Sample Questions 9 - Sol

Q1. $Y = 2, 4, 6 \dots, 2n \qquad X = 1, 2 \dots n$

$Y = 2X \quad Var(X) = \frac{n^2-1}{12} \quad Var(Y) = 4\,Var(X)$

Q2. $\begin{cases} \bar{y} = m\bar{x} + \frac{3}{2} \\ \bar{x} = k\bar{y} + 1 \end{cases} \Rightarrow \begin{cases} 2 = 2m + 3/2 & m = \frac{1}{4} \\ 2 = 2k + 1 & k = \frac{1}{2} \end{cases}$

$\rho^2 = mk = \frac{1}{8} \Rightarrow \rho = \frac{1}{\sqrt{8}}$

Q3. $A, BA, BBA, BBBA, \dots$

$P_A + P_B P_A + P_B^2 P_A + P_B^3 P_A + \dots = \frac{P_A}{1 - P_B} = \frac{P_A}{P_A + P_C}$

Q4. $f(x) = 6(x - x^2) \quad f'(x) = 6(1 - 2x) = 0 \quad \hat{x} = \frac{1}{2}$

Q5 $Cov(X, Y) = E(XY) - E[X]E[Y]$

$f(x)$ is symmetric

$E(X^3) - E(X)E(X^2) = 0 \Rightarrow \rho_{X,Y} = 0$

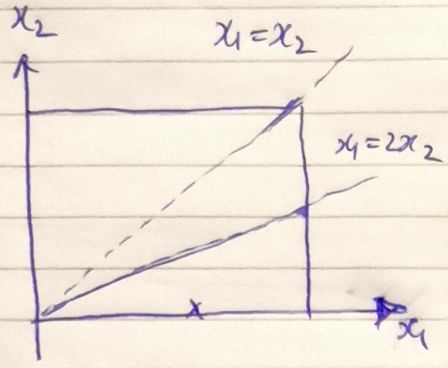
($E(X^3) = 0$, $E(X) = 0$)

Q6. $1 + 1 + 1 - + \dots + 1 = n \rightarrow 2n - n = n$

$n = n \quad k = n \quad m = n \qquad C^{k-1}_{n+k-1} = C^{n-1}_{2n-1}$

Q7 $Y = F(X) \Rightarrow Y \sim U(0,1) \Rightarrow Var(Y) = \frac{(1-0)^2}{12} = \frac{1}{12}$

Q8. $f(x_1, x_2) = 4x_1x_2$



$$P(X_1 < X_2 \mid X_1 < 2X_2) = \frac{P(X_1 < X_2)}{P(X_1 < 2X_2)} = \frac{\int_0^1 \int_0^{\frac{x_1}{2}} 4x_1x_2 \, dx_2 \, dx_1}{\int_0^1 \int_0^{x_1} 4x_1x_2 \, dx_2 \, dx_1} = \frac{\cdots}{\cdots}$$

Q9) $F_Y(y) = P(Y \le y) = P(|X| \le y) = P(-y \le X \le y) = \int_{-y}^{y} \frac{3x^2}{2} dx$

$$= \int_0^y 3x^2 dx = y^3$$

$$f_Y(y) = \frac{dF_Y(y)}{dy} = 3y^2$$

Q10. As long as we don't know what happend before the sixth taken chip, the probability remains the same $P(\text{the sixth one is red}) = P(\text{the first one is red}) = \frac{5}{9}$