

MAU22C00: TUTORIAL 20 SOLUTIONS

TURING MACHINES

1) Consider the language over the binary alphabet $A = \{0, 1\}$ given by $L = \{0^m 1^{2m} \mid m \in \mathbb{N}\}$.

(a) Write down the algorithm of a Turing machine that recognizes L . Process the following strings according to your algorithm: ϵ , 01, 011, and 010.

(b) Draw the transition diagram of the Turing machine from part (a) carefully labelling the initial state, the accept state, the reject state, and all the transitions specified in your algorithm.

Solution: 1) (a) Here is the algorithm for recognising $L = \{0^m 1^{2m} : m \in \mathbb{N}\}$.

- (1) If there is a blank in the first cell, ACCEPT. If there is anything else, apart from 0, then REJECT.
- (2) If 0 is in the current cell, delete it, then move right to the first 1.
- (3) If there is no first 1, REJECT. Otherwise change 1 to x .
- (4) Move to the leftmost non blank symbol. If 0, go to step 2. If 1, REJECT. If x , go to step 5. If y , go to step 6.
- (5) Delete x , move right to the nearest 1. If none, REJECT. Otherwise change it to y and go to step 4.
- (6) Move right to the rightmost non blank character. If anything but y is found, REJECT. Otherwise, ACCEPT.

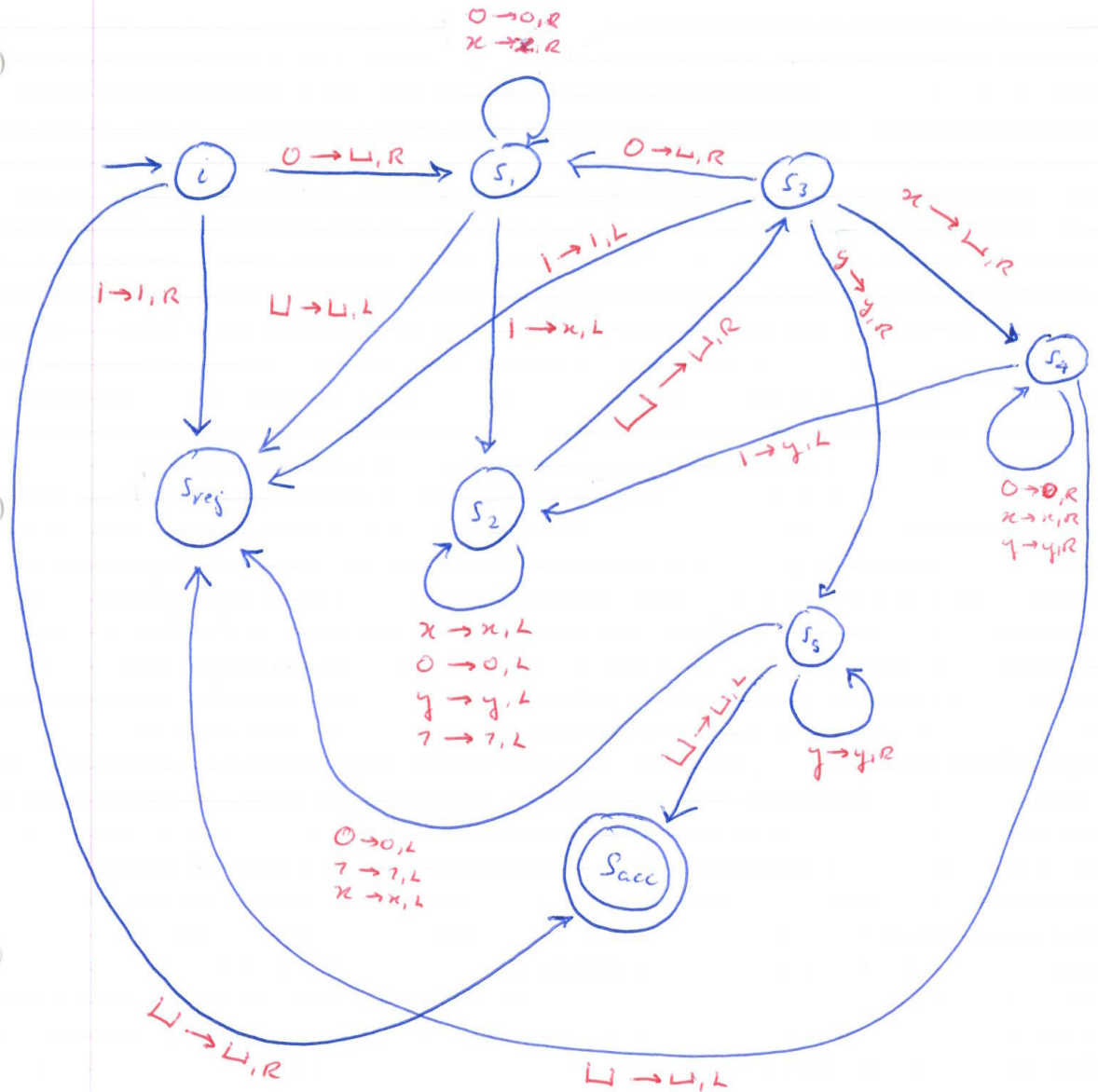
Here is how the following strings are treated:

- ϵ is accepted immediately.
- $01 \rightarrow \sqcup 1 \rightarrow \sqcup x \rightarrow \sqcup \rightarrow \text{REJECT}$.
- $011 \rightarrow \sqcup 11 \rightarrow \sqcup x 1 \rightarrow \sqcup \sqcup 1 \rightarrow \sqcup \sqcup y \rightarrow \text{ACCEPT}$.
- $010 \rightarrow \sqcup 10 \rightarrow \sqcup x 0 \rightarrow \sqcup \sqcup 0 \rightarrow \text{REJECT}$.

(b) Here is transition diagram for

$$T = (\{i, s_1, s_2, s_3, s_4, s_5, s_{\text{acc}}, s_{\text{rej}}\}, \{0, 1\}, \{0, 1, x, y, \sqcup\}, t, i, s_{\text{acc}}, s_{\text{rej}})$$

along with an example of an accepted string:



E.g.

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 |
| L | 0 | 1 | 1 | 1 | 1 |
| | 0 | x | 1 | 1 | 1 |
| | 0 | x | 1 | 1 | 1 |
| L | x | 1 | 1 | 1 | |
| | x | x | 1 | 1 | |
| | x | x | 1 | 1 | |
| L | x | 1 | 1 | | |
| | x | y | 1 | | |
| | x | y | 1 | | |

s_1
 s_2
 s_3
 s_1
 s_2
 s_3
 s_4
 s_2
 s_3

L
1
1
1
1
1
1
1
1
1

s_4
 s_2
 s_3
 $s_{acc}!$