CHROMOSCALE

Unique Language for Sounds Colors and Numbers

Themes:

Elementary arithmetic
Base 7, The Harmonic Base
Chromatic Numbers in Base 7
Unique Language in Chromatic Scale
Sound Color Number Wheels
Chromatic Numbers between 0 to 10
Complementary Sounds Colors and Numbers
Additive and Subtractive Sounds and Colors
Chromatic Geometry
CHROMOSCALE - Cyclic Order of Chromotones
CHROMOSCALE - Chromotone Fractions
Circle of Fifths
CHROMOSCALE Harmonics and Full MIDI Note Range
Sound & Color APP

Attached files:

“Hexagonal Lattice” and “Colour Ramping for Data Visualisation”
Prof. Paul Bourke – IVEC@UVA

“An Orthogonal Oriented Quadrature Hexagonal Image Pyramid”
Prof. Andrew B. Watson – Ames Research Center

Paolo Di Pasquale
Lighting Designer
Rome, Italy

chromoscale @ gmail.com

Elementary Arithmetic in BASE 10

Used since the time of the Sumerians, the Base 10 System born because the man it found simplifies the calculations through the hands. Historically other number base systems have been used, but Humans insist on using Base 10 because it is the most convenient for ten fingered beings. Base 10 is the international standard of today and represents what is necessary in the mathematical language to every action of our life.

I think it is unnecessary for anyone, but for starters, I have to say that the numbers of Base 10 are: 0, 1, 2, 3 , 4, 5, 6, 7, 8 and 9.

The more common operations are :

+ ADDITION
- SUBTRACTION
X MULTIPLICATION
÷ DIVISION

In any case, the most simple operations as the most complex and articulated can be developed with whichever numerical Base. These operations can be used in Base 10 or in any different Base systems (binary, octal, hexadecimal, etc.)

This is the numerical table from 1 to 100 of BASE 10

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Adding the numbers that compose BASE 10 obtains the following Multiplication Times Table

1	2	3	4	5	6	7	8	9	0
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

In this second Table are the numbers marked in yellow on the Table on the left

1	2	3	4	5	6	7	8	9	0
2	4	6	8	0	2	4	6	8	0
3	6	9	2	5	8	1	4	7	0
4	8	2	6	0	4	8	2	6	0
5	0	5	0	5	0	5	0	5	0
6	2	8	4	0	6	2	8	4	0
7	4	1	8	5	2	9	6	3	0
8	6	4	2	0	8	6	4	2	0
9	8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	0	0

In the yellow Table

1 is present 4 times
2 is present 12 times
3 is present 4 times
4 is present 12 times
5 is present 9 times
6 is present 12 times
7 is present 4 times
8 is present 12 times
9 is present 4 times
0 is present 8 times (inside)

In order to understand better, we begin to analyze the position of numbers in the previous Yellow Table.
We note that on the centre of the Table there is the number 5

1	2	3	4	5	6	7	8	9	0
2	4	6	8	0	2	4	6	8	0
3	6	9	2	5	8	1	4	7	0
4	8	2	6	0	4	8	2	6	0
5	0	5	0	5	0	5	0	5	0
6	2	8	4	0	6	2	8	4	0
7	4	1	8	5	2	9	6	3	0
8	6	4	2	0	8	6	4	2	0
9	8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	0	0

In this Table (the same of previous Yellow Table) there are numbers presents in different quantity as follow :

1=4 times 2=12 times 5=9 times
3=4 times 4=12 times 0=8 times
7=4 times 6=12 times
9=4 times 8=12 times

1	2	3	4	5	6	7	8	9	0
2	4	6	8	0	2	4	6	8	0
3	6	9	2	5	8	1	4	7	0
4	8	2	6	0	4	8	2	6	0
5	0	5	0	5	0	5	0	5	0
6	2	8	4	0	6	2	8	4	0
7	4	1	8	5	2	9	6	3	0
8	6	4	2	0	8	6	4	2	0
9	8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	0	0

Wherever you trace the opposite diagonal lines within the Table you get the same sequence of numbers.

Examples:
see diagonals of 5-2-1-2-5
see diagonals of 7-2-5-6-5-2-7
see diagonals of 3-8-5-4-5-8-3

1	2	3	4	5	6	7	8	9	0
2	4	6	8	0	2	4	6	8	0
3	6	9	2	5	8	1	4	7	0
4	8	2	6	0	4	8	2	6	0
5	0	5	0	5	0	5	0	5	0
6	2	8	4	0	6	2	8	4	0
7	4	1	8	5	2	9	6	3	0
8	6	4	2	0	8	6	4	2	0
9	8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	0	0

As indicated above, the numerical sequence of vertical and horizontal lines are composed with the same numbers. This rule does not apply to number 5 which is coupled and accompanied with the 0.

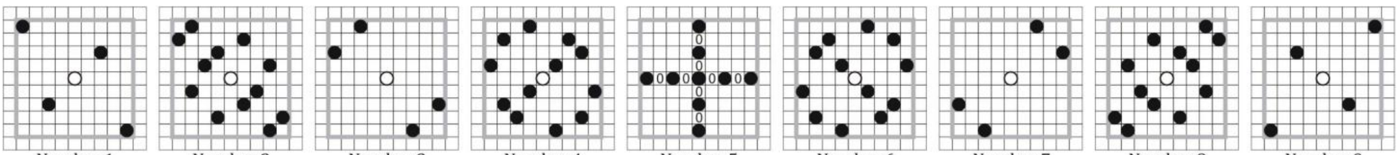
Example:
see lines of 3-6-9-2-5-8-1-4-7

1	2	3	4	5	6	7	8	9	0
2	4	6	8	0	2	4	6	8	0
3	6	9	2	5	8	1	4	7	0
4	8	2	6	0	4	8	2	6	0
5	0	5	0	5	0	5	0	5	0
6	2	8	4	0	6	2	8	4	0
7	4	1	8	5	2	9	6	3	0
8	6	4	2	0	8	6	4	2	0
9	8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	0	0

Number 5 and number 0 are not balanced as the other numbers. Their characteristic breaks the equilibrium of numbers inside this magic table belonging to the Base 10.

Number 5 is 9 times present
Number 0 is 8 times present (inside)

Now we explore the graphic position of numbers on the previous yellow table



Number 1 Number 2 Number 3 Number 4 Number 5 Number 6 Number 7 Number 8 Number 9

The positions of number 1 are equals but opposites to the positions of number 9
The positions of number 2 are equals but opposites to the positions of number 8
The positions of number 3 are equals but opposites to the positions of number 7
The positions of number 4 are equals but opposites to the positions of number 6
The positions of number 5 are different from those filled by the number 0

BASE 7

The Base 7 system is composed with the numbers 0 , 1 , 2 , 3 , 4 , 5 , 6

This is the table from 1 to 100 in BASE 7

1	2	3	4	5	6	10
11	12	13	14	15	16	20
21	22	23	24	25	26	30
31	32	33	34	35	36	40
41	42	43	44	45	46	50
51	52	53	54	55	56	60
61	62	63	64	65	66	100

Adding the numbers that compose BASE 7 obtains the following Multiplication Times Table

1	2	3	4	5	6	10
2	4	6	11	13	15	20
3	6	12	15	21	24	30
4	11	15	22	26	33	40
5	13	21	26	34	42	50
6	15	24	33	42	51	60
10	20	30	40	50	60	100

In this second Table are the numbers marked in yellow on the Table on the left

1	2	3	4	5	6	0
2	4	6	1	3	5	0
3	6	2	5	1	4	0
4	1	5	2	6	3	0
5	3	1	6	4	2	0
6	5	4	3	2	1	0
0	0	0	0	0	0	0

In the yellow Table

1 is present 6 times
2 is present 6 times
3 is present 6 times
4 is present 6 times
5 is present 6 times
6 is present 6 times
six numbers six times presents

In order to understand better, we begin to analyze the position of numbers in the previous Yellow Table.

1	2	3	4	5	6	0
2	4	6	1	3	5	0
3	6	2	5	1	4	0
4	1	5	2	6	3	0
5	3	1	6	4	2	0
6	5	4	3	2	1	0
0	0	0	0	0	0	0

In this Table (the same of previous Yellow Table) there are numbers presents in the same amount

1=6 times   2=6 times   3=6 times
4=4 times   5=6 times   6=6 times

number 0 is only on the perimeter

1	2	3	4	5	6	0
2	4	6	1	3	5	0
3	6	2	5	1	4	0
4	1	5	2	6	3	0
5	3	1	6	4	2	0
6	5	4	3	2	1	0
0	0	0	0	0	0	0

Wherever you trace the opposite diagonal lines within the Table you get the same sequence of numbers.

Examples:
see diagonals of 2-2
see diagonals of 3-1-1-3
see diagonals of 5-1-2-1-5

1	2	3	4	5	6	0
2	4	6	1	3	5	0
3	6	2	5	1	4	0
4	1	5	2	6	3	0
5	3	1	6	4	2	0
6	5	4	3	2	1	0
0	0	0	0	0	0	0

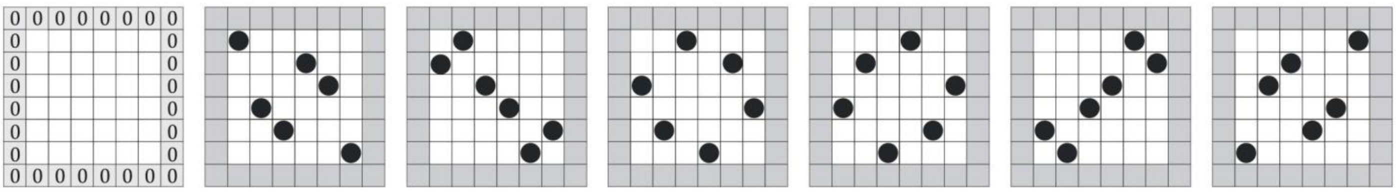
As indicated above, the numerical sequence of vertical and horizontal lines are composed with the same numbers.

Example:
see lines of 3-6-2-5-1-4

1	2	3	4	5	6	0
2	4	6	1	3	5	0
3	6	2	5	1	4	0
4	1	5	2	6	3	0
5	3	1	6	4	2	0
6	5	4	3	2	1	0
0	0	0	0	0	0	0

Number 0 is not present inside the Table.

Now we explore the graphic position of numbers on the previous yellow table



Number 0 Number 1 Number 2 Number 3 Number 4 Number 5 Number 6

The positions of number 1 are equals but opposites to the positions of number 6
The positions of number 2 are equals but opposites to the positions of number 5
The positions of number 3 are equals but opposites to the positions of number 4
Number 0 is not present inside, but only on the perimeter

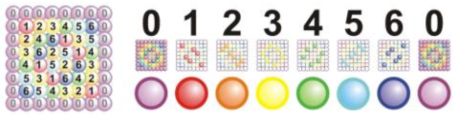
Same characteristics and harmonies all ready noticed in Base 10.
But inside the yellow table, using a Base 7 composed from 0, 1, 2, 3 , 4 , 5 , 6, we have this splendid result :
Six numbers presents , six times present, while the 0 is present only on the external perimeter, nearly to form a line of border.

This is the harmonic and magnificent numerical equilibrium of Base Seven.

BASE 7

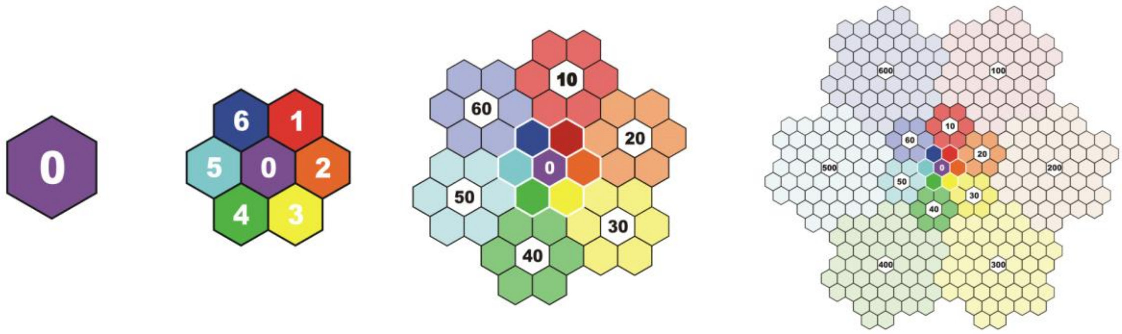
The Harmonic and chromatic Base

Numerical Table of Base 7 associating numbers with the seven colors of the rainbow.



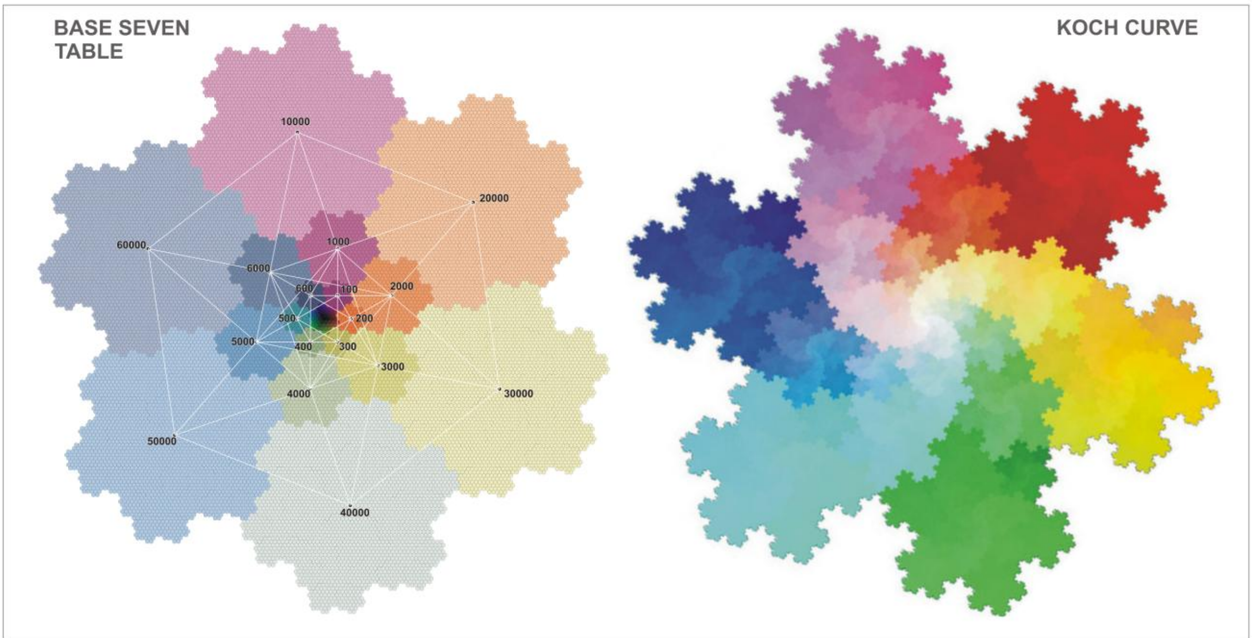
Hexagonal Table of Base 7 associating numbers with colors.
System where the numbers will be chromatic with the Base 7 characteristics and harmonies.

