MICROWAVE ENGINEERING

TRANSMISSION LINE THEORY (MODULE 2)

Module Learning Outcomes

- Define the term Transmission Line.
- Derive the mathematical expression for propagation constant.
- Derive the mathematical expression for characteristic impedance, reflection coefficient, voltage standing wave ratio.
- Use Smith's Chart to solve transmission line problems.

INTRODUCTION

- The term transmission line in electromagnetics is commonly reserved for those structures which are capable of guiding TEM waves.
- Transmission lines are a special class of the more general electromagnetic waveguide.
- TEM waves can only exist in structures which contain two or more separate conductors. For example, coaxial lines, parallel plates, and two-wire lines.

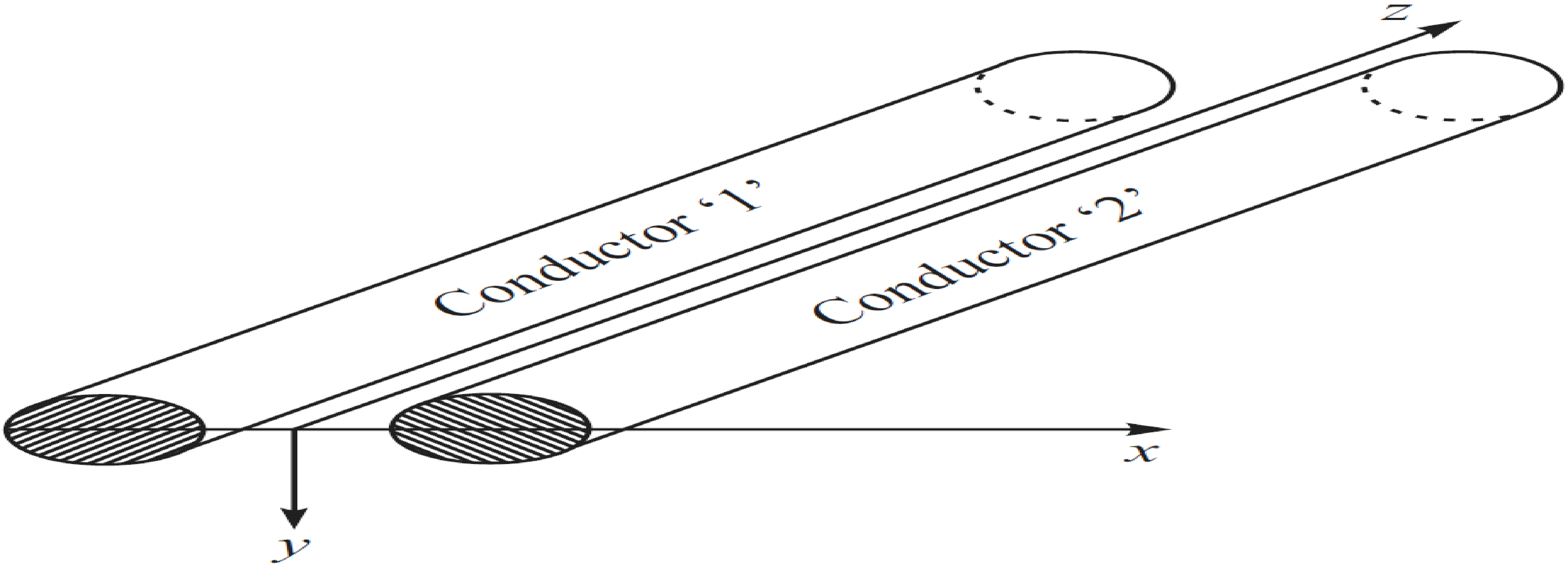
Types of Transmission Lines

- At low frequencies, electromagnetic signal power can be easily transmitted from one point in a system to another without much loss using a conducting wire.
- But at higher frequencies, conducting wires dissipate electromagnetic signals as radiations (antenna).
- The radiation loss increases as the square of the frequency.

Types of Transmission Lines

- The tendency of interconnecting wires to radiate may be curbed to some extent by making the return current flow through another wire placed very close and parallel to the interconnecting wire, thus forming the two-wire transmission line.
- This arrangement can be used satisfactorily only up to 300MHz.

