

# EE5705 Electric Drives in Sustainable Energy Systems

## Course Project 3

Pei Xu

May 14, 2016

### 1 Introduction

Doubly fed induction generators (DFIGs) are a kind of induction machines, who is widely used in wind generation. The major advantage of DFIG is its sufficiently wide speed range, which is able to make the combined wind turbine run at the optimum coefficient of performance  $C_p^{opt}$ .

This report focuses on the modeling of DFIG with a combined wind turbine in  $dq$  domain and the modeling of controllers based on  $dq$  domain, as described in Chapter 3, 5 and 7 in [2].

The report is organized as the following. First, a detailed mathematical model of induction machines in  $dq$  domain is provided. Then, the report, based on the characteristics of DFIG, offers a modification to the general mathematical model and provides the vector control model of DFIG. And, a basic model for the wind turbine and its torque controller is provided then. Finally, a set of simulation is conducted, in order to test the performance of the designed controllers.

### 2 General Mathematical Model of Induction Machine in $dq$ Domain

#### 2.1 Assumptions

Similarly to Project 1, the study to the model of induction machines are based on the below assumptions.

- The airgap is uniform.
- Eddy currents, friction and windage losses are all negligible.
- Stator and rotor windings are identical.
- The studied induction machine works under the condition in which the magnetic fields are unsaturated.
- The studied induction machine is magnetic linear.

- The studied induction machine has a squirrel-cage rotor.
- The input voltages and currents of the studied induction machine are sinusoidal balanced.

## 2.2 dq Winding Representation

For stator windings, in three-phase frame, assuming phase  $a$  is the reference phase, the stator current space vector  $\vec{i}_s$  and the magnetomotive force space vector  $\vec{F}_s$  at time  $t$  are defined as

$$\vec{i}_s^a(t) = i_a(t) + i_b(t)e^{j2\pi/3} + i_c(t)e^{j4\pi/3} \quad (2.1)$$

and

$$\vec{F}_s^a(t) = \frac{N_s}{p} \vec{i}_s^a(t) \quad (2.2)$$

where  $i_a(t)$ ,  $i_b(t)$  and  $i_c(t)$  are the phase currents of the stator windings at time  $t$ ,  $N_s$  is the equivalent turns of each stator winding, and  $p$  is the number of poles.

Supposing that the magnetomotive force at any instant time is produced by a group of two orthogonal windings,  $i_{sd}$  and  $i_{sq}$ , each of which has  $kN_s$  equivalent turns, then we have

$$\frac{kN_s}{p}(i_{sd}(t) + j i_{sq}(t)) = \frac{N_s}{p} \vec{i}_s^d(t) \quad (2.3)$$

where  $\vec{i}_s^d$  is  $\vec{i}_s$  using  $d$ -axis as the reference axis and is obtained as

$$\vec{i}_s^d(t) = \vec{i}_s^a(t)e^{-j\theta_{da}(t)} \quad (2.4)$$

in which  $\theta_{da}(t)$  is the intersection angle between  $d$ -axis and  $a$ -axis at time  $t$  in electrical space.

Here we choose

$$k = \sqrt{3/2} \quad (2.5)$$

such that the winding magnetizing inductance in the  $dq$  domain satisfies

$$dq \text{ winding magnetizing inductance} = k^2 L_{m,1\phi} = L_m \quad (2.6)$$

where  $L_m$  is the per-phase magnetizing inductance in the three-phase frame,  $L_{m,1\phi}$  is single-phase magnetizing inductance that is the same in both stator and rotor based on the assumptions.

Therefore,  $i_{sd}$  and  $i_{sq}$  can be obtained as

$$i_{sd} = \sqrt{\frac{2}{3}} \times \text{Re}\{\vec{i}_s^d(t)\} \quad (2.7)$$

and

$$i_{sq} = \sqrt{\frac{2}{3}} \times \text{Im}\{\vec{i}_s^d(t)\} \quad (2.8)$$

where  $\text{Re}\{\vec{i}_s^d(t)\}$  and  $\text{Im}\{\vec{i}_s^d(t)\}$  are respectively the real and imaginary part of  $\vec{i}_s^d(t)$ ; and thus the relationship between  $dq$  currents and  $abc$  currents can be expressed as

$$\mathbf{i}_{s,dq} = \mathbf{T}_{s,abc \rightarrow dq} \mathbf{i}_{s,abc} \quad (2.9)$$

$$\mathbf{i}_{s,abc} = \mathbf{T}_{s,dq \rightarrow abc} \mathbf{i}_{s,dq} \quad (2.10)$$

where  $\mathbf{i}_{s,abc} = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$ ,  $\mathbf{i}_{s,dq} = \begin{bmatrix} i_{sd}(t) \\ i_{sq}(t) \end{bmatrix}$ ,  $\mathbf{T}_{s,abc \rightarrow dq}$  and  $\mathbf{T}_{s,dq \rightarrow abc}$  are the transformation matrices for stator windings that are obtained as

$$\mathbf{T}_{s,abc \rightarrow dq} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta_{da}(t)) & \cos(\theta_{da}(t) - 2\pi/3) & \cos(\theta_{da}(t) + 2\pi/3) \\ -\sin(\theta_{da}(t)) & -\sin(\theta_{da}(t) - 2\pi/3) & -\sin(\theta_{da}(t) + 2\pi/3) \end{bmatrix} \quad (2.11)$$

and

$$\mathbf{T}_{s,dq \rightarrow abc} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta_{da}(t)) & -\sin(\theta_{da}(t)) \\ \cos(\theta_{da}(t) - 2\pi/3) & -\sin(\theta_{da}(t) - 2\pi/3) \\ \cos(\theta_{da}(t) + 2\pi/3) & -\sin(\theta_{da}(t) + 2\pi/3) \end{bmatrix} \quad (2.12)$$

Similarly, for stator voltages, we have

$$\mathbf{v}_{s,dq} = \mathbf{T}_{s,abc \rightarrow dq} \mathbf{v}_{s,abc} \quad (2.13)$$

and

$$\mathbf{v}_{s,abc} = \mathbf{T}_{s,dq \rightarrow abc} \mathbf{v}_{s,dq} \quad (2.14)$$

where  $\mathbf{v}_{s,dq} = \begin{bmatrix} v_{sd}(t) \\ v_{sq}(t) \end{bmatrix}$  and  $\mathbf{v}_{s,abc} = \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$ .

For rotor windings, we have

$$\mathbf{i}_{r,dq} = \mathbf{T}_{r,ABC \rightarrow dq} \mathbf{i}_{r,ABC}, \quad \mathbf{v}_{r,dq} = \mathbf{T}_{r,ABC \rightarrow dq} \mathbf{v}_{r,ABC} \quad (2.15)$$

and

$$\mathbf{i}_{r,ABC} = \mathbf{T}_{r,dq \rightarrow ABC} \mathbf{i}_{r,dq}, \quad \mathbf{v}_{r,ABC} = \mathbf{T}_{r,dq \rightarrow ABC} \mathbf{v}_{r,dq} \quad (2.16)$$

where  $\mathbf{i}_{r,dq} = \begin{bmatrix} i_{rd}(t) \\ i_{rq}(t) \end{bmatrix}$ ,  $\mathbf{v}_{r,dq} = \begin{bmatrix} v_{rd}(t) \\ v_{rq}(t) \end{bmatrix}$ ,  $\mathbf{i}_{r,ABC} = \begin{bmatrix} i_A(t) \\ i_B(t) \\ i_C(t) \end{bmatrix}$ ,  $\mathbf{v}_{r,ABC} = \begin{bmatrix} v_A(t) \\ v_B(t) \\ v_C(t) \end{bmatrix}$ ,

$$\mathbf{T}_{r,ABC \rightarrow dq} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta_{dA}(t)) & \cos(\theta_{dA}(t) - 2\pi/3) & \cos(\theta_{dA}(t) + 2\pi/3) \\ -\sin(\theta_{dA}(t)) & -\sin(\theta_{dA}(t) - 2\pi/3) & -\sin(\theta_{dA}(t) + 2\pi/3) \end{bmatrix} \quad (2.17)$$

and

$$\mathbf{T}_{r,dq \rightarrow ABC} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta_{dA}(t)) & -\sin(\theta_{dA}(t)) \\ \cos(\theta_{dA}(t) - 2\pi/3) & -\sin(\theta_{dA}(t) - 2\pi/3) \\ \cos(\theta_{dA}(t) + 2\pi/3) & -\sin(\theta_{dA}(t) + 2\pi/3) \end{bmatrix} \quad (2.18)$$

in which  $\theta_{dA}(t)$  is the intersection angle between  $d$ -axis and  $A$ -axis at time  $t$  in electrical space.

### 2.3 Flux Leakage Equation

Given that  $dq$  windings are orthogonal, there is no mutual inductance between  $d$  and  $q$  windings. Therefore, for stator windings, due to that the leakage flux caused by rotor currents does not cross the air gap, we have

$$\lambda_{sd} = (L_{ls} + L_m)i_{sd} + L_m i_{rd} \quad (2.19)$$

and

$$\lambda_{sq} = (L_{ls} + L_m)i_{sq} + L_m i_{rq} \quad (2.20)$$

where  $L_{ls}$  is the stator leakage inductance.

Similarly, for rotor windings, we have

$$\lambda_{rd} = (L_{lr} + L_m)i_{rd} + L_m i_{sd} \quad (2.21)$$

and

$$\lambda_{rq} = (L_{lr} + L_m)i_{rq} + L_m i_{sq} \quad (2.22)$$

where  $L_{lr}$  is the rotor leakage inductance.

Combining the above four equations, we obtain the flux leakage equation

$$\begin{bmatrix} \lambda_{s,dq} \\ \lambda_{r,dq} \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{s,dq} \\ \mathbf{i}_{r,dq} \end{bmatrix} \quad (2.23)$$

where  $L_s \triangleq L_{ls} + L_m$ ,  $L_r \triangleq L_{lr} + L_m$  and  $\lambda_{s,dq} = \begin{bmatrix} \lambda_{sd}(t) \\ \lambda_{sq}(t) \end{bmatrix}$ ,  $\lambda_{r,dq} = \begin{bmatrix} \lambda_{rd}(t) \\ \lambda_{rq}(t) \end{bmatrix}$ .

### 2.4 Voltage Equation

According to Ohm's law and Faraday's law, the stator voltage space vector can be expressed as

$$\vec{v}_s^a = R_s \vec{i}_s^a + \frac{d}{dt} \vec{\lambda}_s^a \quad (2.24)$$

where  $R_s$  is the equivalent resistance of each stator winding.

Therefore, we have

$$\vec{v}_s^d e^{j\theta_{da}} = R_s \vec{i}_s^d e^{j\theta_{da}} + \frac{d}{dt} (\vec{\lambda}_s^d e^{j\theta_{da}}) \quad (2.25)$$

Simplifying the above equation, we get

$$\begin{aligned} \vec{v}_s^d &= R_s \vec{i}_s^d + \frac{d\vec{\lambda}_s^d}{dt} + j\vec{\lambda}_s^d \frac{d\theta_{da}}{dt} \\ &= R_s \vec{i}_s^d + \frac{d\vec{\lambda}_s^d}{dt} + j\omega_d \vec{\lambda}_s^d \end{aligned} \quad (2.26)$$

where

$$\omega_d = \frac{d\theta_{da}}{dt} \quad (2.27)$$

Given that  $\vec{v}_s^d = v_{sd} + j v_{sq}$ ,  $\vec{i}_s^d = i_{sd} + j i_{sq}$  and  $\vec{\lambda}_s^d = \lambda_{sd} + j \lambda_{sq}$  we have

$$\mathbf{v}_{s,dq} = R_s \mathbf{i}_{s,dq} + \frac{d}{dt} \boldsymbol{\lambda}_{s,dq} + \omega_d \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \boldsymbol{\lambda}_{s,dq} \quad (2.28)$$

Similarly, for rotor windings, we have

$$\mathbf{v}_{r,dq} = R_r \mathbf{i}_{r,dq} + \frac{d}{dt} \boldsymbol{\lambda}_{r,dq} + \omega_{dA} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \boldsymbol{\lambda}_{r,dq} \quad (2.29)$$

where  $R_r$  is the equivalent resistance of each stator winding and

$$\omega_{dA} = \frac{d\theta_{dA}}{dt} \quad (2.30)$$

## 2.5 Torque Equation

Given that  $dq$  windings are orthogonal, the produced torque on the  $d$ -axis is caused by the flux that is due to the  $q$ -axis winding. Referring to Chapter 10 of [1], we have

$$dT_{em,d}(\xi) = \left[ \frac{p}{2} \cdot r \cdot (\hat{B}_{rq} \cos \xi) \cdot l \cdot i_{rd} \cdot \left( \frac{\sqrt{3/2} N_s}{p} \cos \xi \right) \right] d\xi \quad (2.31)$$

where  $\hat{B}_{rq} \cos \xi$  is the flux density caused by rotor windings on  $q$ -axis at angle  $\xi$ ,  $r$  is the rotor radius, and  $l$  is the length of rotor.

