqrt{1 - \frac{2}{n^2}}} = \frac{3}{\sqrt{1 + \sqrt{1}}} = \frac{3}{2} \end{split}$$