6(a).When $tf_{ij} > 0$, what are the maximum and minimum values of the corresponding tf'_{ij} values based on the above transformation? Please specify what cases the max and min value achieves.

Given:

- $tf'_{ij} = tf'_{ij} * (1 + \sum_0^j \frac{p_{ij} log p_{ij}}{log m})$ where $p_{ij} = \frac{tf_{ij}}{gf_i}$
- $0 < tf_{ij} <= k$
- $gf_i = \sum_1^m tf_{ij}$
  -above represents the number of times ith term appears in all documents
- $1 < m$
  -above represents max number of documents

Proof:

Min:

-Occurs when $tf_{ij} = 1$ (aka only 1 of ith term found in each document)

Step 1: Assume $tf_{ij} = 1$ and solve for p_{ij}

$p_{ij} = \frac{tf_{ij}}{\sum_1^m tf_{ij}} = \frac{1}{\sum_1^m 1} = \frac{1}{m}$

Step 2: plug in the values for p_{ij} and tf_{ij} into the tf'_{ij} formula

$$\begin{aligned} tf'_{ij} &= tf_{ij} * (1 + \sum_0^j \frac{p_{ij} log p_{ij}}{log m}) \\ &= 1 * (1 + \sum_0^j \frac{\frac{1}{m} log \frac{1}{m}}{log m}) \\ &= 1 + j[\frac{\frac{1}{m} log \frac{1}{m}}{log m}] \\ &= 1 + \frac{-j}{m} \\ &= \frac{m-j}{m} \end{aligned}$$

Ans: $= \frac{m-j}{m}$

Max:

-Occurs when $tf_{ij} = k$ (aka only k(max frequency) of ith term is found in each document)

Step 1: Assume $tf_{ij} = k$ and solve for p_{ij}

$p_{ij} = \frac{tf_{ij}}{\sum_1^m tf_{ij}} = \frac{k}{\sum_1^m k} = \frac{k}{km} = \frac{1}{m}$

Step 2: plug in the values for p_{ij} and tf_{ij} into the tf'_{ij} formula

$$\begin{aligned} tf'_{ij} &= tf_{ij} * (1 + \sum_0^j \frac{p_{ij} log p_{ij}}{log m}) \\ &= k * (1 + \sum_0^j \frac{\frac{1}{m} log \frac{1}{m}}{log m}) \\ &= k * (1 + \frac{-j}{m}) \\ &= \frac{k(m-j)}{m} \end{aligned}$$

Ans: $= \frac{k(m-j)}{m}$

b.)Briefly explain the purpose for this transformation in the context of natural language processing.

Ans: The purpose of this transformation is to get the average occurrence of the ith term found from the first to the jth document.