

**MAU22C00: TUTORIAL 14 PROBLEMS**  
**GRAPH THEORY**

- 1) For what type of  $p$  and  $q$  does the complete bipartite graph  $K_{p,q}$  have a Hamiltonian circuit? Justify your answer.
- 2) Let  $(V, E)$  be the graph with vertices  $a, b, c, d, e$ , and  $f$  and edges  $ab, ac, bc, bd, cd, de, df$ , and  $ef$ .
  - (a) Does this graph have a Hamiltonian circuit? Justify your answer.
  - (b) Is this graph a tree? Justify your answer.
  - (c) If it is not a tree, how many distinct spanning trees does it have?
- 3) Consider the statement “A graph  $(V, E)$  is a tree  $\iff \#(E) = \#(V) - 1$ . ” What hypothesis is needed for this equivalence to be true? Give an example to show why this hypothesis is necessary.

Recall that

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

read as “ $n$  choose  $k$ ” gives the number of distinct combinations of  $k$  objects taken out of a possible  $n$  objects for  $n \geq k \geq 0$  with the convention  $0! = 1$ .

- 4) Consider the complete graph  $K_n$  for  $n = 2, 3, 4$ . In each of the three cases
  - (a) Is this graph a tree? Justify your answer.
  - (b) If it is not a tree, how many distinct spanning trees does it have?(Hint: How many edges does  $K_n$  have?)