The hexagon has many interesting properties, and the result is a polygonal table constructible with elementary geometry.



Number 0 **10 Numbers** **100 Numbers** **1000 Numbers**

We obtain a shape of hexagram similar to the Koch Snowflake



about Hexagonal Table in Base 7 see also:

page 15 – 19
“Hexagonal Lattice”
by Prof. Paul Bourke – IVEC@UWA
http://paulbourke.net/texture_colour/

page 22 - 40
NASA
“An Orthogonal Oriented Quadrature Hexagonal Image Pyramid”
by Prof. Andrew B. Watson – Ames Research Center
http://vision.arc.nasa.gov/publications/OrthogonalHexagonal.pdf

Unique Language in “Chromatic Scale”

Correspondence of Sounds, Colors and Numbers in Base 7

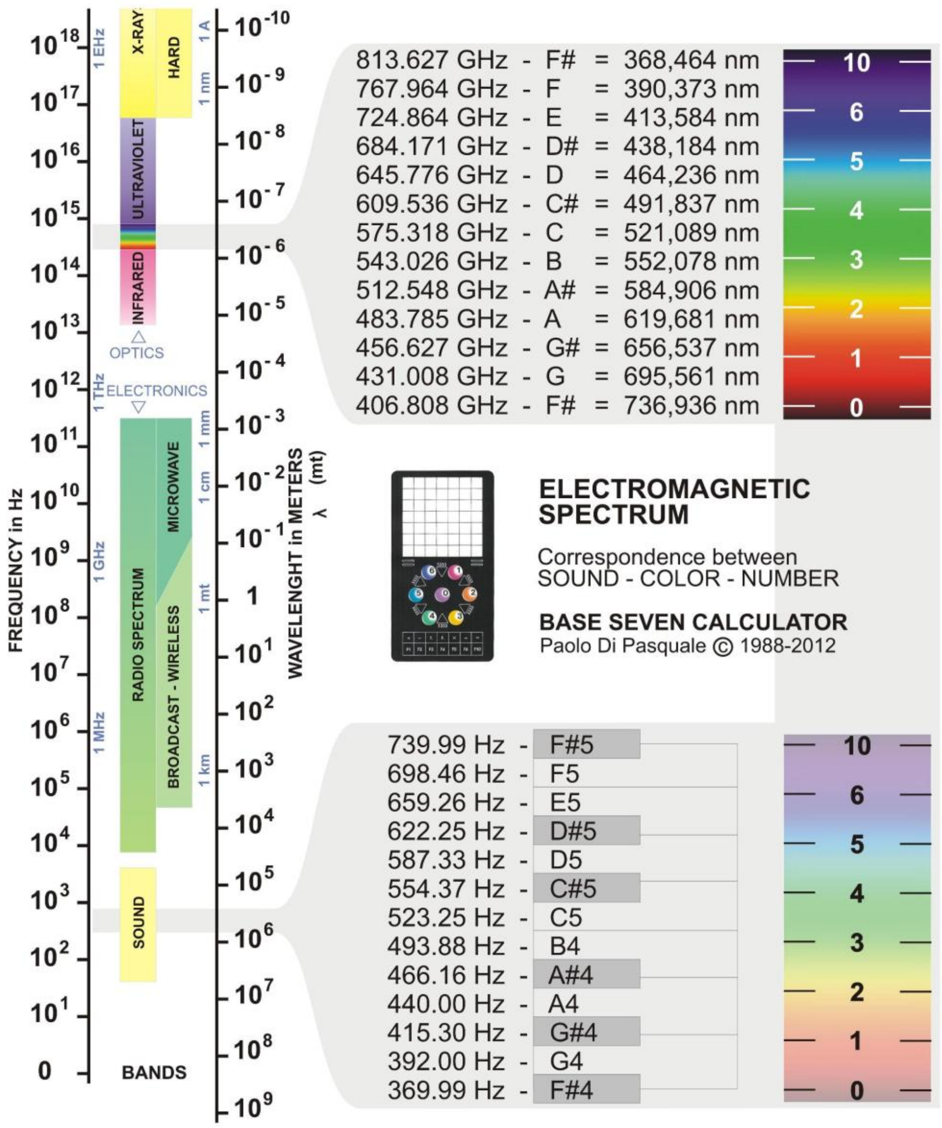
It should be emphasized that to pass from one NOTE to the next octave of the same NOTE is enough to double the frequency in Hz. Theoretically, if we double the frequency of the INFRARED we will get ULTRAVIOLET frequency.

Based on this principle, we proceed by multiplying 2 of the frequency in Hertz of a TONE until we get close to values in GHz of the visible spectrum. Furthermore, if we multiply by 2^40 the frequency of 369,994 Hz of F# we obtain 406.813 GHz, frequency that corresponds at the border-line of the INFRARED.

We proceed with this system for all the TONES that make up the "Chromatic Scale" by completing the following Table:

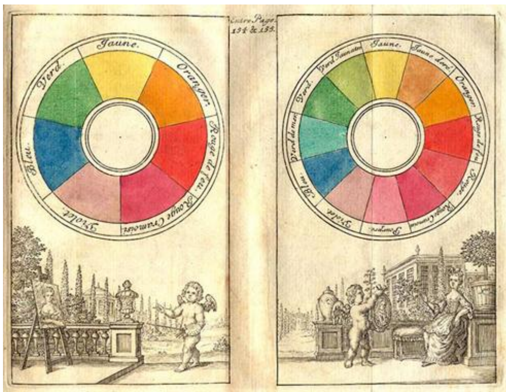
SOUND COLOR CORRESPONDENCE CHART

FA#	F#	369,994 Hz.	x	2^40	=	406.813 GHz.	=	736,929 nm.
SOL	G	391,995 Hz.	x	2^40	=	431.003 GHz.	=	695,568 nm.
SOL#	G#	415,304 Hz.	x	2^40	=	456.632 GHz.	=	656,529 nm.
LA	A4	440,000 Hz.	x	2^40	=	483.785 GHz.	=	619,681 nm.
LA#	A#	466,163 Hz.	x	2^40	=	512.552 GHz.	=	584,901 nm.
SI	B	493,883 Hz.	x	2^40	=	543.030 GHz.	=	552,073 nm.
DO	C	523,251 Hz.	x	2^40	=	575.320 GHz.	=	521,087 nm.
DO #	C#	554,365 Hz.	x	2^40	=	609.531 GHz.	=	491,841 nm.
RE	D	587,329 Hz.	x	2^40	=	645.775 GHz.	=	464,236 nm.
RE#	D#	622,253 Hz.	x	2^40	=	684.175 GHz.	=	438,180 nm.
MI	E	659,255 Hz.	x	2^40	=	724.858 GHz.	=	413,587 nm.
FA	F	698,456 Hz.	x	2^40	=	767.961 GHz.	=	390,374 nm.
FA#	F#	739,988 Hz.	x	2^40	=	813.626 GHz.	=	368,464 nm.

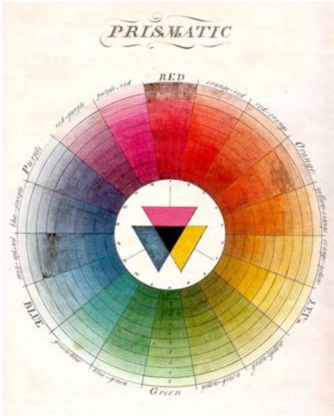


Sound-Color Wheels

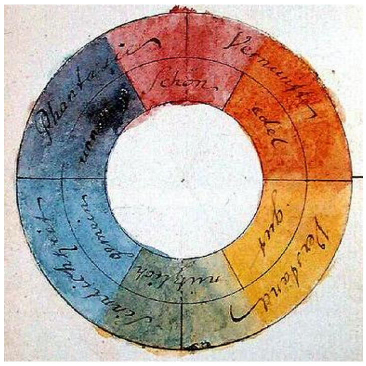
A wheel with primary and secondary colors is traditional in the science as in the arts.
Sir Isaac Newton developed the first circular diagram of colors around the 1704. Since then, any color wheel which presents a logically arranged sequence of pure hues has merit.



1704 – Isaac Newton

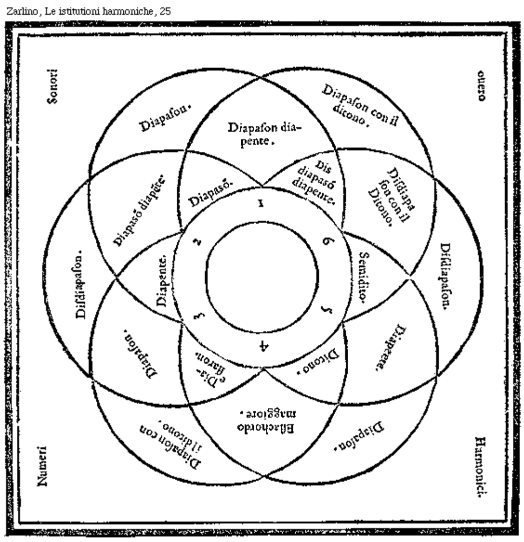


1776 – Moses Harris

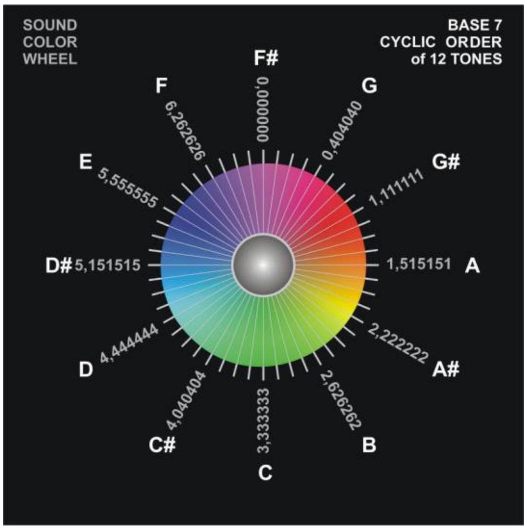


1810 – Wolfgang Goethe

Before the first circular diagram of colors, Gioseffo Zarlino in 1558 drew in his book “Le Istitioni Harmoniche” the first wheel of sounds titled “Numeri sonori”



This is the Sound Color Numbers Wheel in Base 7



Chromatic Numbers between 0 and 10 in Base 7

10/15 = 0,40404040… (in Base 10 7/12 = 0,58333333….)

One Octave of “Equal Tempered Scale” is composed by 12 Tones.

Number 12 in Base 10 correspond at number 15 in Base 7

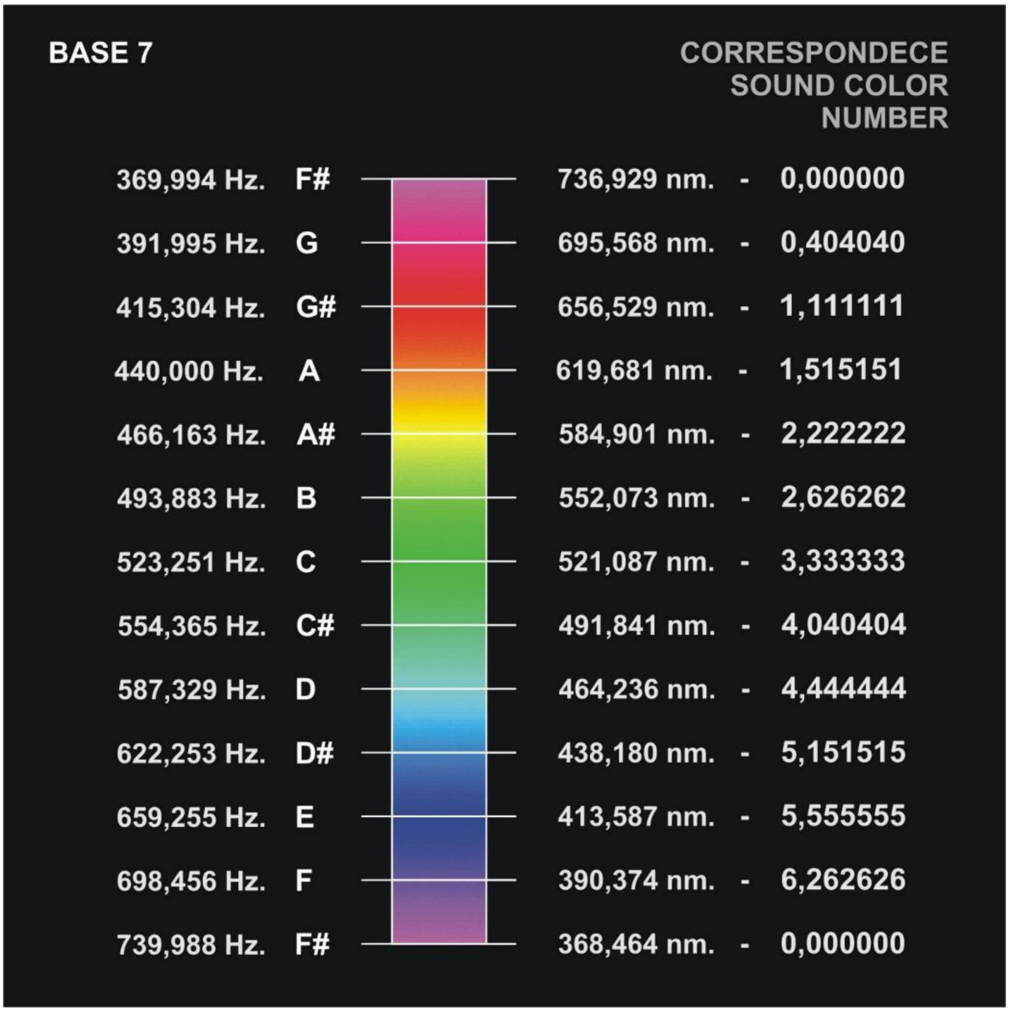
First Octave in Base 7 is from 0 to 10
(F#0 = 0 and F#1 = 10)