Therefore,

$$T_{em,d} = 2 \int_{\xi=-\pi/2}^{\xi=\pi/2} dT_{em,d}(\xi) = \frac{p}{2} \left( \pi \frac{\sqrt{3/2} N_s}{p} r l \hat{B}_{rq} \right) i_{rd} \quad (2.32)$$

Given that  $\hat{B}_{rq} = \mu_0 \hat{H}_{rq} = \mu_0 \frac{\hat{F}_{rq}}{l_g}$ ,  $\hat{F}_{rq} = \left( \frac{\sqrt{3/2} N_r}{p} \right) \frac{\lambda_{rq}}{L_m}$  and  $N_r = N_s$  where  $\mu_0$  is the permeability of air,  $l_g$  is the length of air gap,  $\hat{H}_{rq}$  is the peak field intensity of rotor windings along the  $q$ -axis,  $\hat{F}_{rq}$  is the peak magnetomotive force of rotor windings along the  $q$ -axis,  $N_r$  is the equivalent turns of each rotor winding, and  $N_s$  is the equivalent turns of each stator winding, the above equation can be further simplified, and  $T_{em,d}$  can be expressed as

$$T_{em,d} = \frac{p}{2} (L_m i_{sq} + L_r i_{rq}) i_{rd} = \frac{p}{2} \lambda_{rq} i_{rd} \quad (2.33)$$

Similarly, the produced torque on the  $q$ -axis can be expressed as

$$T_{em,q} = -\frac{p}{2} (L_m i_{sd} + L_r i_{rd}) i_{rq} = -\frac{p}{2} \lambda_{rd} i_{rq} \quad (2.34)$$

Therefore, the electromagnetic torque can be expressed as

$$T_{em} = T_{em,d} + T_{em,q} = \frac{p}{2} (\lambda_{rq} i_{rd} - \lambda_{rd} i_{rq}) \quad (2.35)$$

or

$$T_{em} = \frac{p}{2} L_m (i_{sq} i_{rd} - i_{sd} i_{rq}) \quad (2.36)$$

## 2.6 Angular Equations

The intersection angle between  $a$ -axis and  $A$ -axis in electrical radians,  $\theta_m$ , satisfies

$$\theta_{dA} = \theta_{da} - \theta_m \quad (2.37)$$

$$\frac{d}{dt}\theta_{dA} = \omega_{dA} \quad (2.38)$$

and

$$\theta_m = \int \omega_m(t)dt + \theta_0 \quad (2.39)$$

where  $\omega_m$  is the electrical rotational speed and  $\theta_0$  is the the initial angle by which rotor and stator coils are offset in electrical space.

Therefore, according to Eqn. 2.27 and 2.30 we have

$$\omega_d = \omega_{dA} + \omega_m \quad (2.40)$$

Based on the assumptions, we have

$$\frac{d\omega_{mech}}{dt} = \frac{T_{em} - T_L}{J} \quad (2.41)$$

where  $\omega_{mech}$  is the mechanically rotational speed and  $T_L$  is the load torque and  $J$  is the moment of inertia.

Given that

$$\omega_m = \frac{p}{2}\omega_{mech} \quad (2.42)$$

where  $p$  is the number of poles of the given induction machine, we have the angular acceleration equation

$$\frac{d\omega_m}{dt} = \frac{p}{2} \frac{T_{em} - T_L}{J} \quad (2.43)$$

## 3 $dq$ Model for DFIGs

### 3.1 Assumptions

Besides the assumptions described in Section 2.1, the model of DFIG is also based on the following assumptions:

- The stator voltage is applied by the power grid with balanced sinusoidal three phase voltage.
- The rotor voltage is controllable through power electronics.
- The stator and rotor leakage inductances and resistances are quite small and even negligible.

### 3.2 Selection of $d$ -Axis

Due to that the stator voltage is applied by the power grid and thus is uncontrollable, the purpose of modeling DFIGs in  $dq$  domain is to control DFIG (its speed, electromagnetic torque and so on) through the two decoupling axial components of rotor current or voltage in  $dq$  domain. Usually, the  $d$ -axis is chosen to be aligned with the rotor flux-leakage space vector or the direction stator voltage. In this project, the direction of the stator voltage  $\vec{v}_s^a$  is chosen as the  $d$ -axis.

When the  $d$ -axis is aligned with  $\vec{v}_s^a$ , we have the following equations

$$|\vec{v}_s^a| = v_{s,d} \quad (3.44)$$

$$v_{s,q} = 0 \quad (3.45)$$

$$\theta_{da} = \angle \vec{v}_s^a \quad (3.46)$$

i.e.

$$\vec{v}_s^a = v_{s,d} \angle \theta_{da} \quad (3.47)$$

and the rotational speed of  $d$ -axis is equal to the rotational speed of  $\vec{v}_s$  in electrical space, i.e.

$$\omega_d = \omega_{\vec{v}_s} \quad (3.48)$$

Let  $\vec{v}_s^a = v_\alpha + jv_\beta$ . Given that the stator voltage is applied by the power grid balanced sinusoidal three phase voltage  $v_a \angle 0^\circ$ ,  $v_b \angle 120^\circ$  and  $v_c \angle -120^\circ$ , we have

$$v_\alpha = v_a \cos 0^\circ + v_b \cos 120^\circ + v_c \cos(-120^\circ) \quad (3.49)$$

and

$$v_\beta = v_a \sin 0^\circ + v_b \sin 120^\circ + v_c \sin(-120^\circ) \quad (3.50)$$

Writing Eqn. 3.49 and 3.50 in the form of matrix, we can obtain

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} \widehat{V}_s \\ 0 \end{bmatrix} \quad (3.51)$$

where  $\widehat{V}_s$  is the peak value of the phase voltage applied to the stator. According to  $\vec{v}_s^a = v_{s,d} \angle \theta_{da}$ , we have

$$\theta_{da} = \tan^{-1} \frac{v_\beta}{v_\alpha} \quad (3.52)$$

and

$$\omega_d = \omega_{syn} = 2\pi f_{syn} \quad (3.53)$$

where  $f_{syn}$  is the frequency of the grid.

Given that for DFIGs, the stator resistances are quite small, we have

$$Rs \approx 0 \quad (3.54)$$

And, in steady state,

$$\frac{d}{dt} \lambda_{s,dq} \approx 0 \quad (3.55)$$

Therefore, from Eqn. 2.28, in steady state, we have

$$v_{sd} \approx -\omega_d \lambda_{sq} \quad (3.56)$$

and

$$v_{sq} \approx \omega_d \lambda_{sd} \quad (3.57)$$

i.e.

$$\lambda_{sq} \approx -\frac{v_{sd}}{\omega_d} \quad (3.58)$$

and

$$\lambda_{sd} \approx \frac{v_{sq}}{\omega_d} \quad (3.59)$$

Given that  $v_{sq} = 0$ , we have

$$\lambda_{sd} \approx 0 \quad (3.60)$$

Given that  $\lambda_{sd} = L_m i_{rd} + L_s i_{sd}$ , we have

$$i_{sd} \approx -\frac{L_m}{L_s} i_{rd} \quad (3.61)$$

### 3.3 Selection of Control Variables

Given that the stator real and reactive power can be obtained as

$$P_s + jQ_s = (v_{sd} + jv_{sq})(i_{sd} + ji_{sq})^H \quad (3.62)$$

where the superscript  $H$  denotes the conjugate of the given complex-valued vector, we have

$$P_s = v_{sd}i_{sd} + v_{sq}i_{sq} \quad (3.63)$$

$$Q_s = v_{sq}i_{sd} - v_{sd}i_{sq} \quad (3.64)$$

Similarly, for the rotor real and reactive power  $P_r + jQ_r = (v_{rd} + jv_{rq})(i_{rd} + ji_{rq})^H$ , we have

$$P_r = v_{rd}i_{rd} + v_{rq}i_{rq} \quad (3.65)$$

$$Q_r = v_{rq}i_{rd} + v_{rd}i_{rq} \quad (3.66)$$

Given that  $v_{sq} = 0$  and  $i_{sd} \approx -\frac{L_m}{L_s} i_{rd}$ , we have

$$P_s = v_{sd}i_{sd} \approx -\frac{L_m}{L_s} v_{sd}i_{rd} \quad (3.67)$$

Given that  $v_{sq} = 0$  and  $\lambda_{sq} = L_m i_{rq} + L_s i_{sq} \approx -\frac{v_{sd}}{\omega_d}$ , we have

$$Q_s = -v_{sd}i_{sq} \approx v_{sd}\left(\frac{v_{sd}}{\omega_d L_s} + \frac{L_m}{L_s}\right)i_{rq} \quad (3.68)$$



Given that  $\lambda_{sq} = L_s i_{sq} + L_m i_{rq} \approx -\frac{v_{sd}}{\omega_d}$ ,  $\lambda_{sd} = L_m i_{rd} + L_s i_{sd}$ ,  $\lambda_{sd} \approx 0$ ,  $\lambda_{sq} \approx -\frac{v_{sd}}{\omega_d}$  and  $i_{sd} \approx -\frac{L_m}{L_s} i_{rd}$ , we have

$$\begin{aligned} T_{em} &= \frac{p}{2} L_m (i_{sq} i_{rd} - i_{sd} i_{rq}) \\ &= \frac{p}{2} \frac{L_m}{L_s} [(\lambda_{sq} - L_m i_{rq}) i_{rd} - (\lambda_{sd} - L_m i_{rd}) i_{rq}] \\ &\approx -\frac{p L_m v_{sd}}{2 \omega_d L_s} i_{rd} \end{aligned} \quad (3.69)$$

Given that  $v_{sd} = \sqrt{\frac{2}{3}} \widehat{V}_s$  is a constant, from Eqn. 3.67, 3.68 and 3.69, it can be recognized that we can control the DFIG's stator real power  $P_s$  and electromagnetic  $T_{em}$  through adjusting  $i_{rd}$  and control the DFIG's stator reactive power  $Q_s$  through adjusting  $i_{rq}$ . In summary, the  $d$ -axis rotor current is employable to control the stator active power as well as the electromagnetic torque and the  $q$ -axis rotor current is employable to control the stator flux as well as the stator reactive power. Obviously, it is able to control the DFIG through the two decoupling components  $i_{rd}$  and  $i_{rq}$ , and our control purpose is reachable.

The reference signal can be obtained from the desired electromagnetic torque (or the desired stator real power) and the desired stator reactive power, as shown below

$$i_{rd}^* \approx -\frac{L_s}{L_m v_{sd}} P_s^* \quad (3.70)$$

$$i_{rd}^* \approx -\frac{2 L_s \omega_d}{p L_m v_{sd}} T_{em}^* \quad (3.71)$$

$$i_{rq}^* \approx -\frac{L_s v_{sd}}{L_m L_s \omega_d} + \frac{L_s}{L_m v_{sd}} Q_s^* \quad (3.72)$$

### 3.4 Controller Design for DFIG

In the control strategy mentioned in the last subsection,  $i_{rd}$  and  $i_{rq}$  is chosen as the reference signals, and the input signals to the DFIG model is  $\mathbf{v}_{r,abc}$ , which can be calculated from  $v_{sd}$  and  $v_{sq}$ .

From Eqn. 2.29, and  $\frac{d}{dt} \boldsymbol{\lambda}_{r,dq} = 0$ , the  $d$ - and  $q$ -axis rotor voltages can be expressed as the summation of two components, as shown below

$$v_{rd} = \underbrace{R_r i_{rd} + \sigma L_r \frac{d}{dt} i_{rd}}_{v'_{rd}} - \underbrace{\omega_d A \sigma L_r i_{rq}}_{v_{rd,comp}} \quad (3.73)$$

$$v_{rq} = \underbrace{R_r i_{rq} + \sigma L_r \frac{d}{dt} i_{rq}}_{v'_{rq}} + \underbrace{\omega_d A \sigma L_r i_{rd}}_{v_{rq,comp}} \quad (3.74)$$

where

$$\sigma \triangleq 1 - \frac{L_m^2}{L_s L_r} \quad (3.75)$$

From

$$\mathbf{v}'_{r,dq} = R_r \mathbf{i}_{r,dq} + \sigma L_r \frac{d}{dt} \mathbf{i}_{r,dq} \quad (3.76)$$

where  $\mathbf{v}'_{r,dq} = \begin{bmatrix} v'_{rd} \\ v'_{rq} \end{bmatrix}$ , we have

$$\mathbf{i}_{r,dq}(s) = \frac{1}{R_r + s\sigma L_r} \mathbf{v}'_{r,dq}(s) \quad (3.77)$$

in which  $s$  is the complex number frequency parameter.

