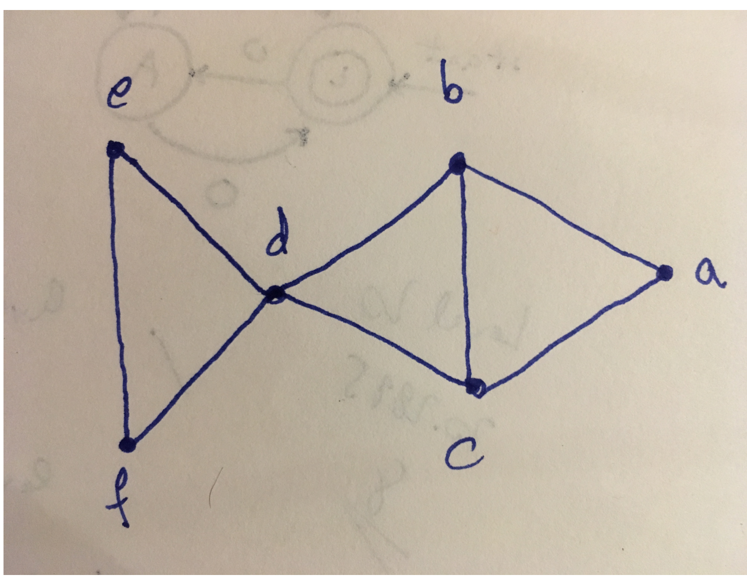
MAU22C00: TUTORIAL 12 SOLUTIONS
GRAPH THEORY

1) Let (V, E) be the graph with vertices a, b, c, d, e, and f and edges ab, ac, bc, bd, cd, de, df, and ef.

(a) Draw this graph.
(b) Is this graph connected? Justify your answer.
(c) What is the minimum number of edges you would have to remove for the resulting subgraph to have two connected components? Justify your answer.
(d) What about three connected components? Justify your answer.
(e) What about four connected components? Justify your answer.
(f) What about five connected components? Justify your answer.
(g) Give an example of a shortest possible circuit in the graph. Justify your answer.
(h) Give an example of a longest possible circuit in the graph. Justify your answer.

Solution: Let (V, E) be the graph with vertices a, b, c, d, e, and f and edges ab, ac, bc, bd, cd, de, df, and ef.

(a) Here is the graph:



(b) The graph is connected as there is a walk from every vertex to every other vertex.

(c) Two edges: removing de and ef gives the component consisting of the vertex e alone and the component consisting of abcdf.

(d) Three edges: removing de, ef, and df from the original graph gives the component consisting of vertex e alone, the component consisting of vertex f alone, and the component consisting of abcd.

(e) Five edges: the three we removed before (de, ef, and df) as well as the two edges bd and cd to disconnect vertex d from abc.

(f) Seven edges: besides the five edges we removed (de, ef, df, bd, and cd), we also need to disconnect one vertex from the triangle abc by removing for example ab and ac.

(g) A circuit has a minimum of three vertices as it cannot be trivial, it cannot repeat edges, and it must close up. Any three-vertex circuit in this graph is thus an example of a shortest possible circuit: defd, bcdb, or abca.

(h) abdefdca is an example of a longest possible circuit in this graph. We cannot use edge bc without repeating one other edge, which would not give us a circuit as a circuit is a trail (and cannot repeat edges).