Solution for
Sample Questions - 8

Q1. We know

$$r = \frac{\sum (x_i - \bar{x})(y_i - y_i)}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}}$$

$$\Rightarrow 0.75 = \frac{S_{xy}}{\sqrt{9 \times 16}} \Rightarrow 0.75 = \frac{S_{xy}}{12}$$

$$\Rightarrow S_{xy} = -9$$

The regression model is presented as

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x. \quad \text{where } \hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$$

and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

Therefore: $\hat{\beta}_1 = \frac{-9}{9} = -1$ and

$$\hat{\beta}_0 = 6 - (-1)(4) = 10$$

$$\Rightarrow \hat{y} = 10 + x$$

Q2.

We know $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ or $\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$.

This mean the line $y = \hat{\beta}_0 + \hat{\beta}_1 x$ passes through the point $(\bar{x}, \bar{y})$, in the case of this question $(1, 9)$.

In addition, The question states the the line passes through the point $(9, 9)$. The line that passes through two points with same y and different xs is a horizontal line and Therefor $\hat{\beta}_1 = 0$

Q3. For two regression models $\begin{cases} \hat{y} = \beta_0 + \beta_1 x \text{ and} \\ \hat{x} = \alpha_0 + \alpha_1 y \end{cases}$

We have.

1) $\hat{\beta}_1 = \dfrac{S_{xy}}{S_{xx}}$ 2) $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

3) $\hat{\alpha}_1 = \dfrac{S_{xy}}{S_{yy}}$ 4) $\hat{\alpha}_0 = \bar{x} - \hat{\alpha}_1 \bar{y}$

We know $\beta_0 = -2$ $\beta_1 = 2$ therefore from (2)

$$-2 = \bar{y} - (2)(3) \Rightarrow \bar{y} = 4$$

Using 4) $\boxed{\hat{\alpha}_0 = 3 - \hat{\alpha}_1 \times 4}$ $\circledast$

Also, $r_{xy} = 0.5 = \dfrac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}}$ and $\hat{\beta}_1 = \dfrac{S_{xy}}{S_{xx}} = 2$

from 1) & 3) $\hat{\alpha}_1 \hat{\beta}_1 = \dfrac{S_{xy}^2}{S_{xx} S_{yy}} = r^2 \Rightarrow 2\hat{\alpha}_1 = 0.25$

$$\Rightarrow \hat{\alpha}_1 = \frac{0.25}{2} = 0.125$$

$\Rightarrow$ using $(\circledast)$ $\hat{\alpha}_0 = 3 - 4(0.125) = \underline{2.5}$

Q4. $\hat{y} = \hat{\beta}_1 x$

$$E = \sum (y_i - \hat{y})^2 = \sum (y_i - \hat{\beta}_1 x_i)^2$$

diffrentiate E with respect to $\hat{\beta}_1$ and put it $= 0$

$$\frac{dE}{d\hat{\beta}_1} = -2 \sum (y_i - \beta_1 x_i) x_i = 0$$

$$\Rightarrow \sum x_i y_i = \beta_1 \sum x_i^2 \Rightarrow \boxed{\hat{\beta}_1 = \frac{\sum x_i y_i}{\sum x_i^2}}$$

Q5. By assuming $z = \frac{1}{y}$ we have

$$\frac{1}{\hat{z}} = \frac{x}{\beta_0 + \beta_1 x} \Rightarrow \hat{z} = \frac{\beta_0 + \beta_1 x}{x} = \frac{\beta_0}{x} + \beta_1$$

now by choosing $\frac{1}{x} = t$ We have

$\hat{z} = \beta_0 t + \beta_1$ which is an SLR Model

(we can now call $\beta_0 \to \alpha_1$ & $\beta_1 \to \alpha_0$)

therefore The Transformations are

$Z = \frac{1}{Y}$ and $T = \frac{1}{X}$

Q6. $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$

$$\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x} \Rightarrow \underline{10 = \hat{\beta}_0 + 10\hat{\beta}_1} \quad | \; *$$

$$-0.8 = r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = (-0.8)^2 = \frac{S_{xy}^2}{S_{xx} \cdot S_{yy}} = \frac{S_{xy}^2}{S_{xx} \cdot 4 S_{xx}}$$

$$\Rightarrow (-0.8)^2 = \frac{1}{4} \frac{S_{xy}^2}{S_{xx}^2} = \frac{1}{4} \left(\hat{\beta}_1\right)^2$$

$$\Rightarrow \hat{\beta}_1^2 = 4(-0.8)^2 \Rightarrow \hat{\beta}_1 = -1.6$$

$$* \Rightarrow \hat{\beta}_0 = 26$$

Be careful, as $r < 0$, $\hat{\beta}_1$ must be < 0

and therefore it is -1.6 not 1.6.