One tone is 10/15
10/15 = 0,40404040…

The corresponding numbers of Tones in an Octave are between 0 and 10

0,00000000 **F#**

0,00000000 + 0,40404040 = 0,40404040 **G**	0,40404040 + 0,40404040 = 1,11111111 **G#**
1,11111111 + 0,40404040 = 1,51515151 **A**	1,51515151 + 0,40404040 = 2,22222222 **A#**
2,22222222 + 0,40404040 = 2,62626262 **B**	2,62626262 + 0,40404040 = 3,33333333 **C**
3,33333333 + 0,40404040 = 4,04040404 **C#**	4,04040404 + 0,40404040 = 4,44444444 **D**
4,44444444 + 0,40404040 = 5,15151515 **D#**	5,15151515 + 0,40404040 = 5,55555555 **E**
5,55555555 + 0,40404040 = 6,26262626 **F**	6,26262626 + 0,40404040 = 10,00000000 **F#** **(1 octave up)**



about **RGB, CMY, HSL** and
Colour Ramping for Data Visualisation
see attached files, page 20 - 21
by Prof. Paul Bourke

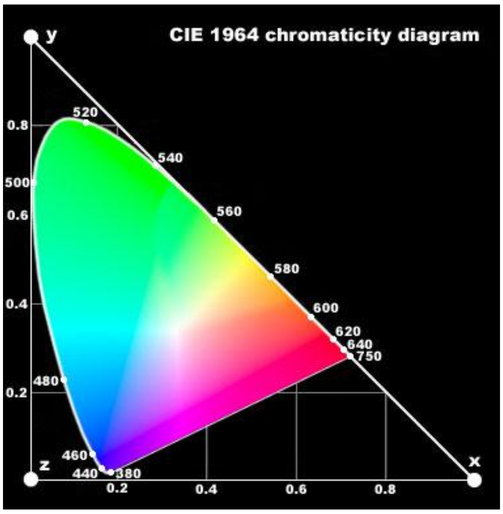
Complementary Sounds, Colors and Numbers

In color theory, two colors are called **complementary** when mixed in proper proportion,
they produce a neutral color (grey, white, or black).

In roughly-percentual color models, the neutral colors (grey, white, or black) lie around a central axis.

For example, in the HSV color space,
complementary colors (as defined in HSV) lie opposite each other on any horizzontal cross-section.

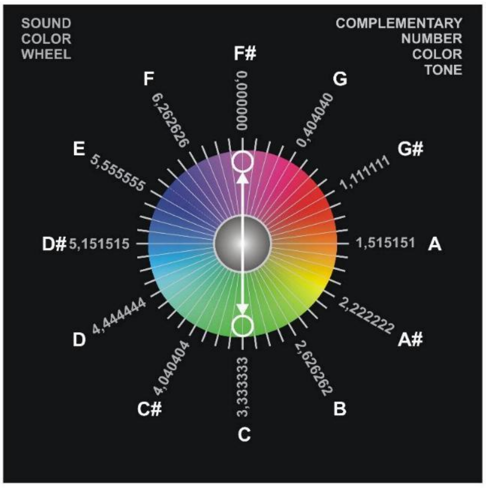
Thus, in CIE 1964 Color Space, a color space of a particular dominant wavelength can be mixed with
a particular amount of the “complementary” wavelength to produce a neutral color (grey or white)



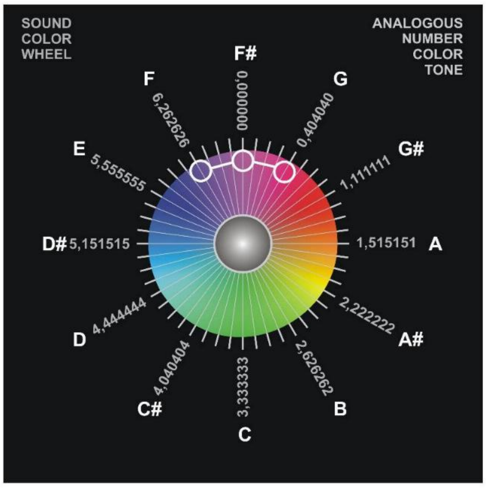
In the RGB color model (and derived models such as HSV),
Primary colors and secondary colors are paired in this way

RED – CYAN
GREEN – MAGENTA
BLUE – YELLOW

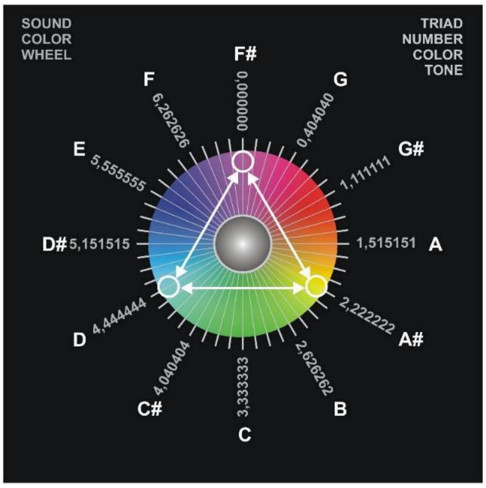
Also, we can apply the same theory of colors for sounds and numbers in Base 7.



Complementary



Analogous



Triad

Complementary
Colors and Sounds that are opposite each other on the wheel are considered to be complementary

Analogous
Analogous scheme use Colors and Sounds that are next to each other on the wheel

Triad
A triadic scheme uses Colors and Sounds that are evenly spaced around the wheel

Additive and Subtractive in Base 7

Primary colors are sets of colors that can be combined to make a useful range of colors.
For human applications, three primary colors are usually used since human color vision is trichromatic.
Additive color primaries are the secondary subtractive colors, or vice versa.

Primary colors are not a fundamental property of light but are related to the physiological response of the eye to light. Fundamentally, light is a continuous spectrum of the wavelengths that can be detected by the human eye, an infinite-dimensional stimulus space. However, the human eye normally contains only three types of color receptors, called cone cells. Each color receptor responds to different ranges of the color spectrum. Humans and other species with three such types of color receptors are known as trichromats. These species respond to the light stimulus via a three-dimensional sensation, which generally can be modeled as a mixture of three primary colors.
Before the nature of colorimetry and visual physiology were well understood, scientists such as Thomas Young, James Clark Maxwell, and Hermann von Helmholtz expressed various opinions about what should be the three primary colors to describe the three primary color sensations of the eye. Young originally proposed red, green, and violet, and Maxwell changed violet to blue; Helmholtz proposed "a slightly purplish red, a vegetation-green, slightly yellowish (wave-length about 5600 tenth-metres), and an ultramarine-blue (about 4820)".

http://en.wikipedia.org/wiki/Primary_color

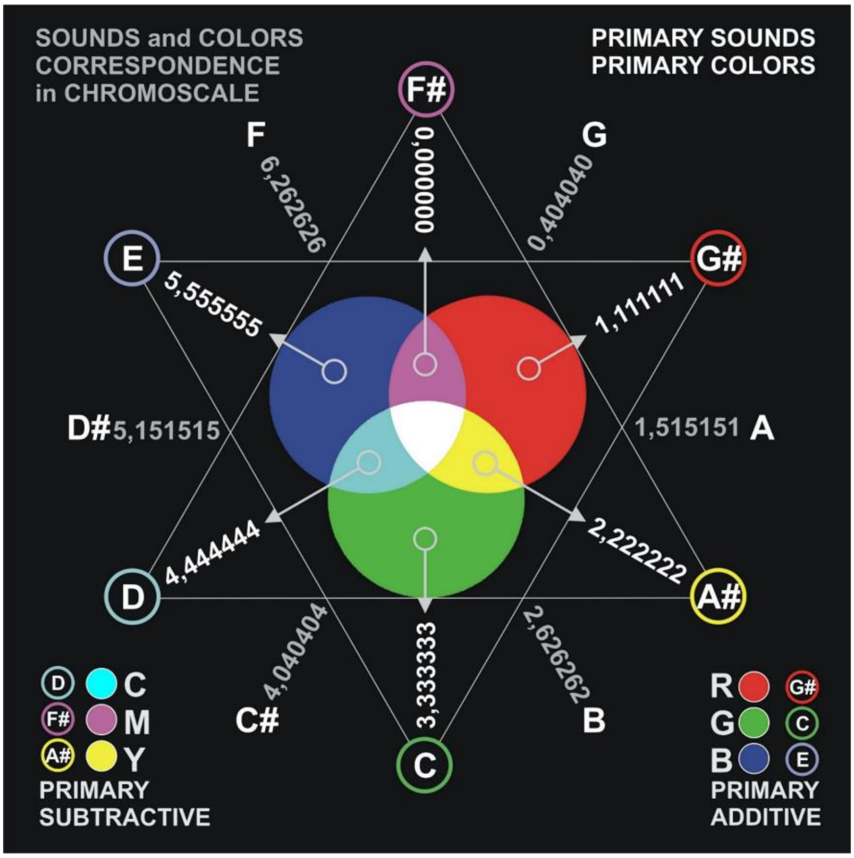
Correspondence of Sounds, Colors and Numbers
regarding Primary Additive and Primary Subtractive

PRIMARY ADDITIVE

G#	RED	**1,1111111**	656,529 nm.	415,304 Hz.
C	GREEN	**3,3333333**	521,087 nm.	523,251 Hz.
E	BLUE	**5,5555555**	413,587 nm.	659,255 Hz.

PRIMARY SUBTRACTIVE

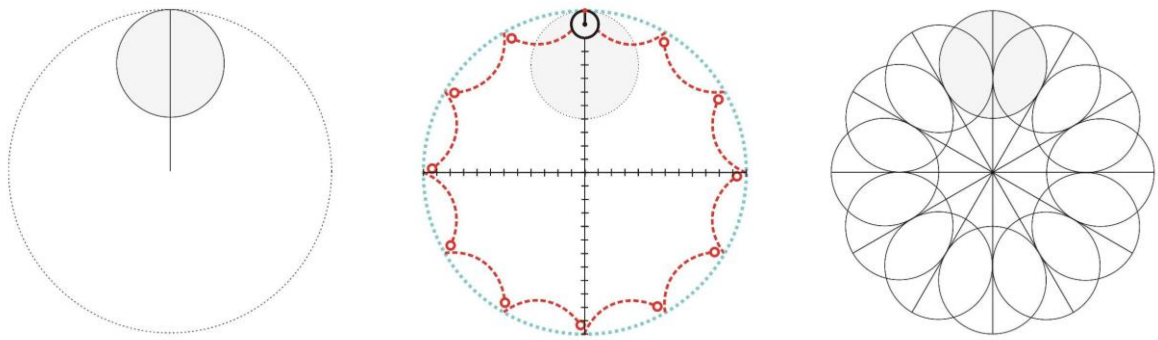
F#	MAGENTA	**0,0000000**	736,929 nm.	369,994 Hz.
A#	YELLOW	**2,2222222**	584,901 nm.	466,163 Hz.
D	CYAN	**4,4444444**	464,236 nm.	587,329 Hz.



Primary ADDITIVE Sounds = **G#, C, E**
Primary SUBTRACTIVE Sounds = **D, F#, A#**

Chromatic Geometry

WHEEL OF TONES

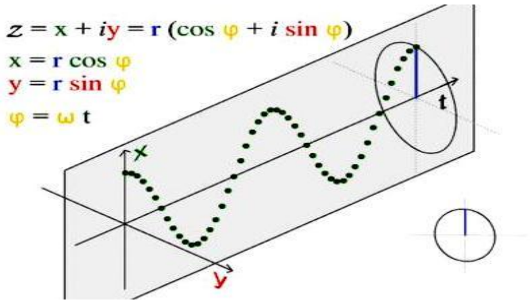


1 TONE
OF THE SCALE

Representation of Hypocycloid
in EQUAL TEMPERED SCALE

12 TONES
in EQUAL TEMPERED SCALE

The wheel of sounds in “equal tempered scale” is representable by an hypocycloid curve with twelve cusps.
In geometry, a hypocycloid is a special plane curve generated by the trace of a fixed point on a small circle that rolls within a large circle. The red curve is the hypocycloid traced as the smaller black circle rolls around inside the larger blue circle.
(parameters are R = 12, r = 1 and so k = 12)



The sine wave or sinusoid is a mathematical curve that described a smooth repetitive oscillation. The sine wave is important in physic because it retains its waveshape when added to another sine wave of the same frequency and arbitrary phase and magnitude.
It is the only periodic wave form that has this property.
This wave pattern occurs often in nature, including, sound waves, light waves and ocean waves.

To the human ear, a sound that is made up of more than one sine wave will either sound “noisy” or will have detectable harmonics. This may be described as a different timbre. (http://en.wikipedia.org/wiki/Sine_wave)

The correspondence of Sounds, Colors and Base 7 Numbers respects these mathematical and physical principles.