Types of Transmission Lines

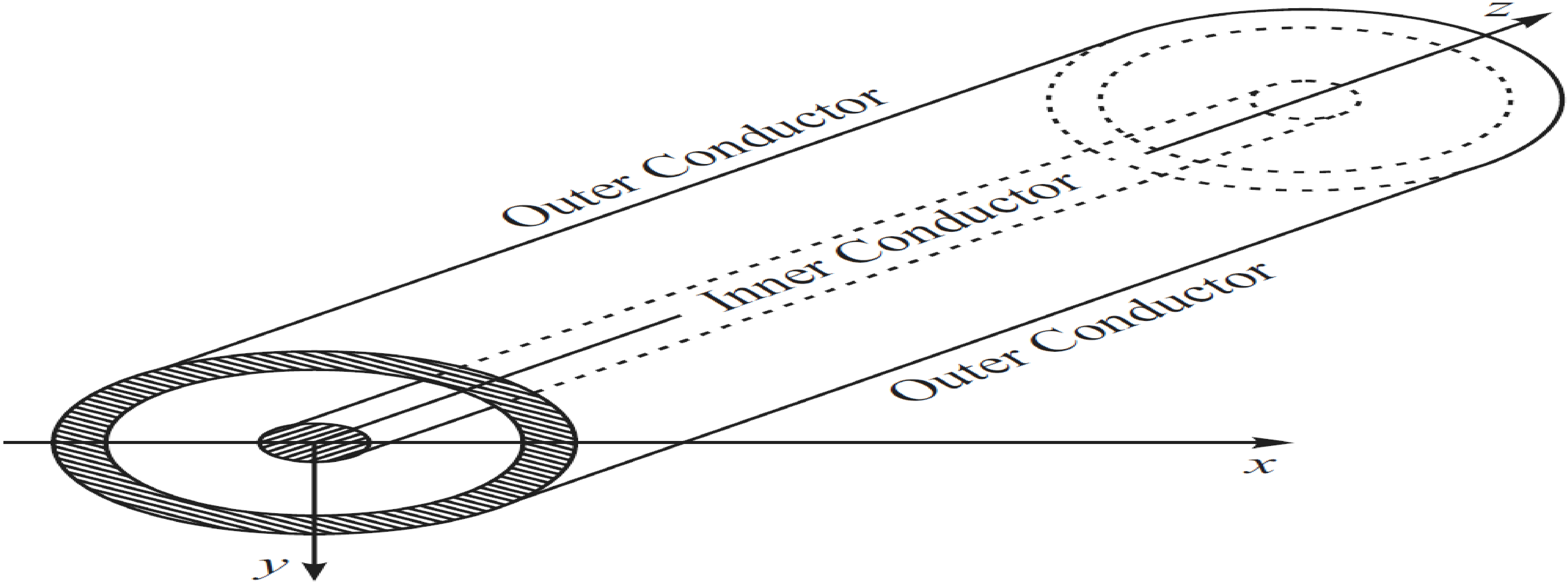


Geometry of a two-conductor or two-wire transmission line

Types of Transmission Lines

- At some frequencies higher than 300MHz, the radiation losses may be made negligible by employing a circular cylindrical conducting tube, placed coaxially with an inner conductor as the conductor for carrying the return currents. The resulting wave guiding structure, is called the Coaxial Line.

