STU22004 – Sample Questions 7

Q1. X & Y are independent Poisson RVs with means λ and μ. If $Z = X + Y$, find $f(x|Z = z)$.

Q2. If two point are randomly chosen in $(0,1)$ and their locations are shown by X and Y, what is $P\left(|X - Y| < \frac{1}{4}\right)$.

Q3. If the joint probability distribution of X and Y is given by $f_{X,Y}(x,y) = 2e^{-2x-y},\ x > 0\,, y > 0$ find the correlation coefficient of X and Y.

Q4. If X and Y are iid Exponential random variables, find $P\left(\frac{X}{X+Y} < a\right)$.

Q5. X and Y have joint Uniform distribution in square (0,0), (0,2), (2,0), (2,2). $P(X^2 + Y^2 \leq 1) =?$

Q6. X and Y are independent and similar Geometric RVs. Find $P(X = Y)$.

Q7. Random variable X has pdf and cdf of $f(x)$ and $F(x)$, respectively. If we choose $Y = F(X)$ as a function of X, what is the probability distribution of Y.

Q8. If $f(x,y) = ke^{-x}, 0 < y < x < \infty$, find $E[Y|x]$.

Q9. If X and Y have joint Uniform distribution in $D \equiv \{(x,y), |x| + |y| < 1\}$, find $f(x)$.

Q10. If $X = X(t)$ is the number of customers arriving to a service system in (0,t), find the correlation coefficient between $X = X(t)$ and $Y = X(\alpha t)$.

Q11. If $f(x,y) = k(|x| + |y|)\,, 1 \leq x^2 + y^2 \leq 2$, find $P(0 < Y < X)$.

Q12. The annual number of insurance claims for a car has Poisson distribution with mean n. If the lifetime of a car has Normal distribution $N(\mu, \sigma^2)$, what is the average number of claims during the lifetime of a car?

Q13. If $X_1, \dots, X_n$ are iid random variables, find the correlation coefficient of X_1 and $\bar{X}$.

Q14. If $X \sim N(0,1)$, find the pdf for random variable $Y = X^2$.

Q15. Assume $X \sim U(0,a)$ and $Y = \begin{cases} X & X < \frac{a}{2} \\ \frac{a}{2} & X \geq \frac{a}{2} \end{cases}$. Find $E[Y]$.

Q16. Show that the error (MES) of $E[Y|x]$ equals $Var[Y|x]$.

Q17. Show that $E_X\left[E_Y[Y\,|X]\right] = E_Y[Y]$.

Q18. Show that $Var[Y] = E_X[\,Var[Y\,|X]\,] + Var\left[E_Y[Y|X]\right]$.

Q19. If

$$Y = \sum_{i=1}^{N} X_i$$

where N is a random variable and X_is are iid, find $E[Y]$ and $Var[Y]$ in terms of moments of X and N.

Q20. The number of customers arriving into a supermarket during t minutes, has a Poisson distribution with mean βt. The time that each customer spends in the shop has exponential distribution

$$f(t) = \alpha e^{-\alpha t} \quad t > 0 \ , \qquad E[T] = \frac{1}{\alpha} \quad , \qquad Var[T] = \frac{1}{\alpha^2}$$

What is the probability that while a customer is in the shop k customer arrive?

Q21. In Q20, what is the average number of customers arriving while a customer is shopping?

Q22. In Q20, what is the variance of the number of customers arriving while a customer is shopping?

Q23. For n independent/identical random variables X_i $(i = 1, \ \dots, n)$, with pdf $f_X(x)$, we define

$$U = min(X_1, \dots, X_n) \quad , \quad V = Max\ (X_1, \dots, X_n)$$

Prove that the pdf of U and V are then given by:

$$f_U(u) = nf_X(u)[1 - F_X(u)]^{n-1}$$

and

$$f_V(v) = nf_X(v)[F_X(v)]^{n-1}$$

Q24. $X_1, \dots, X_n$ are iid random variables with $U(a, b)$ distribution. Find pdf of U and V as in Q23.

Q25. $X_1, \dots, X_n$ are iid random variables with exponential distribution. Find pdf of U as in Q23.

Q26. If Z_1 and Z_2 independently have $N(0,1)$ distribution, find the marginal distribution of $X = \mu_X + \sigma_X Z_1$ and $Y = \mu_Y + \sigma_Y\left(\rho Z_1 + \sqrt{1-\rho^2}\, Z_2\right)$.

Q27. In Q26, find the $Cov(X, Y)$, and thus correlation coefficient of X and Y.

Q28. In Q26, find the joint pdf for X and Y.

Q29. In Q26, show that $E[Y|x] = \mu_Y + \rho_{X,Y}\frac{\sigma_Y}{\sigma_X}(x - \mu_X)$ and $Var[Y|x] = \sigma_Y^2\ (1 - \rho^2)$.

Q30. If $f(x, y) = C\ exp\ (-8x^2 - 6xy - 18y^2)$, find:

I) C.
II) $P(\, X < \ -2 \mid Y = 4)$.