CHAPTER 5

5.1 (a) 0.08 (b) 0.28 (c) 0.30 (d) 0.40

(e)

$$f_1(x_1) = \begin{cases} 0.42, & x_1 = 5 \\ 0.30, & x_1 = 10 \\ 0.28, & x_1 = 20 \\ 0 & \text{otherwise} \end{cases} \qquad f_2(x_2) = \begin{cases} 0.37, & x_2 = 0 \\ 0.23, & x_2 = 10 \\ 0.40, & x_2 = 20 \\ 0, & \text{otherwise} \end{cases}$$

(f) 0.58 (g) 0.60

5.3 (a) $f_1(x) = 1/3$, $x = 1, 2, 3$; $f_2(y) = 1/4$, $y = 1, 2, 3, 4$

5.5

(a) $f_1(x_1) = \begin{cases} 0.22, & x_1 = 0 \\ 0.36, & x_1 = 1 \\ 0.42, & x_1 = 2 \\ 0, & \text{otherwise} \end{cases}$ (b) $f_2(x_2) = \begin{cases} 0.3, & x_2 = 0 \\ 0.4, & x_2 = 1 \\ 0.3, & x_2 = 2 \\ 0, & \text{otherwise} \end{cases}$

(c) $f_1(0|0) = 0.2$, $f_1(1|0) = 0.3$, $f_1(2|0) = 0.5$, $f_1(0|1) = 0.2$, $f_1(1|1) = 0.425$, $f_1(2|1) = 0.375$, $f_1(0|2) = 4/15$, $f_1(1|2) = 1/3$, $f_1(2|2) = 0.4$

(d) $f_2(0|0) = 3/11$, $f_2(1|0) = f_2(2|0) = 4/11$, $f_2(0|1) = 0.25$, $f_2(1|1) = 17/36$, $f_2(2|1) = 5/18$, $f_2(0|2) = f_2(1|2) = 5/14$, $f_2(2|2) = 2/7$ (e) 0.58 (f) 0.7 (g) 0.4 (h) 0.23

5.7

(a)

x	Event	$P(X = x)$
2	{(1,1)}	1/36
3	{(1,2), (2,1)}	1/18
4	{(1,3), (2,2), (3,1)}	1/12
5	{(1,4), (2,3), (3,2), (4,1)}	1/9
6	{(1,5), (2,4), (3,3), (4,2), (5,1)}	5/36
7	{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}	1/6
8	{(2,6), (3,5), (4,4), (5,3), (6,2)}	5/36
9	{(3,6), (4,5), (5,4), (6,3)}	1/9
10	{(4,6), (5,5), (6,4)}	1/12
11	{(5,6), (6,5)}	1/18
12	{(6,6)}	1/36

(b)

$$F(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1/36 & \text{if } 2 \le x < 3 \\ 1/12 & \text{if } 3 \le x < 4 \\ 1/6 & \text{if } 4 \le x < 5 \\ 5/18 & \text{if } 5 \le x < 6 \\ 5/12 & \text{if } 6 \le x < 7 \\ 7/12 & \text{if } 7 \le x < 8 \\ 13/18 & \text{if } 8 \le x < 9 \\ 5/6 & \text{if } 9 \le x < 10 \\ 11/12 & \text{if } 10 \le x < 11 \\ 35/36 & \text{if } 11 \le x < 12 \\ 1 & \text{if } x \ge 12 \end{cases}$$

(c) 1/3

5.9 (a) $f(x, y) = 6e^{-2x-3y}$, $x > 0$, $y > 0$ (b) independent

(c) $f_1(x|y) = 2e^{-2x}$, $x > 0$ (d) $e^{-9} - e^{-13}$ (e) X has exponential distribution with mean 1/2 and Y has exponential distribution with mean 1/3. (f) $Var(X) = 1/4$, $Var(Y) = 1/9$ (g) 0