Types of Transmission Lines



Geometry of a coaxial line

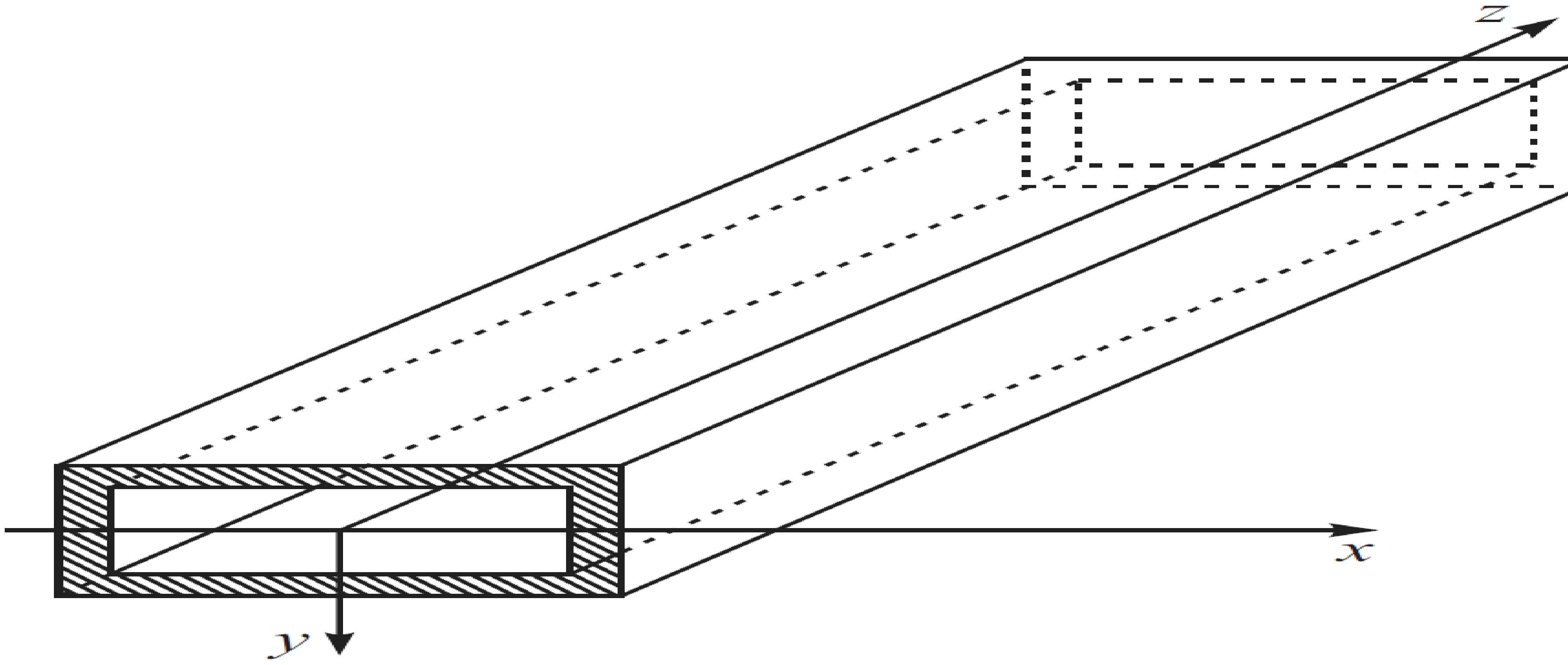
Types of Transmission Lines

- The fields in the coaxial line remain confined to the annular region between the inner and the outer conductor and hence, no radiation takes place. The field configuration inside the coaxial line also corresponds to the TEM mode.
- The coaxial line is quite acceptable as a transmission structure up to 3000 MHz above which losses due to imperfections of the dielectric and conductor, become appreciable and cannot be neglected.

Types of Transmission Lines

- These losses above 3000 MHz can be reduced considerably by dispensing with the centre conductor and using merely a hollow metal tube of circular or rectangular cross section. Such as system is called the Waveguide.
- In waveguides, the TEM mode cannot exist, instead, a longitudinal component of either the electric field or the magnetic field or both fields must exist.

Types of Transmission Lines



Geometry of a rectangular, hollow waveguide

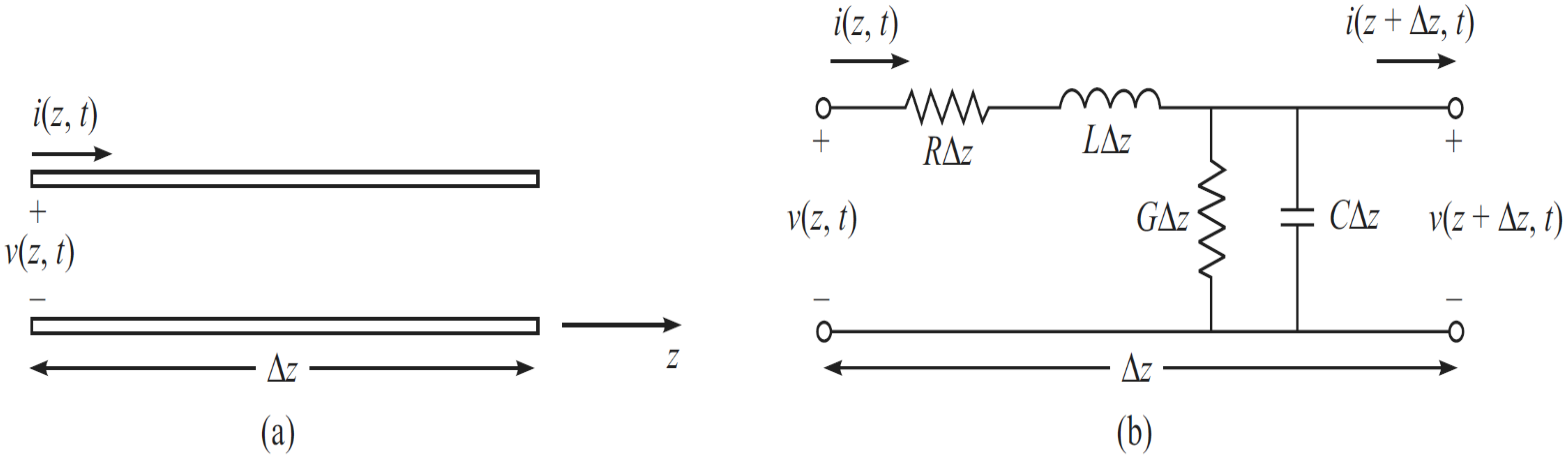
The Lumped-Element Circuit Model for a Transmission Line