BASE 7 - CHROMATIC FRACTIONS
in “EQUAL TEMPERED SCALE”

FA# F#	SOL G	SOL# G#	LA A	LA# A#	SI B	DO C	DO# C#	RE D	RE# D#	MI E	FA F	FA# F#
0/15	10/15	20/15	30/15	40/15	50/15	60/15	100/15	110/15	120/15	130/15	140/15	150/15
0			20/11			40/11			60/11			10
0			10/4			20/4			30/4			10
0						10/2						10
0/12	7/12	14/12	21/12	28/12	35/12	42/12	49/12	56/12	63/12	70/12	77/12	84/12

(BASE 10 CORRESPONDENCE)

Chromatic Numbers between 0 and 10

0,00000000 **F#**

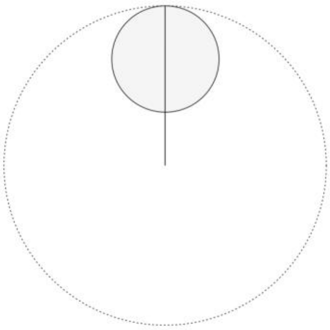
0,00000000 + 0,40404040 = 0,40404040 **G**
1,11111111 + 0,40404040 = 1,51515151 **A**
2,22222222 + 0,40404040 = 2,62626262 **B**
3,33333333 + 0,40404040 = 4,04040404 **C#**
4,44444444 + 0,40404040 = 5,15151515 **D#**
5,55555555 + 0,40404040 = 6,26262626 **F**

0,40404040 + 0,40404040 = 1,11111111 **G#**
1,51515151 + 0,40404040 = 2,22222222 **A#**
2,62626262 + 0,40404040 = 3,33333333 **C**
4,04040404 + 0,40404040 = 4,44444444 **D**
5,15151515 + 0,40404040 = 5,55555555 **E**
6,26262626 + 0,40404040 = 10,00000000 **F#**
(1 octave up)

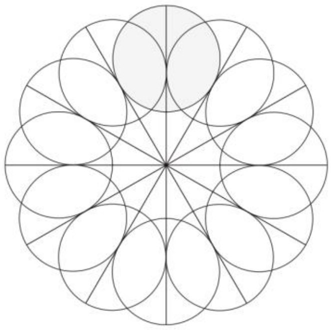
CHROMOSCALE ©

Cyclic Order in Base 7

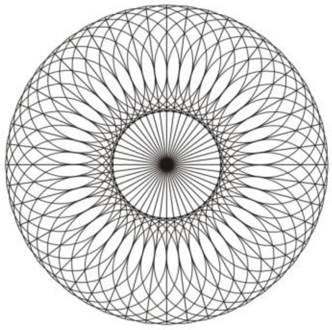
WHEELS OF 1 - 12 - 48 TONES



1 TONE
OF THE SCALE



12 TONES
in EQUAL TEMPERED SCALE



48 TONES
in CHROMOSCALE

Cyclic Number is an integer in which cyclic permutation of the digits are successive multiples of the number.
Every proper multiple of a cyclic number (that is, a multiple having the same number of digits) is a rotation.

In Base 10 the first "prime number" that produces cyclic numbers is the 7
***b* = 10, *p* = 7** the cyclic number is 0,142857142857….

A cyclic order is a way to arrange a set of objects in a circle.
Set with a "Cyclic Order" is called a cyclically ordered set or simply a cycle.

Monotòne function

The "cyclic order = arranging in a circle" idea works because any subset of a cycle is itself a cycle.
In order to use this idea to impose cyclic orders on sets that are not actually subsets of the unit circle in the plane, it is necessary to consider functions between sets.
A function between two cyclically ordered sets, $f : X \to Y$, is called a *monotonic function* or a *homomorphism* if it pulls back the ordering on Y: whenever $[f(a), f(b), f(c)]$, one has$[a, b, c]$.
Equivalently, f is monotone if whenever $[a, b, c]$ and $f(a)$, $f(b)$, and $f(c)$ are all distinct, then $[f(a), f(b), f(c)]$.

Chromotone Numbers in Base 7

$10 \div (10^2-1) = 0{,}10101010$................
$10 \div (10^3-1) = 0{,}01001001001$..........
$10 \div (10^4-1) = 0{,}00100010001$..........
$10 \div (10^5-1) = 0{,}00010000100001$....

Based on this principle, born the **CHROMOSCALE** in **48 CHROMOTONES**

CHROMOSCALE

BASE 7 - CHROMOTONE NUMBERS

$$\frac{10}{(10^2-1)} = 0{,}10101010$$

0,000000	1,010101	2,020202	3,030303	**4,040404**	5,050505	6,060606
0,101010	**1,111111**	2,121212	3,131313	4,141414	**5,151515**	6,161616
0,202020	1,212121	**2,222222**	3,232323	4,242424	5,252525	**6,262626**
0,303030	1,313131	2,323232	**3,333333**	4,343434	5,353535	6,363636
0,404040	1,414141	2,424242	3,434343	**4,444444**	5,454545	6,464646
0,505050	**1,515151**	2,525252	3,535353	4,545454	**5,555555**	6,565656
0,606060	1,616161	**2,626262**	3,636363	4,646464	5,656565	**6,666666**

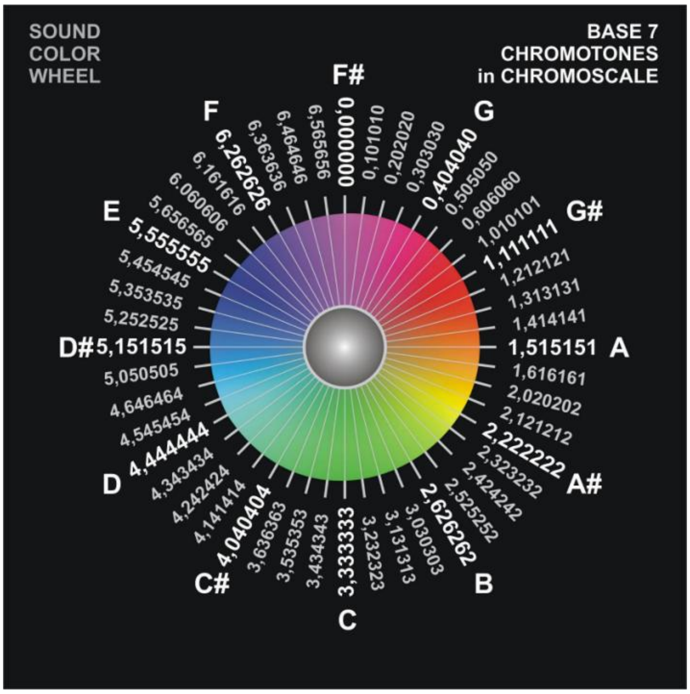
The first known occurrence of explicitly infinite sets is in Galileo's last book Two New Sciences.
Galileo argues that the set of squares is the same size as $\mathbb{S} = \{1,4,9,16,25,...\}$ is the same size as $\mathbb{N} = \{1,2,3,4,5,\ldots\}$ because there is a one-to-one correspondence : 1↔1, 2↔4, 3↔9, 4↔16, 5↔25, ………
And yet, as he says, $\mathbb{S}$ is a proper subset of $\mathbb{N}$ and $\mathbb{S}$ even gets less dense as the numbers get larger.

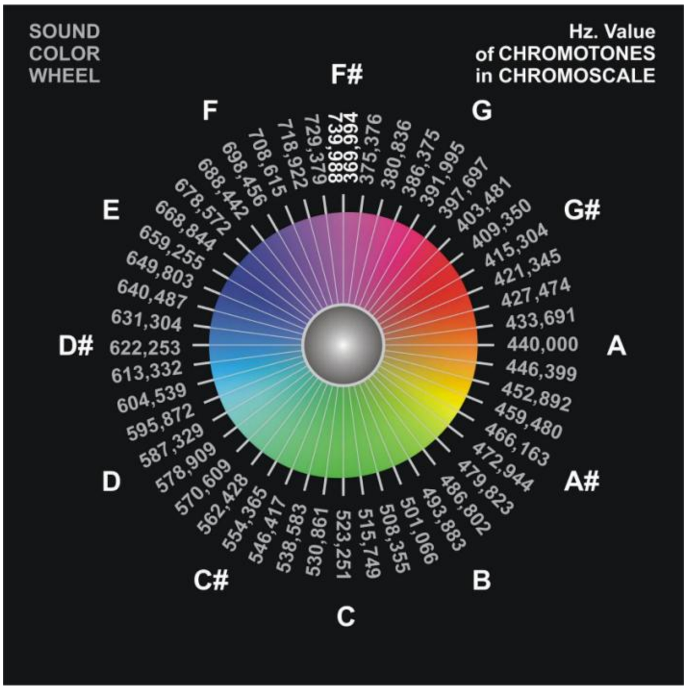
CHROMOSCALE

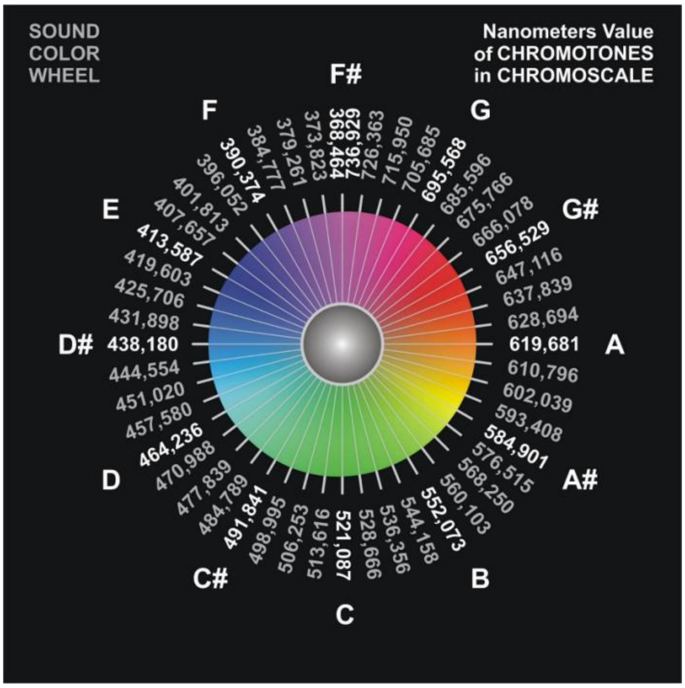
CHROMOTONE FRACTIONS

0/66	100/66	200/66	300/66	**400/66**	500/66	600/66
10/66	**110/66**	210/66	310/66	410/66	**510/66**	610/66
20/66	120/66	**220/66**	320/66	420/66	520/66	**620/66**
30/66	130/66	230/66	**330/66**	430/66	530/66	630/66
40/66	140/66	240/66	340/66	**440/66**	540/66	640/66
50/66	**150/66**	250/66	350/66	450/66	**550/66**	650/66
60/66	160/66	**260/66**	360/66	460/66	560/66	**660/66**

Chromotones in CHROMOSCALE

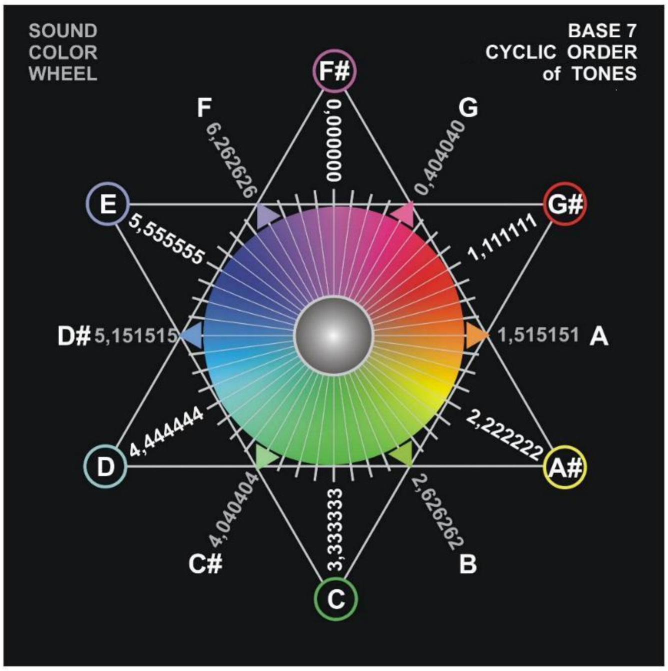


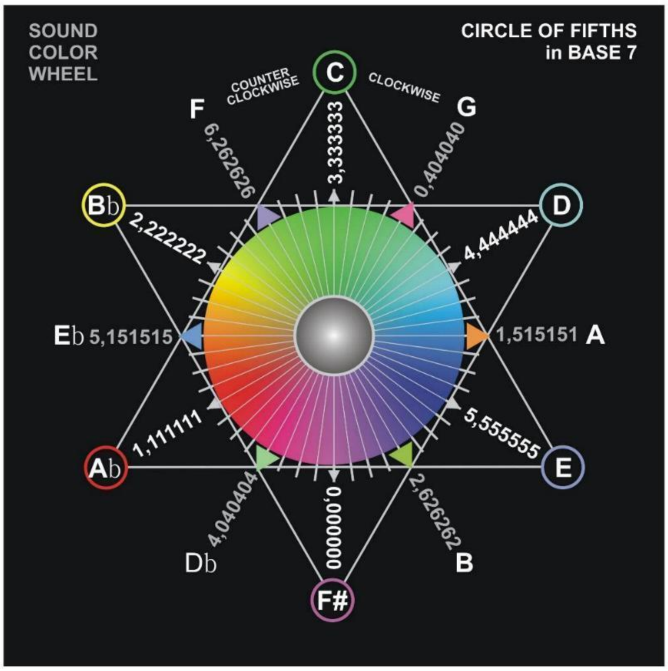




CHROMOSCALE with its **Chromotones** give the possibility to develop the musical composition with harmonic microtonality and just intonation

CIRCLE of FIFTHS





The **Circle of Fifths** is a sequence of pitches or key tonalities, represented as a circle,
in which the next pitch is found seven semitones higher than the last.

