

# The shore power deployment problem for maritime transportation

Lingxiao Wu, Shuaian Wang\*

Department of Logistics & Maritime Studies, The Hong Kong Polytechnic University, Kowloon, Hong Kong



## ARTICLE INFO

### Keywords:

Container shipping  
Green shipping  
Port operations  
Shore power

## ABSTRACT

In this paper, we study a shore power deployment problem in a container shipping network. The aim of the problem is to develop a subsidy program for a government that achieves the utmost reduction of at-berth emissions from ships in the network. We formulate the problem as a mathematical model that captures the involved relationships among the government, container ports, and shipping lines. The model is hard to solve because it involves a multi-phase process that does not have a closed-form solution. To solve the problem, we develop a tailored labeling algorithm. Extensive numerical experiments are conducted, and the results demonstrate the applicability and efficiency of the solution method for solving practical instances. The results also demonstrate that the solutions delivered by our algorithm to the problem can significantly reduce the at-berth emissions from ships in the shipping network.

## 1. Introduction

Ports have long been the main gateways for global trade and are critical to economies around the world (Qu and Meng, 2012). However, they are major sources of ship pollution, cargo handling equipment emissions, and noise (McArthur and Osland, 2013; Wang et al., 2019). When berthing at ports, ships use their diesel auxiliary engines to generate electricity for hoteling, unloading, and loading activities, and they emit huge amounts of greenhouse gases (GHGs), sulfur dioxide (SO<sub>2</sub>), nitrogen oxides (NO<sub>x</sub>), particulate matter (PM), and other harmful pollutants (Merk, 2014). These emissions can cause adverse impacts on the environment (e.g., climate change, acid rain, water contamination) and contribute to significant health problems for local communities including premature mortality, cardiovascular and respiratory diseases (Sharma, 2006).

One effective measure to reduce at-berth emissions is to provide electricity to the ships from the shore-side national grid while turning off the ships' auxiliary diesel engines at ports, which can significantly reduce the emissions of air pollutants and GHGs. U.S. Environmental Protection Agency (2017) summarized 13 studies on the benefits of shore power (SP) at different ports in America, Europe, and Asia, and found that the reductions in emissions of CO<sub>2</sub> and pollutants (e.g., SO<sub>2</sub>, NO<sub>x</sub>, and PM) from ships at most ports were between 60% and 80%. To date, shore power facilities have been set up at more than 30 ports, mostly in Europe and North America. Many Asian ports are planning to deploy SP to curb their increasingly serious port emissions.

Several issues must be tackled before SP can be adopted. A major issue is the expensive retrofitting process—shore-side electricity supply infrastructure (SPI) is required at ports, and ships should be retrofitted with equipment that enables the connection to SP (SPE). In particular, it typically takes around 2 million US dollars to build up SPI at a port, and retrofitting a container ship with SPE can require 0.3 to 0.5 million US dollars' investment. Another fundamental and inherent issue is the “chicken and egg” dilemma (Winkel et al., 2016): ports do not install SPI until ships are SPE ready, while ships wait for SPI to be set up at ports prior to being retrofitted. This dilemma also exists in the investment in electric vehicle infrastructure (Zhang et al., 2019; Qu et al., 2020).

\* Corresponding author.

E-mail address: [wangshuaian@gmail.com](mailto:wangshuaian@gmail.com) (S. Wang).

Therefore, governmental subsidies play a critical role in promoting SP. In addition, the network effect in SP deployment further complicates the problem: as more ports build SPI, ships with SPE will have more opportunities to use SP and this will reduce the cost for shipping companies (for ships with SPE, using SP electricity when berthing at ports with SPI is cheaper than generating electricity using the auxiliary engines); as a result, more ships will be retrofitted and then even more ports will install SPI (providing SP electricity to ships brings in extra revenues for a port) (Wang et al., 2015). This procedure will be repeated between ports and shipping lines, and considering the network effect is essential for the promotion of SP.

In practice, many governments provide subsidies to shipping companies and ports that are under their administration (including both state-owned and private shipping companies and ports). These subsidies relate to support for national flags, seafarer employment, the competitiveness of maritime clusters, promoting high quality standards and maintaining maritime connectivity (International Transport Forum, 2019). The types of maritime subsidies include direct subsidies, tax expenditures (e.g., tax exemption), and transfer of financial risk to governments (International Transport Forum, 2019). As estimated by International Transport Forum (2019), at least 3 billion EUR per year is spent on maritime subsidies in 36 countries included in the report. For example, in many large ports, government acts as owners or shareholders and governmental investments and subsidies play a key role in their operations and developments. Besides, in China, South Korea, and the U.S., governmental subsidies are provided to ships under the domestic flags or to state-owned shipping companies (International Transport Forum, 2019). Some European countries (including Italy, the United Kingdom, Sweden, and Norway) offered subsidies to shipping companies in order to reduce congestion and greenhouse gas emissions (International Transport Forum, 2019). In particular, the European Union (EU) has provided financial incentives to attract ships to be equipped with SPE. It also offers subsidies to ports to install SPI under the Marco Polo and Trans-European Transport Network programs (European Commission, 2019). In addition, the United Kingdom's recently released Maritime 2050 strategy clearly states that the government is considering granting subsidies and investments to ports and ships to increase the uptake of SP (Department for Transport, 2019).

Container ships are the most polluting ships among all types of ships (Smith et al., 2014). This paper studies a Shore Power Deployment Problem (SPDP) in a container shipping network. The objective of the problem is to develop a subsidy program for a government that achieves the utmost reduction of at-berth emissions in the network. We formulate the problem as a mathematical model that captures the interaction between government and stakeholders (container ports and shipping lines) and the network effect in SP deployment. The model is difficult to solve because it involves a multi-phase process that does not have a closed-form solution. We prove that the problem is NP-hard. To solve the problem, we develop a tailored labeling algorithm that takes advantage of the network effect in the problem. The great effectiveness of the algorithm is demonstrated through a series of numerical experiments.

Literature on container shipping studies can be divided into the stream of port operations (e.g., Kim and Park, 2004; Du et al., 2011; Song et al., 2016) and the stream of shipping operations (e.g., Dong and Song, 2009; Wang and Meng, 2012b; Ng and Lin, 2018). Bierwirth and Meisel (2015) and Meng et al. (2013) have provided excellent reviews of studies in port operations and shipping operations, respectively. Most SP-related studies have focused on the cost-benefit analysis of whether a port should install SPI (the health benefit from the reduction of emissions versus the installation cost of SPI) by assuming a fixed percentage of ships that visit the port will use SP. Ballini and Bozzo (2015) assumed 60% of all cruise ships visiting Copenhagen used SP and calculated that the total capital cost of establishing SPI in Copenhagen would be recovered by the health benefits in 12–13 years. Zis et al. (2016) analyzed the payback period for a ship to be retrofitted with SPE and found that the payback time depended on the price of fuel, the electricity price, and the time spent at ports. Wang et al. (2015) assumed that 40% of ships visiting the Port of Shenzhen (China) came from ports in Europe and North America and were already equipped with SPE, and they evaluated the potential emissions reduction if the port were to install SP infrastructure. Vaishnav et al. (2016) calculated ships and ports in the U.S. that should be switched to SP to maximize the social benefit. They assumed that port operators and ship owners act in a socially optimal manner. To the best of our knowledge, although governmental subsidization is key to the promotion of SP, there are no existing studies that aim at generating an SP-related subsidization plan for a government. In addition, no studies have considered the network effect in SP deployment in a quantitative manner. Our paper is the first study that considers the SPDP in a container shipping network. By solving this problem, we aim to formulate a subsidization plan for a government whose goal is to minimize the at-berth emissions from ships in the container shipping network. The network effect is also considered in the problem to ensure that the delivered result is applicable to real cases.

For better readability, we summarize notations used in this paper in the Table 1.

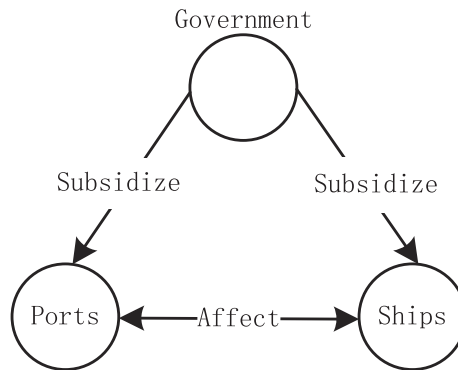
In the following, we formally describe the considered problem in Section 2. We formulate the problem as a mathematical model in Section 3. The complexity of the SPDP is discussed in Section 4. The labeling algorithm for solving the problem is introduced in Section 5. The computational results are reported in Section 6. Finally, we conclude our main findings in Section 7.

## 2. Problem description

Consider a container shipping network. In the network, there are a set of container ports  $P$  that are managed by a government and a set of shipping routes  $R$  (operated by shipping lines under the administration of the government) that call at the ports. The Shore Power Deployment Problem (SPDP) involves three types of players: the government, container port operators (ports), and shipping lines (ships), as shown in Fig. 1. In order to reduce the at-berth emissions from the ships when they are calling at the ports, the government considers to subsidize a set  $P' \subseteq P$  of ports to set up SPI and subsidize ships that sail on a set  $R' \subseteq R$  of routes to be retrofitted with SPE. Given an SP subsidization plan from the government, ports and shipping lines affect each other. In particular, given a set of ports that newly set up SPI in a route, shipping lines will retrofit their ships on this route with SPE if the cost of retrofitting is no larger than the benefits brought by it. Meanwhile, given a set of routes on which ships are newly retrofitted with SPE, ports contained in these routes will build up SPI if the revenue brought by selling SP electricity to ships with SPE outweighs the relevant cost.