- The key difference between circuit theory and transmission line theory is electrical size.
- Circuit analysis assumes that the physical dimensions of a network are much smaller than the electrical wavelength; while transmission lines may be a considerable fraction of a wavelength, or many wavelengths, in size.

The Lumped-Element Circuit Model for a Transmission Line

- Thus, a transmission line is a distributed-parameter network, where voltages and currents can vary in magnitude and phase over its length.
- Thus, a piece of line of infinitesimal length Δz can be modeled as a lumped-element circuit, where the resistance (R) in Ω/m, inductance (L) in H/m, shunt conductance (G) in S/m, and the shunt capacitance (C) in F/m, are the per-unit length quantities.

Types of Transmission Lines



Voltage and current definitions and equivalent circuit for an incremental length of transmission line: (a) Voltage and current definitions (b) Lumped-element equivalent circuit

The Lumped-Element Circuit Model for a Transmission Line

- The series inductance L represents the total self-inductance of the two conductors, and the shunt capacitance C is due to the close proximity of the two conductance.
- The series resistance R represents the resistance due to the finite conductivity of the conductors.
- The shunt conductance G is due to dielectric loss in the material between the conductors.
- R and G represent loss.

The Lumped-Element Circuit Model for a Transmission Line

Using Kirchhoff's voltage law on the outer loop only:

$$v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z, t) = 0$$

(1a)

The Lumped-Element Circuit Model for a Transmission Line

Using Kirchhoff's current law on the inner loop only:

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,\ t)}{\partial t} - i(z+\Delta z,t) = 0$$

(1b)

The Lumped-Element Circuit Model for a Transmission Line

Dividing (1a) and (1b) by Δz and taking the limit as $\Delta z \to 0$, gives the following differential equations:

$$\frac{\partial \mathrm{v}(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial \mathrm{i}(z,t)}{\partial t} \qquad (2a)$$

The Lumped-Element Circuit Model for a Transmission Line

$$\frac{\partial \mathrm{i}(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial \mathrm{v}(z,t)}{\partial t} \quad (2b)$$

Equations 2a and 2b are the time-domain form of the transmission line.

We rewrite equations 2a and 2b as cosine-based phasors as:

The Lumped-Element Circuit Model for a Transmission Line

From complex numbers:

$$Xe^{j\theta} = \mathrm{X}\cos(\theta) + Xj\sin(\theta)$$

$$Xe^{j\omega t} = X\cos(\omega t) + Xj\sin(\omega t)$$

This consists of the real (cosine-based)(Re) and the imaginary (sine-based)(Im) parts.

Taking only the real (cosine-based part) becomes:

$$Re(Xe^{j\omega t}) = \mathrm{Re}(\mathrm{X}\cos(\omega t))$$

Taking only the imaginary (sine-based part) becomes:

$$Im(Xe^{j\omega t}) = Im(Xj\sin(\omega t))$$

For the most part, we would use only the cosine-based, real part of the TEM waves.

