STU22004 – Sample Questions 1

1. In how many ways can 10 people sit in a row, if two of them insist not to be seated beside each other?

$10! - 2! \times 9!$

2. In how many ways can you put n different gifts in k different boxes? $(n \le k)$

$\binom{k}{n} n!$

3. In how many ways can 4 boys and 4 girls sit around a table, no 2 boys or girls adjacent?

$3!\,4!$

4. How many sequences from the letters of word MISSISSIPPI, no 2 "i"s adjacent?

$\frac{7!}{4!\,2!\,1!} \quad C_8^4$

5. In how many ways can you put 70 similar chips in 5 numbered boxes, min 2^i chips in box number i ?

$2+4+8+16+32=62 \quad 70-62=8 \quad C_{n+k-1}^{k-1} = C_{12}^{4}$

6. Redo question 3, if boys and girls are siblings (2 by2), and they do not want to sit adjacent?

$3! \times 2$

7. You put 20 similar chips in 5 numbered boxes. In how many ways a particular box would be empty?

$n=20$
$k=4$ $\rightarrow C_{n+k-1}^{k-1} = C_{23}^{3}$

8. In how many ways can you order 15 different books, if 5 particular ones should be adjacent?

$5! \times 11!$

9. How many 7-digit numbers with no same adjacent digits? $\underline{9}\ \underline{9}\ \underline{9}\ \underline{9}\ \underline{9}\ \underline{9}\ \underline{9} \quad 9^7$

10. In a company, 7 technicians know how to work with a particular machine. 3 machines are used in each work shift. In how many different shifts does a particular technician work? $\binom{6}{2}$

11. Find $\sum_{i=0}^{n}(-1)^i\binom{n}{i}$

$\sum_{i=0}^{n} a^i b^{n-i}\binom{n}{i} = (a+b)^n \qquad a=-1,\ b=1 \Rightarrow \sum_{i=0}^{n}(-1)^i\binom{n}{i} = 0$

12. $\sum_{i=0}^{n} \binom{n}{i}^2$ Choose n People from n Ms & n Fs

$$\binom{2n}{n} = \binom{n}{0}\binom{n}{n} + \binom{n}{1}\binom{n}{n-1} + \dots + \binom{n}{n}\binom{n}{0} \Rightarrow \binom{2n}{n} = \sum_{i=1}^{n} \binom{n}{i}^2$$

13. In how many ways can you write 30 as the sum of 5 non-negative 3 multipliers?

$x_1 + x_2 + \dots + x_5 = 30$ $\quad x_i \geq 0 \quad x_i = 3y_i$

$y_1 + y_2 + \dots + y_5 = 10$ $\quad n = 10, \; k = 5 \quad C_{n+k-1}^{k-1}$

14. Find the number of integer answers for the equation $x + y + z = 10$ if $0 \leq x \leq 6, 0 \leq y \leq 6, 0 \leq z \leq 7$.

1) Assume all are between 0 to 6. $\Rightarrow$ $n = 10$, $k = 3$, $m = 6$

$$N_1 = \sum_{S=0}^{k} (-1)^S \binom{k}{S} \binom{n+k-(m+1)S-1}{k-1}$$

$$= \binom{12}{2} - \binom{3}{1}\binom{5}{2}$$

Some answers are missing, as z could be equal to 7 and we assumed it could not. We now put $z = 7$ to see how many answers are missing.

$x + y + 7 = 10 \Rightarrow x + y = 3$ $\quad n = 3, \; k = 2, \; m = 3$

$$N_2 = C_{n+k-1}^{k-1} = C_4^1$$

$N = N_1 + N_2$