Deriving Bondi-Hoyle accretion rate

Gravitational focusing $\longrightarrow$ accretion radius

Energy per unit mass at ∞

$$\frac{GM}{r_a} \sim \frac{1}{2} v_\infty^2 \Rightarrow r_a \sim \frac{2GM}{v_\infty^2}$$

$$r_{BH} = \frac{2GM}{v_\infty^2 + c_s^2} \quad , \quad \sigma \sim \pi r_{BH}^2$$

$$v_{eff} \sim \sqrt{v_\infty^2 + c_s^2}$$

$$\dot{M} \sim \rho_\infty v_{eff} \, \sigma = \rho_\infty \sqrt{v_\infty^2 + c_s^2} \, \pi r_{BH}^2$$

$$\dot{M} \sim \rho_\infty \sqrt{v_\infty^2 + c_s^2} \, \pi \left(\frac{2GM}{v_\infty^2 + c_s^2} \right)^2$$

$$\dot{M}_{BH} \sim \frac{4\pi G^2 M^2 \rho_\infty}{\left(v_\infty^2 + c_s^2 \right)^{3/2}}$$

we add dimensionless λ

$$\dot{M}_{BH} = 4\pi \lambda \frac{G^2 M^2 \rho_\infty}{\left(v_\infty^2 + c_s^2 \right)^{3/2}}$$

supersonic $v_\infty \gg c_s$

$$\dot{M} \approx 4\pi \frac{G^2 M^2 \rho_\infty}{v_\infty^3}$$

Subsonic $c_s \gg v_\infty$

$$\dot{M} \approx 4\pi \frac{G^2 M^2 \rho_\infty}{c_s^3}$$