1) a) $P(|Y_1 - \mu| \leq 1/2)$

independent random sample $Y_1, \dots, Y_9$ with unknown mean μ, $\sigma^2 = 1$

$$Y_1 \sim N(\mu, 1)$$

$$Y_1 - \mu \Rightarrow \frac{Y_1 - (1)\mu}{(1)(1)} \Rightarrow \frac{Y_1 - (1)\mu}{\sigma(1)} \sim N(0,1)$$

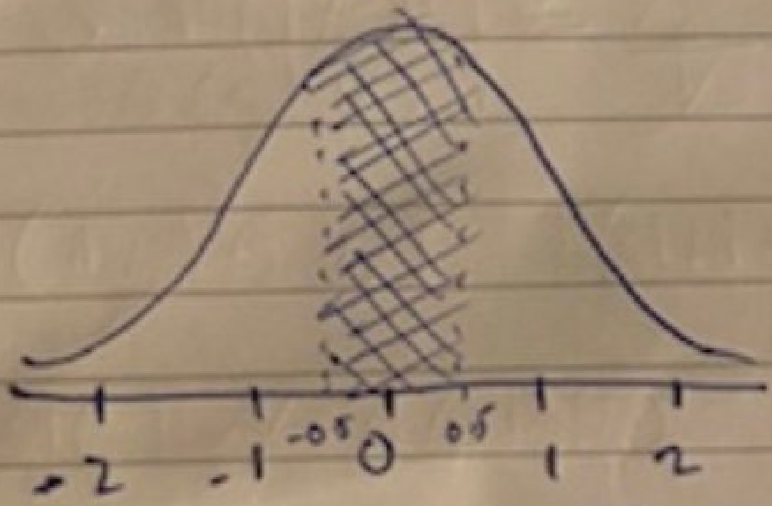
As $Y_1 - \mu$ is already standardised.

$P(|Y_1 - \mu| \leq 1/2)$

$= P(|Z| \leq 0.5) = P(-0.5 \leq Z \leq 0.5)$

where, $Z \sim N(0,1)$ is the standard normal distn.

$= P(Z \leq 0.5)$



1)b) $P(|\bar{Y}-\mu| \le 1/2)$

$\bar{Y} \sim N(\mu, \frac{\sigma^2}{n}) \Rightarrow \bar{Y} \sim N(\mu, \frac{1}{9})$

Standardising $\bar{Y}$, we get

$\frac{\bar{Y}-\mu}{1/3} \sim N(0,1)$ $\quad 1/3$ (as) $\sigma/\sqrt{n}$

$P(|\bar{Y}-\mu| \le 1/2) = P\left(\left|\frac{\bar{Y}-\mu}{1/3}\right| \le \frac{\frac{1}{2}}{\frac{1}{3}}\right)$

$P(|Z| \le 3/2) = P(|Z| \le 1.5)$

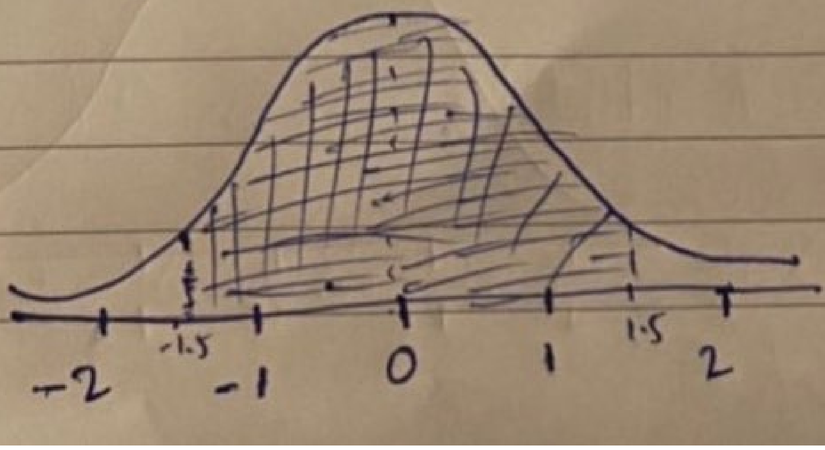
$P(-1.5 \le Z \le 1.5) = P(Z \le 1.5)$
$\quad - P(Z \le -1.5)$

$= P(Z \le 1.5) - (1 - P(Z \le 1.5))$

$= 2P(Z \le 1.5) - 1 \quad = 2(0.9332) - 1$

$= 1.8664 - 1$

$= 0.8664.$



Q2)

Poisson Random Variable

mean — 2 accidents per week.

$\lambda = 2 \times 52 = 104$

$X \sim Poi(\lambda = 104)$

$$P(Y < 100) = \sum_{i=0}^{99} \frac{e^{-\lambda} (\lambda)^i}{i!} = \sum_{i=0}^{99} \frac{e^{-104} (104)^i}{i!}$$

With Central Limit Theorem, we can say Y is sum of 52 IID Random Variables, with mean & Variance $\mu = \sigma^2 = 2$, we can approximate

$X \approx Y \sim N(52*2, 52*2)$

$$P(X < 100) \approx P(Y < 100)$$
$$= P\left(\frac{Y - 104}{\sqrt{104}} < \frac{100-104}{\sqrt{104}}\right) \quad \left(\because z = \frac{Y-\mu}{\sigma}\right)$$
$$= P\left(z < \frac{-4}{\sqrt{104}}\right) = P(z < -0.3922)$$
$$= \quad 0.34827$$