**Table 1**  
Notations.

Indices:	
$p$	Index of ports
$r$	Index of routes
$n$	Index of phases in the network effect
Sets:	
$P$ :	Set of ports in the shipping network
$R$ :	Set of routes in the shipping network
$P_r$ :	Set of ports visited in route $r$
$R_p$ :	Set of routes that visit port $p$
Parameters:	
$h_{rp}$ :	The annual fuel cost of the auxiliary engine of ships on route $r$ when at berth of port $p$ , without using SP
$e_{rp}$ :	The annual cost of using shore electricity of ships on route $r$ when at berth of port $p$ if using SP
$u_{rp}$ :	The annual profit for port $p$ for providing shore power to ships on route $r$
$B$ :	Budget of the government for subsidization
$C_p^1$ :	Cost to set up SPI at port $p$
$C_r^2$ :	Cost of retrofitting the ships on route $r$ with SPE
$a$ :	The coefficient that converts the cost to set up SPI at a port into the annualized cost ( $aC_p^1$ equals the annualized cost of setting up SPI at port $p$ )
$b$ :	The coefficient that converts the cost to retrofit a ship with SPE into the annualized cost ( $bC_r^2$ equals the annualized cost of retrofitting the ships on route $r$ with SPE)
$N$ :	Number of phases after which the network effect reaches its equilibrium
Variables:	
$x_p$ :	= 1, if port $p$ is subsidized for building up SPI; = 0, otherwise
$y_r$ :	= 1, if ships on route $r$ are subsidized for retrofitting with SPE; = 0, otherwise
$\hat{P}_n$ :	Set of ports that have set up SPI in any of the phases 0, 1, ..., $n$ of the network effect, where $n = 0, 1, \dots, N$ . In particular, $\hat{P}_0$ represents the initial set of ports that have set up SPI (all are subsidized by the government)
$\hat{R}_n$ :	Set of routes on which the ships are retrofitted with SPE in any of the phases 0, 1, ..., $n$ of the network effect, where $n = 0, 1, \dots, N$ . In particular, $\hat{R}_0$ represents the initial set of routes on which the ships have been equipped with SPE (all are subsidized by the government)
Auxiliary Variables:	
$\alpha_p(\hat{R})$ :	= 1, if port $p$ finds setting up SPI using its own funding is profitable and thus does so; = 0, otherwise, given a set of routes ( $\hat{R}$ ) on which the ships are SPE-ready
$\alpha_p(\hat{R}) = \begin{cases} 1 & \text{if } \sum_{r \in \hat{R}} u_{rp} \geq aC_p^1 \\ 0 & \text{if } \sum_{r \in \hat{R}} u_{rp} < aC_p^1 \end{cases}$	
$\beta_r(\hat{P})$ :	= 1, if the shipping line operating route $r$ finds that retrofitting ships on this route using its own funding reduces its operating cost and thus does so; = 0, otherwise, given a set of ports ( $\hat{P}$ ) that have set up SPI
$\beta_r(\hat{P}) = \begin{cases} 1 & \text{if } \sum_{p \in P_r} h_{rp} \geq bC_r^2 + \sum_{p \in P_r \cap \hat{P}} e_{rp} + \sum_{p \in P_r \setminus \hat{P}} h_{rp} \\ 0 & \text{if } \sum_{p \in P_r} h_{rp} < bC_r^2 + \sum_{p \in P_r \cap \hat{P}} e_{rp} + \sum_{p \in P_r \setminus \hat{P}} h_{rp} \end{cases}$	

**Fig. 1.** Three players in the SPDP.

Therefore, the SPDP is a two-stage optimization problem, in which the government decides its SP subsidization plan in the first stage and in the second stage, the ports and the shipping lines make decisions in reaction to (i) the subsidization plan from the government in the first stage and (ii) the network effect among ports and shipping lines in the second stage. Note that since the government aims at minimizing the at-berth emissions in the long run, when making the subsidization plan, it should also consider the second-stage decisions made by the ports and the shipping lines.

### 2.1. The government

The aim of the government is to reduce at-berth emissions from the ships in the shipping network by minimizing the amount of fuel consumption of the auxiliary engines when the ships are berthing at the ports in the network. To this end, it decides to subsidize a set of ports to set up SPI and ships on a set of routes to be retrofitted with SPE. The budget of the government for subsidizing ports and ships is denoted by  $B$ .

In this study, there is only one government (e.g., China, the U.S., or EU) that provides subsidies to the ports and shipping lines in the shipping network. The ports and the shipping lines considered in the network are all managed by this government. Therefore, the subsidization plan can be generated in a coordinated fashion among all ports and shipping lines.

Our study can be directly applied in the scenarios where the routes connect ports that are all managed by one government. Such scenarios include the deployment of SP on the ships and ports on the shipping routes in the Yangtze River, China, the deployment of SP on the ships and ports on the shipping routes along China's or the U.S.'s coastline, and the deployment of SP on the ships and ports on the shipping routes within EU.

In addition, by carefully setting the problem parameters, this study can also be applied in scenarios where a shipping route visits ports managed by different governments. For instances, in a shipping route connecting China and the U.S., some ports are managed by China and the others are managed by the U.S. In this sense, the network considered in our problem does not necessarily contain all ports that are visited by the routes in practice, i.e., we only consider ports in the routes that are managed by one particular government. Also note that a shipping route may visit multiple ports in one country (e.g., China or the U.S.). We illustrate the method to handle such routes in the SPDP in Section 2.5.

### 2.2. Ports

The shipping network contains a set  $P$  of container ports. For each port  $p \in P$ , let  $R_p$  denote the set of routes that include it. Let  $C_p^1$  where  $p \in P$  be the one-time set-up cost of building up SPI at port  $p$ . Note that if  $p$  is subsidized by the government, then the set-up cost  $C_p^1$  is afforded by the government, otherwise, the cost is paid by the port operators. The annualized cost of setting up shore power is  $aC_p^1$ , where  $a$  is the coefficient that converts the set-up cost  $C_p^1$  to the annualized cost. Note that  $0 < a < 1$ , because the SPI at a port lasts more than one year.

Ports with SPI obtain electricity from the national grid and provide electricity to berthing ships with SPE. Suppose that the SP electricity is provided to all ports with SPI at an identical unit price from the national grid, and that SP electricity is also sold at an identical unit price to ships with SPE from these ports. Then a port makes profits due to the difference between the unit prices of purchasing and selling electricity. Given a route  $r$ , suppose ships sailing on it have been equipped with SPE. Then, the annual profit of the port  $p$  from providing SP electricity to ships on route  $r$  is denoted by  $u_{rp}$  ( $u_{rp} = 0$  if the route does not include port  $p$ ).

Let  $\hat{R}$  be the set of routes on which ships are retrofitted with SPE. Given  $\hat{R}$ , let  $\alpha_p(\hat{R}) \in \{0, 1\}$  be equal to 1 if port  $p$  finds setting up SPI using its own funding is profitable and thus does so and zero otherwise. Then,  $\alpha_p(\hat{R}) = 1$  if  $\sum_{r \in \hat{R}} u_{rp} \geq aC_p^1$  and zero otherwise.

### 2.3. Shipping routes

A set  $R$  of routes is included in the container shipping network. Consider a shipping route  $r \in R$  on which ships visit a set  $P_r \in P$  of different ports  $p_{r,1}, p_{r,2}, \dots, p_{r,|P_r|}$  and then return to  $p_{r,1}$ . The cost of retrofitting the ships on the route with SPE is  $C_r^2$ . The retrofitting cost of ships on a route is paid by the government if the government decides to subsidize these ships. In comparison, the shipping line that operates a route pays the retrofitting cost of ships on the route, if the ships are not subsidized by the government. Note that the cost of retrofitting a ship is mainly decided by the characteristics of the ship itself (e.g., capacity of the ship). In this paper, we assume that the ships deployed on each shipping route are known and fixed. As a result, the cost of retrofitting all ships on a route is also known and fixed. Further, let  $bC_r^2$  denote the annualized retrofitting cost, where  $b$  is the coefficient that converts the set-up cost  $C_r^2$  to the annualized cost. Note that since the SPE on a ship can be used in more than a year, we set  $0 < b < 1$ .

To simplify the analysis, we assume that there is a fixed ratio between (1) the amount of SP electricity a ship with SPE uses when berthing at a port with SPI and (2) the fuel consumption of the same ship's auxiliary engine when it berths at the same port without using SP. It is also assumed that the ratios are identical for all ships visiting all ports in the network. In other words, we assume that the amount of electricity a ship consumes when at a port keeps unchanged no matter whether the electricity is provided by SP or by its auxiliary engine. This is reasonable because the berthing time of a ship at a port will not be affected by the adoption of SP. In addition, we assume that the fuel-to-electricity conversion rate is a constant for auxiliary engines in all ships. This is also a reasonable assumption since most ships use the same fuel (i.e., Marine Gas Oil, MGO) in their auxiliary engines when berthing at ports (Zis et al., 2016).

