

Deriving hydrostatic equilibrium equations (gas in halo)

Reference: Halo Gas Density (radial) profile

MS-5-CGM-2025

Fluid momentum (Euler)

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla P - \rho \nabla \Phi$$

Steady state $\Rightarrow \frac{\partial \mathbf{v}}{\partial t} = 0$, no bulk flow $\Rightarrow \mathbf{v} = 0$

$$0 = -\nabla P - \rho \nabla \Phi$$

$$\nabla P = -\rho \nabla \Phi$$

spherical symmetry (gas in halo)

$$\nabla \Phi = \frac{d\Phi}{dr} \hat{r}$$

$$\frac{dP}{dr} = -\rho \frac{d\Phi}{dr}$$

$$\frac{d\Phi}{dr} = \frac{GM(<r)}{r^2}$$

$$\frac{dP}{dr} = -\rho(r) \frac{GM(<r)}{r^2}$$