According to Eqn. 3.77, the applied voltages  $v_{sd}$  and  $v_{sq}$  can be obtained via the current loop whose transfer function is

$$K_i(s) = \frac{1}{R_r + s\sigma L_r} \quad (3.78)$$

Here, we adopt a PI controller whose transfer function is

$$K_i(s) = \frac{K_{i,i}}{s} + K_{p,i} \quad (3.79)$$

And, we can develop the expressions for  $K_{i,i}$  and  $K_{p,i}$  as

$$K_{i,i} = \omega_c^2 \sqrt{\frac{R_r^2 + (\omega_c L_r \sigma)^2}{\omega_c^2 + \tan^2(PM - \pi/2 + \arctan(\omega_c L_r \sigma / R_r))}} \quad (3.80)$$

and

$$K_{p,i} = \frac{K_{i,i} \tan(PM - \pi/2 + \arctan(\omega_c L_r \sigma / R_r))}{\omega_c} \quad (3.81)$$

where  $PM$  is the desired phase margin and  $\omega_c$  is the desired crossover frequency. A more detailed discussion of the PI controller design is provided in [1].

Due to that the voltage we obtain from the PI control is  $\mathbf{v}'_{r,dq}$  rather than  $\mathbf{v}_{r,dq}$ , we need a voltage compensator for the applied voltage  $v_{rd}$  and  $v_{rq}$ . The compensation can be calculated from

$$\mathbf{v}_{r,dq,comp} = \omega_{dA} \sigma L_r \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{i}_{r,dq} \quad (3.82)$$

where  $\mathbf{v}_{r,dq,comp} = \begin{bmatrix} v_{rd,comp} \\ v_{rq,comp} \end{bmatrix}$ .

## 4 Modeling of Wind Turbine

### 4.1 Mathematical Model of Wind Turbine

Similarly to the induction machine, for a wind turbine, we have

$$\frac{d\omega_{mech}}{dt} = \frac{T_{turb} - T_{em,gen}}{J_{turb}} \quad (4.83)$$

where  $\omega_{mech}$  here is the mechanically rotational speed of the wind turbine,  $T_{turb}$  is the output torque of the wind turbine,  $T_{em,gen}$  is the electromagnetic torque of the generator combined with the wind turbine and  $J_{turb}$  is the moment of inertia of the system composed of the wind turbine and the combined generator.

For a wind turbine, its output power is associated with  $T_{turb}$ , as shown below

$$P_{out} = P_{turb} = T_{turb}\omega_{mech} \quad (4.84)$$

and its input power is associated with the speed of wind, as shown below

$$P_{in} = P_{wind} = \frac{1}{2}\rho A v_{wind}^3 \quad (4.85)$$

where  $\rho$  is the density of air whose unit is  $\text{kg}/\text{m}^3$  and  $A$  is the swept area whose unit is  $\text{m}^2$ .

The relationship between  $P_{turb}$  and  $P_{wind}$  can be expressed as

$$P_{turb} = C_p P_{wind} = C_p \frac{1}{2} \rho A v_{wind}^3 \quad (4.86)$$

where  $C_p$  is the wind power coefficient that is defined as

$$C_p = \frac{P_{out}}{P_{in}} \quad (4.87)$$

Therefore,  $T_{turb}$  can be expressed as

$$T_{turb} = C_p \frac{1}{2} \rho A \frac{R^3}{\lambda^3} \omega_{wind}^2 \quad (4.88)$$

For a given wind turbine, its wind power coefficient  $C_p$  usually is a function of  $v_{wind}/\omega_{mech}$  and  $\beta$  where  $v_{wind}$  is wind speed and  $\beta$  in general is a constant. Therefore,  $C_p$  can be regarded as a function of  $v_{wind}/\omega_{mech}$ . Define the tip-speed ratio (TSR) as

$$\lambda = \frac{R\omega_{mech}}{v_{wind}} \quad (4.89)$$

$C_p$  can be further regarded as a function of  $\lambda$ . The plot of  $C_p$  versus  $\lambda$  usually has the shape as shown in Fig. 1.

For  $C_p = C_p^{opt}$ , the corresponding TSR is represented as  $\lambda^{opt}$ , i.e.  $C_p(\lambda^{opt}) = C_p^{opt}$ . The relationship between  $\lambda^{opt}$  and  $\omega_{mech}^{opt}$  can be expressed as

$$\lambda^{opt} = \frac{R\omega_{mech}^{opt}}{v_{wind}} \quad (4.90)$$

where  $\lambda^{opt}$  usually is a constant for a given wind turbine and  $\omega_{mech}^{opt}$  usually is dependent on wind speed  $v_{wind}$ .

For a given wind speed, the plot of  $C_p$  versus  $\omega_{mech}$  is shown in Fig.2. From the figure, we can find that there exists a value of  $\omega_{mech}$  at which  $C_p$  has the maximum value. Let  $C_p^{opt}$  denote the maximum value of  $C_p$ , and let  $\omega_{mech}^{opt}$  denote the corresponding value of  $\omega_{mech}$ .

When the wind turbine operates, we hope the wind power coefficient  $C_p$  can keep at  $C_p^{opt}$  such that the wind turbine has the maximum efficiency to convert the input power into the output power. In other words, we hope the mechanically rotational speed of the wind power,  $\omega_{mech}$ , can keep at  $\omega_{mech}^{opt}$  who usually would change according to wind speed.

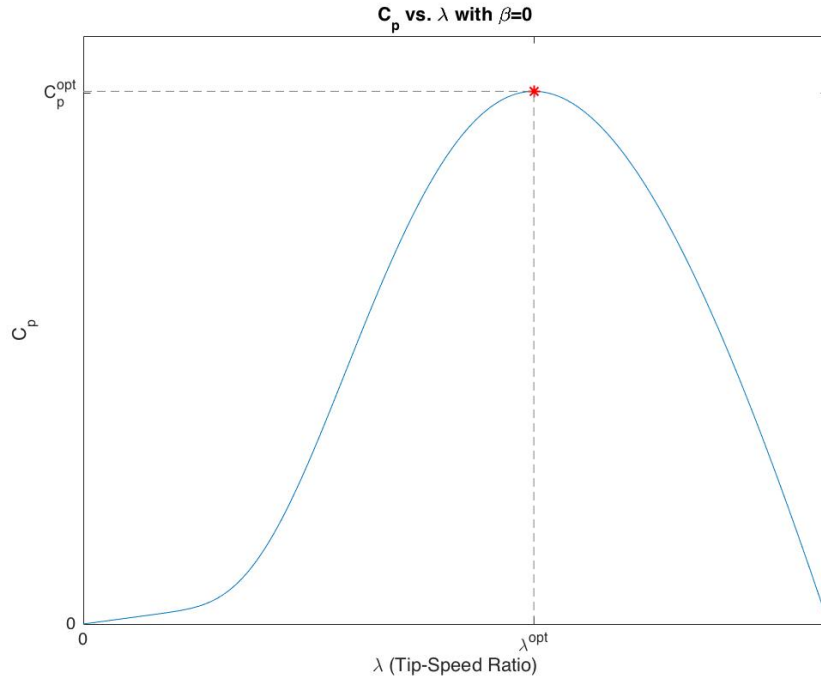


Figure 1: The Shape of the Wind Power Coefficient  $C_p$  versus Tip-Speed Ratio  $\lambda$

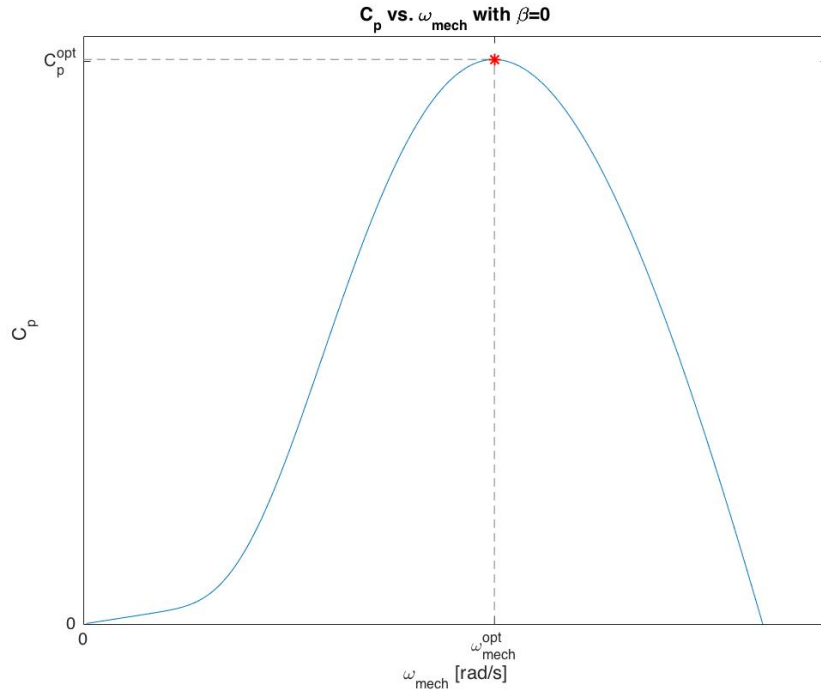


Figure 2: The Shape of the Wind Power Coefficient  $C_p$  versus the Mechanically Rotational Speed  $\omega_{mech}$

## 4.2 Speed-Squared Controller for Wind Turbine

Given Eqn. 4.83,  $\omega_{mech}$  is controllable through adjusting  $T_{em,gen}$ . Here we adopt a speed-squared controller to generate the reference signal  $T_{em,gen}^*$ .

Due to that we hope the wind turbine runs with the maximum wind power coefficient  $C_p^{opt}$ , similarly to Eqn. 4.88, the reference signal of  $T_{em,gen}$  can be obtained as

$$\begin{aligned} T_{em,gen}^* &= \frac{C_p^{opt} \rho A R^3}{2\lambda^{opt}} \omega_{mech}^2 \\ &= K_{opt} \omega_{mech}^2 \end{aligned} \quad (4.91)$$

Therefore, the controller used to generate the reference signal of  $T_{em,gen}$  can be expressed as

$$K_{opt} = C_p^{opt} \frac{1}{2} \rho A \frac{R^3}{\lambda^{opt}} \quad (4.92)$$

who is a proportional controller whose input signal is  $\omega_{mech}^2$  and whose output signal is  $T_{em}^*$ .

## 4.3 Combination of Wind Turbine and DFIG

The DFIG is combined with the wind turbine through a gearbox. The electromagnetic torque is amplified by a given ratio and then affects on the wind turbine as  $T_{em,gen}$ . The system performs in the accordance with Eqn. 4.83.  $\omega_{mech}$  of the wind turbine as the output is amplified by the given ratio and then affects on the DFIG as the mechanically rotational speed of the DFIG. In this combined system, the model of the wind turbine produces  $T_{turb}$  according to the parameters of the turbine and wind speed. This process is described via Eqn. 4.88. The details of the combination of the DFIG and wind turbine is shown in Fig. 8.

# 5 Case Study

## 5.1 Modeling of DFIG Combined with Wind Turbine in MATLAB/Simulink

The MATLAB/Simulink model of the studied DFIG in  $dq$  domain is shown in Fig. 3.



The estimator model, who calculates  $v_{s,dq}$ ,  $i_{s,dq}$ ,  $v_{r,dq}$  and  $i_{r,dq}$  all of which can be used as the feedback signal for the controller and who calculates  $\omega_{dA}$  and  $\theta_{dA}$  which are used for the rotor voltage compensation model and for the calculation of the applied rotor voltage in three-phase frame from the voltage in  $dq$  domain, is shown in Fig. 4. Basically, this estimator model is similar to the DFIG model except for that there is no need to calculate the current and torque in the estimator model.

