Sample Questions 4 – Sol

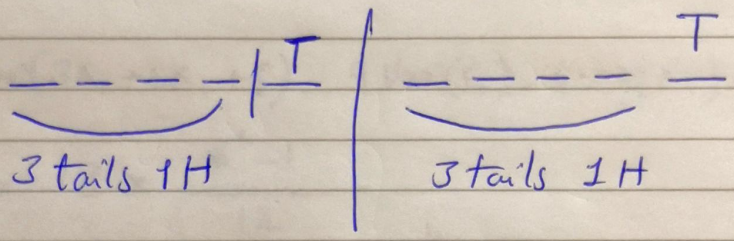
Q1. $P(X=1) = \frac{1}{4} = \lim_{x \to 1^+} F(x) - \lim_{x \to 1^-} F(x) = a - \frac{1}{4}$

$\Rightarrow a = 1/2$

$P(X=2) = \frac{1}{6} = \lim_{x \to 2^+} F(x) - \lim_{x \to 2^-} F(x) = b - \left(\frac{1}{2} + \frac{1}{4}\right)$

$\Rightarrow b = \frac{11}{12}$

Q2. Here is the event:



$\Rightarrow \left[\binom{4}{3} \left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right) \right]^2$

Q3. $np = 3$, $npq = 2$ $\Rightarrow \frac{npq}{np} = \frac{2}{3} \Rightarrow q = \frac{2}{3}$, $p = 1/3$

$np = n\left(\frac{1}{3}\right) = 3 \Rightarrow n = 9$

$P(X \geq 1) = 1 - P(X = 0) = 1 - \left(\frac{2}{3}\right)^9$

Q5 $\hat{x} = 1.82 \Rightarrow \lambda = 2$

$P(X=4) = \frac{e^{-2} 2^4}{4!}$

Q6. $Var\left(\frac{X_2}{\alpha} - \frac{X_1}{\beta}\right) = \frac{1}{\alpha^2} Var(X_2) + \frac{1}{\beta^2} Var(X_1)$

$= \frac{1}{\alpha^2}\beta + \frac{1}{\beta^2}\alpha$

Q7. $\lambda = rt = 4 \times \frac{1}{2} = 2 \quad P(X=0) = \frac{e^{-2} 2^0}{0!} = e^{-2}$

Q8. $Var(3X_1 + 2X_1 + 5) = 9\,Var(X_1) + 4\,Var(X_2) = 13\,Var(X)$

$= 13 \times \frac{6^2 - 1}{12}$

Q9. $X \sim Ber(p) \rightarrow X^k \sim Ber(p)$

$\Rightarrow X = X_1 + X_2^2 + \cdots + X_n^n \sim Binom(n, p)$

$E(X^2) = Var(X) + E^2(X) = npq + n^2p^2$

Q10. Geometric distribution with $p = 1/4$

$E(Trials) = \frac{1}{p} = \frac{1}{1/4} = 4$