Homework 4
CS540 F23

Assignment Goals

- Process real-world data
- Implement hierarchical clustering
- Visualize the clustering process

You are to perform hierarchical clustering on socioeconomic data from various countries. Each country is defined by a row in the data set. We will use a subset of the data to represent each country with a feature vector. For this assignment, you must represent a country with 6 statistics: 'Population', 'Net migration', 'GDP ($ per capita)', 'Literacy (%)', 'Phones (per 1000)', and 'Infant mortality (per 1000 births)'.

After each country is represented as a 6-dimensional feature vector ($x_1, \ldots, x_6$), you need to cluster the countries with hierarchical agglomerative clustering (HAC). This clustering will allow us to visualize which countries have similar socioeconomic situations. Your function should work similarly to scipy.cluster.hierarchy.linkage() with method='complete'.

Program Overview

The data in CSV format can be found in the file countries.csv. Note, that there is no starter code for this assignment. If you feel stuck, refer to the starter code from past assignments. You will have to write a few Python functions for this assignment. Here is a high-level description of each for reference:

1. load_data(filepath) — takes in a string with a path to a CSV file and returns the data points as a list of dicts. Section 0.1
2. calc_features(row) — takes in one-row dict from the data loaded from the previous function then calculates the corresponding feature vector for that country as specified above, and returns it as a NumPy array of shape (6,). The dtype of this array should be float64. Section 0.2
3. hac(features) — performs complete linkage hierarchical agglomerative clustering on the country with the ($x_1, \ldots, x_6$) feature representation, and returns a NumPy array representing the clustering. Section 0.3
4. fig_hac(Z, names) — visualizes the hierarchical agglomerative clustering on the country's feature representation. Section 0.4
5. normalize_features(features) — takes a list of feature vectors and computes the normalized values. The output should be a list of normalized feature vectors in the same format as the input. Section 0.5

You may implement other helper functions as necessary, but these are the functions we are testing. In particular, your final Python file is just a suite of functions, you should not have code that runs outside of the functions. To test your code, you may want a "main" method to put it all together. Make sure, you either delete any testing code running outside functions or wrap it in a 'if __name__=="__main__":'. This is discussed more in Section 0.6.

Program Details

0.1 load_data(filepath)

Summary. [10pts]

- Input: string, the path to a file to be read.
- Output: list, where each element is a dict representing one row of the file read.

Details.

1. Read in the file specified in the argument, filepath. Note, that the DictReader from Python's csv module is useful but, depending on your Python version, this might return OrderedDicts instead of normal dicts. Make sure you convert to dict as appropriate if you choose to use this function.
2. Return a list of dictionaries, where each row in the dataset is a dictionary with the column headers as keys and the row elements as values.

You may assume the file exists and is a properly formatted CSV.

0.2 calc_features(row)

Summary. [10pts]

- Input: dict representing one country.
- Output: NumPy array of shape (6,) and dtype float64. The first element is x_1 and so on with the sixth element being x_6.

Details. This function takes as input the dict representing one country and computes the feature representation ($x_1, \ldots, x_6$). Specifically,

1. x_1 = 'Population'
2. x_2 = 'Net migration'
3. x_3 = 'GDP ($ per capita)'
4. x_4 = 'Literacy (%)'
5. x_5 = 'Phones (per 1000)'
6. x_6 = 'Infant mortality (per 1000 births)'

Note, these stats in the dict may not be float types. Make sure to convert each relevant

stat to float when computing each x_i. Return a NumPy array having each x_i in order: $x_1, \ldots, x_6$. The shape of this array should be (6,). The dtype of this array should be float64. Remember, this function works for only one country, not all of the ones that you loaded in load_data. Make sure you are outputting the exact data structures with appropriate types as specified or you risk a major reduction in points.

0.3 hac(features)

Summary. [50pts]

- Input: list of NumPy arrays of shape (6,), where each array is an $(x_1, \ldots, x_6)$ feature representation as computed in Section 0.2. The total number of feature vectors, i.e. the length of the input list, is n. Note, that we test your code on different n's as stated in Section 0.6).

- Output: numpy array of shape $(n-1) \times 4$. For any i, $Z[i, 0]$ and $Z[i, 1]$ represent the indices of the two clusters that were merged in the ith iteration of the clustering algorithm. Then, $Z[i, 2] = d(Z[i, 0], Z[i, 1])$ is the complete linkage distance between the two clusters that were merged in the ith iteration (this will be a real value, not integer like the other quantities). Lastly, $Z[i, 3]$ is the size of the new cluster formed by the merge, i.e. the total number of countries in this cluster. Note, the original countries are considered clusters indexed by $0, \ldots, n-1$, and the cluster constructed in the ith iteration ($i \geq 1$) of the algorithm has cluster index $(n-1)+i$. Also, there is a tie-breaking rule specified below that must be followed.

