STU22004 – Solutions for Sample Questions 9

Q11. If 40% of people vote for Mr. A, in a random sample, how many samples in average are required to have 20 votes for Mr. A?

$$E[X] = np \Rightarrow 20 = n \times 0.4 \Rightarrow n = 50$$

Q12. If $X \sim B(p)$, find $E[(X-2)(X^2+2X+4)]$.

$$E[(X-2)(X^2+2X+4)] = E[X^3 - 8]$$

$$= E[X^3] - 8 = (1)^3 p + 0^3(1-p) - 8 = p - 8$$

Q13. $X_i s$ are the random lifetime of bulbs and have iid $U(0,1)$ distributions. What is the average lifetime of the bulb which breaks latest among 9 bulbs?

$$T = \max(X_i)$$

$$F_T(t) = P(T \le t) = P(\max(X_i) \le t)$$

$$= [F_X(t)]^n = \left(\frac{t-0}{1-0}\right)^n = t^9$$

$$f_T(t) = \frac{d(t^9)}{dt} = 9t^8 \qquad 0 \le t \le 1$$

$$E[T] = \int t f(t)\, dt = \int_0^1 t \times 9t^8\, dt$$

$$= \int_0^1 9t^9\, dt = 0.9$$

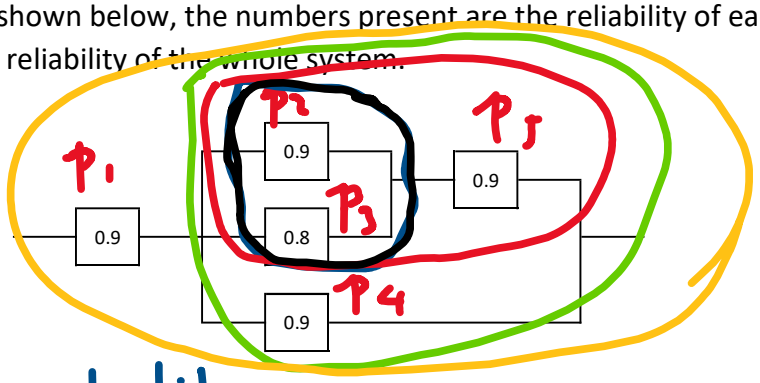
Q14. If $X \sim U\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, find pdf of $Y = \tan(X)$.

- $Y = g(X) = \tan(X) \rightarrow X = g^{-1}(Y) = \tan^{-1}(Y)$
- Y is a 1-to-1 function in $(-\pi/2, \pi/2)$.
- $f_X(x) = \frac{1}{\pi}$

$$f_Y(y) = \left|\frac{d g^{-1}(y)}{dy}\right| f_X(g^{-1}(y))$$

$$= \frac{1}{y^2+1} \frac{1}{\pi} \qquad -\infty < y < +\infty$$

Q15. For the system shown below, the numbers present are the reliability of each component. Find the reliability of the whole system.

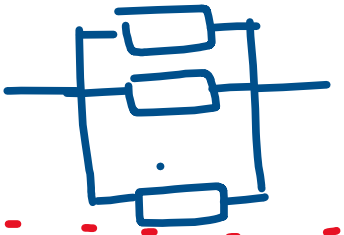


System Reliability = R

Component Reliability = p $q = 1-p$

If n components are series $R = p^n$

If n components are Parallel. $R = 1-q^n$



$$R = p_1\left[1-\left\{1-\left[p_5\left(1-q_2 q_3\right)\right]\right\}\left(1-p_4\right)\right]$$