Econometrics Project: A study of the EUR/USD Exchange rate with the Mundell-Fleming model

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Contents

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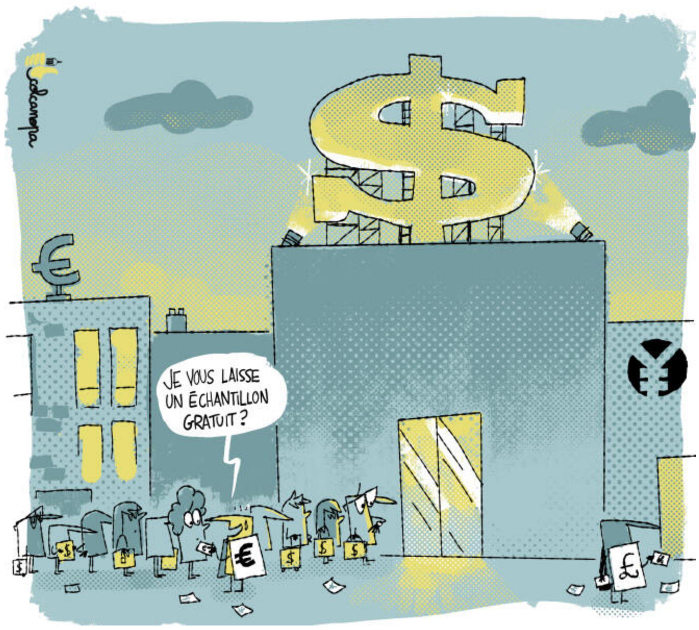
1 Introduction



Until the First World War, gold was the common reference for the means of exchange, ensuring stability between the exchange rates of the different national currencies. Since the collapse of the gold standard, many efforts have been made to maintain a degree of exchange rate stability, but ended up failing and floating currencies became the standard since the early 1970s. The exchange rate risk arising is omnipresent for companies whose investors, customers, suppliers or competitors have a different reference currency. Thus, understanding the dynamics of exchange rates is not only crucial for policymakers but also for investors, businesses, and analysts navigating the complex terrain of international business.

The main objective of this report is thus to study a subject of debate in economics: the evolution of exchange rates as a function of different macroeconomic factors. We base our analysis on the Mundell-Fleming model, which helps understand the impact of economic policies (monetary and fiscal) in an economy open to foreign trade and finance. Mundell-Fleming supports the idea that differentials in exchange rates between two economic zones are influenced by variables such as the interest rate differential, trade surplus or deficits, unemployment rates, etc. Hence, with the help of gretl and econometrics tools, we will assess the model's hypothesis.

2 Definition and descriptive statistics

2.1 Dependent Variable (Y)

We are hence observing the EUR/USD exchange rate data in quarterly average prices from the European Central Bank Data Browser from 1999 to 2023. The variable corresponds to the price of 1 euro expressed in US dollars.

Both the Euro and the US Dollar are floating currencies, which means the exchange rate is driven by the FX market's demand and supply. Quarterly exchange rates are constructed by taking the average daily closing exchange rate during that quarter.

Statistic	**Value**
Mean	1.1882$
Minimum	0.8683$
Maximum	1.5622$
Standard Deviation	0.15615$

Table 1: Y: Main Statistics

2.2 Explanatory Variables

We are studying the evolution of the exchange rate between two currencies in two distinct economic zones (the Euro area and the United States). As such, when picking the explanatory variables, we decided to take a comparable macroeconomic measure between the two economic zones, and study the differential between the two. Our data on the following variables comes from the ECB data browser, FRED St. Louis and rateinflation.com.