The Lumped-Element Circuit Model for a Transmission Line

$$v(z,t) = Re\left[V(z)e^{j\omega t}\right]$$

$$\frac{\partial v(z,t)}{\partial t} = \frac{\partial Re\left[V(z)e^{j\omega t}\right]}{\partial t}$$

$$= Re\left[V(z)\frac{\partial e^{j\omega t}}{\partial t}\right]$$

$$= Re\left[j\omega V(z)e^{j\omega t}\right]$$

$$j\omega V(z) - is\ the\ phasor\ of\ \frac{\partial v(z,t)}{\partial t}$$

The Lumped-Element Circuit Model for a Transmission Line

$$i(z,t) = Re\left[I(z)e^{j\omega t}\right]$$

$$\frac{\partial i(z,t)}{\partial t} = \frac{\partial Re[I(z)e^{j\omega t}]}{\partial t}$$

$$= Re[I(z)\frac{\partial e^{j\omega t}}{\partial t}]$$

$$= Re[j\omega I(z)e^{j\omega t}]$$

$$j\omega I(z) - is\ the\ phasor\ of\ \frac{\partial i(z,t)}{\partial t}$$

The Lumped-Element Circuit Model for a Transmission Line

Similarly,

$$\frac{\partial v(z,t)}{\partial z} = \frac{\partial Re[V(z)e^{j\omega t}]}{\partial z}$$

$$= Re[\frac{\partial V(z)}{\partial z} e^{j\omega t}]$$

$$\frac{\partial i(z,t)}{\partial z} = \frac{\partial Re[I(z)e^{j\omega t}]}{\partial z}$$

$$= Re[\frac{\partial I(z)}{\partial z} e^{j\omega t}]$$

The Lumped-Element Circuit Model for a Transmission Line

Therefore, the transmission-line equations in cosine-based phasor form become:

$$\frac{\partial \mathrm{v}(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial \mathrm{i}(z,t)}{\partial t} \qquad (2a)$$

Recall that in phasor form, $\frac{\partial v(z,t)}{\partial z} = \frac{\partial V(z)}{\partial z}; i(z,t) = I(z); and \frac{\partial i(z,t)}{\partial t} = j\omega I(z)$

Therefore equation 2a becomes:

$$\frac{\partial \mathrm{v}(z)}{\partial z} = -RI(z) - j\omega LI(z)$$

(3a)

The Lumped-Element Circuit Model for a Transmission Line

$$\frac{\partial \mathrm{i}(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial \mathrm{v}(z,t)}{\partial t} \qquad (2b)$$

Recall that in phasor form, $\frac{\partial i(z,t)}{\partial z} = \frac{\partial I(z)}{\partial z}; v(z,t) = V(z); and\, \frac{\partial v(z,t)}{\partial t} = j\omega V(z)$

Therefore equation 2b becomes:

$$\frac{\partial \mathrm{I}(z)}{\partial z} = -GV(z) - j\omega CV(z)$$

(3b)

The Lumped-Element Circuit Model for a Transmission Line

Bringing 3a and 3b together forms the transmission line equations in phasor form:

$$\frac{\partial \mathrm{v}(z)}{\partial z} = -RI(z) - j\omega LI(z)$$

(3a)

$$\frac{\partial \mathrm{I}(z)}{\partial z} = -GV(z) - j\omega CV(z)$$

(3b)

Wave Propagation on a Transmission Line

- The solution of the equations (3a and 3b) is the sinusoidal excited steady-state voltage and current phasor along the transmission line.
- These two first-order coupled equations (3a and 3b) can be combined to give two second-order uncoupled wave equations. Steps:

1. Differentiate both sides of equation 3a.
2. Substitute equation 3b into the differentiated equation 3a.

Wave Propagation on a Transmission Line

1. $$\frac{\partial^2 \mathrm{v}(z)}{\partial z^2} = -(R + j\omega L)\frac{dI(z)}{dz}$$
2. $$\frac{\partial^2 \mathrm{v}(z)}{\partial z^2} = -(R + j\omega L)[-GV(z) - j\omega CV(z)]$$

(4a)

Taking like-terms of equation 4a we have:

$$\frac{\partial^2 \mathrm{v}(z)}{\partial z^2} = -(R + j\omega L)[-(G + j\omega C)V(z)]$$

$$\frac{\partial^2 \mathrm{v}(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)V(z)$$

(5a)

