Open-Book Take-Home Exam — Communication Systems

John Thomas p

Roll number: 72

References: B. P. Lathi — Modern Digital and Analog Communication Systems; Simon Haykin — Communication Systems

Bloom's level: Understanding

Instructions (used)

Data rate = 10 × roll = 720 kbps.

Carrier frequency (when required) = roll × 1 kHz = 72 kHz (or as specified in question text).

Amplitude (when required) = 0.25 × roll = 18 V.

Sampling rate (when required) = 4000 + 100 × roll = 11 200 Hz.

Eb = roll × 10^{-6} J = 72 × 10^{-6} J.

N0 = roll × 10^{-8} W/Hz = 72 × 10^{-8} = 7.2 × 10^{-7} W/Hz.

Assume unity load (R = 1 Ω) where carrier power formulas require a load unless otherwise stated.

Show derivation steps and intermediate calculations.

PART A (6 × 3 = 18 Marks)

Answer ALL questions. Each carries 3 marks.

1. Why modulation is necessary in communication systems? Relate to channel bandwidth. (CO1, PO1)

Answer (concise):

Modulation maps a baseband (low-frequency) information signal onto a high-frequency carrier for efficient transmission. Main reasons:

Radiation & antenna size: Practical antennas require carrier frequency wavelengths; low-frequency signals would need impractically large antennas.

Multiplexing: Different carriers allow multiple signals to share the same physical channel (frequency-division multiplexing).

Channel suitability: Many transmission media (wireless, satellite) operate best at RF; modulating places the signal within usable band.

Noise/interference management & propagation: Certain frequency bands suffer less noise/attenuation; moving to those bands improves link quality.

Relation to bandwidth: The required channel bandwidth depends on modulation type and baseband bandwidth. For AM the modulated spectrum occupies carrier ± baseband bandwidth; for FM bandwidth depends on deviation and modulating bandwidth (Carson's rule). Thus modulation determines spectral occupancy and which channel (bandwidth) is required.

---

2. Differentiate between narrowband and wideband FM; discuss relative advantages in noisy channels. (CO2, PO1, PO2)

Answer (concise):

Narrowband FM (NBFM): modulation index $\beta \ll 1$ (typically $\beta < 0.3$). Spectrum similar to AM: main carrier plus small first-order sidebands. Approximate instantaneous FM ≈ carrier + phase modulation term. Occupies ≈ 2fm bandwidth. Simpler receivers; lower bandwidth usage.

Wideband FM (WBFM): $\beta \gg 1$ (many sidebands); wide spectrum. Carson's rule: $BW \approx 2(\Delta f + fm)$ where Δf is peak deviation. WBFM typically used in FM broadcast (better fidelity).

Advantages in noisy channels: WBFM provides capture effect and improved SNR (FM is largely amplitude noise-immune; discriminator recovers frequency and suppresses amplitude noise), especially when Δf large and when threshold effects are managed by pre-emphasis/de-emphasis. NBFM uses less bandwidth but offers less noise immunity and fidelity.

---

3. Define power spectral density (PSD) and explain its significance in random process analysis. (CO5, PO1, PO2)

Answer (concise):

Power Spectral Density of a (wide-sense) stationary random process describes how the process's average power is distributed over frequency. Formally, if is the autocorrelation function, then

$$S_X(f)=\mathcal{F}\{R_X(\tau)\}=\int_{-\infty}^{\infty} R_X(\tau)\,e^{-j2\pi f\tau}\,d\tau.$$

---

4. Derive expression for signal-to-noise ratio (SNR) in an amplitude-modulated system. (CO2, PO1, PO2)

Answer (concise derivation):

Consider AM (DSB-FC) with carrier and modulating signal normalized so that peak modulation index applies (for single tone). Carrier power (assuming $R = 1\ \Omega$):

$$P_c=\dfrac{A_c^2}{2}.$$

$$P_T = P_c\Big(1+\dfrac{\mu^2}{2}\Big).$$

$$\mathrm{SNR}=\dfrac{P_{\text{signal (sidebands)}}}{P_{\text{noise}}}$$

$$=\dfrac{P_c(\mu^2/2)}{N_0 B_m}.$$

---

5. Explain the sampling theorem's role in digital communications with a sketch. (CO3, PO1, PO2)

Answer (concise):

Nyquist–Shannon sampling theorem: A bandlimited signal with highest frequency can be perfectly reconstructed from uniform samples if sampling frequency (Nyquist rate). In digital comms:

A/D conversion: sampling is first step to convert analog message to discrete-time sequence for quantization and encoding. Choosing ≥ Nyquist avoids aliasing.