2.2.1 X1 - Reference Interest Rate Differential between the US and Euro Area

Director interest rates are used as a tool to stabilize the economy. Hence, variations in these rates can reflect the economic outlook and stability in each region, which has an impact on investor confidence and therefore on the exchange rate. To observe the impact of the director interest rates on the EUR/USD exchange rate, we created a differential which is the difference between the key interest rate from the FED and the one from the ECB. We can compute the following data with gretl :

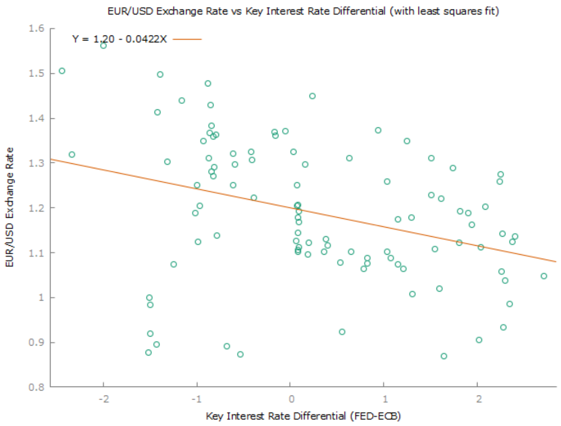


Figure 1: EUR/USD Exchange rate vs Key Interest Rate Differential

Statistic	Value
Mean	0.27950
Minimum	-2.4400
Maximum	2.7200
Standard Deviation	1.2687

Figure 2: X1: Main Statistics

The correlation coefficient is given by:

$$r_{x,y} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i}{\sqrt{(n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2)} \times \sqrt{(n\sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2)}} = -0.343$$

2.2.2 X2 - US and EU unemployment rates

Unemployment rates are key indicators of the economic health of a region. Low unemployment rates often signify a strong and growing economy, while high unemployment may indicate economic challenges. Economic conditions affecting trade flows can influence the supply and demand for currencies, impacting exchange rates. Here, we created the differential by subtracting the US unemployment rate from the Eurozone one. Below are some of the computations for this variable:

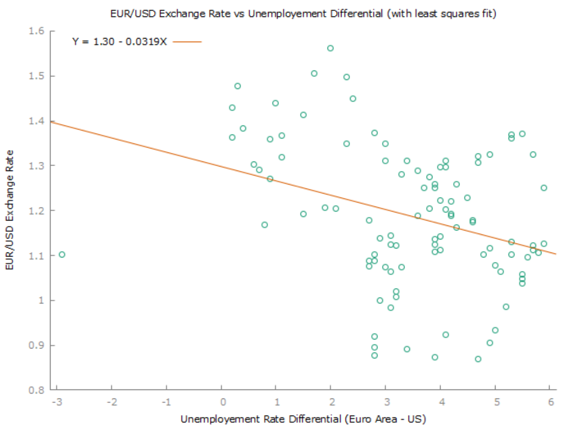


Figure 3: EUR/USD Exchange rate vs Unemployment Rate Differential

Statistic	Value
Mean	3.4470
Minimum	-2.900
Maximum	5.900
Standard Deviation	1.6297

Figure 4: X2: Main Statistics

With the following correlation coefficient :

$$r_{x,y} = -0.3327 \qquad\qquad with\ n = 100$$

2.2.3 X3 - US and EU inflation rates

In the Mundell-Fleming model, the link between inflation and the exchange rate is made through the concept of the Purchasing Power Parity (PPP). This concept stipulates that the exchange rate between two countries with floating exchange rate regimes should adjust in order to equalize the price levels of a basket of identical goods and services in those countries. Hence, if the inflation rate increases in the US against the one in Europe, the US dollar should be depreciated in order to maintain purchasing power parity and limit the decrease in export competitiveness. Here, the differential is the US inflation rate minus the Eurozone one, and we can compute the following data from gretl :

