1) a) The binomial distribution could be used to model this data as each iteration is a bernoulli trail where the outcome is either a success or a failure.

Distribution Parameters:
The number of tests (n), and then
$p \rightarrow$ the probability of a success and the
$q \rightarrow$ probability of a failure.

b) $L(p) = P(X = x_1) \times \dots \times P(X = x_i) = \prod_{i=1}^{n} P(X = x_i)$

Since we have a binomial distribution we can use

$$P(X = x_i) = \binom{n}{x} p^x q^{n-x}$$

As we know $x_1 = 3$, $x_2 = 2$ and
$n = 12$

$\therefore P(X=x_1) = \binom{12}{3} p^3 q^9$

$P(X=x_2) = \binom{12}{2} p^2 q^{10}$

$$\therefore \prod_{i=1}^{n} P(X=x_i) = \binom{12}{3} p^3 q^9 \times \binom{12}{2} p^2 q^{10}$$

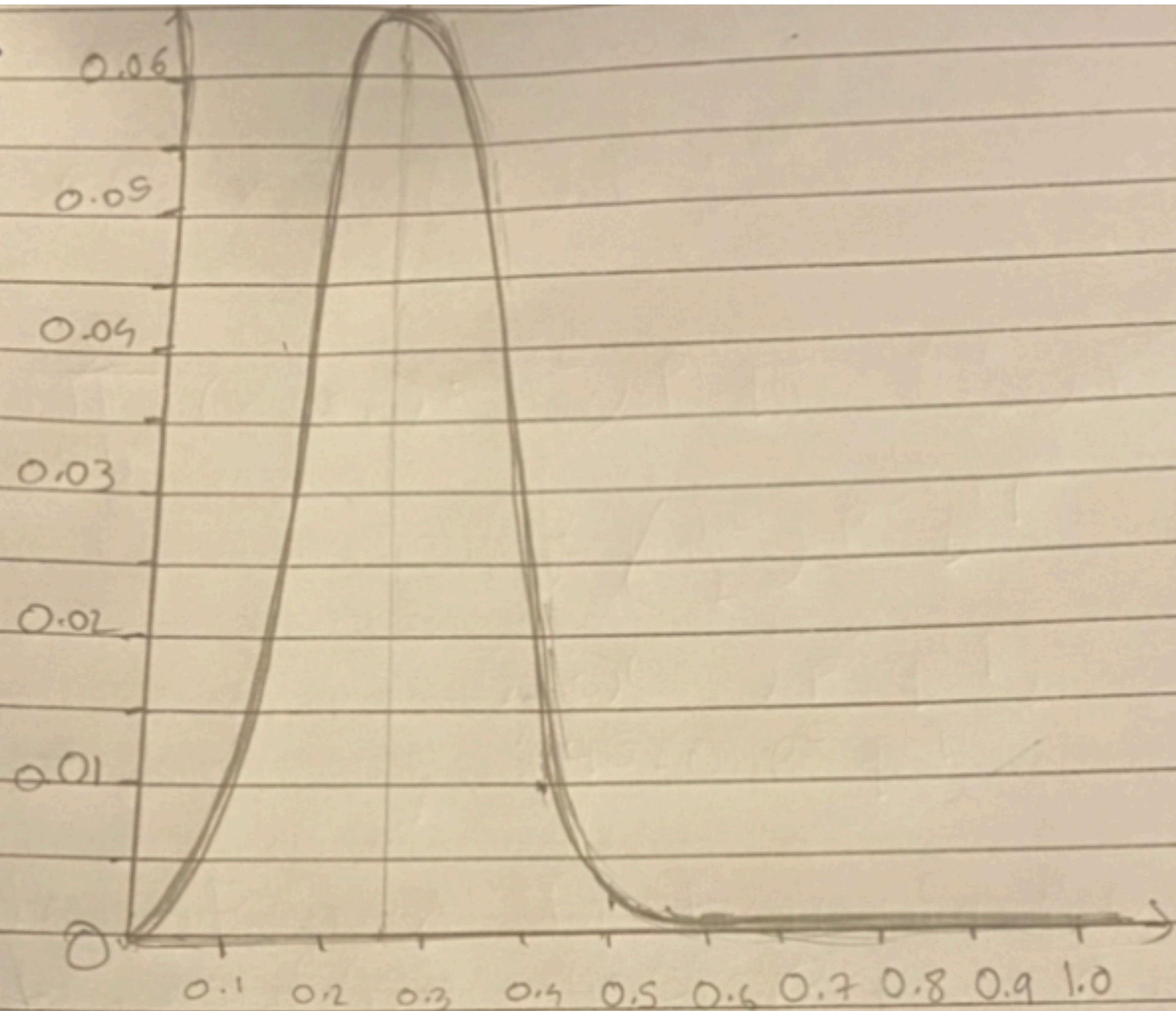
$$= \binom{12}{3}\binom{12}{2} p^5 q^{19}$$

$$= (220)(66)\, p^5 q^{19}$$

$$= 14520\, p^5 q^{19}$$

$\therefore$ The likelihood function for this data is given by $\underline{(14520)p^5 q^{19}}$ or $\underline{(14520)(p^5)(1-p)^{19}}$ where q is the failures and p is the success.

c)



Thee function maxes out at (0.2083, 0.0673)

function : $14520\,(P^5)(1-P)^{19}$

d) Likely hood function

$$L(P) = \prod_{i=1}^{2} \binom{12}{x_i} p^i q^{12-i}$$

$$L(P) = 14520\, P^5 (1-P)^{19}$$

$$L(P) = \log(14520) + 5\log(P) + 19\log(1-P)$$

$$\frac{dL}{dP} = 0 + \frac{5}{P} + \frac{19(-1)}{(1-P)}$$

$$\frac{dL}{dP} = \frac{5}{P} - \frac{19}{1-P}$$

To find maximum value $\Rightarrow \frac{dL}{dP} = 0$

$\Rightarrow 5/P - 19/1-P = 0 \Rightarrow \frac{5}{P} = \frac{19}{1-P}$

$\Rightarrow 5 - 5P = 19P \Rightarrow 5 = 24P$

$\Rightarrow P = 5/24$.

$\therefore$ The maximum Likelihood estimate is $\frac{dL}{dP}$ and its value is $5/24$ or 0.2083