Wave Propagation on a Transmission Line

By comparison o f both sides of equation 5a we have:

$$\frac{\partial^2 \mathrm{v}(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)V(z)$$

$$\frac{\partial^2 \mathrm{v}(z)}{\partial z^2} = \gamma^2 V(z) \qquad (6a)$$

$$(R + j\omega L)(G + j\omega C) = \frac{\partial^2}{\partial z^2} = \gamma^2$$

Where γ is the propagation constant which is equivalent to:

$$\gamma = \sqrt{(\mathrm{R} + \mathrm{j}\omega\mathrm{L})(\mathrm{G} + \mathrm{j}\omega\mathrm{C})} \qquad (7)$$

Wave Propagation on a Transmission Line

Similarly,

$$\frac{\partial^2 I(z)}{\partial z^2} = \gamma^2 I(z) \qquad (6b)$$

Combining equations 6a and 6b gives:

$$\frac{\partial^2 \mathrm{v}(z)}{\partial z^2} = \gamma^2 V(z) \qquad (6a)$$

$$\frac{\partial^2 I(z)}{\partial z^2} = \gamma^2 I(z) \qquad (6b)$$

Wave Propagation on a Transmission Line

The solution to equations 6a and 6b are:

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z} \qquad (8a)$$

$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z} \qquad (8b)$$

$V_o^+ e^{-\gamma z}$: traveling wave propagating in the +z direction

$V_o^- e^{+\gamma z}$: traveling wave propagating in the –z direction

$$e^{-\gamma z} = e^{-(\alpha + j\beta)z} = e^{-\alpha z} e^{-j\beta z}$$

Where α is the attenuation constant in neper/m and β is the phase constant in radians/m.

Wave Propagation on a Transmission Line

- Characteristic Impedance is the ratio of the forward travelling voltage to the forward travelling current wave at any point on the transmission line.
- Characteristic impedance is also the input impedance of an infinite line.
- The characteristic impedance is also called the input impedance of an infinite line.

Wave Propagation on a Transmission Line

It can be shown that:

$$\frac{V_o^+}{I_o^+} = z_o = Characteristic\ Impedance$$

$$\frac{V_o^-}{I_o^-} = -z_o$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Wave Propagation on a Transmission Line

Important fundamental equations to recall:

1. To convert the phasor representation of the voltage back to its time domain representation:

$$v(z,t) = |V_o^+|\cos(\omega t - \beta z + \emptyset^+)\, e^{-\alpha z} + |V_o^-|\cos(\omega t + \beta z + \emptyset^-)\, e^{\alpha z}$$

Wave Propagation on a Transmission Line

Important fundamental equations to recall:

1. The wavelength (λ):

$$\lambda = \frac{2\pi}{\beta}$$

β – the phase constant.

2. The phase velocity (V_p): $V_p = \frac{\omega}{\beta} = \lambda f$

The Lossless Line

- In practical cases, however, the loss of the line is very small and so can be neglected, resulting in a simpler equation.

Set $R = G = \alpha = 0$ and the propagation constant becomes:

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$\beta = \omega\sqrt{LC}$$

$$Z_o = \sqrt{\frac{L}{C}}$$

Transmission on Lines of Finite Length

- In practice, infinite long lines do not exist, but finite lines can be made to behave like infinitely long lines if they are terminated with the characteristic impedance of the line.
- A transmission line which is terminated by its own characteristic impedance, Z_o, is said to be matched or properly terminated (this prevents reflections).

Transmission on Lines of Finite Length

- A line which is terminated in any impedance other than Z_o is said to be unmatched or improperly terminated.

Reflection Coefficient

- Reflections occur due to imperfect termination of transmission lines.
- Reflection coefficient is the ratio of the reflected power to the incident power.

$$\Gamma = \frac{Reflected\ Power}{Incident\ Power}$$

- The reflection coefficient varies between (0 (perfect match) and 1 (perfect mismatch))

Voltage Reflection Coefficient

- Voltage reflection coefficient is a measure of the degree of mismatch between the source and load impedances that causes the reflection of incident waves:

$$\Gamma_v = \frac{V_{reflected}}{V_{incident}}$$

$$\Gamma_I = \frac{I_{reflected}}{I_{incident}}$$

Voltage Reflection Coefficient

$$\Gamma_v = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$-\Gamma_v = \frac{i_r}{i_i} = \frac{-v_r/z_o}{v_i/z_o}$$