5.11 (a) 1

(b)

$$f_1(x) = \frac{2(x+1)}{3}, 0 < x < 1; \; f_2(y) = \frac{4y+1}{3}, 0 < x < 1$$

(c)

$$f_1(x|y) = \frac{2(x+1)}{4y+1}, 0 < x < 1, 0 < y < 1$$

(d) 5/12

5.13

(a)

$$f_1(x) = \frac{12x^2 + 6x}{7}, \; 0 < x < 1 \qquad f_2(y) = \frac{2y+4}{21}, 0 < y < 3$$

(b)

$$F_1(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{4x^2 + 3x^2}{7}, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases} \qquad F_2(y) = \begin{cases} 0, & y \le 0 \\ \dfrac{y^2 + 4y}{21}, & 0 < y < 3 \\ 1, & y \ge 3 \end{cases}$$

(c) 4/7 (d) 17/84 (e) 5/21 (f) not independent

(g)

$$f_2(y|x) = \frac{2(3x+y)}{9(2x+1)}, 0 < y < 3, \;\; 0 < x < 1$$

(h) 1/3

5.15 (a) 0.4

(b)

$$f_1(x) = \begin{cases} 0.3, & x = 1 \\ 0.45, & x = 2 \\ 0.25, & x = 3 \end{cases} \qquad f_2(y) = \begin{cases} 0.6, & y = 1 \\ 0.4, & y = 2 \end{cases}$$

(c)

$$f_1(x|y=2) = \begin{cases} 0.5, & x = 1 \\ 0.375, & x = 2 \\ 0.125, & x = 3 \end{cases}$$

(d) not independent (e) $E(X) = 1.95$, $E(Y) = 1.4$ (f) $Var(X) = 0.5475$, $Var(Y) = 0.24$
(g) -0.3586

5.17 (a) 0.6
(b)

$$f_1(x) = \begin{cases} 0.3 & x = 1 \\ 0.25, & x = 2 \\ 0.45, & x = 3 \end{cases} \qquad f_2(y) = \begin{cases} 0.35 & y = 1 \\ 0.3, & y = 2 \\ 0.35, & y = 3 \end{cases}$$

(c) 1/6 (d) not independent (e) $E(X) = 2.15$, $E(Y) = 2$
(f) $Var(X) = 0.7275$, $Var(X) = 0.7$ (g) -0.14

5.19 (a) $f_1(x) = 3x^2,\ 0 < x < 1$; $f_2(y) = 6y(1-y),\ 0 < y < 1$
(b) $f_1(x|y) = 1/(1-y), 0 < y < x < 1$ (c) 3/4 (d) $E(X) = 3/4$, $E(Y) = 1/2$
(e) $Var(X) = 3/80, Var(Y) = 1/20$ (f) 1/40 (g) 0.5774

5.21 (a) $f_1(x) = 3(x^2+1)/4,\ 0 < x < 1$; $f_2(y) = (9y^2+1)/4,\ 0 < y < 1$
(b)

$$F_1(x) = \begin{cases} 0 & x \le 0 \\ \dfrac{x^3 + 3x^2}{4}, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases} \qquad F_2(y) = \begin{cases} 0 & y \le 0 \\ \dfrac{3y^3 + y}{4}, & 0 < y < 1 \\ 1, & y \ge 1 \end{cases}$$

(c)

$$f_1(x|y) = \frac{3(x^2 + 3y^2)}{9y^2 + 1}, 0 < x < 1, 0 < y < 1$$

$$f_2(y|x) = \frac{x^2 + 3y^2}{x^2 + 1}, 0 < x < 1, 0 < y < 1$$

(d) $E(X) = 9/16$, $E(Y) = 11/16$ (e) $Var(X) = 107/1280$, $Var(Y) = 233/3840$
(f) $-3/256$ (g) 1/15 (h) 7/10

5.23 (a) 3/4 (b) 6

5.25 $E(U) = 9, Var(U) = 106$

5.27 (a) $3\mu_1 - 2\mu_2$ (b) $13\sigma^2$ (c) $Y \sim N(3\mu_1 - 2\mu_2, 13\sigma^2)$

5.29 (a) 0.923 (b) 0.9213 (c) 0.6421 (d) 0.9586