Supposing no ships on  $r$  are equipped with SPE, then the annual fuel cost of the auxiliary engines of all ships on  $r$  when berthing at port  $p \in P_r$  is  $h_{rp}$ . Alternatively, if all ships on  $r$  are equipped with SPE and port  $p \in P_r$  has set up SPI, the annual cost of all ships on  $r$  using SP electricity when berthing at port  $p$  is  $e_{rp}$ . Suppose that  $e_{rp} < h_{rp}$ ,  $\forall r \in R, \forall p \in P$ . Several aspects regarding the parameter settings here are worth mentioning. First, in this paper, we only consider two cases regarding the ships on a route: all ships on the route are equipped with SPE or none of the ships are equipped with SPE. This is because ships deployed on the same route are typically of the same type (Wang and Meng, 2012a; Ng, 2017). Hence, from the perspective of shipping lines, if it is preferable (or cost-effective due to the lower costs of using SP at ports) to retrofit one ship on a particular route, retrofitting other ships on the route will also be preferable. Therefore, treating all ships on a route as a whole will not lead to sub-optimal retrofitting decisions for

shipping lines. Second, given a fixed fleet of ships on a shipping route  $r$  and the fixed amount of electricity each ship on  $r$  consumes at a port  $p$ ,  $e_{rp}$  is decided by the SP electricity price, and  $h_{rp}$  is decided by the fuel price. In practice,  $e_{rp} < h_{rp}$  holds in many ports, including the ports in many European countries (Transport Malta, 2014; Kanellakis, 2016; Gutierrez Saenz, 2019), ports in the U.S. (Vaishnav et al., 2016), and Port of Shenzhen in China (Peng, 2016). In addition, European Commission (2017) is also considering offering lower taxation rates on shore-supplied power.

Let  $\hat{P}$  be the set of ports with SPI. Given  $\hat{P}$ , let  $\beta_r(\hat{P}) \in \{0, 1\}$  be equal to 1 if the shipping line operating route  $r$  finds that retrofitting ships on this route using its own funding reduces its operating cost and thus does so and zero otherwise. Then,  $\beta_r(\hat{P}) = 1$  if  $\sum_{p \in P_r} h_{rp} \geq bC_r^2 + \sum_{p \in P_r \cap \hat{P}} e_{rp} + \sum_{p \in P_r \setminus \hat{P}} h_{rp}$  and zero otherwise.

#### 2.4. Network effect and its long-term equilibrium

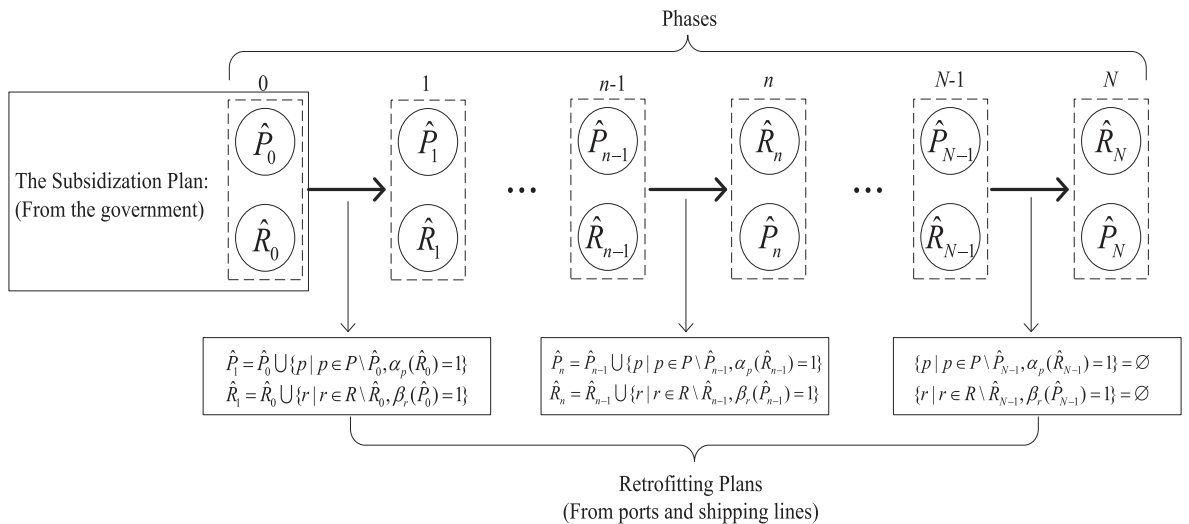
Given a subsidization plan from the government, let  $\hat{P}_0$  and  $\hat{R}_0$  be the initial set of ports deployed with SPI and the initial set of routes on which the ships are retrofitted with SPE after the subsidization. The network effect can be described as follows. To begin with, because ships sailing on routes in  $\hat{R}_0$  are equipped with SPE, some ports may find it profitable to set up SPI and providing electricity to ships that are SPE-ready (we only need to consider ports  $p \in P \setminus \hat{P}_0$ , since ports  $p \in \hat{P}_0$  have already set up SPI and will not take any action). In particular, for each  $p \in P \setminus \hat{P}_0$  if  $\alpha_p(\hat{R}_0) = 1$ , then port  $p$  will set up SPI (through its own investment). Define  $\hat{P}_1 = \hat{P}_0 \cup \{p | p \in P \setminus \hat{P}_0, \alpha_p(\hat{R}_0) = 1\}$ . Similarly, consider a route  $r \in R \setminus \hat{R}_0$ . The shipping line that operates the route may find that it is favorable to invest on retrofitting ships on  $r$  with SPE (i.e.,  $\beta_r(\hat{P}_1) = 1$ ). Define  $\hat{R}_1 = \hat{R}_0 \cup \{r | r \in R \setminus \hat{R}_0, \beta_r(\hat{P}_1) = 1\}$ . To describe the evolvement of the network effect, we introduce the following definitions for *phases* and *the long-term equilibrium* in the network effect.

**Definition 1 (Phases).** Given the initial SP deployment (i.e., the initial set of ports deployed with SPI and the initial set of routes on which the ships are retrofitted with SPE), then (1) all ports in the network without SPI will make their decisions regarding SPI establishment based on the initial set of routes on which the ships are retrofitted with SPE and (2) the operators (i.e., shipping lines) of all routes on which ships are not retrofitted with SPE will decide whether or not to retrofit their ships with SPE based on the initial set of ports with SPI. Given such decisions made by the ports and the shipping lines, a *phase* in the network effect refers to a status in which (1) the SPI has been established on all the ports that decide to set up SPI and (2) the SPE has been set up on the ships that are decided to be retrofitted. Note that the SP deployment in the current phase becomes the initial SP deployment for the next phase.

**Definition 2 (The long-term equilibrium).** The long-term equilibrium of SP deployment in the network effect is a state in which no more ports have incentives to invest in setting up SPI and no more routes on which the shipping lines will be better off by retrofitting their ships with SPE. Note that given a subsidization plan from the government, its utmost emission reduction in the network is achieved at the equilibrium.

The evolvement process of the network effect is demonstrated in Fig. 2. In particular, we define  $\hat{P}_0$  and  $\hat{R}_0$  as the result of the initial phase (Phase 0) which are decided by the government's subsidization plan.  $\hat{P}_1$  and  $\hat{R}_1$  as the result of the first phase (Phase 1) of the network effect. Now consider the following two cases. First, if  $\hat{P}_1 = \hat{P}_0$  and  $\hat{R}_1 = \hat{R}_0$ , then no ships on routes  $r \in R \setminus \hat{R}_1$  will be retrofitted and no ports  $p \in P \setminus \hat{P}_1$  will invest on SPI. Otherwise, we can generate  $\hat{P}_2$  and  $\hat{R}_2$ , which are the result of the second phase (Phase 2) of the network effect. In particular,  $\hat{P}_2 = \hat{P}_1 \cup \{p | p \in P \setminus \hat{P}_1, \alpha_p(\hat{R}_1) = 1\}$ , and  $\hat{R}_2 = \hat{R}_1 \cup \{r | r \in R \setminus \hat{R}_1, \beta_r(\hat{P}_1) = 1\}$ .

This procedure repeats until the  $N$ -th ( $N \geq 1$ ) phase (or Phase  $N$ ) of the network effect such that:



$$\hat{P}_N = \hat{P}_{N-1}, \quad (1)$$

$$\hat{R}_N = \hat{R}_{N-1}, \quad (2)$$

where  $\hat{P}_N$  and  $\hat{R}_N$  are the sets of ports with SPI and routes on which ships are retrofitted with SPE at the  $N$ -th phase. We refer to the state in which these conditions hold as the long-term equilibrium of SP deployment in the network effect, and  $\hat{P}_N$  and  $\hat{R}_N$  are the sets of ports with SPI and routes on which ships are retrofitted with SPE in the equilibrium, respectively. Note that in the long-term equilibrium, SP devices are installed in the ports and on the ships if (1) they are subsidized to set up SP devices by the government or (2) they find that setting up SP devices is profitable (or cost-effective) due to the network effect.

