

MAU22C00: ASSIGNMENT 5
DUE MONDAY, APRIL 12 BEFORE MIDNIGHT
UPLOAD ON BLACKBOARD

Please attach a cover sheet with a declaration confirming that you know and understand College rules on plagiarism. Details can be found on <http://tcd-ie.libguides.com/plagiarism/declaration>.

Each of the following problems is worth 5 points:

- (1) Let $A = \mathbb{N} \times \mathbb{Z} \times \mathbb{Q} \times \mathbb{C}$. Is A finite, countably infinite or uncountably infinite? Justify your answer.
- (2) Let A be the set of points in \mathbb{R}^2 whose polar coordinates (r, θ) satisfy the equation $r^2 = (\sin(\theta) - 1)^2$. Is A finite, countably infinite or uncountably infinite? Justify your answer.
- (3) Let $A = \{(x, y) \in \mathbb{C}^2 \mid x^6 - 3x^2 + 1 = 0\}$. Is A finite, countably infinite or uncountably infinite? Justify your answer.
- (4) Let

$$A = \{(x, y) \in \mathbb{R} \times \mathbb{R}^+ \mid 1 + xy = 0\} \cap \left\{ (x, y) \in \mathbb{R}^2 \mid \frac{(x-7)^2}{25} + \frac{(y+4)^2}{9} = 1 \right\}.$$

\mathbb{R}^+ stands for all positive real numbers. Consider $\mathcal{P}(A)$, the power set of A . Is $\mathcal{P}(A)$ finite, countably infinite or uncountably infinite? Justify your answer.

- (5) Let A consist of all 2×2 matrices with entries in the real numbers \mathbb{R} and determinant equal to 1. Is A finite, countably infinite or uncountably infinite? Justify your answer.
- (6) Let $A = \{(x, y, z) \in \mathbb{R}^3 \mid 3x - y + 2z = 0 \text{ and } x + 2y + 3z = 0\}$. Is A finite, countably infinite or uncountably infinite? Justify your answer.
- (7) Let $A = \{0, 1\}$. Is the language $[(0 \cup \epsilon)^* \circ (1 \cup \epsilon)] \cap (A \circ A)^*$ finite, countably infinite, or uncountably infinite? Justify your answer.
- (8) Let A be a countably infinite alphabet. Is A^* finite, countably infinite or uncountably infinite? Justify your answer.
- (9) Let $A = \{0, 1, 2, 3, 4, 5\}$. Let the language L consist of all even length strings containing at least three odd letters. Is L finite, countably infinite or uncountably infinite? Justify your answer.
- (10) Does there exist a sequence $\{x_1, x_2, x_3, \dots\}$ of languages over a finite alphabet A such that x_i is not a regular language $\forall i \geq 1$? Justify your answer.