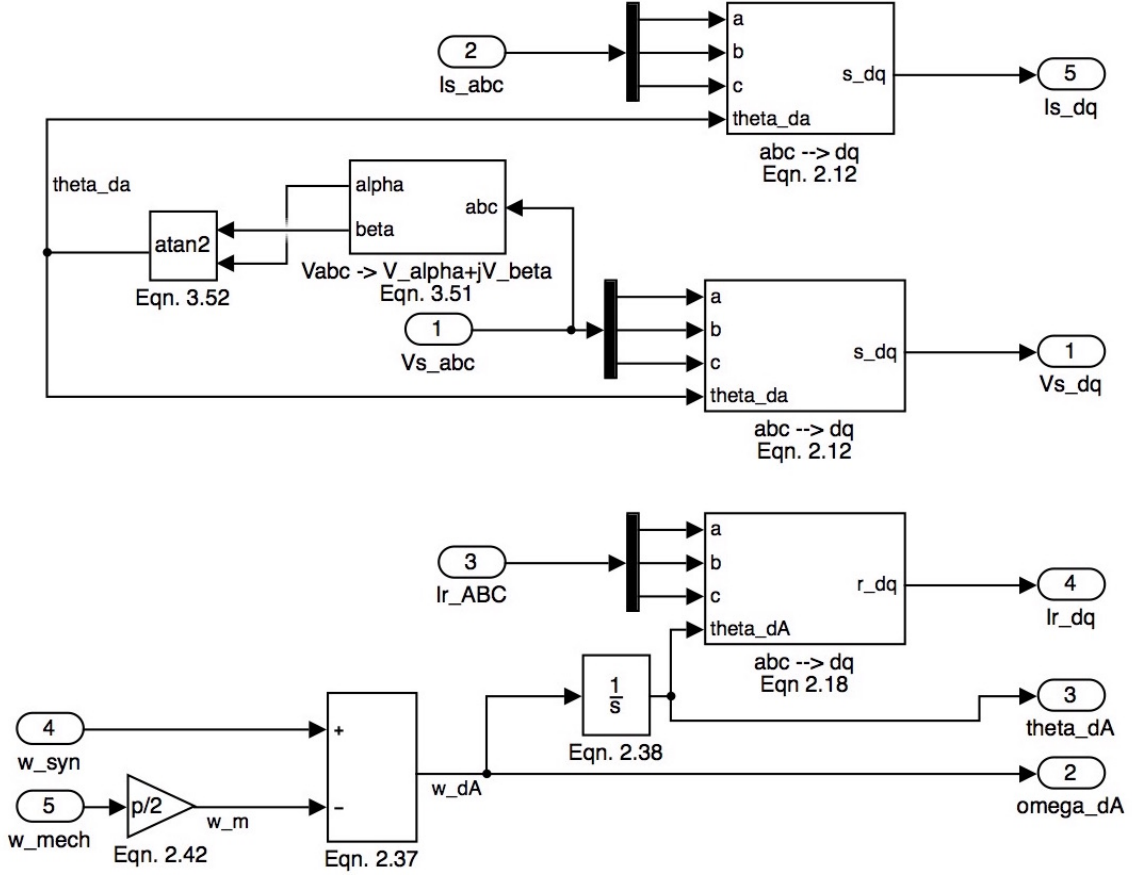


Figure 4: Model of Estimator Model

The detail of the DFIG controller and rotor voltage compensation model is shown in Fig. 5.

For the controller, the reference signal  $i_{rq}^*$  is controlled through adjust the reference signal  $Q_s^*$ , while the reference signal  $i_{rd}^*$  is controlled directly. The process of generating  $i_{rq}^*$  from  $Q_s^*$  is shown in Fig. 6

The model of the studied wind turbine is shown in Fig. 7.

The DFIG and wind turbine is combined in the form shown in Fig. 8. The stator voltage

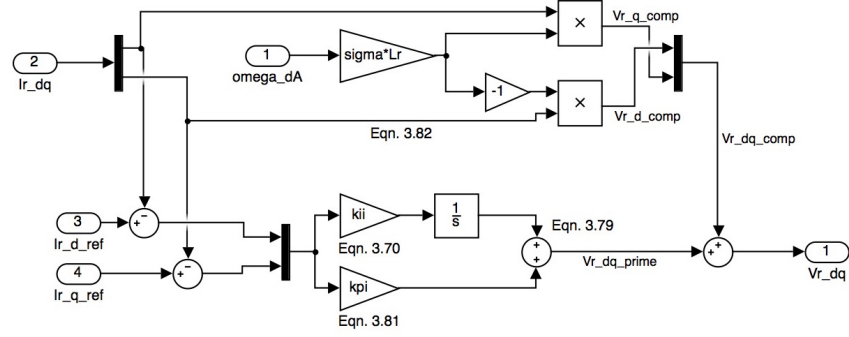


Figure 5: Model of the DFIG Controller with Rotor Voltage Compensation

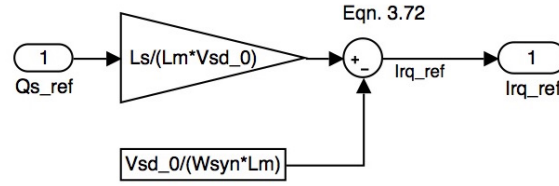


Figure 6: Model of Generating  $i_{rq}^*$  from  $Q_s^*$

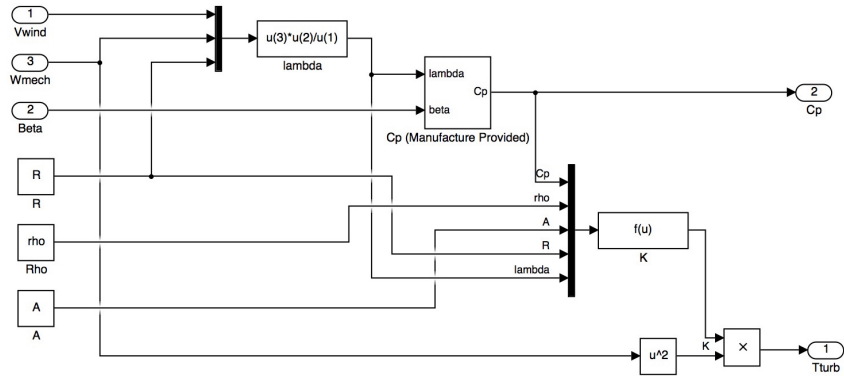


Figure 7: Model of the Studied Wind Turbine



applied to the DFIG is balanced sinusoidal three-phase voltage, which satisfies that

$$\begin{aligned}\bar{V}_a &= v_\phi^{rated} \angle 0^\circ \\ \bar{V}_b &= v_\phi^{rated} \angle 120^\circ \\ \bar{V}_c &= v_\phi^{rated} \angle -120^\circ\end{aligned}\quad (5.93)$$

where  $v_\phi^{rated}$  is the rated phase voltage. The applied rotor voltage is calculated from  $v_{rd}$  and  $v_{rq}$  obtained from the controller in Fig. 5. The gear ratio is set as

$$GearRatio = \frac{\omega_{mech}^{rated}}{\omega_{mech}^{opt}} \quad (5.94)$$

where  $\omega_{mech}^{rated} = (1 - s)\omega_{syn}/(p/2)$  is the rated mechanically rotational speed of the DFIG and  $s$  is the slip of the DFIG at full (rated) load, and  $\omega_{mech}^{opt}$  is obtained from the given  $C_p$  model at  $v_{wind} = 12$  m/s. The detail of the speed-squared controller is shown in Fig. 9.

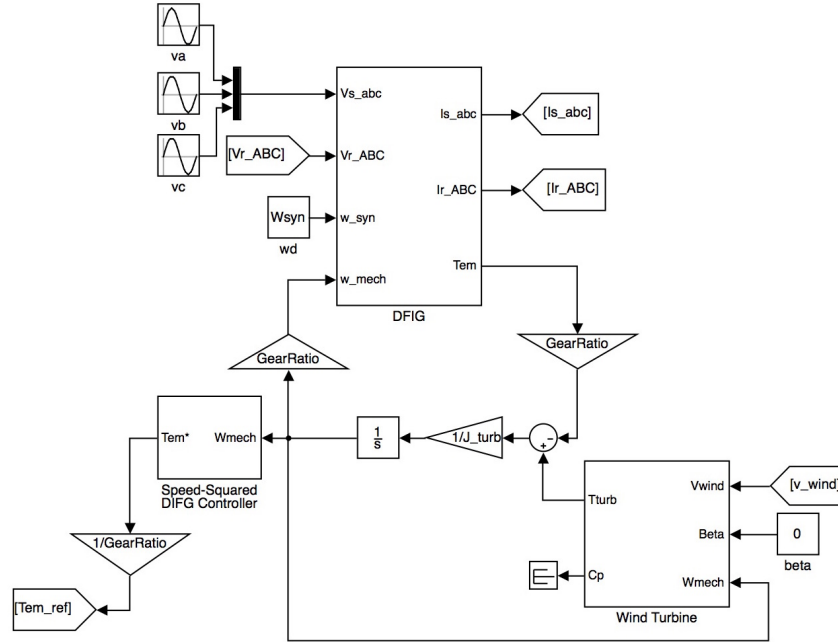


Figure 8: Combination of the DFIG and the wind turbine

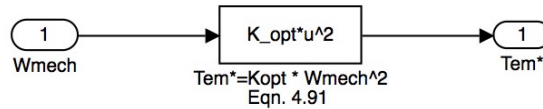


Figure 9: Model of the Speed-Squared Controller for the Wind Turbine

We employ Eqn. 3.71 to generate the reference signal of  $i_{rd}$  from  $T_{em}^*$  generated via the speed-squared controller. The detail of generating  $i_{rd}^*$  is shown in Fig. 10.

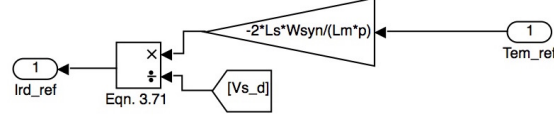


Figure 10: Model of Generating the Reference Signal  $i_{rd}^*$

## 5.2 Simulation and Numerical Analysis

### 5.2.1 Specifications of the Studied DFIG and Wind Turbine

The studied DFIG is based on an example provided in [2], while the studied wind turbine is based on a GE wind turbine. The involved specifications of the DFIG are listed in Table 1 and the specifications of the wind turbine are listed in Table 2.

Table 1: Specifications of the Studied DFIG

Rated Line Voltage (RMS Value)	690 V	$R_s$	2.0 m $\Omega$
Rated Frequency	$f_{syn} = 60$ Hz	$R_r$	1.5 m $\Omega$
Poles	$p = 6$	$X_{ls}$	50 m $\Omega$
Full Load Slip	$s = 1\%$	$X_{lr}$	47 m $\Omega$
Moment of Inertia	$J = 75$ kg·m <sup>2</sup>	$X_m$	860 m $\Omega$

Table 2: Specifications of the Studied Wind Turbine

Swept Area	$A = 3904$ m <sup>2</sup>	Rotor Diameter	$2R = 70.5$ m
Density of Air	$\rho = 1.2$ kg/m <sup>3</sup>	System Inertia	$J_{turb} = 2.4 \times 10^6$ kg·m <sup>2</sup>

### 5.2.2 Simulation for DFIG Controller Performance

In this simulation, we only exam the performance of the DFIG controller shown in Fig. 5 and do not combine the wind turbine with the DFIG. The simulation starts from the rated state of DFIG. Then the reference signal,  $i_{rd}^*$ , from the rated value of  $i_{rd}$  directly decreases to  $i_{rd}/2$  at time  $t = 0.1$  s; the reference signal,  $Q_s^*$ , from the rated value of  $Q_s$  directly decreases to 0 at time  $t = 0.5$  s.

The simulation results are shown in Fig. 11, 12 and 13.

As described in the last section, the control of  $i_{rd}$  is realized through directly controlling the reference signal  $i_{rd}^*$ , while the control of  $i_{rq}$  is realized through controlling the reference signal  $Q_s^*$ . From Fig. 11 and 12, we can spot that the controller described in Section 3.4 can effectively adjusting  $i_{rd}$  and  $i_{rq}$ . Besides, from the two figure, it can be recognized that although the change in one of  $i_{rd}$  and  $i_{rq}$  could cause the fluctuation of the other one, the fluctuation is very limited and thus the two quantities basically are decoupling. That is to say, we can control  $P_s$  and  $Q_s$  dependently through adjusting  $i_{rd}$  and  $i_{rq}$  respectively.