**Lemma 1.**  $\alpha_p(\hat{R}') \leq \alpha_p(\hat{R}'')$  if  $\hat{R}' \subset \hat{R}''$ .

**Proof.** Considering  $\hat{R}' \subset \hat{R}''$ , we have  $\sum_{r \in \hat{R}'} u_{rp} \leq \sum_{r \in \hat{R}''} u_{rp}$ . It can be readily seen that  $\alpha_p(\hat{R}') \leq \alpha_p(\hat{R}'')$ .  $\square$

**Lemma 2.**  $\beta_r(\hat{P}') \leq \beta_r(\hat{P}'')$  if  $\hat{P}' \subset \hat{P}''$ .

**Proof.** Let  $D(\hat{P}) = \sum_{p \in P_r \cap \hat{P}} e_{rp} + \sum_{p \in P_r \setminus \hat{P}} h_{rp}$ . It is sufficient to prove that  $D(\hat{P}') \geq D(\hat{P}'')$ . To see this, consider  $D(\hat{P}') - D(\hat{P}'') = \sum_{p \in P_r \cap (\hat{P}' \setminus \hat{P}'')} e_{rp} - \sum_{p \in P_r \cap (\hat{P}'' \setminus \hat{P}')} e_{rp} + \sum_{p \in P_r \cap (\hat{P}' \setminus \hat{P}'')} h_{rp} - \sum_{p \in P_r \cap (\hat{P}'' \setminus \hat{P}')} h_{rp}$ . Since  $\hat{P}' \subset \hat{P}''$ , we have  $D(\hat{P}') - D(\hat{P}'') = \sum_{p \in P_r \cap (\hat{P}' \setminus \hat{P}'')} (h_{rp} - e_{rp}) \geq 0$ .  $\square$

**Proposition 1.** Given any subsidization plan from the government, the equilibrium of the network can be obtained after at most  $\min\{2|P|, 2|R|\}$  phases.

**Proof.** Suppose the equilibrium of the network effect with  $P \neq \emptyset$  and  $R \neq \emptyset$  is reached after  $N$  phases. Let  $\hat{P}_0$  and  $\hat{R}_0$  denote the sets of ports with SPI and routes on which the ships are retrofitted, respectively, at 0-th (initial) phase. If at the first phase, we have  $\hat{P}_1 = \hat{P}_0$  and  $\hat{R}_1 = \hat{R}_0$ , then the network effect has achieved its equilibrium (i.e.,  $N = 1$ ). Otherwise, suppose that at stage  $n \geq 2$ , we have  $\hat{P}_n = \hat{P}_{n-2}$ . It is easy to infer that  $\hat{P}_n = \hat{P}_{n-1} = \hat{P}_{n-2}$ . Then, from Lemma 2, we have  $\hat{R}_{n-1} = \hat{R}_n$ . Therefore, we have  $\hat{P}_n = \hat{P}_{n-1}$  and  $\hat{R}_n = \hat{R}_{n-1}$ , which indicate  $N = n$ . Similarly, suppose that at stage  $\tilde{n} \geq 2$ , we have  $\hat{R}_{\tilde{n}} = \hat{R}_{\tilde{n}-2}$ . Then by following the same logic, we have  $\hat{P}_{\tilde{n}} = \hat{P}_{\tilde{n}-1}$  and  $\hat{R}_{\tilde{n}} = \hat{R}_{\tilde{n}-1}$ , which indicate  $N = \tilde{n}$ .

Consider the following cases:

- (I) Suppose we have  $\hat{P}_1 = \hat{P}_0$  and  $\hat{R}_1 = \hat{R}_0$ . Then,  $N = 1$ , and  $N \leq \min\{2|P|, 2|R|\}$ .
- (II) Suppose we have  $\hat{P}_2 = \hat{P}_1$  and  $\hat{R}_2 = \hat{R}_1$ . Then,  $N = 2$ , and  $N \leq \min\{2|P|, 2|R|\}$ .
- (III) Let  $k$  be any positive integer such that  $k \geq 2$ , and suppose  $N = 2k$  or  $2k - 1$ . Then, it is necessary to have (1)  $\hat{P}_3 \supset \hat{P}_1$ ,  $\hat{P}_5 \supset \hat{P}_3, \dots$ ,  $\hat{P}_{2k-1} \supset \hat{P}_{2k-3}$ , and (2)  $\hat{R}_3 \supset \hat{R}_1$ ,  $\hat{R}_5 \supset \hat{R}_3, \dots$ ,  $\hat{R}_{2k-1} \supset \hat{R}_{2k-3}$ , which are equivalent to (a)  $|\hat{P}_3| - |\hat{P}_1| \geq 1$ ,  $|\hat{P}_5| - |\hat{P}_3| \geq 1, \dots$ ,  $|\hat{P}_{2k-1}| - |\hat{P}_{2k-3}| \geq 1$ , and (b)  $|\hat{R}_3| - |\hat{R}_1| \geq 1$ ,  $|\hat{R}_5| - |\hat{R}_3| \geq 1, \dots$ ,  $|\hat{R}_{2k-1}| - |\hat{R}_{2k-3}| \geq 1$ . If  $|\hat{P}_1| = 0$  or  $|\hat{R}_1| = 0$ , it is easy to infer that  $N = 1$ , therefore in this case, we have  $|\hat{P}_1| \geq 1$  and  $|\hat{R}_1| \geq 1$ . Following this, we have  $|\hat{P}_3| \geq 2$ ,  $|\hat{R}_3| \geq 2, \dots$ ,  $|\hat{P}_{2k-1}| \geq k$ , and  $|\hat{R}_{2k-1}| \geq k$ . Considering  $|\hat{P}_{2k-1}| \leq |P|$  and  $|\hat{R}_{2k-1}| \leq |R|$ , we have  $k \leq \min\{|P|, |R|\}$ . Therefore,  $N \leq 2k \leq \min\{2|P|, 2|R|\}$ .

Summarizing the above results gives us  $N \leq \min\{2|P|, 2|R|\}$ .  $\square$

## 2.5. Extensions

In this section, we show how to handle routes that visit ports managed by multiple governments in the SPDP. Suppose a government is considering subsidizing a set of ports  $P$  and ships on a set of shipping routes  $R$  to be retrofitted with SP devices. We refer to this government as the *target government*. Now consider a route  $r \in R$  that visits a set of ports denoted by  $P_r^+$ . Assume that  $P_r^+$  is composed of ports managed by different governments (including this target government). Let  $\hat{P}_r^+$  denote the set of ports in  $P_r^+$  that have deployed SPI.

Our aim is to construct an artificial route denoted by  $r'$  to replace  $r$  in the SPDP. To this end, we first partition  $P_r^+$  into two subsets:  $P_{r1}^+$  and  $P_{r2}^+$ . Particularly,  $P_{r1}^+ = P_r^+ \cap P$  and  $P_{r2}^+ = P_r^+ \setminus P$ . The parameters associated with  $r'$  are set as follows. First, let  $P_{r'} = P_{r1}^+$ . Second, let  $e_{r'p} = e_{rp}$ ,  $h_{r'p} = h_{rp}$ , and  $u_{r'p} = u_{rp}$  for each  $p \in P_{r'}$  and  $C_{r'}^2 = C_r^2$ . Finally, given the set of ports in  $P$  that have deployed SPI ( $\hat{P}$ ), we redefine the auxiliary variable  $\beta_{r'}(\hat{P})$  associated with  $r'$  to be

$$\beta_{r'}(\hat{P}) = \begin{cases} 1 & \text{if } \sum_{p \in P_{r'}^+} h_{rp} \geq bC_{r'}^2 + \sum_{p \in P_{r'}^+ \cap (\hat{P} \cup \hat{P}_{r'}^+)} e_{rp} + \sum_{p \in P_{r'}^+ \setminus (\hat{P} \cup \hat{P}_{r'}^+)} h_{rp} \\ 0 & \text{if } \sum_{p \in P_{r'}^+} h_{rp} < bC_{r'}^2 + \sum_{p \in P_{r'}^+ \cap (\hat{P} \cup \hat{P}_{r'}^+)} e_{rp} + \sum_{p \in P_{r'}^+ \setminus (\hat{P} \cup \hat{P}_{r'}^+)} h_{rp}. \end{cases}$$

The parameters of  $r'$  satisfy three properties that allow the correct incorporation of  $r$  into the SPDP (which is replaced by  $r'$  in the problem). First, the target government manages all ports on route  $r'$ . Second, the cost to retrofit ships on  $r'$  and once they are retrofitted, the revenue generated to each port  $p \in P_{r'}^+ \cap P$  remain unchanged when compared with the “real” scenario. Third, the



new auxiliary variable  $\beta_r(\hat{P})$  ensures that the ships on  $r'$  will be retrofitted by a shipping line (with or without governmental subsidies) if and only if the shipping line that operates the “real” route  $r$  finds that retrofitting ships on route  $r$  using its own funding reduces its operating cost.

### 3. Model formulation

In this section, we formulate the SPDP as a mathematical model. The model is difficult to solve. This is because, in this model, we have to depict the network effect in SP deployment, which is, in essence, a multi-phase process without a closed-form solution.

In the problem, we assume that there are no ports that have SPI and no routes on which the ships are retrofitted with SPE, before the government makes its subsidization decision. Note that our model and the solution method proposed in the following section can also be used to solve the SPDP in which this assumption does not hold, after small adaptations.