The Sound Color Wheel on the left shows the Base 7 Cyclic Numbers,
while on the right the wheel shows the Circle of Fifths in **C Major**.

At the top of Circle the key of **C Major** has non sharp or flats.
Starting from the apex and proceeding clockwise by ascending fifths,
we can subtract the value of **B** (2,626262) to obtain the key of **G** that has one sharp.
From the key of **G**, we proceed to subtract 2,626262 to obtain the key of **D** that has two sharps,
and so on.

Similarly, if we proceed counterclockwise from the apex by descending fifths,
we add 2,626262 to obtain the key of **F** that has one flat,
we continue to add 2,626262 to obtain the key of **Bb** that has two flats,
and so on.

We can construct geometrically Circle of Fifths, Fourths and Thirds with simple algebraic formulas.
The same rule can be applied by including the Chromotones of **CHROMOSCALE**.

CIRCLE OF FIFTHS

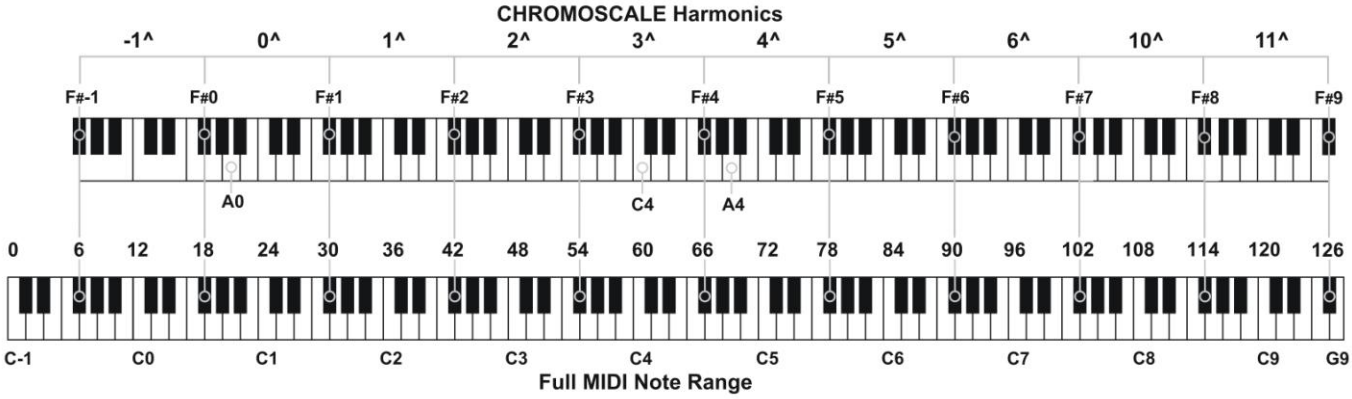
IN EQUAL TEMPERED SCALE
BASE 7 - FRACTIONS

COUNTER CLOCKWISE ← → CLOCKWISE

FA#	DO#	SOL#	RE#	LA#	FA	DO	SOL	RE	LA	MI	SI	FA#
F#	C# Db	G# Ab	D# Eb	A# Bb	F	C	G	D	A	E	B	F#
0,0000	4,0404	1,1111	5,1515	2,2222	6,2626	3,3333	0,4040	4,4444	1,5151	5,5555	2,6262	10,0000
0/66	400/66	110/66	510/66	220/66	620/66	**330/66**	40/66	440/66	150/66	550/66	260/66	660/66
0/33	200/33	40/33	240/33	110/33	310/33	**150/33**	20/33	220/33	60/33	260/33	130/33	330/33
0/15	100/15	20/15	120/15	40/15	140/15	**60/15**	10/15	110/15	30/15	130/15	50/15	150/15
0			60/11			**40/11**			20/11			10
0			30/4			**20/4**			10/4			10
0						**10/2**						10
0/12	49/12	14/12	63/12	28/12	77/12	42/12	7/12	56/12	21/12	70/12	35/12	84/12

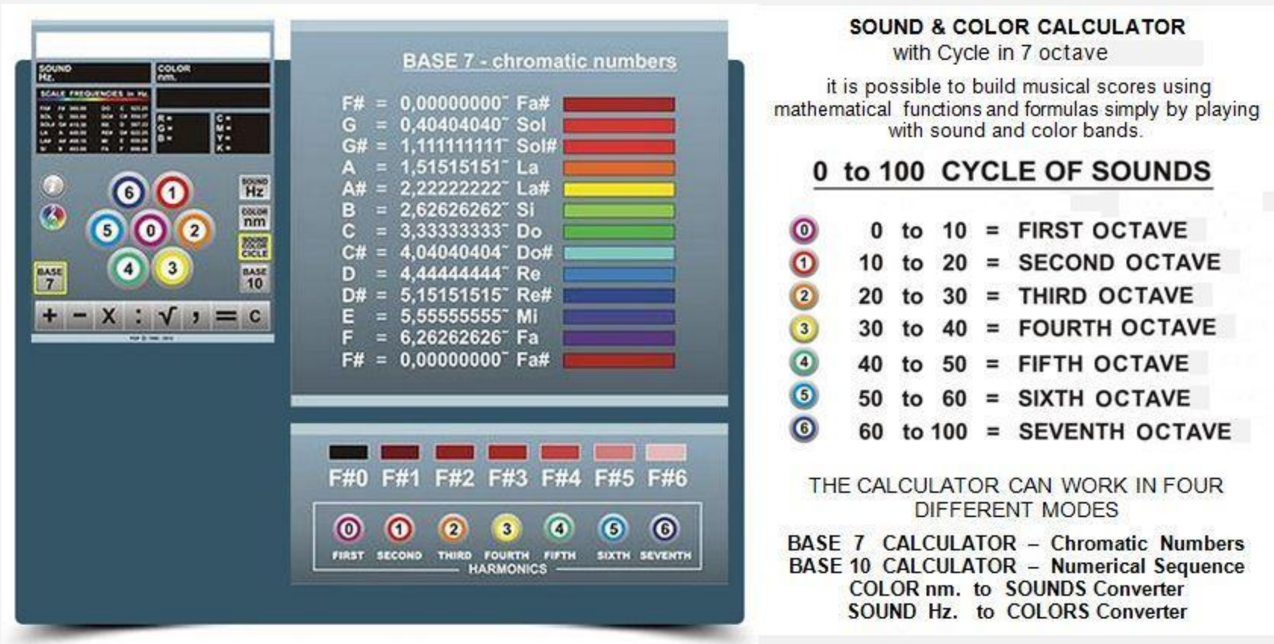
(BASE 10 CORRESPONDENCE)

CHROMOSCALE Octaves and Full MIDI Note Range



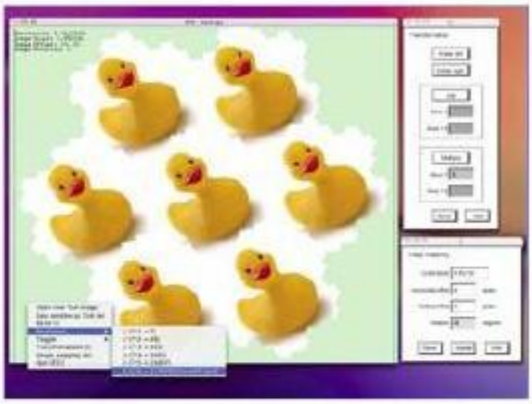
Sound & Color APP

This Calculator converts numbers and related transaction in numeric form, it shows the corresponding value of sounds in Hz., Colors in Nanometers, RGB, CMYK, XYZ, and the device emits the resulting sound.



about
SOUND & COLOR APP
please visit
https://itunes.apple.com/us/app/sound-color/id579920437?l=it&ls=1&mt=8

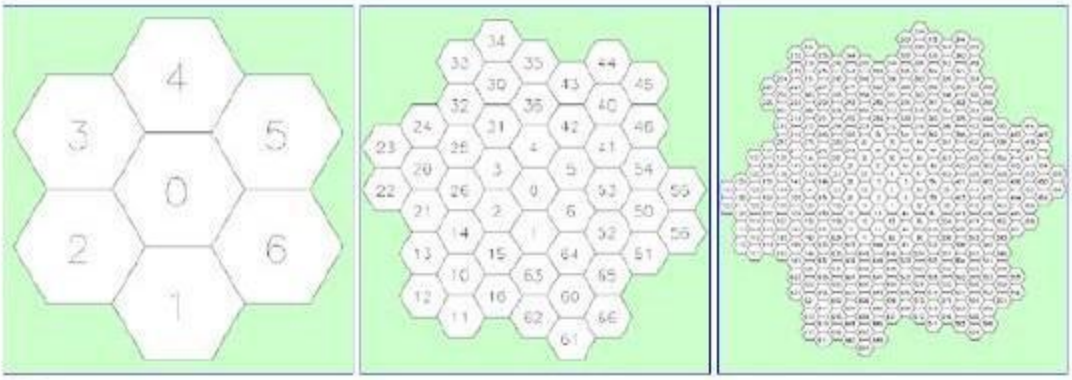
Hexagonal Lattice



Written by Paul Bourke
December 1997, Updated February 2004

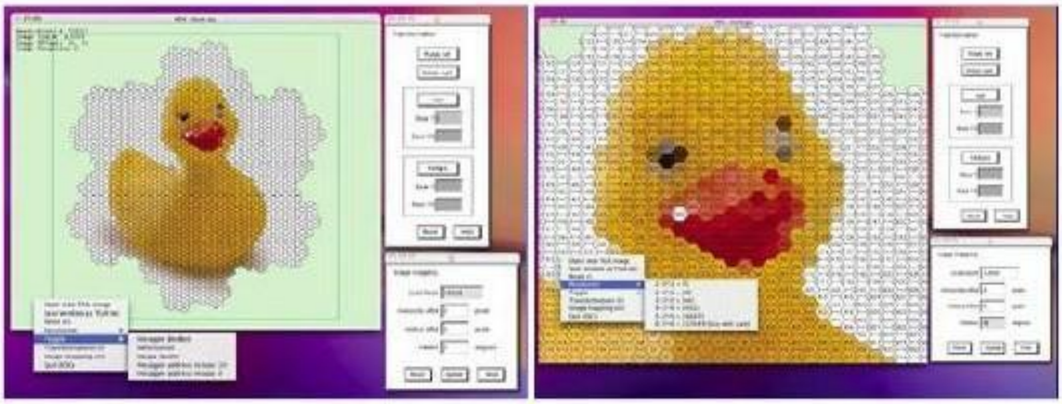
C libraries: hexlib.h, and hexlib.c

Of the three 2 dimensional shapes (equilateral triangle, rectangle, and hexagon) that can be used to tile the plane without holes, the hexagon is the most complex and has many interesting properties. In what follows, an indexing system will be described for a hexagonal tiling called a Spiral Honeycomb Mosaic (SHM). A SHM consists of groups of 7^n hexagons ($n > 0$) called super-hexagons. It uses a base 7 numbering system for the hexagonal mesh, this is illustrated below for $n = 1, 2$, and 3.



Software

An application has been developed to explore operations in SHM space. It is based upon X-Windows and OpenGL and is currently available for Mac OS-X and Linux (by request). Note that before running the Mac OS-X version check that X-Windows is running. Download: macosx.tar.gz



Interface

The interface is straightforward, to find out about any command line options type "hex -h". The left mouse button pans the image, the middle button rolls, and the right button presents a list of menu options. In order to position/rotate/scale the hexagon to the image see the image mapping dialog box. To experiment with SHM operations see the transformation dialog box.

```
>hex -h
Usage: hex [options]
Options
     -h        this text (help)
     -i s      load input TGA file
     -v        verbose mode
Key commands
    -,+        zoom in/out
      r        reset
      w        write TGA image of window contents
   1..6        set resolution
    ESC        quit
Mouse buttons
   left        translate
 middle        rotate
  right        menus
```

Addition and Multiplication

The two basic arithmetic operations can be defined for the SMH, addition and multiplication. These operations act on the addresses of the SHM and result in translation (addition) and rotation/scaling (multiplication) when applied to images represented with the "pixels" of the SHM. The following two C snippets (HexAdd() and HexMul()) implement addition and multiplication. An important characteristics of the operations defined in this way is they are bijective, that is, it is a one to one mapping of each address. Every address (hexagonal pixel) maps uniquely to another address so that no information is lost.

Examples

- This is an example of repeatedly multiplying an image on an order 5 SHM by 10_{hex} or 7_{dec}
- This is the same example at two powers of 7 greater resolution showing the "superduck".
- This example shows repeated addition by 6666_{hex}.

Notes - Spirals

The curve through powers of 7 (10_{hex}) is an equiangular spiral described by

$$r = a \exp^{(b\ \mathrm{theta})}$$

Where r is the radius and theta the angle to the x axis. For example, for the spiral through 1_{hex}, 10_{hex}, 100_{hex}, 1000_{hex}, ... the parameters a and b are

a = sqrt(3)
b = $\log_e$(sqrt(7)) / arctan(sqrt(3) / 2) = 1.3632084.

Since the angle in the above case is taken from the negative y (imaginary) axis, the curve would be traced by

$$x = - r \sin(\mathrm{theta})$$
$$y = - r \cos(\mathrm{theta})$$

The angle between each successive multiples of 10_{hex} is arctan(sqrt(3) / 2) = 40.893395°, the ratio of two successive radii is sqrt(7) = 2.6457513.

