MAU22C00: TUTORIAL 19 PROBLEMS

1) Is $\{x \in \mathbb{R}^+ \mid \log x \in \mathbb{R} \setminus \mathbb{Q}\}$ finite, countably infinite, or uncountably infinite? Justify your answer. The set $\mathbb{R}^+$ is the set of all positive real numbers.

2) Is $\bigcup_{n=1}^{10} \left\{ \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = n^2\} \cap \{(x,y) \in \mathbb{R}^2 \mid y^2 - x^4 = 0\} \right\}$ finite, countably infinite, or uncountably infinite? Justify your answer.

3) Let $A = \{0,1\}$. Is $(0^* \circ 1^*) \cap \{A^* \circ 11 \circ A^*\}$ finite, countably infinite, or uncountably infinite? Justify your answer.

4) Prove that the language generated by a regular expression is countable. Give an example of a regular expression that generates a finite language and another example of a regular expression that generates a countably infinite language. Justify your answers.

5) Consider the language over the binary alphabet $A = \{0,1\}$ given by $L = \{0^m 1^{2m} \mid m \in \mathbb{N}\}$.

(a) Use the Pumping Lemma to show L is not a regular language.

(b) Is the language L finite, countably infinite, or uncountably infinite? Justify your answer.

(c) A language L' over the same alphabet $A = \{0,1\}$ is called a *sublanguage* of L if $L' \subset L$. Let $\mathcal{C}$ be the set of sublanguages of L. Is $\mathcal{C}$ finite, countably infinite, or uncountably infinite? Justify your answer.