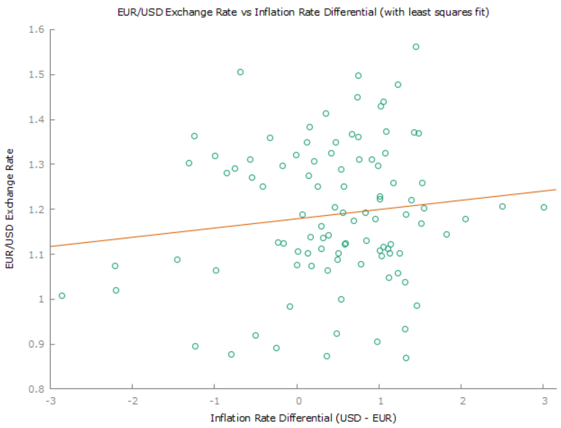


Figure 5: EUR/USD Exchange rate vs Inflation Rate Differential

Statistic	Value
Mean	0.45420
Minimum	-2.8537
Maximum	3.0147
Standard Deviation	0.96200

Figure 6: X3: Main Statistics

The correlation coefficient is given by:

$$r_{x,y} = 0.1266 \qquad\qquad with\ n = 100$$

2.2.4 X4 - Differential between the US 10 years Treasury Bond yield and the 10 years German bond yield

Both the US 10 year Treasury yield and the German 10 year Bond yield are regarded as the safest investments, with both securities being rated AAA by rating agencies. Because both Treasury notes are regarded as risk-free investments, a sizable yield spread (a difference between both yields) can influence capital inflows/outflows from and to both economic zones, which will have an impact on the exchange rate according to the Mundell-Fleming model. The differential here is the US 10yr Treasury yield minus the German 10yr Bund yield. Here are some computations of the main statistics of this variable:

With the following correlation coefficient : $r_{x,y} = -0.2493$.

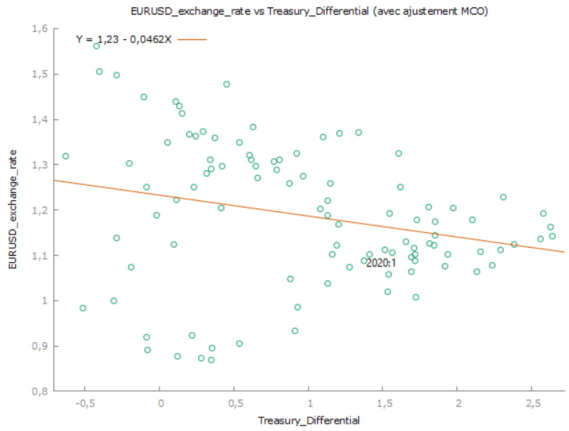


Figure 7: EUR/USD exchange rate vs Treasury yield differential

Statistic	Value
Mean	0.9662
Minimum	-0.6279
Maximum	2.6429
Standard Deviation	0.8432

Figure 8: X4: Main Statistics

3 Econometric Model and First OLS estimation

3.1 First Estimation

Model 1: OLS, using observations 1999:1–2023:4 ($T = 100$)
Dependent variable: EUR/USD Exchange Rate

Table 2: Model 1: The OLS Regression results

Variables	***Coefficient***	***Std. Error***	***t-ratio***	***p-value***
const	1.25982	0.0355026	35.49	0.0000
X1_Interest Rate Differential	−0.0363237	0.0129111	−2.813	0.0059
X2_Unemployment Rate Differential	−0.0232334	0.00988636	−2.350	0.0208
X3_Inflation Rate Differential	0.0410264	0.0152875	2.684	0.0086

Statistic	**Value**	**Statistic**	**Value**
Mean dependent var	1.188212	S.D. dependent var	0.156153
Sum squared resid	1.897461	S.E. of regression	0.140589
R^2	0.213969	Adjusted R^2	0.189405
$F(3, 96)$	8.710847	P-value(F)	0.000036
Log-likelihood	56.33881	Akaike criterion	104.6776
Schwarz criterion	94.25694	Hannan-Quinn	100.4602
ρ	0.932754	Durbin-Watson	0.138269

Thus the equation of the first model is given by :

$$\boxed{(\hat{y}) = 1.256 - 0.0363 \times X_1 - 0.023 \times X_2 + 0.041 \times X_3} \tag{1}$$

The model shows a negative relationship between the EUR/USD exchange rate with key interest rates and unemployment rates, and a positive one with inflation, which seems to be aligned with the theory of the Mundell-Fleming model.