In Fig. 13, the actual value  $T_{em}$  is the electromagnetic torque generated by the DFIG; the

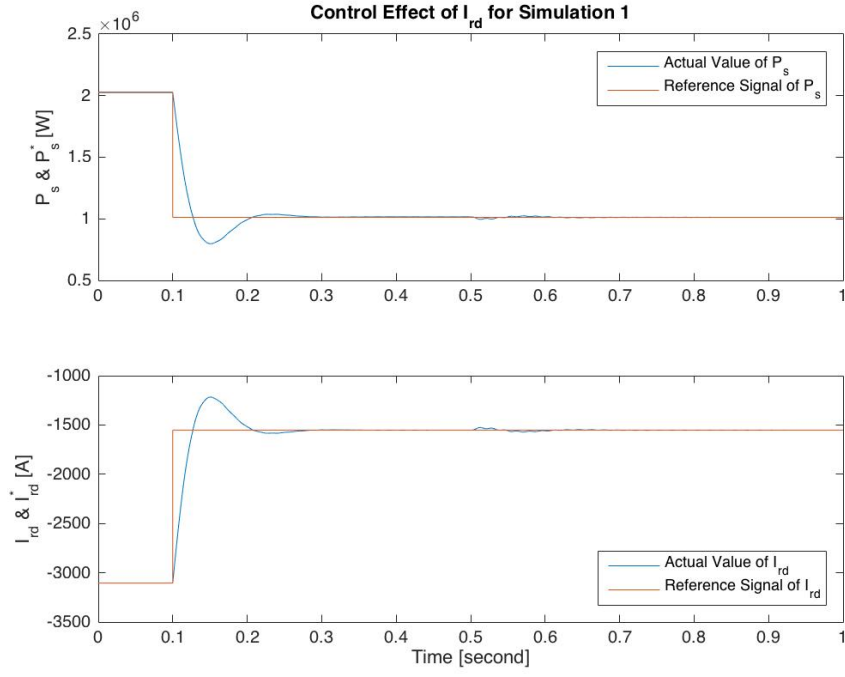


Figure 11: Simulation Results of the Control Effect of  $i_{rd}$  for Simulation 1

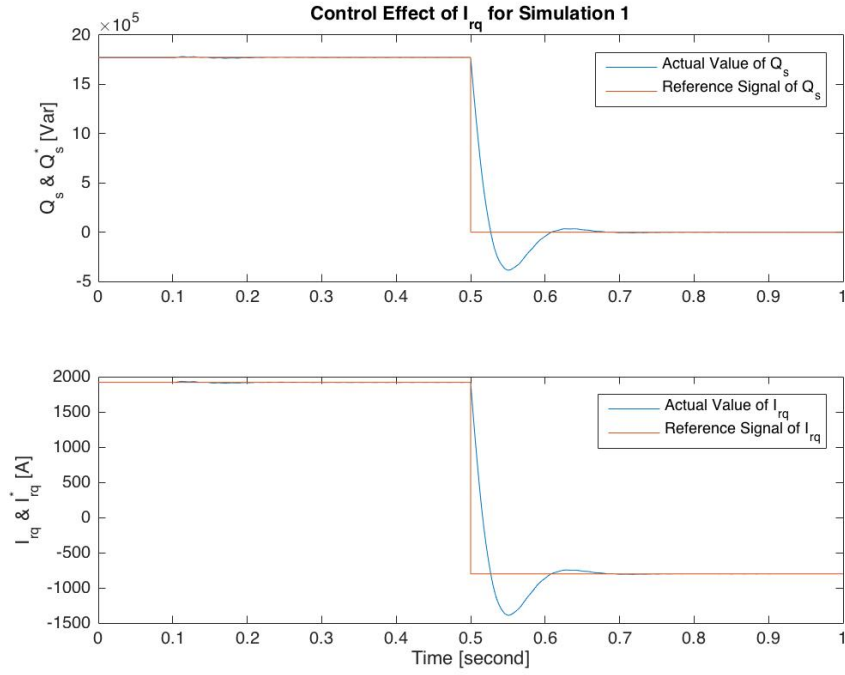


Figure 12: Simulation Results of the Control Effect of  $i_{rq}$  for Simulation 1

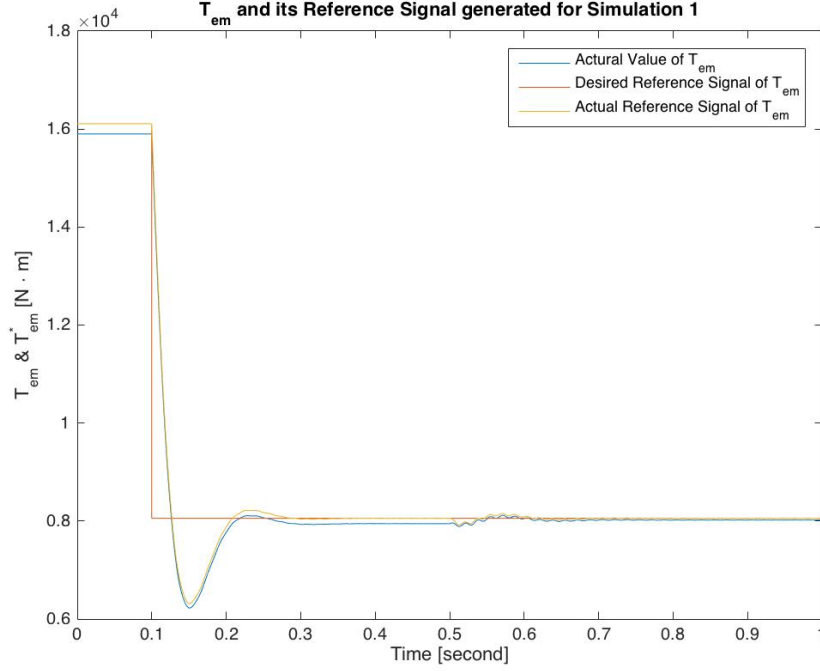


Figure 13: Simulation Results of the Control Performance of  $T_{em}$  for Simulation 1

desired reference signal is calculated via

$$T_{em,1}^* \approx -\frac{pL_m v_{sd}}{2\omega_d L_s} i_{rd}^* \quad (5.95)$$

which is the value that we hope  $T_{em}$  to be, while the actual reference signal is calculated via

$$T_{em,2}^* \approx -\frac{pL_m v_{sd}}{2\omega_d L_s} i_{rd} \quad (5.96)$$

which is the actual reference signal the model generated.

From Fig. 13, we can find that in steady state  $T_{em,1}^*$  is identical to  $T_{em,2}^*$  basically and that in dynamic state  $T_{em,2}^*$  is more close to  $T_{em,1}^*$  compared to  $T_{em}$ . This means the actually generated reference signal meet our desire. Besides, it can be spotted that there is a gap between the actual electromagnetic torque and the desired value. This is caused by the losses of approximation in the calculation of  $T_{em}^*$ .

### 5.2.3 Simulation for Speed-Squared Controller Performance

In this simulation, we assume that the DFIG always is able to generate the desired electromagnetic torque, i.e.  $T_{em,gen} = T_{em}^*$ , and thus we can evaluate the performance of the speed-squared controller for the wind turbine.

Fig. 14 and 15 respectively show  $C_p$  versus TSR  $\lambda$  and that versus  $\omega_{mech}$  at different wind speeds. From these two figures, we can point out  $C_p^{opt}$ ,  $\lambda^{opt}$ , and  $\omega_{mech}^{opt}$  at different wind speeds. The shape of the curve of  $C_p$  is consistent with our analysis in Section 4.1.

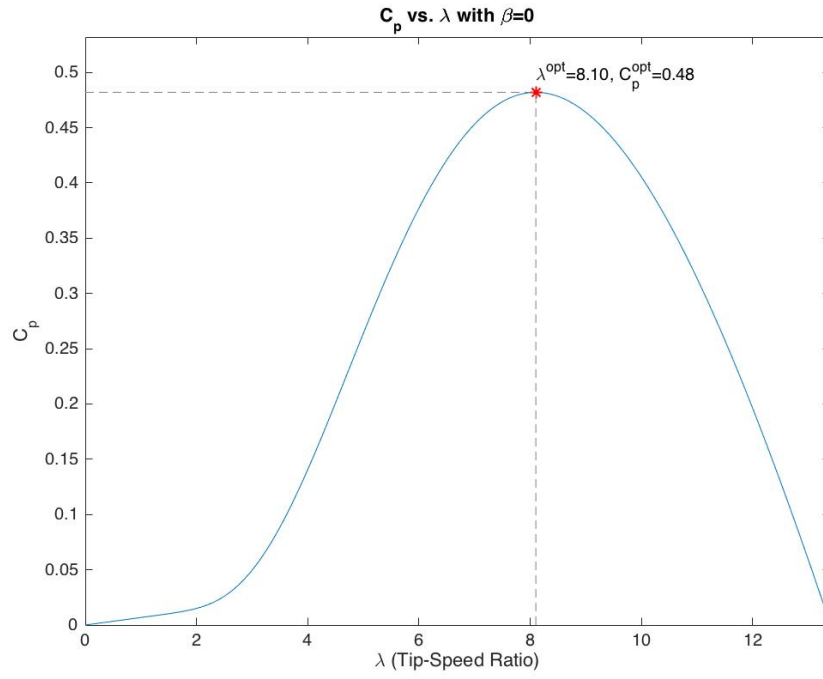


Figure 14: Curve of  $C_p$  versus  $\lambda$

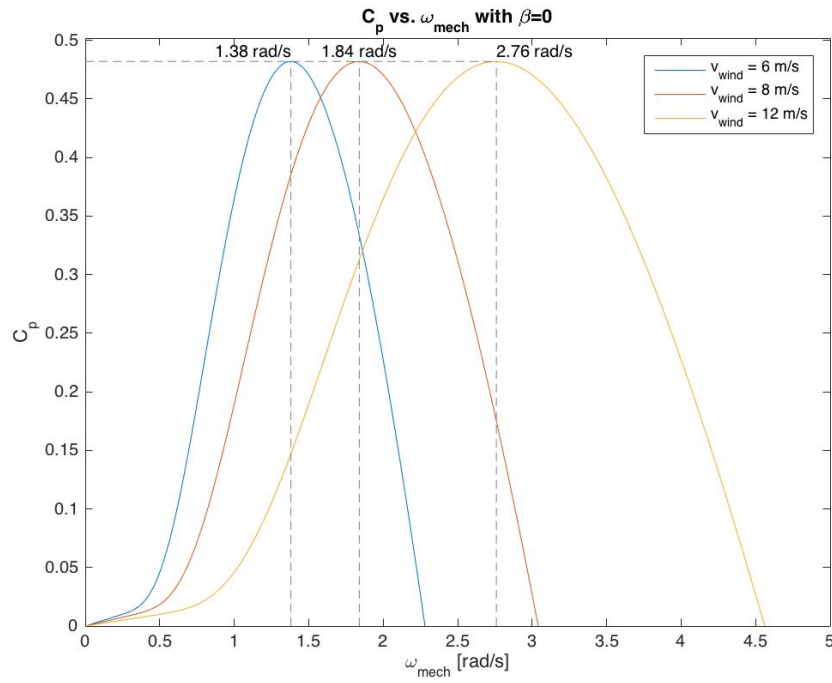


Figure 15: Curve of  $C_p$  versus  $\omega_{\text{mech}}$  at Different Wind Speeds

Fig. 16 shows the performance of the speed-squared controller. From the figure, we can find that the controller can effectively adjust the mechanically rotational speed  $\omega_{mech}$  to remain at the corresponding  $\omega_{mech}^{opt}$  as the change in  $v_{wind}$ .

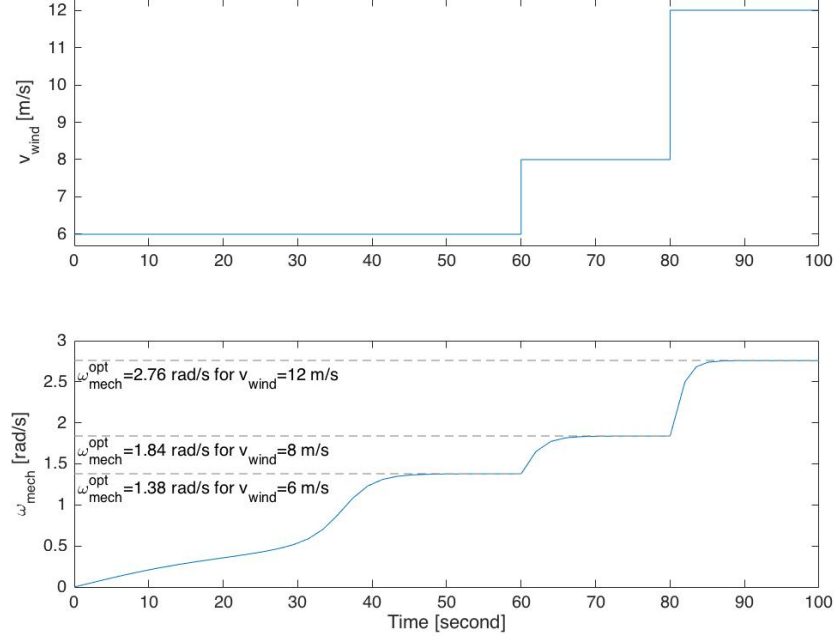


Figure 16: Simulation Results of the Control Performance of the Speed-Squared Controller for Simulation 2

#### 5.2.4 Simulation for the Combined DFIG and Wind Turbine with Certain Wind Speed and Controlled Reference Signal of $Q_s$

In this simulation, we combine the model of the DFIG and that of the wind turbine together as we described in the former sections.

During the simulation,  $v_{wind}$  is set to be 12 m/s. Given that the gear ratio  $GR$  is set according to  $\omega_{mech}^{opt}$  at  $v_{wind} = 12$  m/s, the mechanically rotational speed of the DFIG is expected to be its rated value  $\omega_{mech}^{rated}$ , while the mechanically rotational speed of the wind turbine is expected to be  $\omega_{mech}^{opt}$  at  $v_{wind} = 12$  m/s. From Fig. 15,  $\omega_{mech}^{opt} = 2.76$  rad/s when  $v_{wind} = 12$  m/s.

