## MAU22C00: ASSIGNMENT 6 DUE FRIDAY, APRIL 23 BEFORE MIDNIGHT UPLOAD ON BLACKBOARD

Please write down clearly both your name and your student ID number on everything you hand in. Please attach a cover sheet with a declaration confirming that you know and understand College rules on plagiarism. Details can be found on http://tcd-ie.libguides.com/plagiarism/declaration.

This assignment may be uploaded onto Blackboard up to Friday, April 30 before midnight without any lateness penalty.

- 1) (20 points)
  - (a) Let L be the language over the alphabet  $A = \{a, l, p\}$  consisting of all words containing both a and l. Write down the algorithm of a Turing machine that decides L. Process the following strings according to your algorithm: p, al, pap, pla, and aapppla.
  - (b) Write down the transition diagram of the Turing machine from part (a) carefully labelling the initial state, the accept state, the reject state, and all the transitions specified in your algorithm.
- 2) (10 points) Let the alphabet  $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . Write down the algorithm of an enumerator that prints out EXACTLY ONCE every string in the language

$$L = \{3m + 1 \mid m \in \mathbb{N}\}$$

that is EVEN.

- 3) (20 points)
  - (a) The emptiness testing problem for phrase structure grammars (PSG's) is given by the language

 $E_{PSG} = \{ \langle G \rangle \mid G \text{ is a phrase structure grammar and } L(G) = \emptyset \}.$ 

Is it possible to modify the marking argument used in lecture to prove that the emptiness testing problem for context-free grammars  $E_{CFG}$  is Turing-decidable in order to prove that  $E_{PSG}$  is also Turing-decidable? Justify your answer.

(b) Is  $E_{PSG}$  a finite set, a countably infinite set, or an uncountably infinite set? Justify your answer. You may assume without proof the fact that if G is a phrase structure grammar, then L(G) is a Turing-recognisable language.