3.2 Global Significance of the model

The hypothesis:

$$\begin{cases} H_0 : a1 = a2 = a3 = 0 \\ H_1 : \text{at least one coefficient is different from 0} \end{cases}$$

The test statistic: $F^* = \frac{\frac{R^2}{k}}{\frac{1-R^2}{n-k-1}} = \frac{\frac{0.214}{3}}{\frac{1-0.214}{96}} = 8.712$

Decision rule: F* is to be compared to $F^{0.05*}(k, n-k-1) = F^{0.05*}(96)$ =1.66. F* is larger than 1.66, then H_0 is rejected for a significant level $\alpha = 5\%$. Hence the model is significant.

4 Initial Tests

4.1 Coefficient a1 (T-test)

The hypothesis:

$$\begin{cases} H_0 : a_1 = 0 \\ H_1 : a_1 \neq 0 \end{cases}$$

Test statistic: Under H_0, $t^*_{\hat{a}_1} = \frac{\hat{a}_1 - a_1}{\hat{\sigma}_{\hat{a}_1}} = \frac{-0.0363-0}{0.0129} = -2.813$

Decision rule: $|t^*_{\hat{a}_1}|$ is to be compared to $t^{\alpha/2}(n-k-1) = t^{0.025}(96) = 1.984$ from the Student table. $|t^*_{\hat{a}_1}| = 2.814$ is greater than $t^{0.025}(96)$, then H_0 is rejected for a significant level $\alpha = 5\%$. Hence $\hat{a}_1$ is significantly different from 0.

4.2 Coefficient a2 (T-test)

The hypothesis :

$$\begin{cases} H_0 : a_2 = 0 \\ H_1 : a_2 \neq 0 \end{cases}$$

Test statistic: Under H_0, $t^*_{\hat{a}_2} = \frac{\hat{a}_2 - a_2}{\hat{\sigma}_{\hat{a}_2}} = \frac{-0.023-0}{0.010} = -2.350$

Decision rule:$|t^*_{\hat{a}_2}|$ is greater than $t^{\alpha/2}(n-k-1) = t^{0.025}(96) = 1.984$. Hence H_0 is rejected for a significant level $\alpha = 5$, thus $\hat{a}_2$ is significantly different from 0.

4.3 Coefficient a3 (T-test)

The hypothesis:

$$\begin{cases} H_0 : a_3 = 0 \\ H_1 : a_3 \neq 0 \end{cases}$$

Test statistic:Under H_0, $t^*_{\hat{a}_3} = \frac{\hat{a}_3 - a_3}{\hat{\sigma}_{\hat{a}_3}} = \frac{0.041-0}{0.015} = 2.684$

Decision rule: $|t^*_{\hat{a}_3}|$ is greater than $t^{\alpha/2}(n-k-1) = t^{0.025}(96) = 1.984$. Hence, H_0 is rejected for a significant level $\alpha = 5$, thus $\hat{a}_3$ is significantly different from 0.

4.4 Equality Test

We decided to take a_1 and a_3 for the T-test of equality as the inflation and policy rates are generally correlated. Hence, we want to know if they are equal in our current model.