In the simulation,  $Q_s^{ref}$  decreases from the rated value  $Q_s^{rated}$  to 0 at time  $t = 30$  s and then increases from 0 to  $1.2Q_s^{rated}$  at time  $t = 60$  s.

The plot of  $Q_s$ ,  $i_{rd}$ ,  $i_{rq}$  and their reference signals are shown in Fig. 17. From the figure, it can be recognized that we can effectively generate the reference signal  $i_{rq}^*$  through adjusting the reference signal  $Q_s^*$  and that  $Q_s$  and  $i_{rq}$  can be controlled effectively such that they can effectively track their reference signals. On the other hand, we can find that the change in  $Q_s$  or say  $i_{rq}$  results in the fluctuation of  $i_{rd}$  as well as of the actual  $T_{em}$  (as shown in Fig. 18). The fluctuation

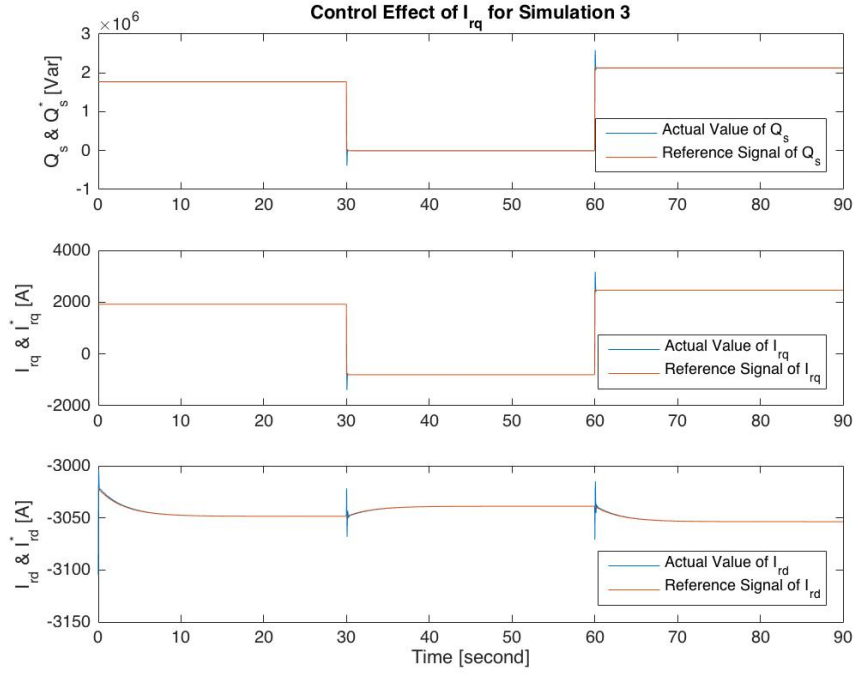


Figure 17: Simulation Results of the Control Effect of  $i_{rq}$  for Simulation 3

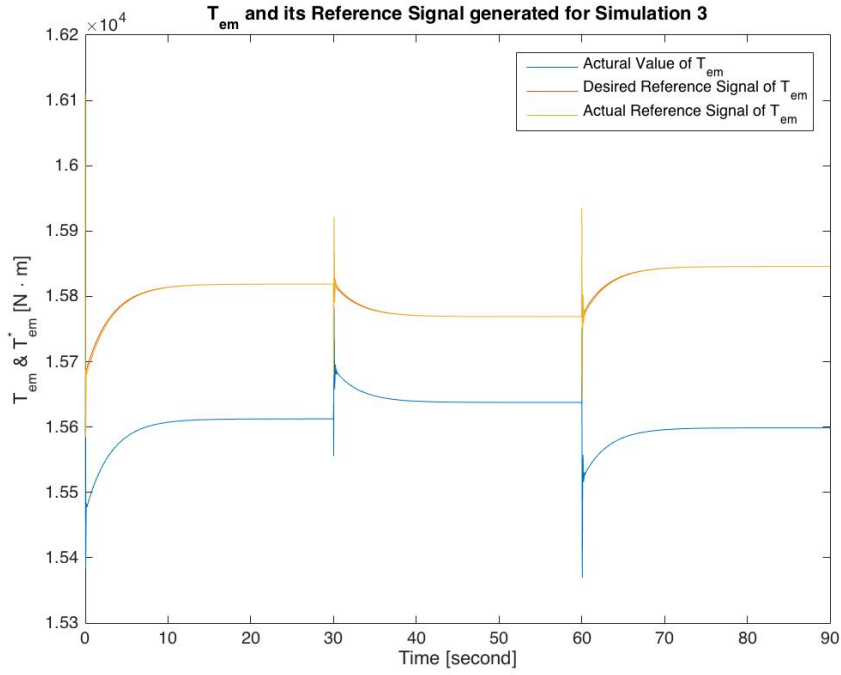


Figure 18: Simulation Results of the  $T_{em}$  and its Reference Signal  $T_{em}^*$  for Simulation 3

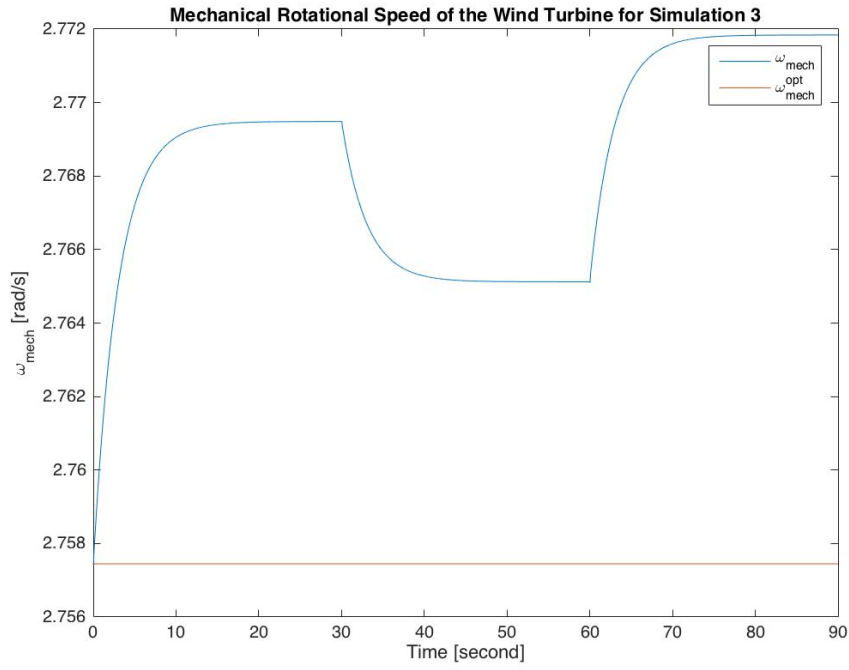


Figure 19: Simulation Results of the Mechanically Rotational Speed of the Wind Turbine for Simulation 3

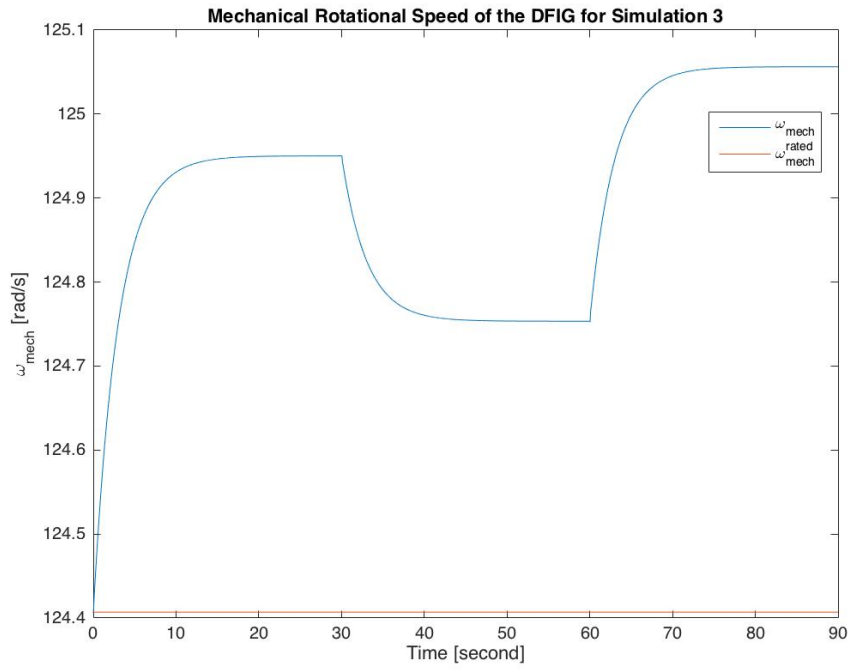


Figure 20: Simulation Results of the Mechanically Rotational Speed of the DFIG for Simulation 3



of  $T_{em}$  further leads to the change in the mechanically rotational speed of the wind turbine (as shown in Fig. 19) such that the speed-squared controller leads to the change in  $i_{rd}^*$  in order to keep the rotational speed at  $\omega_{mech}^{opt}$  for  $v_{wind} = 12$  m/s. The change of the wind turbine's mechanically rotational speed also is reflected by the mechanically rotational speed of the DFIG, as shown in Fig. 20.

From Fig. 19 and 20, we can find that the control system does not strictly keep the mechanically rotational speed of the DFIG or that of the wind turbine at its ideal values ( $\omega_{mech}^{rated}$  and  $\omega_{mech}^{opt}$  for  $v_{wind} = 12$  m/s respectively). However, the gap between the actual value of the ideal value is quite small and acceptable. Hence, the whole control system is still reliable.

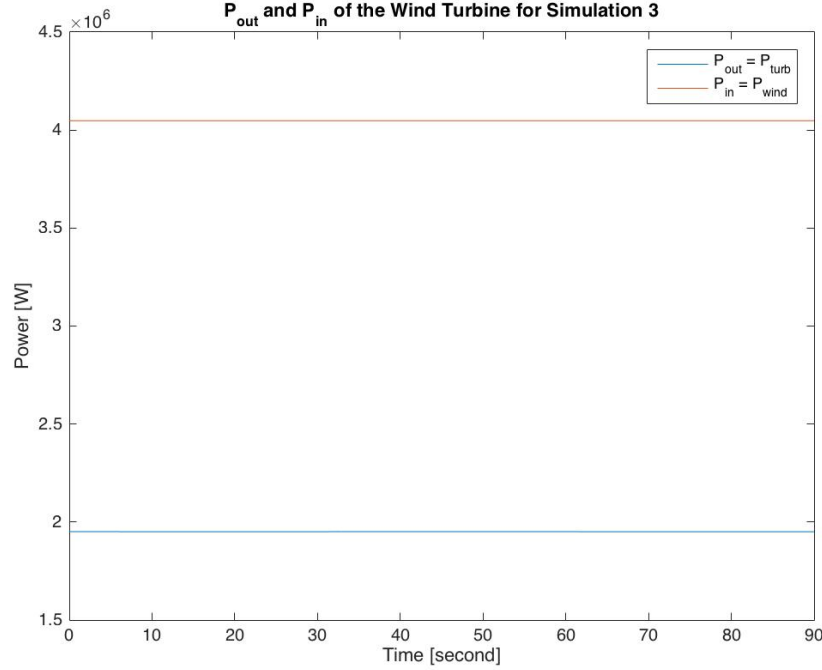


Figure 21: Simulation Results of the Input and Output Power of the Wind Turbine for Simulation 3

Fig. 21 shows the input and output power of the wind turbine. Since  $v_{wind}$  is constant in the simulation, the input power  $P_{wind}$  is constant as well. Given that the change in the mechanically rotational speed of the wind turbine is quite small,  $C_p$  is basically unchanged during the simulation so that there is no change in the output power  $P_{turb}$ .

Fig. 22 shows the real and reactive power of the stator and the rotor of the DFIG. From the figure, it can be spotted that  $P_r$  is around 0 while  $P = P_s + P_r$  is close to the output power of the wind turbine  $P_{out}$ .

