

MAU22C00: TUTORIAL 7 PROBLEM SHEET
HOMOMORPHISMS AND ISOMORPHISMS

1) Let A be a finite set, and let A^* be the set of all words over the alphabet A . Consider (A^*, \circ, ϵ) with the operation of concatenation and empty word ϵ as the identity element. Let $(\mathbb{N}, +, 0)$ be the set of natural numbers with the operation of addition and 0 as the identity element. Let $f : A^* \rightarrow \mathbb{N}$ be the function that assigns to each word $w \in A^*$ its length, $f(w) = |w| \in \mathbb{N}$.

- (a) What type of object is (A^*, \circ, ϵ) in abstract algebra (semigroup, monoid, group)? Justify your answer.
- (b) What type of object is $(\mathbb{N}, +, 0)$ in abstract algebra (semigroup, monoid, group)? Justify your answer.
- (c) Is f a homomorphism? Justify your answer.
- (d) Is f an isomorphism? Justify your answer.

2) Let $(\mathbb{Z}, +, 0)$ be the set of integers with the operation of addition and 0 as the identity element. Let E be the set of even integers, $E = \{2p \mid p \in \mathbb{Z}\}$. Consider $(E, +, 0)$ the set of even integers with the operation of addition and 0 as the identity element. Let $f : \mathbb{Z} \rightarrow E$ be the function $f(n) = 2n$.

- (a) What type of object is $(\mathbb{Z}, +, 0)$ in abstract algebra (semigroup, monoid, group)? Justify your answer.
- (b) What type of object is $(E, +, 0)$ in abstract algebra (semigroup, monoid, group)? Justify your answer.
- (c) Is f a homomorphism? Justify your answer.
- (d) Is f an isomorphism? Justify your answer.