The hypothesis:

$$\begin{cases} H_0 : a_1 + a_3 = 0 \\ H_1 : a_1 + a_3 \neq 0 \end{cases}$$

Test statistic: Under H_0, $t^* = \frac{\hat{a}_1+\hat{a}_3}{\hat{\sigma}_{\hat{a}_1+\hat{a}_3}}$
We have $\hat{\sigma}_{\hat{a}_1} = \frac{\hat{a}_1}{t^*_{\hat{a}_1}} = 0.0129111$ and $\hat{\sigma}_{\hat{a}_3} = \frac{\hat{a}_3}{t^*_{\hat{a}_3}} = 0.0152875$

We generate the covariance matrix of the regression coefficient on Gretl :

Table 3: Coefficient Covariance Matrix

const	**X1**	**X2**	**X3**	
0.0012604	0.00016783	−0.00031518	−5.1247e–005	const
	0.00016670	−5.7039e–005	−3.9205e–005	X1
		9.7740e–005	−1.2749e–005	X2
			0.00023371	X3

Then we can compute:
$\hat{\sigma}_{\hat{a}_1+\hat{a}_3} = (\hat{\sigma}^2_{\hat{a}_1} + \hat{\sigma}^2_{\hat{a}_3} + 2 * \sigma_{\hat{a}_1} * \sigma_{\hat{a}_3} * \text{cov}(\hat{a}_1, \hat{a}_3))^{0.5}$

$\hat{\sigma}_{\hat{a}_1+\hat{a}_3} = ((0.01291)^2 + (0.01529)^2 + 2 \cdot (0.01291) \cdot (0.01529) \cdot (-3.92050e - 05))^{0.5} = 0.0200$
Therefore we find: $t^* = \frac{-0.036+0.041}{0.02} = 0.25$

Decision rule: $|t^*|$ is to be compared to $t^{\alpha/2}(n - k - 1) = t^{0.025}(96) = 1.96$
$|t^*| = 0.25$ Therefore we conclude $\hat{a}_1$ and $\hat{a}_3$ are not significantly different.

4.5 Chow F-Test

One of the biggest flaws in the Mundell-Fleming model is that, after all, exchange rates in floating currencies are subject to market sentiment. Therefore, when looking at long periods of data, we might introduce a good amount of noise into the model. If this theory is validated, breaking our 24-year period into different subsamples should result in different coefficients. For this, we're studying the EUR/USD regression model with data from 1999:1 to 2007:2 (pre-subprime crisis) and 2007:3 to 2023:4 (post-subprime crisis). We chose to define both subsamples as such because we believe that the subprime crisis was an event that changed the assumptions of investors.

We estimate the 2 following regressions (see Appendix for OLS results) :

- **Model** 1_a: Data from Q1 1999 to Q2 2007. n = 34

$$\boxed{(\hat{y}) = 1.615 + 0.035 \times X_1 - 0.0161 \times X_2 + 0.165 \times X_3} \tag{2}$$

- **Model** 1_b: Data from Q3 2007 to Q4 2023. n = 66

$$\boxed{(\hat{y}) = 1.251 - 0.0880 \times X_1 - 0.005 \times X_2 + 0.024 \times X_3} \tag{3}$$

Statistic	**Model 1a**	**Model 1b**
Mean dependent var	1.102297	1.232471
Sum squared resid	0.508728	0.519555
R-squared	0.374699	0.574187
F(3, 30)	5.992296	-
F(3, 66)	-	27.86799

The hypotheses:

$$\begin{cases} H_0 : a_1^1 = a_1^2, \quad a_2^1 = a_2^2, \quad a_3^1 = a_3^2 \\ H_1 : \text{at least one of the coefficients is different} \end{cases}$$

Test statistic:Under H_0,

$$F^* = \frac{\frac{(RRSS-URSS)}{q}}{\frac{URSS}{dF^U}} = 19.44$$

with $dF^U = dF^1 + dF^2 = (n_1 - k - 1) + (n_2 - k - 1) = 92$. Hence, URSS = 1.028283, RRSS = 1.897461, and $q = 4$.

Decision rule:F^* is to be compared to $F_{0.05}(k+1, n-2(k+1)) = F_{0.05}(3+1, 100-2(3+1)) = F_{0.05}(4, 92) = 2.70$
Our aforementioned hypothesis is true; there is a structural change in the model for sub-samples A and B.