Bandwidth & data rate planning: sampling rate determines required digital bit rate after quantization; oversampling may help anti-aliasing and noise performance.

Reconstruction at receiver: proper lowpass reconstruction filter recovers analog waveforms.

(Sketch: time-domain sampling impulse train multiplying analog signal; frequency-domain replica of baseband spectrum centered at multiples of , showing no overlap when .)

---

6. For random process with mean and autocorrelation , explain how stationarity and ergodicity are tested. (CO5, PO1, PO2)

Answer (concise):

Wide-sense stationarity (WSS) tests:

Mean must be constant (independent of t).

Autocorrelation depends only on time difference : . Equivalently independent of t.

Strict stationarity: All joint distributions are invariant under time shifts.

Ergodicity: Time averages over a single realization equal ensemble averages. To test ergodicity in mean: compute time average and compare to ensemble mean . If equal (with probability 1), process ergodic in mean. Similarly for autocorrelation: time-averaged autocorrelation equals ensemble autocorrelation. Practically, ergodicity often tested by verifying WSS and that autocovariance decays properly so time averages converge.

PART B (4 × 10.5 = 42 Marks)

Answer ALL questions. Each question has (a) theoretical ≈70% and (b) numerical ≈30% using roll = 72.

---

7. (a) Explain working principle and block diagram of an AM transmitter and receiver. Discuss effect of modulation index on transmitted power.

Answer (7 marks — structured):

AM Transmitter (block diagram & explanation): typical blocks — Message source → Pre-emphasis/processing → Modulator → RF amplifier → Antenna matching → Antenna.

Carrier oscillator: generates carrier .

Modulator (e.g., linear multiplier or balanced modulator + carrier injection): combines message with carrier to produce where is normalized so . For DSB-FC AM, the modulator produces carrier plus symmetrical sidebands at .

RF amplifier & filter: amplify carrier and sidebands; bandpass filtering ensures only intended spectral components transmitted.

Antenna: radiates RF into channel.

AM Receiver (block diagram & explanation): typical blocks — Antenna → RF bandpass filter → RF amplifier → Mixer/local oscillator or direct envelope detector → IF filtering (if superheterodyne) → Demodulator (envelope detector or synchronous detector) → Lowpass filter → Output.

Superheterodyne: incoming RF is downconverted to IF for stable filtering and amplification; then demodulated.

Envelope detector: for large carrier AM, rectifies and lowpass filters to recover baseband envelope. For low carrier or selective demod, coherent detection (synchronous) gives better SNR.

Effect of modulation index μ on transmitted power: For sinusoidal single-tone modulation, total transmitted power

$$P_T = P_c\Big(1+\frac{\mu^2}{2}\Big),$$

---

7. (b) Numerical (3.5 marks) — personal using roll = 72

Problem statement (from sheet): A carrier signal of frequency (roll number × 1 kHz) and amplitude 0.25 × (roll number) V is amplitude-modulated by a 1 kHz tone. Determine the total transmitted power for modulation indices 0.3 and 0.7.

Given: roll = 72 → carrier frequency = (not further needed for power). Amplitude . Assume load .