Online SHM Calculator

Base Conversion

0
◉ Base 7
○ Base 10

Convert to other base

Addition/multiplication

0
◉ Base 7
○ Base 10

◉ Add
○ Multiply
Basis 4

0
◉ Base 7
○ Base 10

Calculate

Other Functions

0

Base 7
Base 10

Basis 4

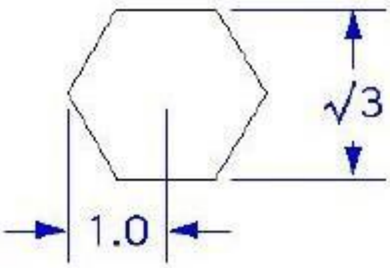
Inverse
Square root(s)
Magnitude

Calculate function

Cartesian

0

Base 7
Base 10



Coordinates

References

Alexander, D. and Sheridan, P.
Proceedings Australian Neuroscience Society (1995)
Algebraic Geometric Model of the Receptive Fields Properties of the Macaque Striate Cortex.

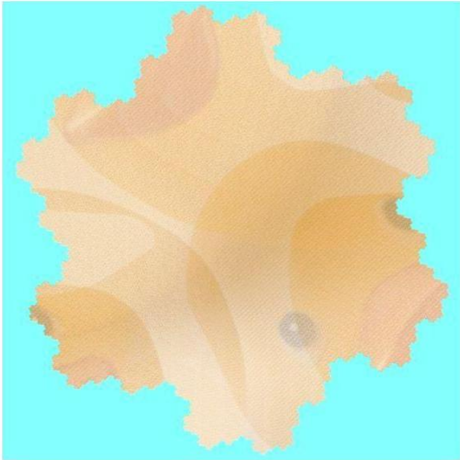
Sheridan, P.
PhD Thesis, University of Technology, Sydney (1996)
Spiral Architecture for Machine Vision

Sheridan, P., Alexander, D.M.
Proceedings of Vision, Recognition, Action: Boston. (1997)
Invariant transformations on a space-variant hexagonal grid

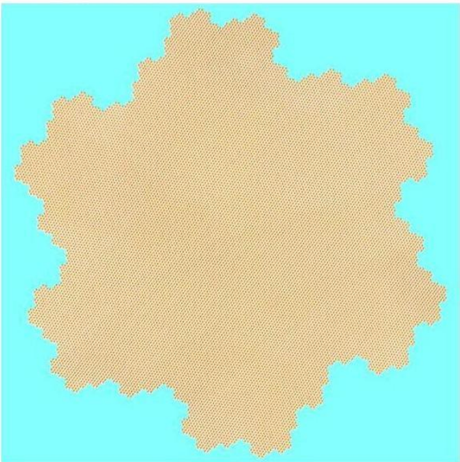
Sheridan, P., Hintz, T., Alexander, D.
Image and Vision Computing, 18 (2000)
Pseudo-invariant image transformations on a hexagonal lattice

Original

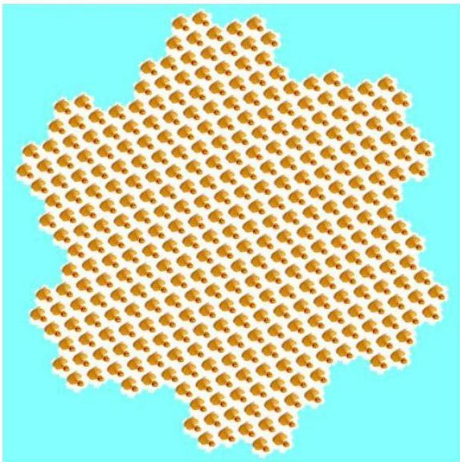
First multiplication by 7



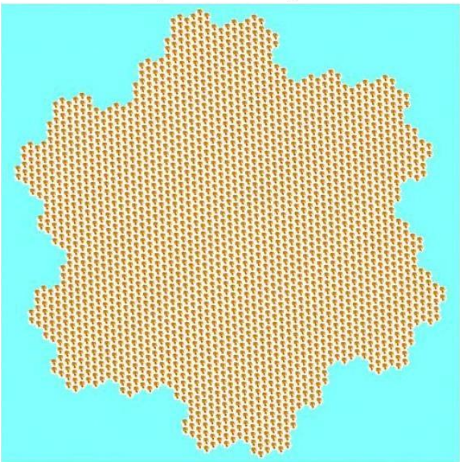
Second multiplication by 7



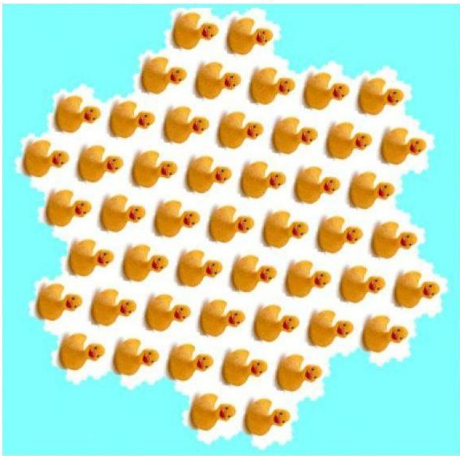
Third multiplication by 7



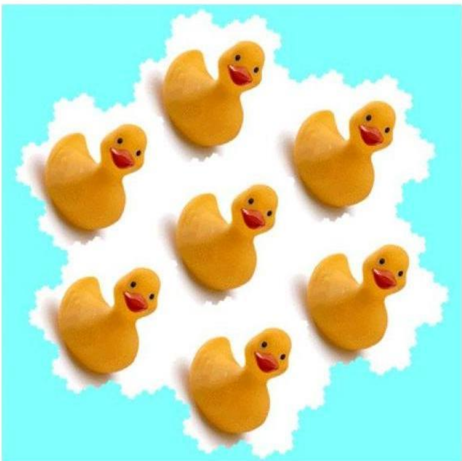
Fourth multiplication by 7



Fifth multiplication by 7



Sixth multiplication by 7



Seventh multiplication by 7
back to the Original

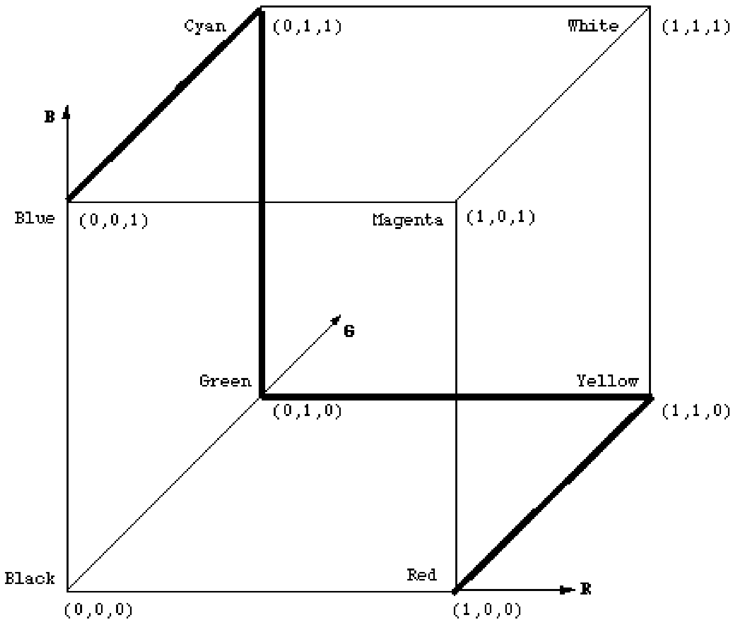
Colour Ramping for Data Visualisation

Written by Paul Bourke
July 1996
Contribution: Ramp.cs by Russell Plume in DotNet C#.

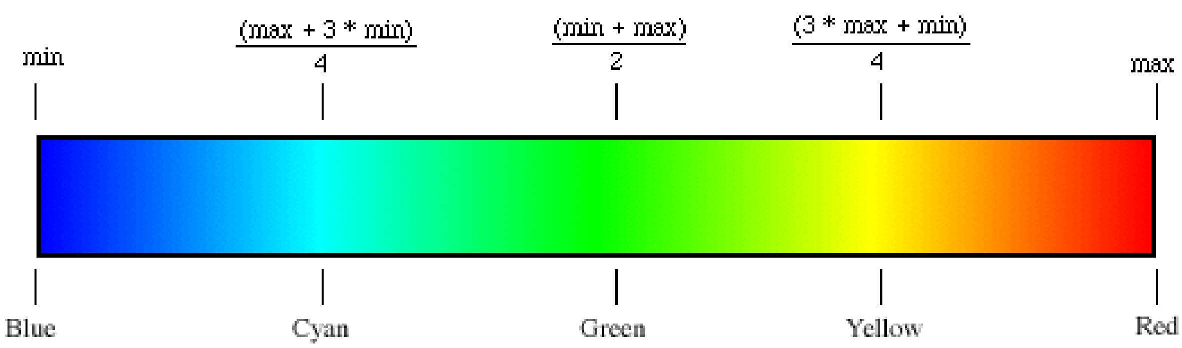
This note introduces the most commonly used colour ramps for mapping colours onto a range of scalar values as is often required in data visualisation. The colour space will be based upon the RGB system.

Colour

The most commonly used colour ramp is often refer to as the "hot- to-cold" colour ramp. Blue is chosen for the low values, green for middle values, and red for the high as these seem "intuitive" bounds. One could ramp between these points on the colour cube but this involves moving diagonally across the faces of the cube. Instead we add the colours cyan and yellow so that the colour ramp only moves along the edges of the colour cube from blue to red. This not only makes the mapping easier and faster but introduces more colour variation. The following illustrates the path on the colour cube.



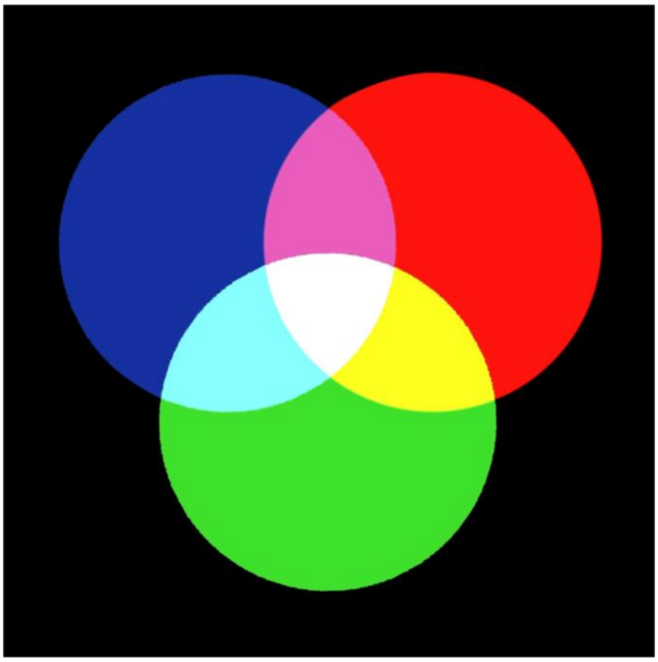
The colour ramp is shown below along with the transition values.



Again there is a linear relationship of the scalar value with colour within each of the 4 colour bands.
In some applications the variable being represented with the colour map is circular in nature in which case a cyclic colour map is desirable. The above can be simply modified to pass through magenta to yield one of many possible circular colour maps.

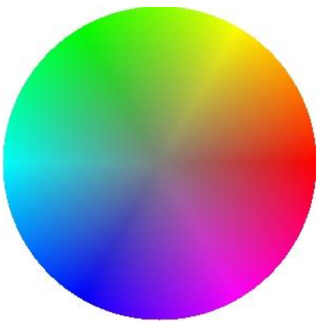
RGB and CMY

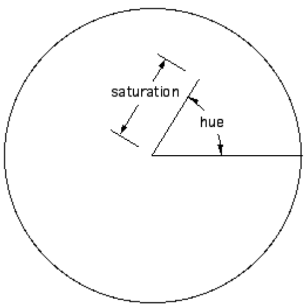
A colour space is a means of uniquely specifying a colour. There are a number of colour spaces in common usage depending on the particular industry and/or application involved. For example as humans we normally determine colour by parameters such as brightness, hue, and colourfulness. On computers it is more common to describe colour by three components, normally red, green, and blue. These are related to the excitation of red, green, and blue phosphors on a computer monitor. Another similar system geared more towards the printing industry uses cyan, magenta, and yellow to specify colour, they are related to the reflectance and absorbance of inks on paper.



HSL, Hue Saturation and Lightness

The HSL colour space has three coordinates: hue, saturation, and lightness (sometimes luminance) respectively, it is sometimes referred to as HLS. The hue is an angle from 0 to 360 degrees, typically 0 is red, 60 degrees yellow, 120 degrees green, 180 degrees cyan, 240 degrees blue, and 300 degrees magenta.
Saturation typically ranges from 0 to 1 (sometimes 0 to 100%) and defines how grey the colour is, 0 indicates grey and 1 is the pure primary colour. Lightness is intuitively what it's name indicates, varying the lightness reduces the values of the primary colours while keeping them in the same ratio. If the colour space is represented by disks of varying lightness then the hue and saturation are the equivalent to polar coordinates (r,theta) of any point in the plane.





http://paulbourke.net/texture_colour/

An Orthogonal Oriented Quadrature Hexagonal Image Pyramid

Andrew B. Watson
Albert J. Ahumada, Jr., Ames Research Center, Moffett Field, California

December 1987

National Aeronautics and
Space Administration

Ames Research Center
Moffett Field, California 94035

AN ORTHOGONAL ORIENTED QUADRATURE HEXAGONAL IMAGE PYRAMID

Andrew B. Watson and Albert J. Ahumada, Jr.

NASA Ames Research Center
Perception and Cognition Group

Abstract

We have developed an image pyramid with basis functions that are orthogonal, self-similar, and localized in space, spatial frequency, orientation, and phase. The pyramid operates on a hexagonal sample lattice. The set of seven basis functions consist of three even high-pass kernels, three odd high-pass kernels, and one low-pass kernel. The three even kernels are identical when rotated by 60° or 120°, and likewise for the odd. The seven basis functions occupy a point and a hexagon of six nearest neighbors on a hexagonal sample lattice. At the lowest level of the pyramid, the input lattice is the image sample lattice. At each higher level, the input lattice is provided by the low-pass coefficients computed at the previous level. At each level, the output is subsampled in such a way as to yield a new hexagonal lattice with a spacing $\sqrt{7}$ larger than the previous level, so that the number of coefficients is reduced by a factor of 7 at each level. We discuss the relationship between this image code and the processing architecture of the primate visual cortex.

