MAU22C00 - TUTORIAL 3

1) (From the 2016-2017 Annual Exam) Let Q denote the relation on the set $\mathbb{Z}$ of integers, where integers x and y satisfy xQy if and only if

$$x - y = (x - y)(x + 2y).$$

Determine the following:

(i) Whether or not the relation Q is *reflexive*;
(ii) Whether or not the relation Q is *symmetric*;
(iii) Whether or not the relation Q is *transitive*;
(iv) Whether or not the relation Q is an *equivalence relation*;
(v) Whether or not the relation Q is *anti-symmetric*;
(vi) Whether or not the relation Q is a *partial order*.

Justify your answers.

2) In lecture we discussed an equivalence relation given by $f : A \to A$ for f any function on a non-empty set A with the relation R defined by $R = \{(x, y) \mid f(x) = f(y)\}$. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \cos x$. What are the equivalence classes determined by the relation R on $\mathbb{R}$, namely for any $x \in \mathbb{R}$ what is $[x]_R$?