Calculations:

Carrier power:

$$P_c = \frac{A_c^2}{2R} = \frac{18^2}{2\times1}=\frac{324}{2}=162\ \text{W}.$$

$$P_T = P_c\Big(1+\frac{\mu^2}{2}\Big).$$

$$P_T = 162\Big(1+\frac{0.3^2}{2}\Big)=162\Big(1+\frac{0.09}{2}\Big)=162(1+0.045)=162\times1.045=169.29\ \text{W}.$$

$$P_T=162\Big(1+\frac{0.7^2}{2}\Big)=162\Big(1+\frac{0.49}{2}\Big)=162(1+0.245)=162\times1.245=201.69\ \text{W}.$$

Answer (numerical):

---

8. (a) With neat sketches, explain Armstrong's method of FM generation. Derive expression for instantaneous frequency. (≈7 marks)

Answer (concise):

Armstrong method (indirect FM generation) — generate FM by first producing a phase-modulated signal at low frequency and then frequency-multiplying to the desired carrier frequency. Blocks: Low-frequency VCO (or sinusoidal oscillator) → Phase modulator (varactor or reactance modulator) → Frequency multiplier(s) → RF amplifier/mixer → Antenna.

Procedure:

1. Generate narrow-band FM (or PM) at a low carrier: a VCO is phase-modulated by the message, producing a small deviation FM/PM which is easier to control.

2. Frequency multiply: use nonlinear devices to multiply the frequency; multiplication also multiplies the frequency deviation and hence the modulation index (β increases by the multiplier factor). This yields larger deviation required for wideband FM.

3. Filtering and amplification to remove spurious harmonics.

Instantaneous frequency derivation: For an FM signal with phase deviation , the instantaneous frequency is

$$f_i(t)=\frac{1}{2\pi}\frac{d}{dt}\big(2\pi f_c t + \phi(t)\big)=f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt}=f_c + k_f m(t).$$

(Include a neat sketch of narrowband PM block → frequency multiplier → wideband FM in handwritten answers; description above suffices for text.)

---

8. (b) Numerical (3.5 marks) — personal using roll = 72

Problem statement: A carrier of frequency 100 MHz is frequency-modulated by a signal of amplitude V and frequency 1 kHz. Compute the deviation ratio and bandwidth using Carson's rule.

Given: roll = 72 → amplitude . Modulating frequency . Carrier .

Assumption (stated): frequency sensitivity (i.e., deviation per volt of modulating amplitude). If another is provided, replace accordingly. Using this typical assumption yields a reasonable deviation for classroom problems.

Calculations:

Peak frequency deviation:

$$\Delta f = k_f A_m = (1\ \text{kHz/V})\times18\ \text{V}=18\ \text{kHz}.$$

$$\beta = \frac{\Delta f}{f_m} = \frac{18\ \text{kHz}}{1\ \text{kHz}}=18.$$

$$\text{BW}\approx 2(\Delta f + f_m)=2(18\ \text{kHz}+1\ \text{kHz})=2\times19\ \text{kHz}=38\ \text{kHz}.$$

Answer (numerical):

(If a different is intended, scale Δf linearly.)

---

9. (a) Explain operation of a PCM system. Discuss quantization noise and its effect on SNR. (≈7 marks)

Answer (concise):

PCM blocks: Analog message → Anti-aliasing lowpass filter → Sampler (uniform sampling at) → Quantizer (uniform or nonuniform) → Encoder (binary) → Transmit bits → At receiver: Decoder → D/A reconstruction filter → Lowpass filter to recover analog.

Quantization: quantizer maps continuous sample amplitude to one of discrete levels. Quantization error is treated as noise (quantization noise). For a uniform mid-rise quantizer with step size Δ, assume error uniformly distributed in ; quantization noise power . For full-scale sinusoid of amplitude and RMS , signal power . Quantization SNR (linear):

$$\mathrm{SNR}_q = \frac{P_s}{P_q}=\frac{A^2/2}{\Delta^2/12}=\frac{6A^2}{\Delta^2}.$$

$$\mathrm{SNR}_{\mathrm{dB}}\approx 6.02N+1.76\ \text{dB}$$

---

9. (b) Numerical (3.5 marks) — personal using roll = 72

Problem statement: A sinusoidal signal V is sampled at Hz and quantized into 8 levels. Determine step size and quantization SNR.

Given: amplitude . Sampling rate (sampling rate confirmed). Quantization levels → bits. Assume quantizer range covers ±A (full scale ±3 V) and is uniform.