Introduction

A digital image is usually represented by a set of two-dimensionally periodic spatial samples, or pixels. Many schemes exist to transform these pixels into alternative image codes that may be useful for compression or progressive transmission. subband codes are a class of transform in which the image is partitioned into sub-images corresponding to separate bands of resolution or spatial frequency (Vetterli, 1984; Woods and O'Neil, 1986). Closely related are pyramid codes, in which each band-pass sub-image is sub-sampled by a common factor, so that the number of pixels in each level of the pyramid is reduced by that factor relative to the preceding level (Tanimoto and Pavlidis, 1975; Burt and Adelson, 1983, Watson, 1986). Several schemes have been devised that also partition the image by orientation. These include quadrature mirror filters (Vetterli, 1984; Woods and O'Niel, 1986; Gharavi and Tabatabai, 1986; Mallat, 1987), and a pyramid modeled on human vision (Watson, 1987a,b). Recently, a number of orthogonal pyramid codes have been developed (E. H. Adelson, Eero Simoncelli, and Rajesh Hingorani , Orthogonal pyramid transforms for image coding, *SPIE Proceedings on Visual Communication and Image Processing II*, 1988). These have the virtues that they are invertible, that they preserve the total number of coefficients, and that they allow simple forward and inverse transformation algorithms.

We are interested in image codes that share properties with the coding scheme used by the primate visual cortex (A. B. Watson, Cortical algotecture, in *Vision: Coding and Efficiency*, C. B. Blakemore, Ed., Cambridge University Press, Cambridge England, 1988). These properties include a subband structure, relatively narrow-band tuning in both spatial frequency and orientation, relatively high spatial localization, both odd and even (quadrature) kernels, and self-similarity. We have also been intrigued by the fact that the image sample lattice in primate vision is approximately hexagonal, rather than rectangular. Guided by these observations, we have derived an orthogonal oriented quadrature hexagonal image pyramid.

Our code is a shift-invariant linear transformation, in which each new coefficient is a linear combination of image samples. The linear combination can be defined by a kernel of weights specifying the spatial topography of the linear combination. We have considered kernels that occupy a point and the hexagon of six nearest neighbors on a hexagonal lattice.

Constraints

We have derived a set of kernels under the following constraints:

(1) The kernels are expressed on a hexagonal sample lattice.
(2) There are seven mutually orthogonal kernels, one low-pass and six high-pass.
(3) Each kernel has seven weights (taps) corresponding to a point and its six nearest neighbors in the hexagonal lattice.
(4) The low-pass kernel has equal values at all taps.
(5) Two high-pass kernels have an axis of symmetry running through the center sample and between samples on the outer ring (at an angle of 30°).
(6) Of these two kernels, one is even about the axis of symmetry, the other is odd.
(7) The remaining four high-pass kernels are obtained by rotating the odd and even kernels by 60° and 120°.
(8) Each kernel has a norm (square root of sum of squares of taps) of one.

With respect to constraint (5), we have determined that there is no solution when the common axis of symmetry is at 0° (on the sample lattice of the outer ring). Note also that constraints (2) and (4) oblige the even kernels, as well as the odd, to have zero DC response (the weights sum to 0).

Under the symmetry constraints, the kernel coefficients can be written as shown in Fig. 1.

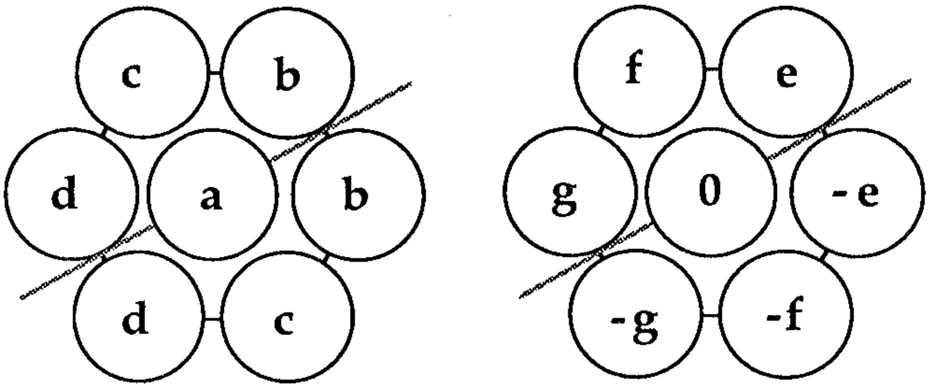


Fig. 1. Even and odd high-pass kernels with symmetry axis at 30°.

One even and one odd kernel are shown. The low-pass kernel (not shown) is simply a constant *h*at each tap. We construct a set of seven equations in these seven unknowns that express the constraints of orthogonality and unit norm. They are:

$a^2 + 2b^2 + 2c^2 + 2d^2 = 1$	(unit norm)	[1]
$2e^2 + 2f^2 + 2g^2 = 1$	(unit norm)	[2]
$a + 2b + 2c + 2d = 0$	($\perp$ to low-pass)	[3]
$a^2 + b^2 + d^2 + 2bc + 2cd = 0$	($\perp$ to self-rotation)	[4]
$a^2 + 2bc + 2bd + 2cd = 0$	($\perp$ to self-rotation)	[5]
$e^2 + g^2 - 2ef - 2fg = 0$	($\perp$ to self-rotation)	[6]
$2eg - 2ef - 2fg = 0$	($\perp$ to self-rotation)	[7]

Subtracting equations [4] and [5], and [6] and [7], shows that

$$b = d \qquad [8]$$

$$e = g \qquad [9]$$

Thus while not explicitly assumed, we see that both odd and even filters must also be symmetrical about the 120° axis.

Further simplifications lead to the following solution for the coefficients of the odd filter:

$$e = \sqrt{2}/3 \qquad [10]$$

$$f = e/2 = \frac{1}{\sqrt{2}\,3} \qquad [11]$$

For the even filter, we find:

$$a = \sqrt{2/7} \qquad [12]$$

But two solutions emerge for b and c:

$$b = \frac{-(1 + 1/\sqrt{7})}{\sqrt{2}\,3} \qquad [13]$$

$$c = \frac{(2 - 1/\sqrt{7})}{\sqrt{2}\,3} \qquad [14]$$

and

$$b = \frac{(1 - 1/\sqrt{7})}{\sqrt{2}\,3} \qquad [15]$$

$$c = \frac{-(2 + 1/\sqrt{7})}{\sqrt{2}\,3} \qquad [16]$$

We will call the first solution the even filter of type 0, and the second solution, type 1. The three kernels are shown in Fig. 2.

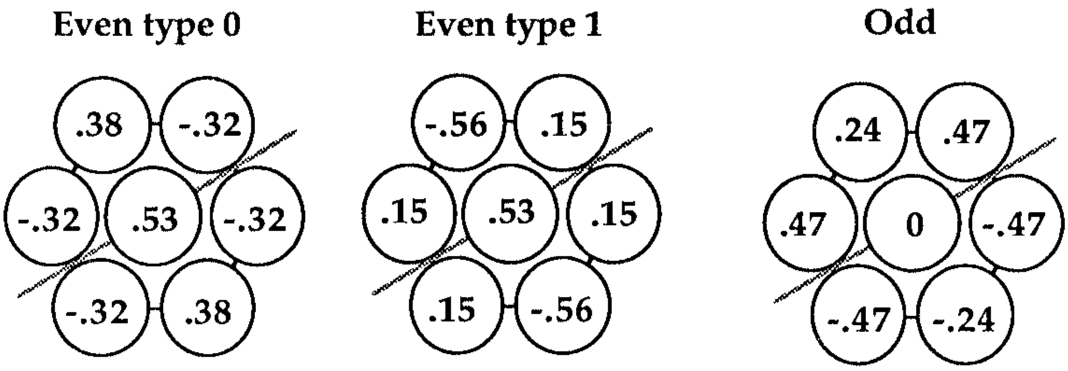


Fig. 2. Values for the two types of even kernel and one odd kernel.

The value of each coefficient h in the low-pass kernel is given directly by the unit norm constraint,

$$h = 1/\sqrt{7} \qquad [17]$$

Filter spectra

One of our objectives was to create subband filters that were somewhat narrowband and oriented. The filter spectra are easily derived. Each kernel consists of a central impulse at the origin, surrounded by 3 pairs of symmetric impulses. These transform in the frequency domain into a constant plus three sinusoids at angles of 0°, 60°, and 120°. The constant is the value of the central coefficient, while each sinusoid has an amplitude twice that of the corresponding coefficient. For the even kernels, the sinusoids are in cosine phase, for the odd kernels, they are in sine phase. The example spectra shown in Fig. 3 demonstrate from their half amplitude response that they are oriented and high-pass. In the pyramid they will become band-pass through convolution with the low-pass kernel at preceding levels.

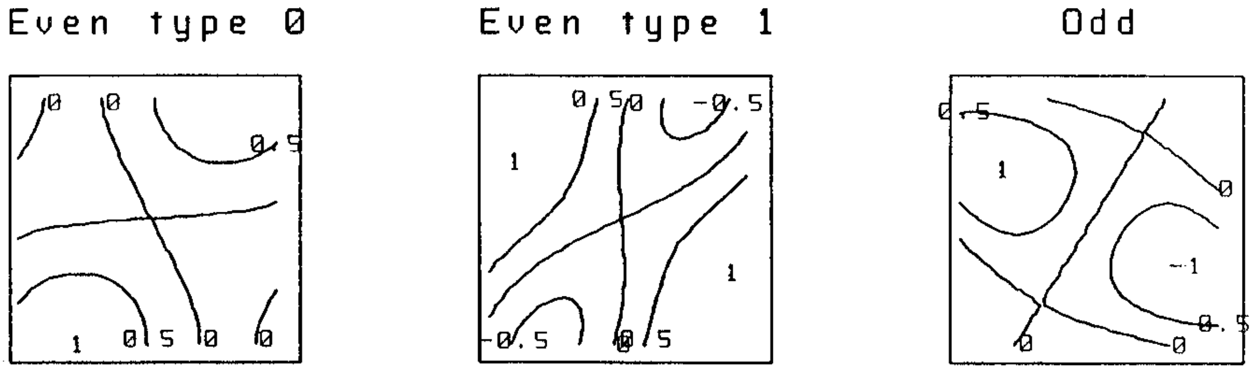


Fig. 3. Spectra of the two types of even kernel and the odd kernel. The origin is at the center of each figure and the spectrum extends to plus and minus 1. These are contour plots of continuous spectra. The discrete spectrum would have a hexagonal shape and a hexagonal sample lattice.

Axes of symmetry and orientation

We define the *orientation* of a kernel as the orientation of the peak of the frequency spectrum, that is, the orientation of a sinusoidal input at which the kernel gives the largest response. An interesting feature of the resulting kernels is that while the axis of symmetry was fixed at 30°, the orientation of the type 0 even kernel is actually orthogonal to this axis at 120°. This places its orientation axis on the hexagonal lattice. In contrast, the orientation of the type 1 even kernel and the odd kernel are equal to the initial axis of symmetry at 30°. Thus if it is desired to have quadrature pairs with equal orientation, the type 1 even kernel must be used.

Subsampling

One virtue of the scheme we have described is that it leads directly to an oriented resolution pyramid, as illustrated in Fig. 4.

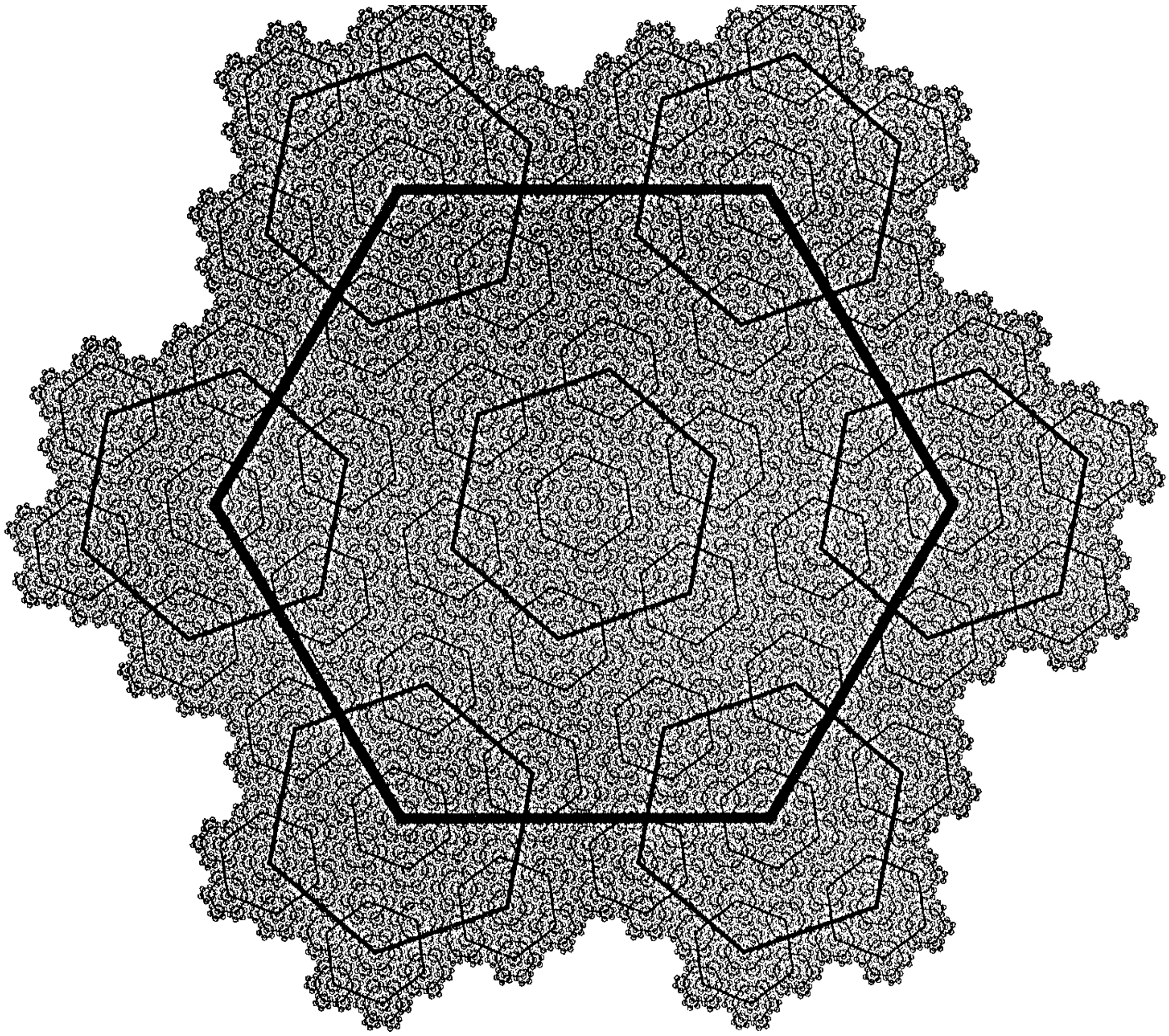


Fig. 4. Construction of the hexagonal pyramid. The image sample lattice is given by the vertices of the smallest hexagons. At each level, sub-images are generated by application of the kernels to the low-pass coefficients from the previous level.