Distance. Using complete-linkage, perform the hierarchical agglomerative clustering algorithm as detailed in the lecture. Use the standard Euclidean distance function to calculate the distance between two points. You may implement your own distance function or use numpy.linalg.norm(). Other distance functions might not work as expected so check your results on gradescope! You are liable for any reductions in points you might get for using a package distance function.

Outline. Here is one possible path you could follow to implement hac()

1. Number each of your starting data points from 0 to $n-1$. These are their original cluster numbers.

2. Create an $(n-1) \times 4$ array or list. Iterate through this array/list row by row. For each row,

   (a) Determine which clusters are closest and put their numbers into the first and second elements of the row, $Z[i, 0]$ and $Z[i, 1]$. The first element listed, $Z[i, 0]$ should be the smaller of the two cluster indexes.

   (b) The complete-linkage distance between the two clusters goes into the third element of the row, $Z[i, 2]$

   (c) The total number of countries in the new cluster goes into the fourth element, $Z[i, 3]$

If you merge a cluster containing more than one country, its index (for the first or second element of the row) is given by n + the row index in which the cluster was created.

3. Before returning the data structure, convert it into a NumPy array if it isn't one already.

For this method to run efficiently when n is large you should create a distance matrix at the beginning of the function to avoid having to recalculate the distances between points. You should be able to run HAC efficiently with all 176 countries. For example, if you have a NumPy array called distance_matrix then distance_matrix[3,4] is equal to the Euclidean distance between the countries at index 3 and 4 in the features input. You can compare your output with scipy.spatial.distance_matrix, but please note we are using Euclidean distance for this assignment.

Tie Breaking. When choosing the next two clusters to merge, we pick the pair having the smallest complete-linkage distance. In the case that multiple pairs have the same distance, we need additional criteria to pick between them. We do this with a tie-breaking rule on indices as follows: Suppose $(i_1, j_1), \ldots, (i_h, j_h)$ are pairs of cluster indices with equal distance, i.e., $d(i_1, j_1) = \cdots = d(i_h, j_h)$, and assume that $i_t < j_t$ for all t (so each pair is sorted). We tie-break by picking the pair with the smallest first index, i. If there are multiple pairs having first index i, we need to further distinguish between them. Say these pairs are $(i, t_1), (i, t_2), \ldots$ and so on. To tie-break between these pairs, we pick the pair with the smallest second index, i.e., the smallest t value in these pairs. Be aware that this tie-breaking strategy may not produce identical results to linkage().

0.4 fig_hac(Z, names)

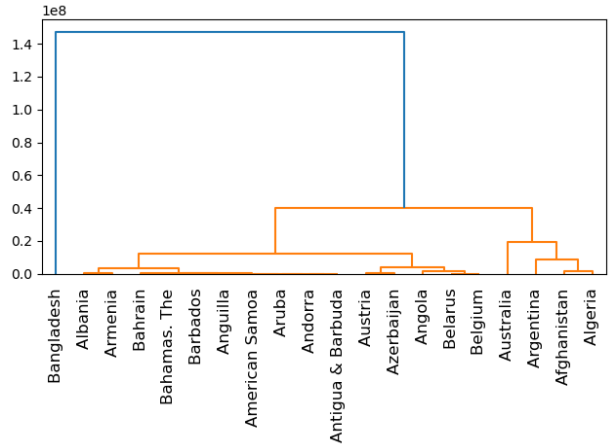
Summary. [10pts]

- Input: NumPy array Z output from hac, and list of string names corresponding to country names with length n.
- Output: A matplotlib figure with a graph that visualizes the hierarchical clustering.

