

6(a).

Given:

- $tf'_{ij} = tf_{ij} * (1 + \sum_0^j \frac{p_{ij} \log p_{ij}}{\log m})$ where $p_{ij} = \frac{tf_{ij}}{gf_i}$
- $0 < tf_{ij} \leq k$
- $gf_i = \sum_1^m tf_{ij}$
-above represents the number of times ith term appears in all documents
- $1 < m$
-above represents max number of documents

Proof:

Min:

-Occurs when $tf_{ij} = 1$ (aka only 1 of ith term found in each document)

Step 1: Assume $tf_{ij} = 1$ and solve for p_{ij}

$$p_{ij} = \frac{tf_{ij}}{\sum_1^m tf_{ij}} = \frac{1}{\sum_1^m 1} = \frac{1}{m}$$

Step 2: plug in the values for p_{ij} and tf_{ij} into the tf'_{ij} formula

$$\begin{aligned} tf'_{ij} &= tf_{ij} * (1 + \sum_0^j \frac{p_{ij} \log p_{ij}}{\log m}) \\ &= 1 * (1 + \sum_0^j \frac{\frac{1}{m} \log \frac{1}{m}}{\log m}) \\ &= 1 + j \left[\frac{\frac{1}{m} \log \frac{1}{m}}{\log m} \right] \\ &= 1 + \frac{-j}{m} \\ &= \frac{m-j}{m} \end{aligned}$$

$$\text{Ans:} = \frac{m-j}{m}$$

Max:

-Occurs when $tf_{ij} = k$ (aka only k(max frequency) of ith term is found in each document)

Step 1: Assume $tf_{ij} = k$ and solve for p_{ij}

$$p_{ij} = \frac{tf_{ij}}{\sum_1^m tf_{ij}} = \frac{k}{\sum_1^m k} = \frac{k}{km} = \frac{1}{m}$$

Step 2: plug in the values for p_{ij} and tf_{ij} into the tf'_{ij} formula

$$\begin{aligned}
 tf'_{ij} &= tf_{ij} * (1 + \sum_0^j \frac{p_{ij} \log p_{ij}}{\log m}) \\
 &= k * (1 + \sum_0^j \frac{\frac{1}{m} \log \frac{1}{m}}{\log m}) \\
 &= k * (1 + \frac{-j}{m}) \\
 &= \frac{k(m-j)}{m}
 \end{aligned}$$

$$\mathbf{Ans:} = \frac{k(m-j)}{m}$$

b.) **Ans:** The purpose of this transformation is to get the average occurrence of the i th term found from the first to the j th document.