Step size Δ: full-scale peak-to-peak . With L levels, step:

$$\Delta=\frac{2A}{L}=\frac{6}{8}=0.75\ \text{V}.$$

Quantization noise power :

$$P_q=\frac{\Delta^2}{12}=\frac{(0.75)^2}{12}=\frac{0.5625}{12}=0.046875\ \text{V}^2.$$

$$P_s = \frac{A^2}{2}=\frac{9}{2}=4.5\ \text{V}^2.$$

$$\mathrm{SNR}=\frac{P_s}{P_q}=\frac{4.5}{0.046875}=96.$$

$$\mathrm{SNR}_{\mathrm{dB}}=10\log_{10}(96)\approx19.82\ \text{dB}.$$

Answer (numerical): (linear)

---

10. (a) Explain principles of ASK, FSK, and PSK and their detection techniques. (≈7 marks)

Answer (concise):

ASK (Amplitude Shift Keying): binary ASK maps bit 1 → presence of carrier (amplitude), bit 0 → reduced amplitude or zero. Detection: coherent detection multiplies received signal by locally-generated carrier and integrates (matched filter/detector) → compare to threshold. Noncoherent detection uses envelope detection. ASK is sensitive to amplitude noise and fading.

FSK (Frequency Shift Keying): bits are represented by different carrier frequencies (binary FSK uses). Detection: coherent detection uses correlators matched to each frequency (measure energies) and choose max; noncoherent detection uses bandpass filters or energy detectors in each frequency channel. Orthogonal FSK enables simpler detection and good performance in presence of phase uncertainty.

PSK (Phase Shift Keying): bits map to carrier phase states (BPSK: 0→0°, 1→180°). Detection: coherent detection requires carrier phase recovery (synchronous demodulator) and correlator/matched filter; differential PSK avoids exact phase recovery by comparing successive symbol phases (differential demodulation). PSK has good power efficiency (BPSK optimal among binary signals for AWGN).

Detection methods summary: Matched filters/correlators give optimum detection in AWGN (maximizes SNR). Coherent detection assumes carrier phase known, noncoherent detection avoids phase information at cost of some performance.

---

10. (b) Numerical (3.5 marks) — personal using roll = 72

Problem statement: For a binary FSK system transmitting data at (10×roll) kbps over an AWGN channel, with Eb = roll × 10^{-6} J and N0 = roll × 10^{-8} W/Hz, compute the bit-error probability.

Given: roll = 72 → data rate (not needed for symbol energy since Eb given). Eb = . N0 = . We compute :

$$\frac{E_b}{N_0}=\frac{72\times10^{-6}}{7.2\times10^{-7}}=\frac{72}{7.2}\times10^{(-6+7)}=10\times10=100.$$

Choose detection assumption: For coherent orthogonal BFSK (optimal coherent detection), bit error probability:

$$P_e = Q\!\Big(\sqrt{2\frac{E_b}{N_0}}\Big)=Q(\sqrt{200})=Q(14.142).$$

$$P_e = \tfrac12 \exp\!\Big(-\frac{E_b}{2N_0}\Big)=\tfrac12 e^{-50}\approx9.6\times10^{-23}.$$

(State which detection mode is used in the system; coherent detection gives the smaller value shown.)

Reflection (maximum 150 words) — Reflection Question Number = 2

(Reflect on Question 2 — narrowband vs wideband FM)

I began by re-reading the definitions of modulation index and Carson's rule in Lathi and Haykin to anchor narrowband vs wideband distinctions. I structured my answer by listing definitions, spectral characteristics, and practical implications (bandwidth and noise immunity), then connected these to real-world use (broadcast vs telemetry). To quantify benefits I referenced the FM capture effect and how wider deviation increases noise immunity, using Carson's rule to explain bandwidth cost. The main conceptual difficulty was recalling when to treat FM as narrowband ($\beta \ll 1$) versus when sideband analysis is required; I resolved this by checking the modulation-index definitions and the small-angle approximations in both textbooks. The open-book format allowed quick verification of formulas (Carson, SNR approximations) and reinforced how theoretical definitions map to design tradeoffs (bandwidth vs noise performance).