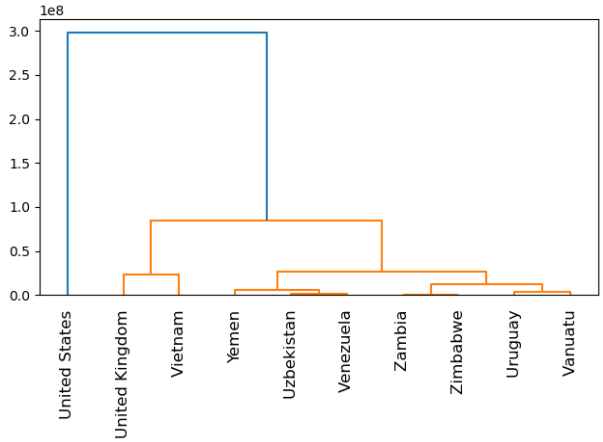
Details. Begin by initializing a figure with 'fig = plt.figure()`. Then use dendrogram in the SciPy module with `labels=names` and `leaf_rotation = ???`. Your plot will likely cut off the x labels. For the graph to look like the below examples you will need to call tight_layout() on the figure.

The visualizations for the first 20 and final 10 countries.

N = 20



N = 10



Discussion. Notice that the figures above cluster the countries almost exclusively by population. The values for population are much larger than the values for other columns in the data. Therefore, the population disproportionately contributes to the Euclidean distance between feature vectors. Ideally, all six statistics should contribute to the clustering, so you will create a function to normalize the data in the next section.

0.5 normalize_features(features)

Summary. [20pts]

- Input: The input to this function will be a list of the feature vectors output from calc_features. Each feature vector is a NumPy array with shape (6,) and dtype float64.
- Output: The output should have an identical format to the input: a list of NumPy arrays with shape (6,) and dtype float64. However, the statistic values in the feature vectors should be replaced with their normalized values.

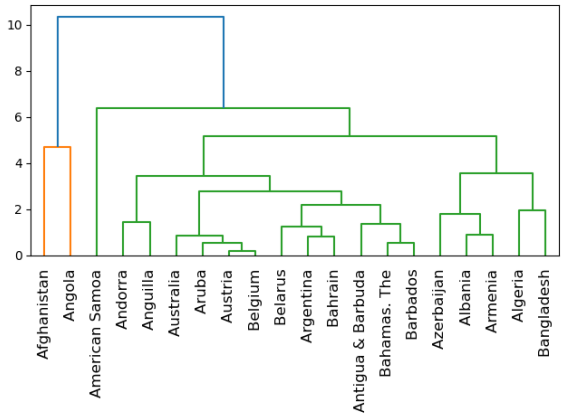
Details. For each of the 6 statistics, calculate the mean and standard deviation across the input data. Use the equation below to calculate the normalized feature values for each data point.

$$\frac{x-\mu}{\sigma}$$

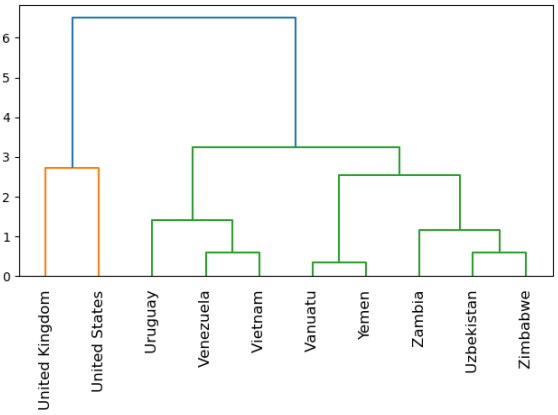
x represents the original value, μ represents the column's mean, and σ represents the column's standard deviation. Applying HAC on the normalized data should produce the plots below.

The visualizations for the first 20 and final 10 countries applying hac to normalized data.

N = 20



N = 10



Discussion. These results seem to cluster countries with similar socioeconomic statuses. Normalizing the data creates a similar range of values for all statistics, so they are able to equally contribute to the Euclidean distance.

0.6 Testing

To test your code, try running the following lines in a main method or in a jupyter notebook for various choices of *n*:

```
data = load_data("countries.csv")
country_names = [row["Country"] for row in data]
features = [calc_features(row) for row in data]
features_normalized = normalize_features(features)
n = ???
Z_raw = hac(features[:n])
Z_normalized = hac(features_normalized[:n])
fig = fig_hac(Z_raw, country_names[:n])
plt.show()
```

To help you test your code, we have provided the correct output of hac using normalized data for the first 50 countries in 'output.txt'. You can also compare your output of fig_hac to the graphs provided in this writeup.

For the hac function, we will test on both small and large values of *n* up to 204. With the entire dataset, your code should not take more than 15 seconds to run. We will test on n <= 30 for fig_hac.

Submission Details

- Please submit a file named hw4.py on Gradescope
- All code should be contained in functions or under a ‘if __name__=="__main__":’
- Be sure to remove all debugging output before submission.
- The assignment is due Tuesday, October 10 at 9:30 am.