5 Refined Model

When running the first couple of models on our EUR/USD exchange rate estimations, two big flaws were identified.

Following our Chow F-Test, we recognized that looking at data series over a long span of time introduced noise into the model. This is logical as after all, the valuation of a currency pair is dependent on the market ”sentiment” which is not easily quantifiable. We decided to narrow down the scope of the model, and focus only on post-2007 crisis data. In doing so, we also decided to switch the unemployment differential variable for the treasury differential, as we saw the latter had a much higher correlation with the EUR/USD Exchange rate for the 2007:2 - 2023:4 period.

Secondly, we have the problem of the “reactivity” of the economy. Data such as the inflation rate is typically published with a certain “lag” (Think about the ECB/FED publishing the April monthly inflation in May or June). Similarly, director interest rates take a certain time to have a quantifiable impact on the economy (Hence why forward guidance and expectation management are important for a central bank).

To tackle this issue, we decided to add a 1-period lag to all 3 variables in the new model.

Introduced the above-mentioned changes resulted in a model with a much higher coefficient of determination, higher global significance, and higher individual t-ratios for the coefficients.

Table 4: Model 2: OLS, using observations 2007:4–2023:4 (T = 65)

	Coefficient	**Std. Error**	***t*-ratio**	**p-value**
const	1.43726	0.0235610	61.00	0.0000
X1_Interest_Rate_Diff_(t-1)	0.0498732	0.0160951	3.099	0.0029
X3_Inflation_Rate_Diff_(t-1)	0.0522482	0.00851722	6.134	0.0000
X4_Treasury_Yield _Diff_(t-1)	−0.194738	0.0209577	−9.292	0.0000

Statistic	**Value**	**Statistic**	**Value**
Mean dependent var	1.230297	S.D. dependent var	0.136923
Sum squared resid	0.263742	S.E. of regression	0.065754
R^2	0.780191	Adjusted R^2	0.769381
$F(3, 61)$	72.17134	P-value(F)	4.79e-20
Log-likelihood	86.75209	Akaike criterion	165.5042
Schwarz criterion	156.8066	Hannan-Quinn	162.0724
ρ	0.694056	Durbin-Watson	0.620449

6 Tests on refined model

6.1 Linear Restriction Test - Testing the refined model against literature

We'll be comparing our new coefficients with those obtained in the following publication by the European Commission Directorate-General for Economic and Financial Affairs: McCoy, 2020, Euro-US Dollar Exchange Rate Dynamics at the Effective Lower Bound, ISBN 978-92-79-77384-6

T-test: The hypothesis:

$$\begin{cases} H_0: & \begin{matrix} \hat{a}_1 = 0.06 \\ \hat{a}_3 = 0.044 \\ \hat{a}_4 = -0.13 \end{matrix} \quad H_1: \begin{matrix} \hat{a}_1 \neq 0.06 \\ \hat{a}_3 \neq 0.044 \\ \hat{a}_4 \neq -0.13 \end{matrix} \end{cases}$$

To test the hypothesis, we compare the F statistic between the restricted and unrestricted model.
Test statistic: Under H_0,

$$F^* = \frac{\frac{(RRSS-URSS)}{q}}{\frac{URSS}{(n-k-1)}} \sim F(q, n-k-1)$$

Decision rule: If $|F^*|$ is smaller than $F^{0.05}(q, n-k-1)$ We can conclude the H_0 is validated. Plugin the linear restriction on Gretl, we obtain the following $F^*(3, 61) = 21.35$ which is larger than $F^{0.05}(3, 61) = 2.75$

Therefore we conclude H_0 is not validated. This does not mean our results are strictly different from those obtained in literature, many factors can affect the regression coefficient on a topic such as this. We did not find another study that constructed a regression model similar to ours (Same period, same frequency of data, same variables, etc.). But we content ourselves of having found regression coefficients that are at least similar to those obtained in other studies.

