

Shortest path problems:

Two of the United States' most-visited national parks are Yellowstone National Park (located mostly in Wyoming, but extending slightly across the border into Idaho and Montana) and Yosemite National Park (located in California). Like many other national parks, Yellowstone and Yosemite are located away from major cities and thoroughfares, so that driving to and from either one may involve either roundabout travel or travel on smaller, slower roadways.

A company specializing in nature tours offers two-location camping trips at Yellowstone and Yosemite. Customers drive to their choice of either of the two parks, and the company provides tours of the parks, as well as bus service between the parks. Every few days a bus drives from one park to the other. A bus from Yosemite to Yellowstone, for example, might have some Yosemite-based campers on their way to the second part of their tour, and also some Yellowstone-based campers who are heading back at the end of their tour.

Using the Rand McNally road atlas, the company has identified all of the major roads that might be used as part of the bus route between the two parks. The roads include, in decreasing order of size and speed, interstate highways, U.S. highways, and state highways. Interstate highways are denoted by an "I" before the highway number, U.S. highways by "US" before the highway number, and state highways by the state's abbreviation (for example, "CA") before the highway number. A map of the highways, as well as a table of the city-to-city connections, is included below.

The company's transportation director is unsure of the best path for the tour buses to take. Being a new, financially shaky company, they purchased older, heavily used buses. Therefore, minimizing the mileage that the buses travel is an important consideration. On the other hand, if the shortest-mileage path results in longer travel times then the company's customers might be dissatisfied, so minimizing the travel time is also an important consideration.

Part I

In the data in the table and map, we assume that travel distances and times are the same in both directions along each arc. Is this a reasonable assumption? Why or why not? Based on this assumption, explain why, if you have found a shortest path from Yellowstone to Yosemite, reversing the path gives the shortest path from Yosemite to Yellowstone.

Part II

Use Dijkstra's algorithm to calculate the shortest-mileage route between Yellowstone and Yosemite. Show your work.

Part III

Use the Dijkstra's algorithm to calculate the shortest-time route between Yellowstone and Yosemite. Show your work.

Part IV

Instead of simply minimizing the total distance or the total time, the company might rather strike a balance between the two by taking either the shortest-distance route subject to a time constraint (for example, no more than 18 hours) or the shortest-time route subject to a distance constraint (for example, no more than 900 miles). Recall from class that if we want to consider two measures of cost, we can use the multilabel algorithm. Use the multilabel algorithm to find the set of paths that the company should consider. Show your work.

Part V

Are the paths you found in Part II and Part III part of your answer for Part IV? Will this always be the case, why?

Location	X-Coord	Y-Coord
Yellowstone N.P., WY	46	30
Livingston, MT	44	35
Idaho Falls, ID	40	26
Butte, MT	38	36
Missoula, MT	32	40
Boise, ID	24	26
Salt Lake City, UT	40	15
Twin Falls, ID	30	22
Winnemucca, NV	17	16
Wells, NV	28	16
Ely, NV	28	8
Reno, NV	8	10
Bishop, CA	14	2
Sacramento, CA	2	6
Yosemite N.P., CA	10	4

Road Segment Endpoints		Highway(s)	Time (Min)	Distance (Miles)
Yellowstone N.P., WY	Livingston, MT	US-89	84	59
Yellowstone N.P., WY	Idaho Falls, ID	US-20	128	100
Livingston, MT	Butte, MT	I-90	100	104
Butte, MT	Idaho Falls, ID	I-15	210	205
Butte, MT	Missoula, MT	I-90	110	119
Missoula, MT	Boise, ID	US-12, US-95	475	371
Idaho Falls, ID	Salt Lake City, UT	I-15	206	208
Idaho Falls, ID	Twin Falls, ID	I-15, I-86, I-84	155	161
Boise, ID	Twin Falls, ID	I-84	128	131
Boise, ID	Winnemucca, NV	US-95	303	256
Twin Falls, ID	Wells, NV	US-93	141	118
Salt Lake City, UT	Wells, NV	I-80	174	181
Salt Lake City, UT	Ely, NV	I-15, US-6	262	241
Wells, NV	Winnemucca, NV	I-80	162	175
Wells, NV	Ely, NV	US-93	180	140
Winnemucca, NV	Reno, NV	I-80	153	164
Ely, NV	Bishop, CA	US-6	337	283
Reno, NV	Bishop, CA	US-395	255	205
Reno, NV	Sacramento, CA	I-80	152	133
Reno, NV	Yosemite N.P., CA	US-395, CA-120	225	154
Sacramento, CA	Yosemite N.P., CA	CA-99, CA-120	277	193
Bishop, CA	Yosemite N.P., CA	US-395, CA-120	132	65