Let  $x_p$  ( $p \in P$ ) be the decision variable which is equal to 1, if port  $p$  is subsidized by the government. Let  $y_r$  ( $r \in R$ ) be the decision variable which is equal to 1, if ships on route  $r$  are subsidized by the government. Let  $N = \min\{2|P|, 2|R|\}$ . According to Proposition 1, the equilibrium of the network effect can be obtained after at most  $N$  phases (i.e., at  $N$ -th phase). Let  $\hat{P}_n$ ,  $n = 0, 1, \dots, N$  be the decision variable that represents the set of ports that set up SPI in any of the phases  $0, 1, \dots, n$  of the network effect, and let  $\hat{R}_n$ ,  $n = 0, 1, \dots, N$  be the decision variable that represents the set of routes on which the ships are retrofitted with SPE in any of the phases  $0, 1, \dots, n$  of the network effect. The SPDP can be formulated as the following model.

$$(M1) \max Z = \sum_{r \in \hat{R}_N} \sum_{p \in P_r \cap \hat{P}_N} e_{rp}, \quad (3)$$

subject to:

$$\sum_{p \in P} C_p^1 x_p + \sum_{r \in R} C_r^2 y_r \leq B, \quad (4)$$

$$x_p \in \{0, 1\} \quad \forall p \in P, \quad (5)$$

$$y_r \in \{0, 1\} \quad \forall r \in R, \quad (6)$$

$$\hat{P}_0 = \{p | x_p = 1, p \in P\}, \quad (7)$$

$$\hat{R}_0 = \{r | y_r = 1, r \in R\}, \quad (8)$$

$$\hat{P}_{n+1} = \hat{P}_n \cup \{p | \alpha_p(\hat{R}_n) = 1, p \in P \setminus \hat{P}_n\}, \quad \forall n \in \{0, 1, \dots, N-1\}, \quad (9)$$

$$\hat{R}_{n+1} = \hat{R}_n \cup \{r | \beta_r(\hat{P}_n) = 1, r \in R \setminus \hat{R}_n\}, \quad \forall n \in \{0, 1, \dots, N-1\}, \quad (10)$$

$$\hat{P}_N = \hat{P}_{N-1}, \quad (11)$$

$$\hat{R}_N = \hat{R}_{N-1}. \quad (12)$$

The objective function (3) maximizes the total cost of ships for using SP electricity in the shipping network in a year. As described in Section 2, since the SP electricity is provided to all ships at an identical unit price at all ports, maximizing the cost of using SP electricity is equivalent to maximizing the usage amount of SP electricity. Further, there is a fixed ratio between the usage amount of SP electricity and the amount of fuel consumption of the auxiliary engines for all ship berthing at all ports in the network. Therefore, maximizing the usage amount of SP electricity is equivalent to minimizing the at-berth fuel consumption from the ships. Constraint (4) ensures the total expenditure of the subsidization does not exceed the budget. Constraints (5) and (6) require the decision variables  $x_p$ 's and  $y_r$ 's to be binary. Constraints (7)–(12) formulate the network effect of SP deployment. In particular, Constraints (7) and (8) calculate the initial set of ports with SPI and the initial set of routes on which the ships are retrofitted with SPE, respectively. The relationship between two consecutive phases is depicted in Constraints (9) and (10). Finally, the equilibrium result of the network effect is given in Constraints (11) and (12).

### 4. Complexity of the problem

In this section, we show that the SPDP is NP-hard. To do so, we show the decision version of the SPDP is NP-hard. That is, given a set of ports and a set of shipping routes and all parameters  $B$ ,  $u_{rp}$ ,  $h_{rp}$ ,  $e_{rp}$ ,  $C_p^1$ ,  $C_r^2$ ,  $a$ , and  $b$ , it cannot be determined in polynomial time whether the objective value  $Z$  of the problem is no smaller than a given constant  $\Gamma$  unless  $P = NP$ .

We prove the NP-hardness of the SPDP by reducing a well-known NP-hard problem—the Knapsack Problem—to a decision version of the SPDP.

**Theorem 1.** *The SPDP is NP-hard.*

**Proof.** We transform the Knapsack Problem to the decision version of the SPDP. The Knapsack Problem can be stated as follows. There is a set  $I$  of given items to be packed in a knapsack of capacity  $K$ . Each item  $p$  has a profit  $f_p$  and a weight  $w_p$ . The problem asks

whether there exists a packing method such that a subset of items whose total weight does not exceed  $K$  and whose total profit is no less than a constant  $F$  are packed in the knapsack.

Given an arbitrary instance of the Knapsack Problem, we construct a corresponding instance of the SPDP. In the instance, there is only one route  $r$  ( $|R| = 1$ ) that contains all ports in  $P$  (i.e.,  $P_r = P$ ). Specifically, we set other parameters in the problem as follows.

$$B = K, \quad (13)$$

$$P = I, \quad (14)$$

$$C_p^1 = w_p, \quad \forall p \in P, \quad (15)$$

$$C_r^2 = 0, \quad (16)$$

$$e_{rp} = f_p, \quad \forall p \in P, \quad (17)$$

$$aC_p^1 > u_{rp}, \quad \forall p \in P, \quad (18)$$

$$\sum_{p \in P_r} h_{rp} < bC_r^2 + \sum_{p \in P_r} e_{rp}, \quad (19)$$

$$\Gamma = F. \quad (20)$$

Clearly, this transformation can be completed in polynomial time. We will show that there exists a feasible solution to the constructed instance of SPDP if and only if the answer to the Knapsack Problem is “yes”.

Suppose the answer to the Knapsack Problem is “yes”. Let  $I^*$  denote the items selected to be packed in the knapsack. Clearly, we have (i)  $\sum_{p \in I^*} w_p \leq K$ , and (ii)  $\sum_{p \in I^*} f_p \geq F$ . Then consider the following solution ( $\$$ ) to the constructed instance of the SPDP. In  $\$,$  the government subsidizes ships on route  $r$ . Then, corresponding to each  $p \in I^*$ , the government subsidizes port  $p$  to set up SPI. The feasibility of  $\$$  to the SPDP instance can be verified as follows. First, given Eq. (16), the total cost of this subsidization plan equals  $\sum_{p \in I^*} C_p^1 = \sum_{p \in I^*} w_p \leq K$ . Considering  $B = K$ , the total cost does not exceed the budget. Second, let  $\hat{P}_0 = I^*$  and  $\hat{R}_0 = R$  be the initial set of ports with SPI and routes on which the ships are retrofitted with SPE. Considering the network effect among ports and routes, suppose that the equilibrium of the network effect is obtained after  $N$  phases. Let  $\hat{P}_N$  and  $\hat{R}_N$  be the set of ports with SPI and the set of routes on which the ships are retrofitted in the equilibrium. We have  $I^* \subseteq \hat{P}_N$  and  $R = \hat{R}_N$ . Further,  $Z = \sum_{r \in \hat{R}_N} \sum_{p \in P_r \cap \hat{P}_N} e_{rp}$ . Hence, we have  $Z \geq \sum_{p \in P_r \cap I^*} e_{rp}$ . Since  $P_r = P$  and  $e_{rp} = f_p$ , we have  $Z \geq \sum_{p \in I^*} f_p = F$ . Therefore,  $\$$  is feasible to the constructed instance of the SPDP.

Conversely, suppose that there exists a feasible solution to the constructed instance of the SPDP such that  $Z \geq \Gamma$ . Let  $P^*$  denote the set of ports that are subsidized by the government to set up SPI. Since  $C_r^2 = 0$ , we have  $\sum_{p \in P^*} C_p^1 \leq B$ , which is equivalent to:

$$\sum_{p \in P^*} w_p \leq K. \quad (21)$$

Let  $\hat{P}_0 = P^*$  and  $\hat{R}_0$  be the set of ports subsidized by the government to set up SPI and the set of routes on which the ships are subsidized to be retrofitted with SPE, respectively. Suppose that the equilibrium of the network effect is obtained after  $N$  phases. Let  $\hat{P}_N$  and  $\hat{R}_N$  be the set of ports with SPI and the set of routes on which the ships are retrofitted in the equilibrium. Considering  $Z \geq \Gamma$ , we have  $\sum_{r \in \hat{R}_N} \sum_{p \in \hat{P}_N \cap P_r} e_{rp} \geq F$ . Supposing  $F > 0$  (the case such that  $F \leq 0$  is trivial), it is easy to infer that  $\hat{R}_N = R$ . Because  $P_r = P$ , and  $f_p = e_{rp}$ , we have:

$$\sum_{p \in \hat{P}_N} f_p \geq F. \quad (22)$$

Further, given (18) and (19), it is easy to infer that  $\hat{R}_N = \hat{R}_0 = R$  and that  $\hat{P}_N = \hat{P}_0 = P^*$ , which imply that (22) is equivalent to:

$$\sum_{p \in P^*} f_p \geq F. \quad (23)$$

Therefore, given (21) and (23), we can construct a feasible solution to the Knapsack Problem by packing items  $p \in P^*$  into the knapsack. This completes the proof.  $\square$

**Remark 1.** In the proof of Theorem 1, the constructed instance of the SPDP has only one shipping route and only the route and the ports that are included in the subsidization plan will set up SPI. Therefore, the SPDP is NP-hard even if there is only one shipping route and the network effect between the ports and routes is not considered.

## 5. Solution method

In this section, we propose the solution method for the SPDP. We solve the problem by using a labeling algorithm, in which all feasible subsidization plans from the government are considered implicitly and the optimal deployment plan is generated dynamically. For the ease of exposition, we introduce the following notations. We define  $\Psi := P \cup R$ , and define  $\bar{C}_i = \begin{cases} C_i^1, & \text{if } i \in P, \\ C_i^2, & \text{if } i \in R, \end{cases}$  for each



$i \in \Psi$ . In what follows, we first introduce the method to derive the long-term equilibrium of the network effect given a set of ports with SPI and a set of routes on which the ships are retrofitted with SPE in Section 5.1. The procedures of the labeling algorithm are presented in Section 5.2. We propose several dominance rules for the algorithm in Section 5.3.

