Deriving Jeans Potential Energy

uniform sphere

$$\rho = \frac{3M}{4\pi R^3}, \quad r = 0 \text{ To } R$$

$$M(r) = \frac{4}{3}\pi r^3 \rho, \quad dM = 4\pi r^2 \rho dr$$

$$dU = -\frac{GM(r)dM}{r}$$

$$dU = -\frac{G}{r}\left(\frac{4}{3}\pi r^3 \rho\right)(4\pi r^2 \rho dr)$$

$$dU = -\frac{16}{3}\pi^2 G \rho^2 r^4 dr$$

$$U = \int_0^R dU = -\frac{16}{3}\pi^2 G \rho^2 \int_0^R r^4 dr$$

$$\int_0^R r^4 dr = \frac{R^5}{5}$$

$$U = -\frac{16}{15}\pi^2 G \rho R^5, \quad \rho = \frac{3M}{4\pi R^3}, \quad \rho^2 = \frac{9M^2}{16\pi^2 R^6}$$

$$U = -\frac{16}{15}\pi^2 G\left(\frac{9M^2}{16\pi^2 R^6}\right) R^5$$

$$\boxed{U = -\frac{3}{5}\frac{GM^2}{R}}$$