Return Loss

- Incident power (P_{inc}) and reflected power (P_{ref}) can be related using the magnitude of the voltage reflection coefficient (Γ). Since $\Gamma = \frac{V_{ref}}{V_{inc}}$, it follows that:

$$\frac{P_{ref}}{P_{inc}} = \frac{v_{ref}^2/R_{load}}{v_{inc}^2/R_{load}} = \Gamma^2$$

The return loss gives the amount of power reflected from a load and is calculated as:

$$\text{Return Loss (dB)} = -10\log\Gamma^2 = -20 log\Gamma$$

Mismatched Loss

- The amount of power transmitted to the load (P_L) is determined from:

$$P_L = P_{inc} - P_{ref} = P_{inc}(1 - \Gamma^2)$$

The fraction of the incident power not reaching the load because of mismatches and reflections is:

$$\frac{P_{load}}{P_{incident}} = \frac{P_L}{P_{inc}} = 1 - \Gamma^2$$

Hence, the mismatch loss (or reflection loss) is calculated as:

$$\text{ML (dB)} = \text{-10}\log(1 - \Gamma^2)$$

Transmission Coefficient

- The transmission coefficient (τ_v) if defined as the ratio of the load voltage (v_l) to the incident voltage (v_i).

$$\tau_v = \frac{v_l}{v_i} = 1 + \Gamma_v = \frac{2z}{z+1}$$

Voltage Standing Wave Ratio (VSWR)

- It is an indication of the degree of mismatch in a termination. It is the ratio of the maximum voltage to the minimum voltage.

$$VSWR = \frac{v_{max}}{v_{min}} = \frac{1 + \Gamma_v}{1 - \Gamma_v}$$

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

- The input impedance for lossy media:

$$Z_{in} = Z_o \left[\frac{Z_L + jZ_o tanh\gamma l}{Z_o + jZ_L tanh\gamma l} \right]$$

- The input impedance for lossless media:

$$Z_{in} = Z_o \left[\frac{Z_L + jZ_o tan\beta l}{Z_o + jZ_L tan\beta l} \right]$$

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

- The voltage reflection coefficient (at the load):

$$\Gamma_L = \frac{V_o^- e^{\gamma l}}{V_o^+ e^{-\gamma l}}$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

- The voltage reflection coefficient at any point on the line is the ratio of the reflected voltage wave to that of the incident wave:

$$\Gamma(z) = \frac{V_o^- e^{\gamma z}}{V_o^+ e^{-\gamma z}} = \frac{V_o^-}{V_o^+} e^{2\gamma z}$$

Where $z = l - l'$

$$\Gamma(z) = \frac{V_o^- e^{\gamma z}}{V_o^+ e^{-\gamma z}} = \frac{V_o^-}{V_o^+} e^{2\gamma l} e^{-2\gamma l'} = \Gamma_L e^{-2\gamma l'}$$

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

- The current reflection coefficient at any point on the line is the negative of the voltage reflection coefficient at that point:

$$\frac{I_o^- e^{\gamma l}}{I_o^+ e^{-\gamma l}} = -\Gamma_L$$

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

- The standing wave ratio (S):

$$s = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$|\Gamma_L| = \frac{S - 1}{S + 1}$$

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

- The standing wave ratio (S):

$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = sZ_o$$

$$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_o}{s}$$

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

- A transmission line is used in transferring power from the source to the load. The average input power at a distance l from the load is given by:

$$P_{ave} = \frac{|V_o^+|^2}{2Z_o}(1 - |\Gamma|^2) = P_i - P_r = P_t$$

The factor $\frac{1}{2}$ is needed because we are dealing with the peak values instead of rms values.

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

- For a shorted line $(Z_L = 0)$:

The input impedance:

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o tan\beta l}{Z_o + jZ_L tan\beta l}\right]$$

becomes,

$$Z_{sc} = Z_{in}\ |_{Z_L=0} = jZ_o tan\beta l$$

$$\Gamma_L = -1, s = \infty$$

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

- For open-circuited line ($Z_L = \infty$):

