1. The probability function of random variable X is given as

$$P(a_k) = P(b_k) = \frac{A}{(k+1)(k+2)} \quad , \quad x \in \{a_0, b_0, a_1, b_1, \dots\}$$

   Find A.

2. If $E[X] = 1$, $Var[X] = 2$ and $Y = aX + b$, find $E[XY]$.

3. A box contains 2^{100} chips which $\binom{100}{x}$ of them are numbered x. If you take a random chip, what is the mean number observed on this chip?

4. If $F(x) = \begin{cases} A & x < -1 \\ B & -1 \le x < 0 \\ C & 0 \le x < 2 \\ D & x \ge 2 \end{cases}$, $P(X = 0) = \frac{1}{6}$, find $P(-\frac{1}{2} < X < \frac{3}{2})$.

5. Rolling a die, $P(X = 6) = \theta$, $P(X = 1) = \frac{1}{3} - \theta$, and all other sides are equiprobable, find $E[X]$.

6. If random variable X gets the values $\{-1, 0, 1\}$ and $f(0) = \frac{1}{4}$, find $E[X^2]$.

7. If

$$F(x) = 1 - \frac{1}{2^{x+1}} - \frac{1}{2^{[x]+1}}$$

   find $P(X = 1)$.

8. If $m_X(t) = \left(\frac{1}{4} + \frac{3}{4}e^t\right)^{12}$, find $Var[X]$.

9. A bus has k passengers and there are n stations in front of the bus. Probability that a passenger gets off the bus is the same for all stations. The bus stops at a station only if at least 1 passenger wants to get off. Find the average number of stops.

10. If random variable X has the cumulative probability function of $F(x)$, and Y is defined as $Y = sgn(X)$, find $E[Y]$.