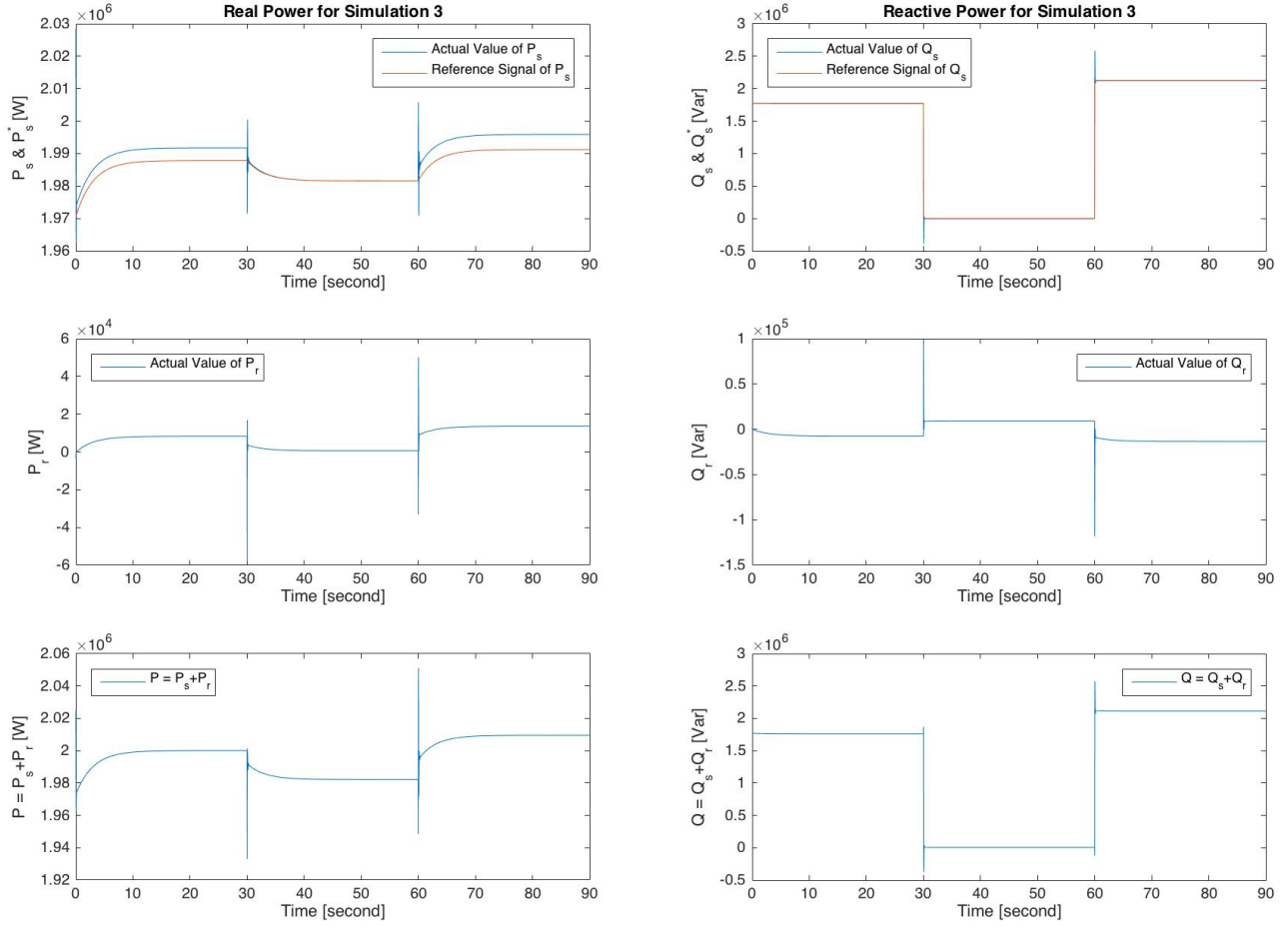


Figure 22: Simulation Results of the Real and Reactive Power of the DFIG for Simulation 3

### 5.2.5 Simulation for the Combined DFIG and Wind Turbine with Different Wind Speed

In this simulation,  $Q_s^*$  is set to be the rated value  $Q_s^{rated}$ , whereas the wind speed  $v_{wind}$  is changed. And, we hope the control system can adjust the mechanically rotational speed of the wind turbine,  $\omega_{mech}$ , such that the wind turbines can always run at  $\omega_{mech}^{opt}$ .

The initial value of  $v_{wind}$  is 12 m/s, and the DFIG and wind turbine both operate at the rated condition. Then  $v_{wind}$  decreases to 9 m/s at time  $t = 30$  s, and then decreases to 6 m/s at time  $t = 60$  s.

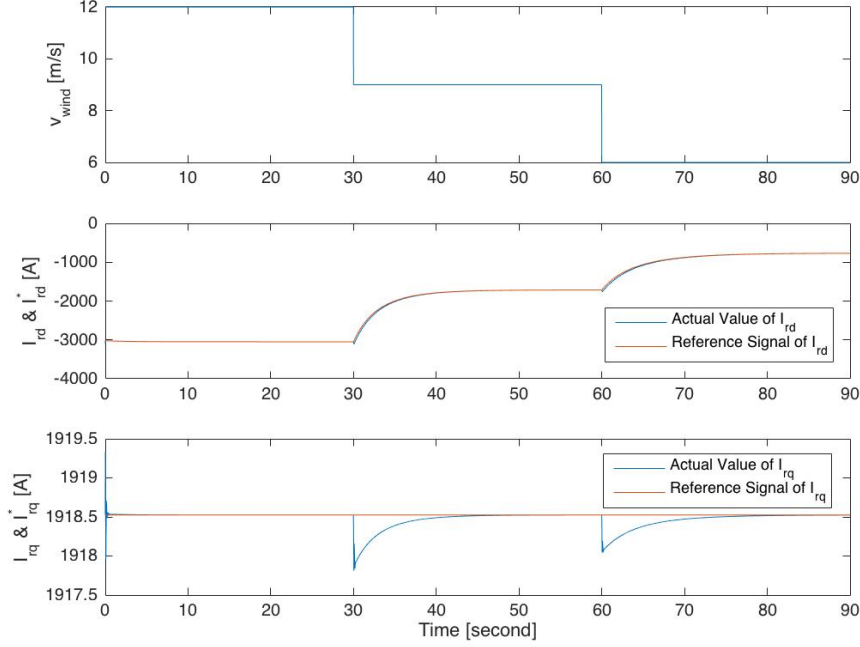


Figure 23: Simulation Results of the Control Effect of  $i_{rd}$  for Simulation 4

From Fig. 23, we can spot that as the change in  $v_{wind}$ , our control system can rapidly respond and generate the reference signal  $i_{rd}$  to control the DFIG's  $T_{em}$  (as shown in Fig. 24) and further to control the mechanically rotational speed of the wind turbine (as shown in Fig. 25).

Although the mechanically rotational speed of the wind turbine is controlled to keep at  $\omega_{mech}^{opt}$  according to the different wind speed, as shown in Fig. 25, the mechanically rotational speed of the DFIG is not always keep at its rated value  $\omega_{mech}^{rated}$ , as shown in Fig. 26. Given that the gear ratio is set according to the ratio of  $\omega_{mech}^{rated}$  and  $\omega_{mech}^{opt}$  at  $v_{wind} = 12$  m/s, only when  $v_{wind} = 12$  m/s can the mechanically rotational speed of the DFIG be kept at or say close to  $\omega_{mech}^{opt}$ .

Besides, from Fig. 26, the mechanically rotational speed of the DFIG change dramatically as the wind speed changes. Due to the characteristic of the DFIG, DFIG can run in a sufficiently wide speed range so that it usually is adaptable to the dramatic change in the mechanically rotational speed.

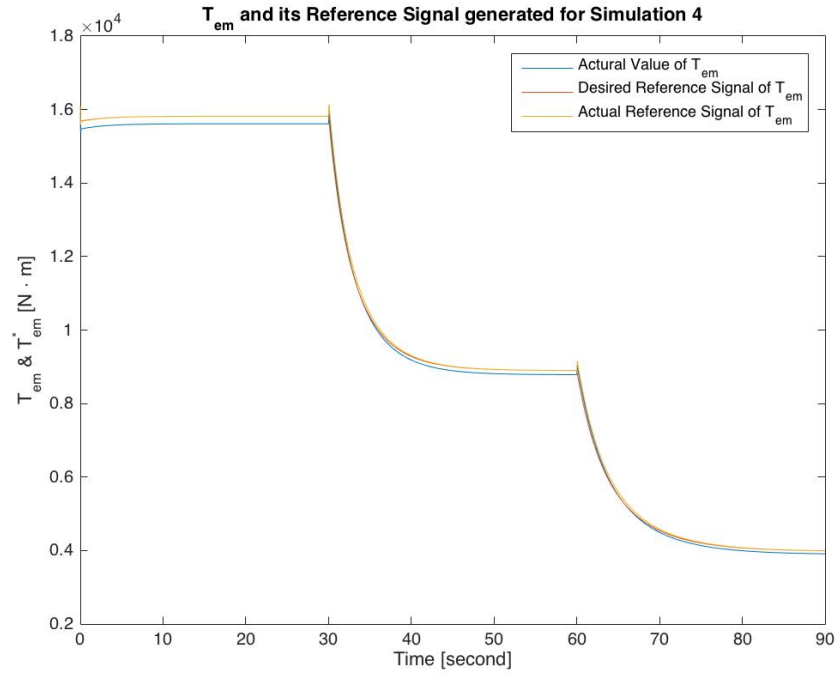


Figure 24: Simulation Results of the  $T_{em}$  and its Reference Signal  $T_{em}^*$  for Simulation 4

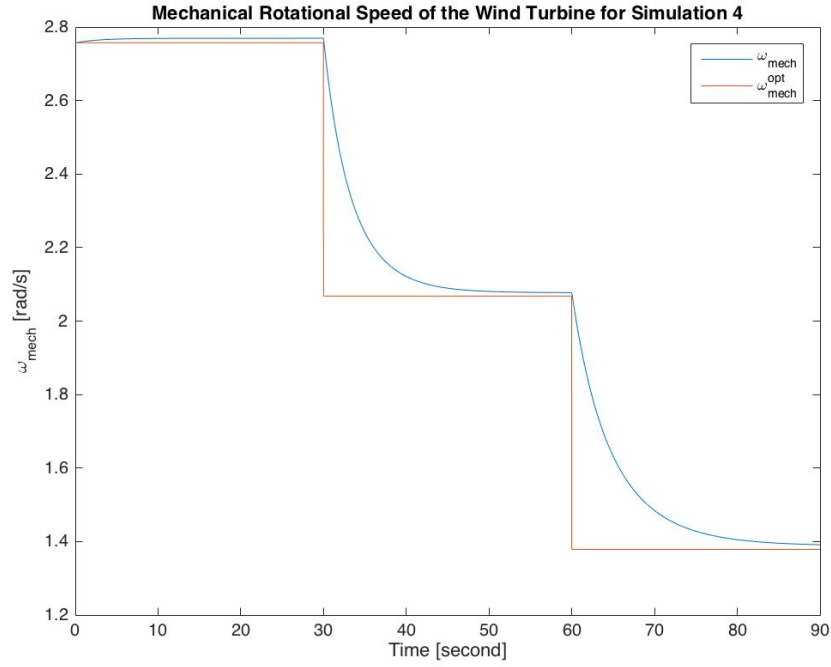


Figure 25: Simulation Results of the Mechanically Rotational Speed of the Wind Turbine for Simulation 4

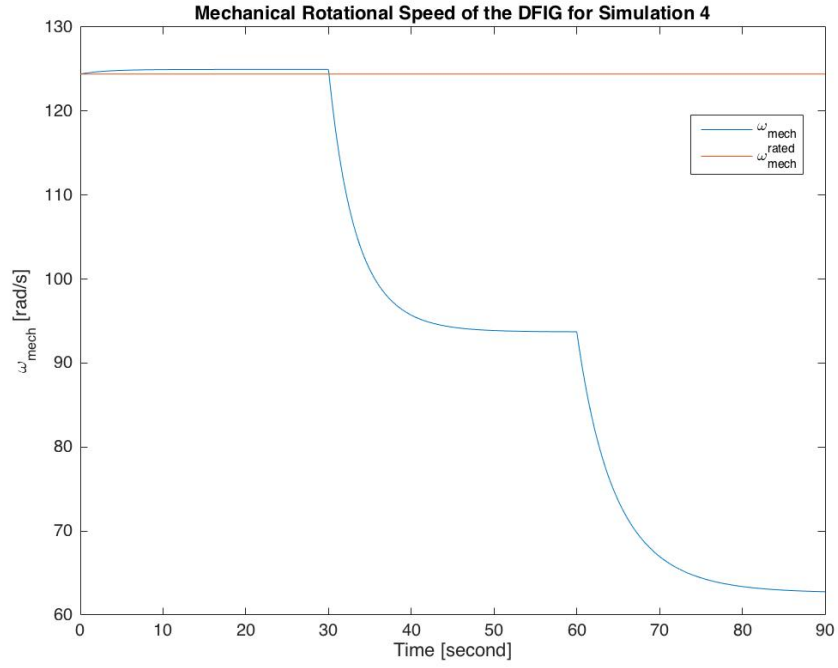


Figure 26: Simulation Results of the Mechanically Rotational Speed of the DFIG for Simulation 4