### 5.1. Deriving the long-term equilibrium

Let  $\hat{P}_0$  and  $\hat{R}_0$  denote the initial set of ports with SPI and the initial set of routes on which the ships are retrofitted, respectively. For example, in a subsidization plan,  $\hat{P}_0$  is the set of ports that are subsidized to set up SPI and  $\hat{R}_0$  is the set of routes on which the ships are retrofitted with SPE. Note that  $\sum_{p \in \hat{P}_0} C_p^1 + \sum_{r \in \hat{R}_0} C_r^2 \leq B$ , if  $\hat{P}_0$  and  $\hat{R}_0$  are the set of ports subsidized by the government and the set of routes on which ships are subsidized by the government, respectively. Also, note that subsidizations from the government are only provided to the ports and ships in the initial phase (Phase 0) of the entire network effect. Let  $\tilde{\Omega} = \hat{P}_0 \cup \hat{R}_0$ . Then, procedure  $F(\tilde{\Omega})$  which is shown in Algorithm 1 finds the long-term equilibrium SP deployment denoted by  $\hat{\Omega}$ . Here,  $\hat{\Omega} = \hat{P} \cup \hat{R}$ , where  $\hat{P}$  is the set of ports with SPI and  $\hat{R}$  is the set of routes on which the ships are retrofitted with SPE in the long-term equilibrium.

**Algorithm 1.** The long-term SP deployment equilibrium calculation procedure ( $F(\tilde{\Omega})$ ).

```

Input:  $\tilde{\Omega}$ ;
Output:  $\hat{\Omega}$ ;
1:  $\hat{P} = P \cap \tilde{\Omega}$ ,  $\hat{R} = R \cap \tilde{\Omega}$ ;
2: while True do
3:    $\hat{P}' = \hat{P}$ ,  $\hat{R}' = \hat{R}$ ;
4:   for  $r \in R \setminus \hat{R}$  do
5:     if  $\beta_r(\hat{P}) = 1$  then
6:        $\hat{R} = \hat{R} \cup \{r\}$ ;
7:     end if
8:   end for
9:   for  $p \in P \setminus \hat{P}$  do
10:    if  $\alpha_p(\hat{R}) = 1$  then
11:       $\hat{P} = \hat{P} \cup \{p\}$ ;
12:    end if
13:  end for
14:  if  $\hat{R} = \hat{R}'$  &  $\hat{P} = \hat{P}'$  then
15:     $\hat{\Omega} = \hat{P} \cup \hat{R}$ ;
16:    Return.
17:  end if
18: end while

```

### 5.2. Procedures of the labeling algorithm

To begin with, to define a label, we introduce some notations related to a partial subsidization plan. Given a partial deployment plan, let  $\hat{P}_0$  and  $\hat{R}_0$  respectively denote the set of ports that are subsidized and the set of routes on which the ships are subsidized in the plan, and let  $\Psi := \hat{P}_0 \cup \hat{R}_0$ . In addition, let  $\bar{B}$  denote the remaining budget, i.e.,  $\bar{B} = B - \sum_{i \in \Psi} \bar{C}_i$ . Further, let  $\hat{P}$  and  $\hat{R}$  denote the set of ports with SPI and the set of routes on which the ships are retrofitted in the long-term equilibrium, respectively. It is easy to infer that  $\hat{P}_0 \subseteq \hat{P}$ , and  $\hat{R}_0 \subseteq \hat{R}$ . Finally, define  $\hat{\Omega} = \hat{P} \cup \hat{R}$ . In the algorithm, a label  $L = (\Psi, \bar{B}, \hat{\Omega})$  is associated with a partial deployment plan such that (1) the ports and routes in  $\Psi$  are subsidized to deploy SPI or the ships on which are retrofitted, (2) the remaining budget is  $\bar{B}$ , and (3) the set  $\hat{\Omega}$  of ports and routes set up SPI or the ships on which are retrofitted in the long-term equilibrium.

The algorithm starts from an initial label  $L_0 = (\emptyset, B, \emptyset)$ . The extension of a label  $L = (\Psi, \bar{B}, \hat{\Omega})$  is as follows. First, we define a candidate pool denoted by  $\Phi$  for extending  $L$  as  $\Phi = \{i | \bar{C}_i \leq \bar{B}, i \in \Psi \setminus \hat{\Omega}\}$ . Then, for each  $i \in \Phi$ , we extend  $L$  to a new label  $L' = (\Psi', \bar{B}', \hat{\Omega}')$ , where  $\Psi' = \Psi \cup \{i\}$ ,  $\bar{B}' = \bar{B} - \bar{C}_i$ , and  $\hat{\Omega}' = F(\hat{\Omega} \cup \{i\})$ .

A label  $L = (\Psi, \bar{B}, \hat{\Omega})$  is terminated if its candidate pool  $\Phi = \emptyset$ . For a terminated label, we calculate  $Z(L)$  which is the usage amount of SP electricity generated by the subsidization plan ( $\Psi$ ) using the following equation:

$$Z(L) = \sum_{r \in \hat{R}} \sum_{p \in \hat{P} \cap P_r} e_{rp}, \quad (24)$$

where  $\hat{R} = R \cap \hat{\Omega}$ , and  $\hat{P} = P \cap \hat{\Omega}$ .

Let  $\Psi^*$  denote the incumbent optimal subsidization plan found by the algorithm and let  $Z^*$  be the usage amount of SP electricity generated by  $\Psi^*$ . Then, if  $Z(L) > Z^*$ , we update  $\Psi^* = \Psi$ , and  $Z^* = Z(L)$ .

### 5.3. Dominance rules

The performance of a labeling algorithm heavily relies on the efficiency of the dominance rules, which allow one to discard a significant number of labels. For the labeling algorithm, we propose the following dominance rules.

**Proposition 2.** Label  $L_1 = (\bar{\Psi}_1, \bar{B}_1, \hat{\Omega}_1)$  dominates Label  $L_2 = (\bar{\Psi}_2, \bar{B}_2, \hat{\Omega}_2)$  if (1)  $\hat{\Omega}_1 \supseteq \hat{\Omega}_2$ , and (2)  $\bar{B}_1 \geq \bar{B}_2$ .

**Proof.** Considering (1) and (2), it is easy to infer that for any extension  $L'_2 = (\bar{\Psi}'_2, \bar{B}'_2, \hat{\Omega}'_2)$  from  $L_2$ , there exists an extension of  $L_1$ , denoted by  $L'_1 = (\bar{\Psi}'_1, \bar{B}'_1, \hat{\Omega}'_1)$  such that  $\hat{\Omega}'_1 \supseteq \hat{\Omega}'_2$ . Therefore, we have  $Z(L'_1) \geq Z(L'_2)$ . This indicates that for any subsidization plan  $\bar{\Psi}'_2$  generated from extensions of  $L_2$ , there exist some subsidization plans generated from extensions of  $L_1$  that are no worse than  $\bar{\Psi}'_2$ . This completes the proof.  $\square$

**Proposition 3.** Label  $L_1 = (\bar{\Psi}_1, \bar{B}_1, \hat{\Omega}_1)$  dominates Label  $L_2 = (\bar{\Psi}_2, \bar{B}_2, \hat{\Omega}_2)$  if (1)  $\hat{\Omega}_1 \supseteq \hat{\Omega}_2$  and (2)  $\Phi_2 \setminus \hat{\Omega}_1 = \emptyset$ , where  $\Phi_2$  is the candidate pool for extending  $L_2$ .

**Proof.** Given (1) and (2), we have that for any extension  $L'_2 = (\bar{\Psi}'_2, \bar{B}'_2, \hat{\Omega}'_2)$  from  $L_2$ ,  $\hat{\Omega}'_2 \subseteq \hat{\Omega}_1$  (i.e.,  $\bar{\Psi}_2$  cannot be extended to include any  $i \notin \hat{\Omega}_1$ ). It follows that the  $Z(L_1) \geq Z(L'_2)$ . Therefore, subsidization plan  $\bar{\Psi}_1$  is no worse than the subsidization plan generated from any extension from  $L_2$ .  $\square$

**Proposition 4.** Label  $L_1 = (\bar{\Psi}_1, \bar{B}_1, \hat{\Omega}_1)$  dominates Label  $L_2 = (\bar{\Psi}_2, \bar{B}_2, \hat{\Omega}_2)$  if (1)  $\hat{\Omega}_1 \supseteq \hat{\Omega}_2$ , and (2)  $\bar{B}_1 \geq \max_{i \in \Phi_2 \setminus \hat{\Omega}_1} \bar{C}_i$  hold, and (4) or (5) holds:

- (4)  $|\Phi_2 \setminus \hat{\Omega}_1| \geq 2$ , and  $\bar{B}_2 < \min_{i \in \Phi_2 \setminus \hat{\Omega}_1} \bar{C}_i + \min[2]_{i \in \Phi_2 \setminus \hat{\Omega}_1} \bar{C}_i$ ;  
 (5)  $|\Phi_2 \setminus \hat{\Omega}_1| = 1$ .