6.2 Joint Test

We want to do a joint test, based on the results obtained in the unrestricted model, to see if we can write the regression model with a restriction equation involving the 3 coefficients variables.

$$\boxed{\hat{a}_4 + 2 \cdot \hat{a}_1 + 2 \cdot \hat{a}_3 = 0} \tag{4}$$

T-test: The hypothesis:

$$\begin{cases} H_0 : \hat{a}_4 + 2 \cdot \hat{a}_1 + 2 \cdot \hat{a}_3 = 0 \\ H_1 : \hat{a}_4 + 2 \cdot \hat{a}_1 + 2 \cdot \hat{a}_3 \neq 0 \end{cases}$$

We apply the same test as before, obtaing the following output from Gretl: $F^*(1, 61) = 0.1361$ which is smaller than $F^{0.05}(1, 61) = 3.99$
We conclude that the H_0 is validated, and that we can apply the restriction on coefficients without loosing fit.

7 Conclusion

We selected four explanatory variables to capture diverse aspects of the economy. On the financial side, interest rate and treasury yield differentials reflect potential investor movements. On the international trade front, differences in inflation rates and unemployment highlight the relative competitiveness of both economies.

We created a 1st Model that naively takes 3 of these variables, and achieves a good level of significance and fit.

Model 2 demonstrates a substantial improvement in explanatory power compared to Model 1, as evidenced by the significantly higher R-squared and F-statistic.

We saw the biggest improvement in significance and fit when splitting the data into sub-samples and when adding a 1-period lag to all explanatory variables. This validated the 2 initial flaws identified in Model 1: There are structural changes overtime which makes the model fit better in shorter periods, and the exchange rate does not react immediately, lagging the variables helps for a temporal alignment with the dependent variable.

The tests performed in model 2 gaves us indication that we can build a simple restriction equation on all 3 coefficients, and not loose fit.

Because we achieved a substantial improvement in explanatory power with the second model, we might be tempted to say our results indicate a causal estimation. However, it's essential to assess the validity of other assumptions such as colinearity or autocorrelation of errors before interpreting causality. This will be the scope of the second part of the project.

8 Appendix

1. Full statistics for variables

Variable/Sum	X	Y	X^2	Y^2	XY	$corr(x,y)$
X1	27.9500	118.8212	167.1679	143.5988	26.4786	-0.3432
X2	344.7000	118.8212	1451.1100	143.5988	401.1954	-0.3327
X3	45.4200	118.8212	112.2479	143.5988	55.8518	0.1266
X4	96.6203	118.8212	163.7387	143.5988	111.5561	-0.2493

2. OLS Model Results for Chow Test

Table 5: Model 1a: OLS Regression Results

Variable	**Coefficient**	**Std. Error**	**t-ratio**	**p-value**
const	1.61506	0.160130	10.09	3.75×10^{-11}
X1_Interest Rate Differential	0.0349094	0.0260363	1.341	0.1901
X2_Unemployment Rate Differential	−0.160845	0.0449172	−3.581	0.0012
X3_Inflation Rate Differential	0.165354	0.0554211	2.984	0.0056

Table 6: Model 1b: OLS Regression Results

Variable	**Coefficient**	**Std. Error**	**t-ratio**	**p-value**
const	1.25067	0.0238101	52.53	4.07×10^{-53}
X1_Interest Rate Differential	−0.0879570	0.0113007	−7.783	9.41×10^{-11}
X2_Unemployment Rate Differential	−0.00481724	0.00675991	−0.7126	0.4788
X3_Inflation Rate Differential	0.0238969	0.0107095	2.231	0.0293

3. Covariance of coefficients matrix for the refined model

Table 7: Coefficient Covariance Matrix: Model2

const	X1	X2	X4	
0.00055512	0.00030752	5.9426e–005	−0.00045920	const
	0.00025905	5.0760e–005	−0.00029971	X1
		7.2543e–005	−7.7265e–005	X3
			0.00043923	X4