

$$1) a) n=10, \bar{y}=124, s=10$$

$$H_0: \mu = 110 \text{ vs } H_A: \mu \neq 110,$$

$$\alpha = 0.01$$

$$v = n - 1 = 10 - 1 = 9$$

$$\mu_0 = 110$$

Under H_0 , the test statistic $T \sim t(9)$.

$$\text{Observed Test Stat, } T_{\text{obs}} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$T_{\text{obs}} = \frac{124 - 110}{10/\sqrt{10}} = \frac{14}{\sqrt{10}} = 4.427$$

$$\text{Critical Value, } t_{v=9, \alpha/2=0.005} = 3.250$$

$$T_{\text{obs}} \leq -t_{v, \alpha/2} \quad | \quad T_{\text{obs}} \geq t_{v, \alpha/2}$$
$$4.427 \leq -(3.250) \quad | \quad 4.427 \geq 3.250 \quad \checkmark$$

So, we reject the H_0 and accept H_A
as $T_{\text{obs}} \geq t_{v, \alpha/2}$

$$b) n=8, \bar{y}=0.6, s=0.2, \dots$$

$$H_0: \mu=0.5, \text{ vs } H_A: \mu \neq 0.5$$

$$\alpha=0.05.$$

$$v=8-1=7, \quad \mu_0=0.5$$

Under H_0 , Test Statistic $T \sim t(7)$

$$\begin{aligned} T_{obs} &= \frac{\bar{y} - \mu_0}{s/\sqrt{n}} = \frac{0.6 - 0.5}{0.2/\sqrt{8}} = \frac{0.1(2\sqrt{2})}{0.2} \\ &= \sqrt{2} = 1.414 \end{aligned}$$

Critical value, $t_{v=7, \frac{\alpha}{2}=0.025} = 2.365$

$$-t_{v, \alpha/2} < T_{obs} < t_{v, \alpha/2} \Rightarrow -2.365 < 1.414 < 2.365$$

We fail to reject the H_0 as

$$-t_{v, \alpha/2} < T_{obs} < t_{v, \alpha/2}$$

$$c) n=25, \bar{y}=33.4, s=6.8$$

$$H_0: \mu=30, \text{ vs } H_A: \mu>30$$

$$\alpha=0.1, v=24, \mu_0=30$$

Under H_0 , Test Statistic $T \sim t(24)$

$$T_{obs} = \frac{\bar{y} - \mu_0}{s/\sqrt{n}} = \frac{33.4 - 30}{6.8/\sqrt{25}} = \frac{3.4 \times 5}{6.8} = 2.5$$

$$\text{Critical Value, } t_{v=24, \alpha=0.1} = 1.318$$

$$T_{obs} > t_{v, \alpha} \Rightarrow 2.5 > 1.318$$

We Reject the H_0 & accept H_A , as $T_{obs} > t_{v, \alpha}$, upper-tailed test.

$$2) a) \text{ mean } (\mu) = 55$$

$$\text{sd } (\sigma) = 100$$

$$P(X > 70)$$

$$\Rightarrow Z = \frac{70 - \mu}{\sigma} = \frac{70 - 55}{100}$$

$$Z = 0.15$$

$$P(X > 70) = P(Z > 0.15)$$

$$= 1 - P(Z \leq 0.15)$$

$$= 1 - 0.5595$$

$$= 0.4401$$

Probability of any one of these getting a first is 0.4401

2) b) Probability that exactly one of the 10 students will get a first

Given, $n=10$, Probability that anyone will get a first, $p=0.4401$

exactly one $\Rightarrow k=1$

Using Binomial distribution,

$$\begin{aligned} \Pr(k) &= \binom{n}{k} p^k (1-p)^{n-k} \\ \Pr(1) &= \binom{10}{1} (0.4401)^1 (1-0.4401)^{10-1} \\ &= \binom{10}{1} (0.4401) (0.5599)^9 \\ &= (10) (0.4401) (0.00541) \\ &= 0.02381 \end{aligned}$$

Probability that exactly one student gets a first is 0.02381

⇒ c) Probability that the avg. mark of class will be above 60.

$$\Rightarrow P(Y > 60) = P\left(\frac{Y - E[Y]}{\frac{\sigma(Y)}{\sqrt{n}}} > \frac{60 - \mu}{\frac{\sigma^2}{\sqrt{n}}}\right)$$

$$= P\left(Z > \frac{60 - 55}{\frac{100}{\sqrt{10}}}\right)$$

$$= P\left(Z > \frac{5}{10\sqrt{10}}\right) = P\left(Z > \frac{1}{2\sqrt{10}}\right)$$

$$= P(Z > 0.158) = 0.436$$

∴ Probability that the avg. mark of class will be above 60 is 0.436