MAU22C00: ASSIGNMENT 1
DUE BY FRIDAY, OCTOBER 16 BEFORE MIDNIGHT
UPLOAD SOLUTION ON BLACKBOARD

Please write down clearly both your name and your student ID number on everything you hand in. Please attach a cover sheet with a declaration confirming that you know and understand College rules on plagiarism. Details can be found on `http://tcd-ie.libguides.com/plagiarism/declaration`**.**

1) (10 points) Please carry out the following proof in propositional logic following the proof format in tutorial 1: Hypotheses: $P \to (Q \leftrightarrow \neg R)$, $P \vee \neg S$, $R \to S$, $\neg Q \to \neg R$. Conclusion: $\neg R$.
For each line of the proof, mention which tautology you used giving its number according to the list of tautologies posted in folder Course Documents. Solutions based on truth tables or any other method except for the one specified will be given **NO CREDIT**.

2) (10 points) Prove the following statement: If n is any integer, then $n^2 - 3n$ must be even. (Hint: Cases come in handy here. See tautology #26 for the basis of proofs by cases. This proof follows the format of the one given in lecture that $\sqrt{2}$ is not a rational number.)

3) (10 points) Prove via inclusion in both directions that for any three sets A, B, and C

$$A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C).$$

Venn diagrams, truth tables, or diagrams for simplifying statements in Boolean algebra such as Veitch diagrams are **NOT** acceptable and will not be awarded any credit.

4) (10 points) Let $\mathbb{N} \times \mathbb{N}$ be the Cartesian product of the set of natural numbers with itself consisting of all ordered pairs (x_1, x_2) such that $x_1 \in \mathbb{N}$ and $x_2 \in \mathbb{N}$. We define a relation on its power set $\mathcal{P}(\mathbb{N} \times \mathbb{N})$ as follows: $\forall A, B \in \mathcal{P}(\mathbb{N} \times \mathbb{N})$ $A \sim B$ iff $(A \setminus B) \cup (B \setminus A) = C$ and C is a finite set. Determine whether or not $\sim$ is an equivalence relation and justify your answer by checking each of the three properties in the definition of an equivalence relation. Please note that a set C is finite if it has finitely many elements. In particular, the empty set $\emptyset$ has zero elements and is thus finite.