The input impedance:

$$Z_{in} = Z_o \left[\frac{Z_L + jZ_o tan\beta l}{Z_o + jZ_L tan\beta l}\right]$$

becomes,

$$Z_{oc} = \lim_{Z_L \to \infty} Z_{in} = \frac{Z_o}{jtan\beta l} = -jZ_o cot\beta l$$

$$\Gamma_L = 1, s = \infty$$

INPUT IMPEDANCE, STANDING WAVE RATIO, and POWER

Note that:

$$Z_{sc}Z_{oc} = Z_o^2$$

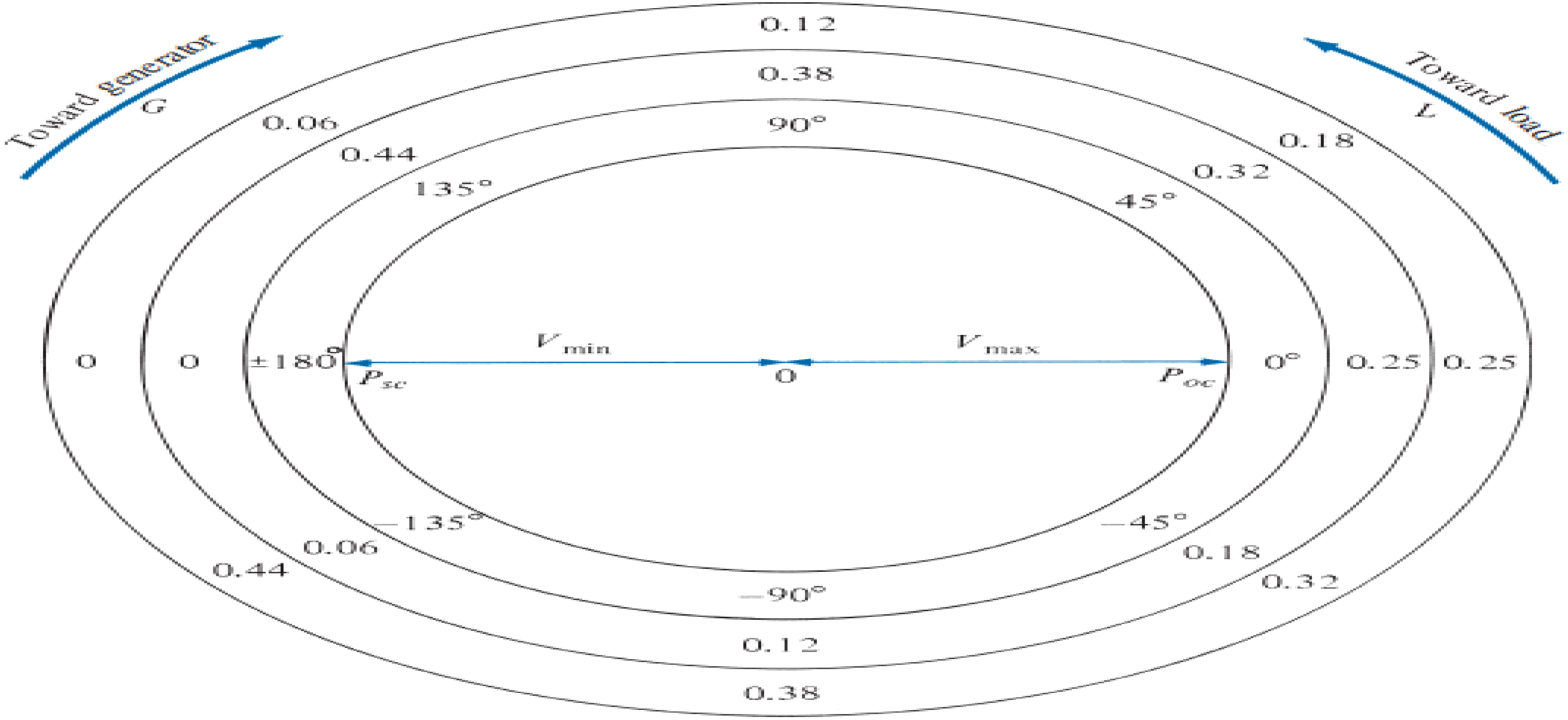
- For matched line $(Z_L = Z_o)$:

The input impedance:

$$Z_{in} = Z_o$$

$$\Gamma_L = 0, s = 1$$

THE SMITH CHART



THE SMITH CHART

- The Smith's chart can be used to get the location of $V_{max}, V_{min,}\ \Gamma$, and s provided we are given Z_o, Z_L, λ, and the length of the line.

- We are going to use a compass, and a plain straightedge.