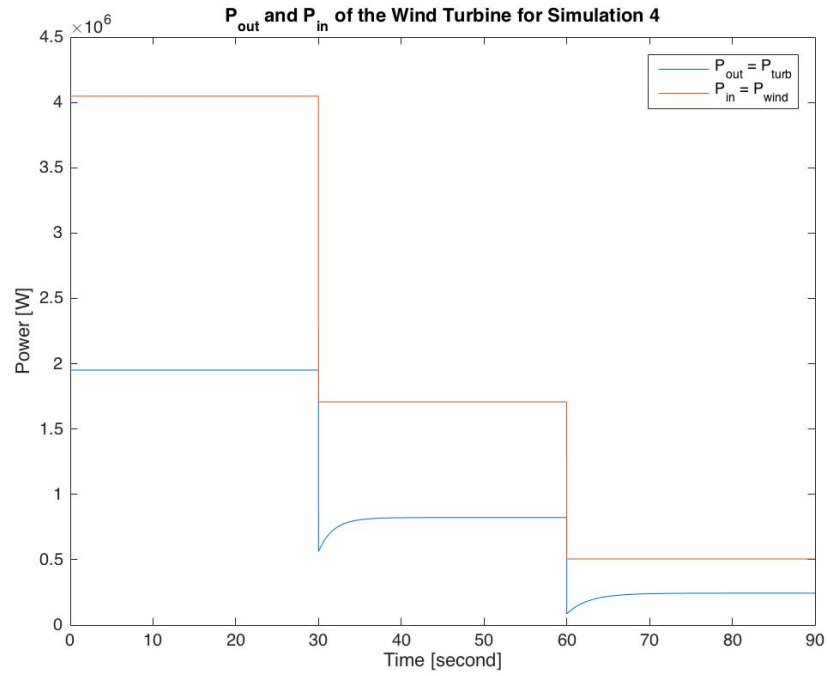


Figure 27: Simulation Results of the Input and Output Power of the Wind Turbine for Simulation 4

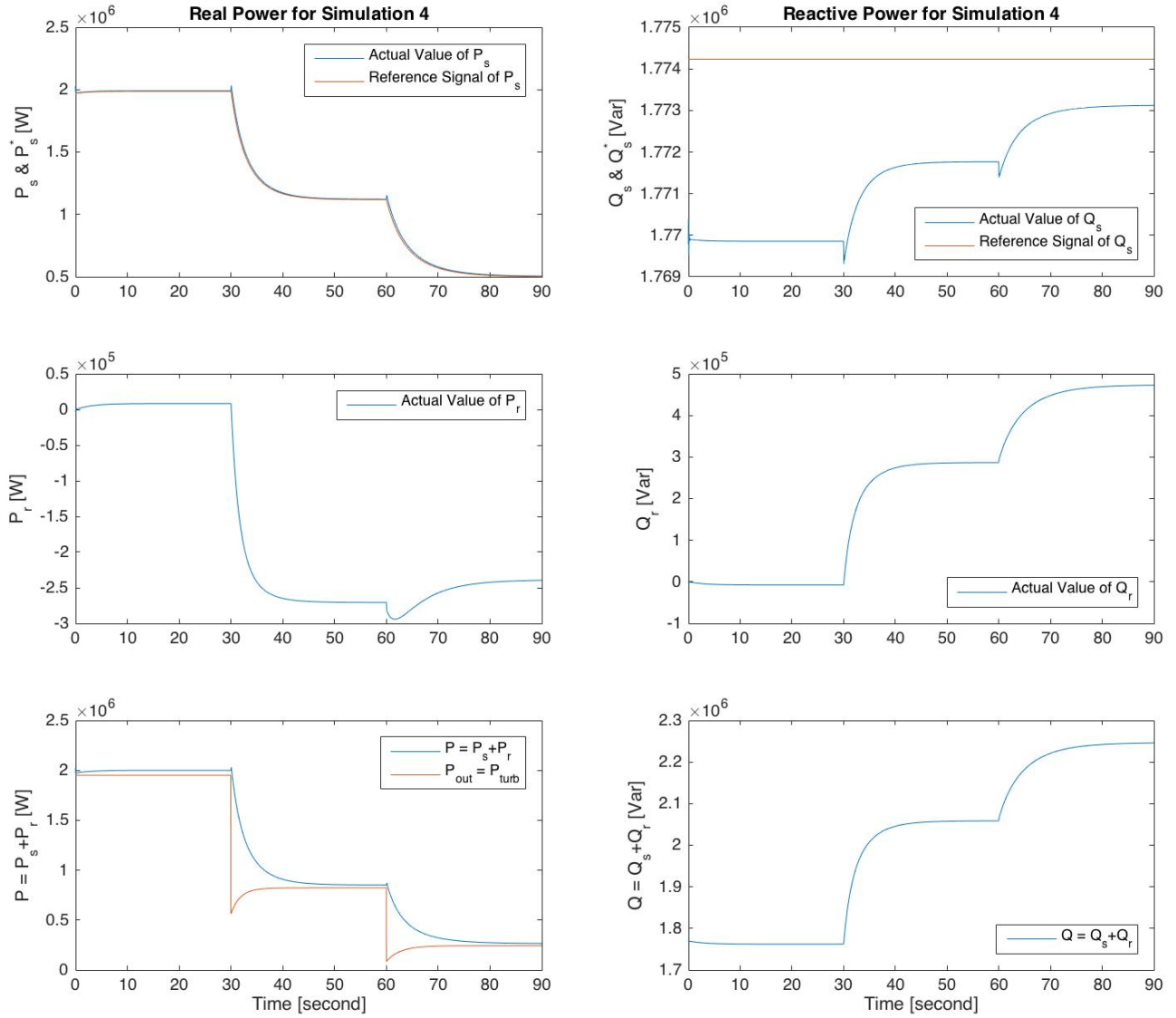


Figure 28: Simulation Results of the Real and Reactive Power of the DFIG for Simulation 4

Fig. 27 shows the input and output power of the wind turbine. Fig. 28 shows the real and reactive power of the DFIG. From the 5th plot in Fig. 28, we can find that in steady state  $P_{turb} \approx P_s + P_r$ . This demonstrates that the power outputted by the wind turbine is consumed by both of rotor and stator.

## 6 Conclusion and Discussion

In this project, the model of doubly fed induction generator (DFIG) and wind turbine in  $dq$  domain and the model of the related controllers are built up in theory as well as in MATLAB/Simulink. From simulation, it is proven that the designed controllers can effectively control DFIG and also can automatically adjust the rotor voltage applied to the DFIG and thus make the wind turbine operate at a optimal condition.

From the analysis in Section 5.2.5, we find that

$$P_{turb} \approx P_s + P_r \quad (6.97)$$

where  $P_{turb}$  is the output real power of the wind turbine and  $P_s$  and  $P_r$  are the real power of the DFIG's stator and rotor respectively. That is to say, the wind turbine's output power is consumed by both of stator and rotor.

From the analysis in Section 5.2.2, although the change in  $Q_s$  could make  $i_{rd}$  fluctuate and thus cause some fluctuation of  $P_s$  and  $T_{em}$  who are associated with  $i_{rd}$  and some other constants, the fluctuation is very small and these quantities would recover rapidly. Similarly, although the change in  $i_{rd}$  could make  $i_{rq}$  as well as  $Q_s$  fluctuate, the influence is quite small and  $Q_s$  can be effectively modulated through the reference signal we give. This conclusion also is demonstrate via Fig. 12, 17 22 and 28 in which the actual value of  $Q_s$  can effectively follow the reference signal  $Q_s^*$ .

From Fig. 19 and Fig. 25, although with the control system, the wind turbine's mechanically rotational speed could be close to the optimal speed  $\omega_{mech}^{opt}$ , which is decided by the wind speed  $v_{wind}$ , there still is a gap between the ideal  $\omega_{mech}^{opt}$  and the actual  $\omega_{mech}$ . This gap is mainly caused by our neglect of  $R_s$ ,  $R_r$ ,  $L_{lr}$  and  $L_{ls}$  in the process of modeling the DFIG. The actual  $\omega_{mech}$  is a little bigger than the ideal  $\omega_{mech}^{opt}$ . For  $v_{wind} = 12$  m/s,  $\omega_{mech} = 2.769$  rad/s in steady state; for  $v_{wind} = 9$  m/s,  $\omega_{mech} = 2.078$  rad/s in steady state; and for  $v_{wind} = 6$  m/s,  $\omega_{mech} = 1.393$  rad/s in steady state. These data are obtained from Fig. 25.

When the mechanically rotational speed is the rated speed  $\omega_{mech}^{rated}$ , the slip of the DFIG is its rated value or the value with full load, i.e. 0.01. However, since the gear ratio is set to be the ratio of  $\omega_{mech}^{rated}$  to the optimal mechanically rotational speed of the wind turbine at  $v_{wind} = 12$  m/s, only when the mechanically rotational speed of the wind turbine is the  $\omega_{mech}^{opt}$  at  $v_{wind} = 12$  m/s can the slip is its rated value. However, due to the existence of the gap between the mechanically rotational speed of the wind turbine and the optimal speed (as mentioned in the last paragraph), the slip is smaller than 0.01 even if  $v_{wind} = 12$  m/s. For  $v_{wind} = 12$  m/s, the slip is 0.005; for  $v_{wind} = 9$  m/s, the slip is 0.25; and for  $v_{wind} = 6$  m/s, the slip is 0.50. These data can be calculated from the mechanically rotational speed of the wind turbine or that of the DFIG.

Besides, as our analysis in Section 3.2,  $v_{sd}$  should be a constant and  $v_{sq}$  should be zero. However, in our model, there exists oscillation of  $v_{sd}$  and  $v_{sq}$ , as shown in Fig. 29. Although the amplitude

of the oscillation is quite small, I wonder if it could influence the control effect on the DFIG. The reason causing the oscillation and the influence of the oscillation may be focused on in the next phase of learning.

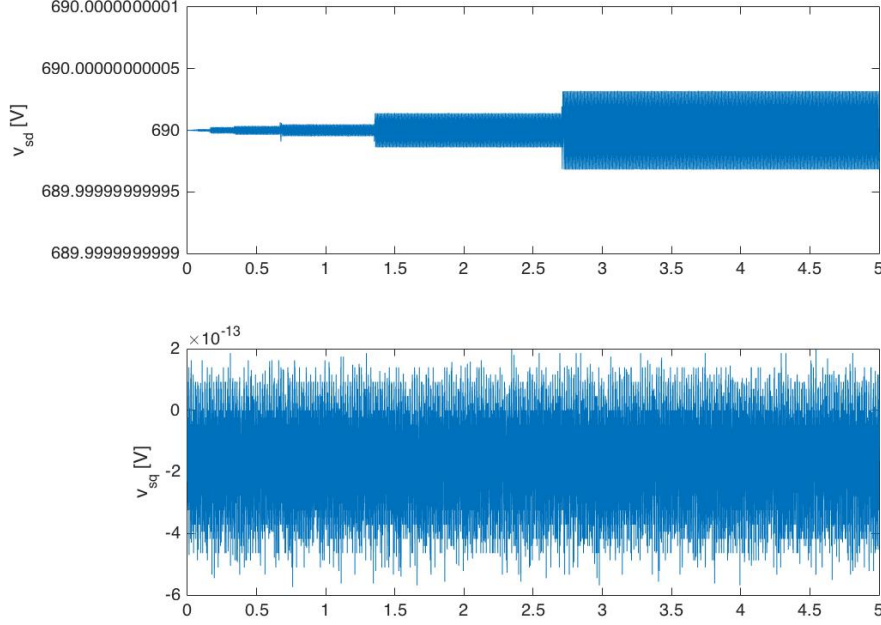


Figure 29:  $v_{sd}$  and  $v_{sq}$  Generated

Additionally, as our analysis in Section 5.2.4 and 5.2.5, there is a gap between our desired reference of the reference signal  $T_{em}^*$  generated through adjusting the reference signal  $i_{rd}^*$ , as shown in Fig. 18 and 24. This gap exists due to that  $R_s$ ,  $R_r$ ,  $L_{ls}$  and  $L_{lr}$  are not really zero and thus that there in fact is no strictly proportional relationship between  $T_{em}$  and  $i_{rd}$ . In summary, although our control system cannot accurately control the electromagnetic torque of the DFIG as well as the mechanically rotational speed of the wind turbine, the control effect is acceptable. Therefore, it is effective to use the controller designed according to the approximation relation between  $T_{em}$  and  $i_{rd}$  and that between  $Q_s$  and  $i_{rq}$ .

## References

- [1] N. Mohan, *Electric Machines and Drives*. 2012.
- [2] N. Mohan, *Advanced Electric Drives: Analysis, Control, and Modeling Using MATLAB/Simulink®*. 2014.