Here,  $\Phi_2$  is the candidate pool for extending  $L_2$ , and  $\min[2]_{i \in \Phi_2 \setminus \hat{\Omega}_1} \bar{C}_i$  returns the second smallest  $\bar{C}_i$  from  $\Phi_2 \setminus \hat{\Omega}_1$ .

**Proof.** Suppose  $L'_2 = (\bar{\Psi}'_2, \bar{B}'_2, \hat{\Omega}'_2)$  is extended from  $L_2$ . Since (4) or (5) holds, for any  $L'_2$ , we have  $|\bar{\Psi}'_2 \setminus \hat{\Omega}_1| \leq 1$ .

First consider the case  $|\bar{\Psi}'_2 \setminus \hat{\Omega}_1| < 1$ , that is  $\bar{\Psi}'_2 \setminus \hat{\Omega}_1 = \emptyset$ . Then according to Proposition 3,  $L'_2$  is dominated by  $L_1$ .

Then, consider the case  $|\bar{\Psi}'_2 \setminus \hat{\Omega}_1| = 1$ . Let  $\{I\} = \bar{\Psi}'_2 \setminus \hat{\Omega}_1$ . Considering (2), we have  $\bar{C}_I \leq \bar{B}_1$ . This indicates that there exists a label  $L'_1 = (\bar{\Psi}'_1, \bar{B}'_1, \hat{\Omega}'_1)$  that is extended from  $L_1$  such that  $\bar{\Psi}'_1 = \bar{\Psi}_1 \cup \{I\}$ . From (1), it is easy to infer that  $\hat{\Omega}'_1 \supseteq \hat{\Omega}'_2$ . Therefore, for any subsidization plan  $\bar{\Psi}'_2$  generated from extensions of  $L_2$ , there exist some subsidization plans generated from extensions of  $L_1$  that are no worse than  $\bar{\Psi}'_2$ . This completes the proof.  $\square$

## 6. Numerical experiments

In this section, we perform a series of computational experiments to verify the effectiveness of our proposed model and solution method. All the experiments are coded in C++ and are conducted on an Intel Core i7 2.20 GHz PC with 32 GB RAM.

To test the performance of our algorithm, we first generate 12 instances in terms of different input parameters. In particular, the number of ports  $|P|$  is selected from  $\{20, 30, 40\}$  and the number of shipping routes  $|R|$  is selected from  $\{20, 30, 40, 50\}$ . We set the other parameters as follows. The parameters are set using the data provided by Papoutsoglou (2012) and Wang et al. (2015). First, we randomly generate  $C_p^1$  from the uniform distribution  $U(1.5, 2.5)$  (million dollars). Second, for each shipping route  $r$ , the cost for retrofitting a ship on this route with SPE (denoted by  $\hat{c}_r$ ) is generated from  $U(0.3, 0.5)$  (million dollars). We assume that each  $r \in R$  provides a weekly service (i.e., ships on the route call at each port  $p \in P_r$  once a week). It is also assumed that all ships on  $r$  are identical. The number of ships deployed in a route (denoted by  $\eta_r$ , which is an integer) is randomly generated from  $[1, 8]$ . Hence, for each route  $r$ ,  $C_r^2$  is set equal to  $\hat{c}_r \eta_r$ . In addition, we randomly generate  $|P_r|$  (an integer) from  $[2, 6]$ , and the ports in  $P_r$  are randomly selected from  $P$ . Third, we set  $a = 0.025$  and  $b = 0.03$ . Fourth, considering the weekly service routine, the  $h_{rp}$  is set to be  $52\lambda_{rp}$ , where 52 represent the number of weeks in a year and  $\lambda_{rp}$  (which is the cost of at-berth fuel consumption of one ship on route  $r$  when berthing at port  $p$ ) is randomly generated using  $U(3000, 5000)$  (dollars). Then we randomly generate  $e_{rp} = \alpha h_{rp}$  and  $u_{rp} = \beta h_{rp}$ , where  $\alpha$  and  $\beta$  are randomly generated from  $U(0.7, 0.8)$  and  $U(0.1, 0.15)$ , respectively. Finally,  $B$  is randomly generated using  $U(0.05D, 0.1D)$ , where  $D = \sum_{p \in P} C_p^1 + \sum_{r \in R} C_r^2$ .

The computational results are demonstrated in Table 2. Columns 1 and 2 report the number of ports and the number of routes in an instance, respectively. We report the computational time for solving an instance in Column 3. Column 4 presents the number of ports subsidized by the government to set up SPI ( $|\hat{P}_0^*|$ ). Column 5 presents the number of routes on which the ships are subsidized to be retrofitted with SPE ( $|\hat{R}_0^*|$ ). The number of ports with SPI and the number of routes on which the ships are retrofitted with SPE in the long-term equilibrium ( $|\hat{P}_N^*|$  and  $|\hat{R}_N^*|$ ) are reported in Columns 6 and 7, respectively. In Column 8, we report the amount of at-berth fuel consumption at all ports from all ships sailing on routes in  $R$  without using SP in a year, which is calculated by  $OBK = \sum_{r \in R} \sum_{p \in P_r} h_{rp}/q$ , where  $q$  is the unit cost (dollars per ton) of bunkers. Column 9 reports the similar value after the SP deployment among ports and ships has achieved its equilibrium, which is calculated by  $EBK = OBK - \sum_{r \in \hat{R}_N^*} \sum_{p \in P_r \cap \hat{P}_N^*} h_{rp}/q$ , where  $\hat{P}_N^*$  is the set of ports that have set up SPI and  $\hat{R}_N^*$  is the set of routes on which the ships have equipped with SPE in the equilibrium obtained from the optimal subsidization plan delivered by the algorithm. In the experiments, we set  $q = 700$  (dollars per ton). Finally,

**Table 2**  
Computational results.

$ P $	$ R $	Time <sup>a</sup>	$ \hat{P}_0^* $	$ \hat{R}_0^* $	$ \hat{P}_N^* $	$ \hat{R}_N^* $	OBK <sup>b</sup>	EBK <sup>b</sup>	Reduction (%)
20	20	<1	3	1	19	19	23900	872	96.35
20	30	<1	1	0	20	30	35882	0	100.00
20	40	<1	1	0	20	40	48381	0	100.00
20	50	<1	1	0	20	50	57457	0	100.00
30	20	<1	2	3	18	19	20593	3589	82.57
30	30	<1	4	1	28	30	36665	1218	96.68
30	40	<1	3	0	30	40	49530	0	100.00
30	50	<1	1	0	30	50	70074	0	100.00
40	20	2016	4	1	17	15	24468	9077	62.90
40	30	809	4	2	30	29	36823	2980	91.91
40	40	<1	5	1	37	40	48403	583	98.79
40	50	2	2	2	35	49	53777	2708	94.96

<sup>a</sup> Note. In seconds.

<sup>b</sup> Note. In tons.

the last column reports fuel consumption reduction (in percentage) between OBK and EBK, which equals  $\frac{OBK - EBK}{OBK} \cdot 100$ .

We can see from Table 2 that the labeling algorithm efficiently solves all instances (with practical sizes). Besides, the results indicate that promoting SP usage among ports and ships generates tremendous environmental and health benefits. In particular, after SP is adopted, the average amount of at-berth fuel consumption in the instances reduces by 93.86%, and in some instances, the reduction percentages reach 100%. The results also verify the network effect among ports and shipping routes. In particular, the average ratio of between  $|\hat{P}_N^*| + |\hat{R}_N^*|$  (i.e., the total number of ports with SPI and routes on which ships are equipped with SPE in the equilibrium) and  $|\hat{P}_0^*| + |\hat{R}_0^*|$  (i.e., the total number of subsidized ports and routes on which ships are subsidized) is 30.16. The network effect also explains why the solution times of the two largest instances (i.e., the instances with 40 ports and 40 or 50 routes) are shorter than those of the instances with 40 ports and 20 or 30 routes. As a matter of fact, when the container shipping network is denser (i.e., ports are linked by more routes), setting up SPI at particular ports or retrofitting ships on particular routes have higher chances to drive more ports to build up SPI and ships on more routes to deploy SPE, and this reduces the searching space in the labeling algorithm.

## 7. Conclusion

In this paper, we analyzed a Shore Power Deployment Problem that aims at generating a subsidization plan for a government whose goal is to minimize at-berth emissions from ships in a container shipping network. The problem was formulated in a framework that captures the complex relationship between the government, ports, and ships. We showed that the problem is NP-hard. For solving the problem, a tailored labeling algorithm was proposed. We conducted extensive numerical experiments to test the performance of the algorithm. The results demonstrated that the proposed algorithm can efficiently solve problems with practical sizes and that the delivered subsidization plans generate great environmental and health benefits.

Ports are the key nodes in a global supply chain. However, they are also a major source of various pollutants. SP provides a potential cure to the adverse environmental impacts of ports. Due to the huge infrastructure set-up costs, governmental subsidization plays a vital role in SP deployment. We have shown both in theory and by the numerical experience that by utilizing the network effect in the SP deployment, the subsidization plan of a government can lead to a significant reduction in at-berth fuel consumption in a shipping network.