This hexagonal fractal was constructed by first creating the largest hexagon, then placing at each of its vertices a hexagon rotated by $\tan^{-1}(\sqrt{3}/5) \approx 19.1°$ and scaled by $1/\sqrt{7}$. The same procedure is then applied to each of the smaller hexagons, down to some terminating level. The image sample lattice is then a finite-extent periodic sequence with a hexagonal sample lattice defined by the vertices of the smallest hexagons. The sample lattice has 7^6 points, the same as a rectangular lattice of 343^2. The perimeter of this "Gosper flake" is a "Koch curve" with a fractal dimension of $\log 3 / \log \sqrt{7} \approx 1.19$ (Mandelbrot, 1983, p. 46). The program used to create this image is given in Appendix 1.

The hexagonal image sample lattice is tesselated with hexagons with unit radius. Each of the 7 kernels is applied in each hexagon, yielding seven new sub-images, six highpass and one lowpass, each with one seventh as many samples as the original. The six high-pass sub-images form level 0 of the pyramid. The next level is created by again tesselating the plane with hexagons of radius $\sqrt{7}$ whose vertices correspond to the centers of the hexagons at the lower level. The seven kernels are applied to the low-pass coefficients derived at the earlier level. This yields seven new sub-images, each a factor of seven smaller than the sub-images at level 0. This process is repeated until a level is reached at which each sub-image has one sample.

While an image shape like that in Fig. 4 is very natural for this code, any shape that is one period of a hexagonally periodic sequence can be exactly encoded if the number of samples is equal to a power of seven. This includes, for example, a parallelogram with sides of length a power of seven samples. Below we show how the code may be applied to a conventional rectangular image.

The sub-sampling at each level can be formalized as follows (Dudgeon and Mersereau, 1984). The original hexagonal sampling lattice can be represented by a sampling matrix **H**,

$$\mathbf{H} = \begin{bmatrix} 1 & 1/2 \\ 0 & \sqrt{3}/2 \end{bmatrix} \qquad [18]$$

The column vectors of this matrix map from sample to sample, and the location of any sample can be expressed as $\mathbf{x} = (x,y)$,

$$\mathbf{x} = \mathbf{H}\,\mathbf{r} \qquad [19]$$

where $\mathbf{r}$ is an integer vector. Let $\mathbf{S}_n$ be the sampling matrix at level n. Since the sample at each level must be a subset of those at the previous level, the column vectors of $\mathbf{S}_{n+1}$ must be integer linear combinations of the column vectors of $\mathbf{S}_n$. Thus

$$\mathbf{S}_{n+1} = \mathbf{S}_n \mathbf{M} \qquad [20]$$

where $\mathbf{M}$ is an integer matrix. Further, the columns of $\mathbf{S}_{n+1}$ must be $\sqrt{7}$ longer than the columns of $\mathbf{S}_n$ (corresponding to the increasing radii of the hexagons at each successive level). And finally, because the determinant of a sampling matrix determines the factor by which the density of samples is reduced, we know that

$$\det(\mathbf{M}) = 7 \qquad [21]$$

Two matrices which satisfy these conditions are:

$$\mathbf{M}_0 = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \qquad [22]$$

$$\mathbf{M}_1 = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \qquad [23]$$

These generate the only two possible sub-samplings from one level to the next. Then $\mathbf{S}_n$ can be constructed in various ways, the three most obvious being

$$\mathbf{S}_n = \mathbf{H}\mathbf{M}_0^n \qquad [24]$$

and

$$\mathbf{S}_n = \mathbf{H}\,\mathbf{M}_1^n \qquad [25]$$

and

$$\mathbf{S}_n = \mathbf{H}\,\mathbf{M}_0\mathbf{M}_1\mathbf{M}_0\mathbf{M}_1\ldots \quad (n \text{ terms}) \qquad [26]$$

The first scheme (used in Fig. 4) causes a rotation of $\tan^{-1}(\sqrt{3}/5) \approx 19.1°$ in the sample lattice at each level, as does the second scheme, while the third scheme alternates between rotations of 19.1° and -19.1°.

Skewed coordinates

It is well known that hexagonal samples on a cartesian plane can also be viewed as rectangular coordinates on a coordinate frame in which one axis is skewed by 60° (Fig. 6A) (Peterson and Middleton, 1962; Mersereau, 1979).

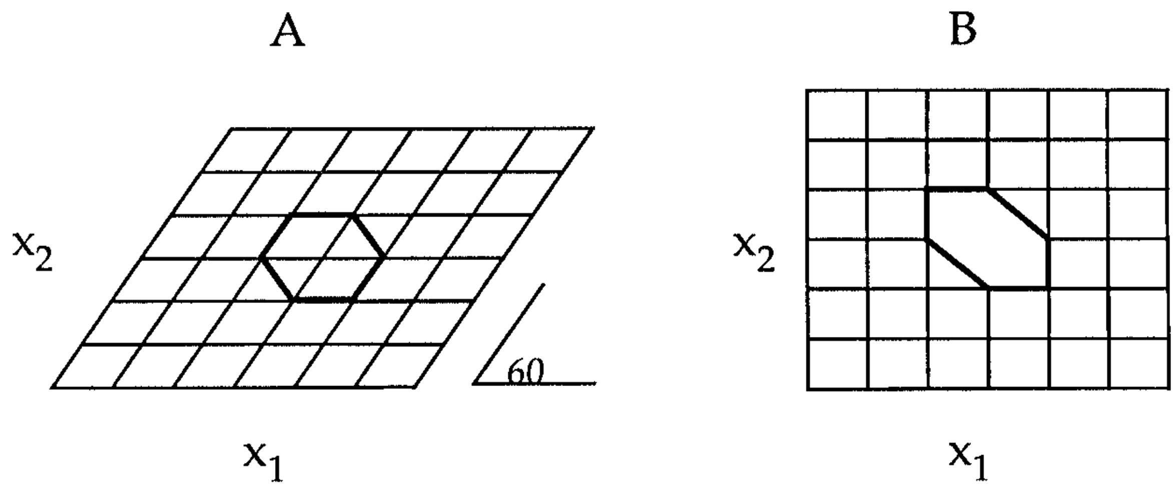


Fig. 6. A) Hexagonal lattice represented as skewed rectangular coordinates. B) De-skewed rectangular coordinates. The hexagon is distorted into an oblique lozenge.

In this coordinate scheme, the sampling matrices are even simpler. They are the same as above (Eq.s 24, 25, and 26) except that we drop the matrix $\mathbf{H}$ from each expression.

This leads to a natural method for application of this coding scheme to conventional rectangular images. When the skewed coordinates are "de-skewed" (Fig. 6B), the hexagon is distorted into an oblique lozenge. The orthogonal pyramid may then be constructed using these lozenges as the shape for each kernel. The kernels will no longer be rotationally symmetric, but for some purposes this may be unimportant. As before, exact coding will be possible so long as the sides of the rectangle are a power of seven.

Biological image coding

One likely role of the primate visual cortex is to encode the retinal image in components that are less correlated than the image pixels themselves. The scheme we have described provides a model for how this might be done. In this context, the initial samples (indicated by the vertices of the smallest hexagons in Fig. 4) correspond to the receptive field centers of retinal ganglion cell inputs. Each hexagon defines the receptive field of a single cortical unit. The coefficients of each basis function describe the weights with which each ganglion cell contributes to the response of the cortical cell. The basis functions defined on the smallest hexagons correspond to the cells tuned to the highest spatial frequencies. Each subsequent level of the pyramid corresponds to cells tuned to lower and lower frequencies. The low-pass basis functions at each level correspond to un-oriented pooling units, which in turn are used to create the high-pass units at the next level. These pooling units may correspond to actual cells, or may simply define which ganglion cells contribute inputs to the high-pass units at each level.

Elsewhere we have introduced the term *chexagon* (cortical hexagon) to describe the generic scheme of construction of cortical receptive fields through combination of retinal ganglion cell inputs laid out on a hexagonal lattice (A. B. Watson, Cortical algotecture, in *Vision: Coding and Efficiency*, C. B. Blakemore, Ed., Cambridge University Press, Cambridge England, 1988). The present chexagon scheme agrees with what is known about cortical cells in several respects. The high-pass filters are tuned for both spatial frequency and orientation. The input lattice of ganglion cells is known to be approximately hexagonal, at least in the foveal region. The shape of the one-dimensional pass-band of each filter, when mutiplied by the pass-band of the ganglion cells, is similar to that of cortical cells. Finally, cortical cells are believed to form quadrature pairs, like the odd and even basis functions described here.

There are on the other hand a number of respects in which this scheme appears to differ from cortical coding. First, the frequency tuning functions of our filters are oriented in the sense of having a strongest response at one orientation, but they have a second lobe of response (of opposite sign) at the orthogonal orientation. Two-dimensional mapping of frequency tuning functions in cortical cells occasionally show such secondary lobes (De Valois, Yund, and Hepler, 1982), but they do not appear to be common. Second, the units we describe change in scale by $\sqrt{7}$ at each level, which might yield rather fewer different scales than are commonly supposed. Third, the 19.1° rotation of the axis of orientation at each scale reduces the degree of rotation invariance of the code, though rotational invariance is not known to hold for the cortical code. Fourth, the tuning functions produced by our scheme are broader in orientation than in spatial frequency. While subject to some debate, it is believed that this is opposite to the aspect ratio of cortical cells.

Finally, the precise crystaline structure of this code is clearly different from the biological heterogeneity of visual cortex. Nonetheless, the cortex is highly regular, and a scheme like ours may be the canonical form from which the actual cortex is a developmental perturbation. These issues are discussed at greater length elsewhere (A. B. Watson, Cortical algotecture, in *Vision: Coding and Efficiency*, C. B. Blakemore, Ed., Cambridge University Press, Cambridge England, 1988). Perhaps the best summary is that while this scheme may not describe exactly the cortical encoding architecture, it is an example of the form such an architecture might take.

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Appendix 1

The following is a program in the Postscript language to draw the chexagon pyramid in Fig. 4. The number of levels drawn is determined by the variable *maxdepth*. On an Apple laser printer, a *maxdepth* of 3 takes about 2 minutes to print. Each greater depth will take a factor of 7 longer.

```
statusdict /jobname (Beau fracthex.ps) put
/#copies 1 def
/timezero usertime def
/showSTATUS { = usertime timezero sub 1000 idiv = (Secs) = flush} def

/depth 0 def
/maxdepth 3 def                                          % maximum levels
/latticeRot 3 sqrt 5 atan def                            % lattice rotation angle
/root7 1 7 sqrt div def                                  % scale change between levels
/negrot {/latticeRot latticeRot neg def} def
/down {/depth depth 1 add def } def                      % increments depth
/up {/depth depth 1 sub def } def                        % decrements depth
/inch {72 mul} def                                       % scale to inches

/hexside {60 rotate 1 0 lineto currentpoint translate } def   % draw one side of a hexagon

/drawhex                                                 % draw unit hexagon
{ gsave
-60 rotate 1 0 moveto 60 rotate currentpoint translate   % move to first vertex
5 { hexside } repeat                                     % draw 5 sides
closepath stroke                                         % draw sixth side
grestore } def

/vertex   % angle is on stack      % go to vertex at angle, draw hexagon pyramid
{/angle exch def
gsave
angle rotate 1 0 translate angle neg rotate
fracthex
grestore
} def
```

```
/fracthex                                            % draw hexagon pyramid
{gsave
root7 dup scale                                      % reduce scale by root 7
2 72 div setlinewidth
down negrot latticeRot rotate drawhex           % move down one level, rotate lattice, draw hex
depth maxdepth le                                    % test if at max level
    {fracthex                                        % recursive call to fracthex
    0 60 300 { vertex } for                          % call vertex at each vertex
    } if
up negrot grestore
} def

gsave                                                % main program
4.25 inch 5.5 inch moveto currentpoint translate     % set origin
6 inch 6 inch scale                                  % set global scale
latticeRot neg rotate                                % set initial orientation
1 setlinejoin
fracthex                                             % do it
grestore
1 inch 1 inch moveto
/Palatino-Roman findfont 34 scalefont setfont        % label
(Chexagon Pyramid) show
showpage
(Elapsed time) showSTATUS
```

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16. Abstract
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