Sample Questions 6 - Sol

Q1. $P(|X-\mu| \geqslant k\sigma) = \int_{|x-\mu|\geqslant k\sigma} f(x)\,dx \leqslant \int_{|x-\mu|\geqslant k\sigma} \frac{|x-\mu|}{k\sigma} f(x)\,dx$

$\leqslant \int_{|x-\mu|\geqslant k\sigma} \frac{(x-\mu)^2}{k^2\sigma^2} f(x)\,dx = \frac{1}{k^2\sigma^2} \int_{|x-\mu|\geqslant k\sigma} (x-\mu)^2 f(x)\,dx$

$\leqslant \frac{1}{k^2\sigma^2} \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\,dx = \frac{1}{k^2\sigma^2} \cdot \sigma^2 = \frac{1}{k^2}$

$\Rightarrow P(|X-\mu| \geqslant k\sigma) \leq \frac{1}{k^2}$

Q2 $E[(X-\alpha)^2] = \int (x-\alpha)^2 f(x)\,dx$

$\frac{d}{d\alpha} \int (x-\alpha)^2 f(x)\,dx = \int \frac{d}{d\alpha} (x-\alpha)^2 f(x)\,dx$

$= -2\int (x-\alpha) f(x)\,dx = 0 \Rightarrow \int x f(x)\,dx = \alpha \int f(x)\,dx$

$\Rightarrow \underline{E[X] = \alpha}$

Q3. $E[T] = 2$ min $\Rightarrow$ 30 cars/h

$\Rightarrow f(x) = \frac{e^{-30}\, 30^x}{x!}$

Q4 $\frac{1}{\lambda} = 2 \Rightarrow \lambda = 1/2 \quad P(X \geqslant 2 \mid X \leqslant 8) = \frac{P(2 \leqslant X \leqslant 8)}{P(X \leqslant 8)}$

$= \frac{\int_2^8 \frac{1}{2} e^{-x/2}\,dx}{\int_0^8 \frac{1}{2} e^{-x/2}\,dx} = \cdots$

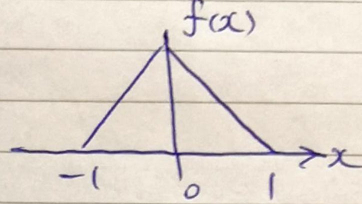
Q5. $\int_0^{\tilde{x}} 2x \, dx = \frac{1}{2} \Rightarrow \tilde{x}^2 = \frac{1}{2} \Rightarrow \tilde{x} = \frac{1}{\sqrt{2}}$

Q6. The chance for each point to be between 2 others is obviously equal to $\frac{1}{3}$

Q7. $E\left[\ln\left(\prod_{i=1}^{n} X_i\right)\right] = E\left[\sum_{i=1}^{n} \ln X_i\right] = \sum_{i=1}^{n} E[\ln X_i]$

$= \sum_{i=1}^{n} \left[\frac{1}{2}\ln(e^{-1}) + \frac{1}{2}\ln(e^{2})\right] = \sum_{i=1}^{n} \frac{1}{2}(-1+2) = \frac{n}{2}$

Q8. $f(x) = 1 - |x|$



This is a symmetric pdf

$\Rightarrow E(X) = 0 \Rightarrow Var(X) = E[X^2]$

$= \int_{-1}^{1} x^2(1-|x|)\, dx = 2\int_0^1 x^2(1-x)\, dx = \ldots$

Q9. This is the MGF of a Normal RV. $X \sim N(3, \frac{1}{2})$
as Normal Dist is symmetric $P(X > 3) = \frac{1}{2}$

Q10. Compare $cx^{d} e^{-x/3}$ to pdf of Gamma dist
Erlang

$f(x) = \frac{\lambda^n}{(n-1)!} x^{n-1} e^{-\lambda x}$ with $E(X) = \frac{n}{\lambda}$ and $Var(X) = \frac{n}{\lambda^2}$

To find n & λ.