Both port-based and ship-based SP devices are expensive and the budget for subsidizing SP deployment from a government is limited. Therefore, identifying the optimal subsidization plan is critical for a government. However, such a problem is very difficult to solve. In this work, we provide a solution method for this important yet challenging problem. In particular, by subsidizing a small proportion of ports and ships to be retrofitted with SP devices, many more ports and ships will be voluntary to set up SPI and SPE in the long-term equilibrium. Considering the huge environmental benefits brought by SP, our study provides theoretical support for governments to devise effective subsidization plans for promoting SP.

The model and algorithm proposed in this work may also provide references to other infrastructure deployment problems in which governmental subsidization is required and the network effect should be considered. Examples include the deployment of charging stations for electronic vehicles in a national road traffic network and the deployment of base stations in a communication network.

There are some potential directions for extending the current study. First, in this study, we only consider the subsidization policy in which the government should decide whether all ships on a shipping route should be subsidized to get retrofitted with SPE or not. However, a better subsidization plan may be possible if more flexibilities are allowed (e.g., the government is allowed to subsidize only part of ships on a route). Similarly, we assume that a shipping line that operates a route can only choose to either get all ships on the route equipped with SPE or let none of the ships equipped with SPE. In practice, a shipping line may also choose to partially retrofit its fleet deployed on a route (a possible reason is the lack of sufficient funds to retrofit the entire fleet). Future studies should consider how to generate subsidization plans that allow more flexibilities in SP deployment. Second, in this study, all parameters are

considered to be deterministic and constant. In practice, the price of SP electricity and the price of fuel for auxiliary engines of ships may change over time and can also be uncertain. Future studies should consider how to generate subsidization plans that are robust against the volatilities and uncertainties in these prices. Finally, the current study only considers the situation that there is only one government that invests in SP deployment in a shipping network. Future studies can extend this study by considering the situation where multiple governments are making their subsidization plans for SP deployment in a network and each of them manages different ports and shipping routes (shipping lines). Hence, an interesting topic is how to coordinate these governments and design a subsidization plan for each of them.

## CRediT authorship contribution statement

**Lingxiao Wu:** Methodology, Software, Writing - original draft. **Shuaian Wang:** Writing - review & editing, Methodology, Conceptualization.

## Acknowledgments

The authors are grateful to the three reviewers for helpful comments. This research was supported by the Research Grants Council of the Hong Kong Special Administrative Region, China (Project number 15201718).

## References

- Ballini, F., Bozzo, R., 2015. Air pollution from ships in ports: the socio-economic benefit of cold-ironing technology. *Res. Transp. Bus. Manage.* 17, 92–98.
- Bierwirth, C., Meisel, F., 2015. A follow-up survey of berth allocation and quay crane scheduling problems in container terminals. *Eur. J. Oper. Res.* 244, 675–689.
- Department for Transport, 2019. Maritime 2050. Technical Report. URL: [https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment\\_data/file/773178/maritime-2050.pdf](https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/773178/maritime-2050.pdf) (accessed November 24, 2019).
- Dong, J.X., Song, D.P., 2009. Container fleet sizing and empty repositioning in liner shipping systems. *Transp. Res. Part E: Logist. Transp. Rev.* 45, 860–877.
- Du, Y., Chen, Q., Quan, X., Long, L., Fung, R.Y., 2011. Berth allocation considering fuel consumption and vessel emissions. *Transp. Res. Part E: Logist. Transp. Rev.* 47, 1021–1037.
- European Commission, 2017. Clean power for transport infrastructure deployment. Technical Report. URL: [https://ec.europa.eu/transport/themes/infrastructure/studies/clean-power-transport-infrastructure-deployment\\_en](https://ec.europa.eu/transport/themes/infrastructure/studies/clean-power-transport-infrastructure-deployment_en) (accessed November 26, 2019).
- European Commission, 2019. Mobility and transport. URL: [https://ec.europa.eu/transport/home\\_en](https://ec.europa.eu/transport/home_en) (accessed November 25, 2019).
- Gutierrez Saenz, J., 2019. Energy analysis and cost estimation of a potential on-shore power supply system in the port of Gävle. <http://www.diva-portal.org/smash/get/diva2:1333376/FULLTEXT01.pdf> (accessed November 26, 2019).
- International Transport Forum, 2019. Maritime subsidies: do they provide value for money?, in: International Transport Forum Policy Papers, Paris, France. URL: <https://www.itf-oecd.org/sites/default/files/docs/maritime-subsidies-value-for-money.pdf> (accessed November 24, 2019).
- Kanellakis, K., 2016. Shore connection – regulations, benefits, success stories. Technical Report. URL: <https://smartports.gr/wp-content/uploads/2018/02/K-KANELAKIS.pdf> (accessed November 26, 2019).
- Kim, K.H., Park, Y.M., 2004. A crane scheduling method for port container terminals. *Eur. J. Oper. Res.* 156, 752–768.
- McArthur, D.P., Osland, L., 2013. Ships in a city harbour: an economic valuation of atmospheric emissions. *Transp. Res. Part D: Transp. Environ.* 21, 47–52.
- Meng, Q., Wang, S., Andersson, H., Thun, K., 2013. Containership routing and scheduling in liner shipping: overview and future research directions. *Transp. Sci.* 48, 265–280.
- Merk, O., 2014. Shipping emissions in ports. In: International Transport Forum, Paris, France. URL: <https://www.itf-oecd.org/sites/default/files/docs/dp201420.pdf> (accessed July 22, 2019).
- Ng, M.W., 2017. Revisiting a class of liner fleet deployment models. *Eur. J. Oper. Res.* 257, 773–776.
- Ng, M.W., Lin, D.Y., 2018. Fleet deployment in liner shipping with incomplete demand information. *Transp. Res. Part E: Logist. Transp. Rev.* 116, 184–189.
- Papoutsoglou, G., 2012. A Cold Ironing Study on Modern Ports, Implementation and Benefits Thriving for Worldwide Ports (Ph.D. thesis). National Technical University of Athens.
- Peng, C., 2016. Application of shore power for ocean going vessels at berth in china. *DEStech Trans. Environ. Energy Earth Sci.*
- Qu, X., Meng, Q., 2012. The economic importance of the Straits of Malacca and Singapore: an extreme-scenario analysis. *Transp. Res. Part E: Logist. Transp. Rev.* 48, 258–265.
- Qu, X., Yu, Y., Zhou, M., Lin, C.T., Wang, X., 2020. Jointly dampening traffic oscillations and improving energy consumption with electric, connected and automated vehicles: a reinforcement learning based approach. *Appl. Energy* 257 114030.
- Sharma, D.C., 2006. Ports in a storm. *Environ. Health Perspect.* 114, 222–231.
- Smith, T., Jalkanen, J., Anderson, B., Corbett, J., Faber, J., Hanayama, S., O'keeffe, E., Parker, S., Johanasson, L., Aldous, L., C, R., Traut, M., Ettinger, S., Nelissen, D., Lee, D., Ng, S., Agrawal, A., Winebrake, J., Hoen, M., Chesworth, S., A, P., 2014. Third IMO GHG study 2014. Technical Report. London, UK.
- Song, D.P., Lyons, A., Li, D., Sharifi, H., 2016. Modeling port competition from a transport chain perspective. *Transp. Res. Part E: Logist. Transp. Rev.* 87, 75–96.
- Transport Malta, 2014. Feasibility study into the possibility of shore side electrical supply for berthing vessels within Maltese harbours. Technical Report.
- U.S. Environmental Protection Agency, 2017. Shore power technology assessment at U.S. ports. Technical Report. URL: <https://www.epa.gov/sites/production/files/2017-05/documents/420r17004-2017-update.pdf> (accessed July 22, 2019).
- Vaishnav, P., Fischbeck, P.S., Morgan, M.G., Corbett, J.J., 2016. Shore power for vessels calling at US ports: benefits and costs. *Environ. Sci. Technol.* 50, 1102–1110.
- Wang, H., Mao, X., Rutherford, D., 2015. Costs and benefits of shore power at the port of Shenzhen. In: The International Council on Clean Transportation. URL: [https://www.wilsoncenter.org/sites/default/files/costs\\_and\\_benefits\\_of\\_shore\\_power\\_at\\_the\\_port\\_of\\_shenzhen.pdf](https://www.wilsoncenter.org/sites/default/files/costs_and_benefits_of_shore_power_at_the_port_of_shenzhen.pdf) (accessed July 22, 2019).
- Wang, S., Meng, Q., 2012a. Liner ship fleet deployment with container transshipment operations. *Transp. Res. Part E: Logist. Transp. Rev.* 48, 470–484.
- Wang, S., Meng, Q., 2012b. Sailing speed optimization for container ships in a liner shipping network. *Transp. Res. Part E: Logist. Transp. Rev.* 48, 701–714.
- Wang, S., Yan, R., Qu, X., 2019. Development of a non-parametric classifier: effective identification, algorithm, and applications in port state control for maritime transportation. *Transp. Res. Part B: Methodol.* 128, 129–157.
- Winkel, R., Weddige, U., Johnsen, D., Hoen, V., Papaefthimiou, S., 2016. Shore side electricity in Europe: potential and environmental benefits. *Energy Policy* 88, 584–593.
- Zhang, D., Liu, Y., He, S., 2019. Vehicle assignment and relays for one-way electric car-sharing systems. *Transp. Res. Part B: Methodol.* 120, 125–146.
- Zis, T., Angeloudis, P., Bell, M.G., Psaraftis, H.N., 2016. Payback period for emissions abatement alternatives: role of regulation and fuel prices. *Transp. Res. Rec.* 2549, 37–44.