MAU22C00: TUTORIAL 18 PROBLEMS
COUNTABILITY OF SETS

For each of the following sets, determine whether it is finite, countably infinite, or uncountably infinite. Justify your answer.

1) $\bigcup_{q \in \mathbb{Q}} L_q$ where $L_q = \{(x, y) \in \mathbb{R}^2 \mid x = q\}$.

2) $\{a^p \mid p \in \mathbb{N} \text{ and } a = e^{q\pi i} \text{ for } q \in \mathbb{Q}\}$

3) $\{a^p \mid p \in \mathbb{N} \text{ and } a = e^{q\pi i} \text{ for } q \in \mathbb{R} \setminus \mathbb{Q}\}$

4) $\{(x, y, z) \in \mathbb{N}^3 \mid x^2 + y^2 = z^2 \text{ and } x, y, z \in \mathbb{N}^*\}$, the Pythagorean triplets that give the lengths of the legs and the hypothenuse of a right triangle.

5) $\{(x, y) \in \mathbb{R}^2 \mid y = x^2 + 1\}$

6) $\mathcal{P}(J_n) \times \mathcal{P}(\mathbb{N})$, where $J_n = \{1, \ldots, n\}$ and $\mathcal{P}(A)$ is the power set of a set A.

7) $\mathbb{R}^n$ for $n \geq 1$.