Name: 8+16.5

ID: ____________________________

Quiz section or time: ____________________________

Stat/Math 390, Spring, Test 3, June 6, 2014; Marzban

Same deal as test 1, ...

Points

7.49 & "rosetta stone"

1. Circle all of the TRUE statements.

a) For a given data set, if a test of $H_0 : \mu_1 - \mu_2 \leq 0$ leads to rejection of H_0, then a test of $H_0 : \mu_2 - \mu_1 \leq 0$ will lead to non-rejection of H_0.

b) A 1-sided CI for $\mu_1 - \mu_2$ cannot be computed, because $\mu_1 - \mu_2$ requires a 2-sample test.

c) A 95% CI for a pop parameter, computed from a specific sample, has a 95% probability of covering the population mean.

d) prob(Type I) + prob(Type II) = 1 e) None of the above

1 8.2 **2.** Suppose we have to decide whether or not to approve a medicine. Suppose you have decided that it's less dangerous to deny a medicine that actually works, and it's more dangerous to approve a medicine that does not work. What are the appropriate hypotheses?

a) H_0 : medicine works, H_1 : medicine does not work.

b) H_0 : medicine does not work, H_1 : medicine works.

c) Based on this information, one cannot decide H_0, H_1.

Type I = (approve / not work)

= (H_1=T / H_0=T)

1 9.? **3.** Suppose there are 10 brands of smart phones, and we are wondering if their mean lifetime depends on the brand. The most appropriate test is the

a) t-test b) chi-squared c) 1-way ANOVA F-test d) F-test of model utility e) None of the above

1 9.? **4.** In the previous problem, suppose we want to see if the mean lifetime in Brand 1 is less than 1 year, AND the mean lifetime in Brand 2 < 2 years, AND, etc. The most appropriate test is

a) t-test b) chi-squared c) 1-way ANOVA F-test d) F-test of model utility e) None of the above

0.5

11 **5.** Circle all of the TRUE statements. In regression,

a) 1-sided CI or PI cannot be computed, because errors are symmetrically distributed about the fit.

b) if the predictor is log-transformed, then CI or PI cannot be computed because S_{xx} is required.

c) collinearity affects the standard deviation of the corresponding regression coefficients.

d) An interaction term cannot be tested with a t-test. e) None of the above.

1 11 **6.** In regression, at a specific value of x, prediction error (not its variance) is generally _____ estimation error (not its variance).

a) less than b) greater than c) equal to d) unrelated to

11.? **7.** We have developed a regression equation for predicting Intracranial Pressure (ICP) from blood pressure at the arm (ABP). Circle all FALSE statements regarding a future person (named X).

a) There is 95% prob that X's CI will cover the true mean ICP for all patients with X's ABP.

b) There is a 95% prob that X's PI will include X's true ICP.

c) There is a 100% prob that the X's PI will include X's predicted ICP.

d) None of the above.

1 11.? **8.** In a multiple regression fit to data, suppose we add one more term to the regression model and find that R^2 remains nearly constant. Then, the F-ratio and the p-value, respectively, will go

a) (down, down) b) (down, up) c) (up, down) d) (up, up) e) Cannot tell

$$F = \frac{R^2/k}{(1-R^2)/(n-(k+1))} = \frac{R^2}{1-R^2} \cdot \left(\frac{n-(k+1)}{k}\right) = \frac{R^2}{1-R^2}\left(\frac{n-1}{k} - 1\right)$$

8.4 **9.** The design of a bridge requires that the deviations about the mean height do not exceed 20cm.

~ 2 a) What are the Type I and Type II errors for this problem?

Type I = (test says deviation < 20 | deviation > 20) "Bad error"

Type II = (test says deviation > 20 | deviation < 20)

~ 2 b) What are the appropriate hypotheses? Clearly define your parameters, if not already defined.

σ = true/pop (standard) deviation

$H_0: \ \sigma > 20$

$H_1: \ \sigma < 20$

~ 3 7.54 **10.** Makers of coin-operated machines need to know about the reflectivity of coins. Reflectivity measurements are made of both sides of 61 coins. The sample mean of the difference between the two sides is 1, and the sample standard deviation of the difference is $3\sqrt{61}$. At what confidence level can one reject a claim that the true difference exceeds 7?

Appropriate conf. interval is upper conf. bound for paired data.

$$\bar{d} + t^* \frac{s_d}{\sqrt{n}} < 7 \Rightarrow 1 + t^* \frac{3\sqrt{61}}{\sqrt{61}} < 7 \Rightarrow t^* < 2$$

$$\text{Conf. level} = \text{prob}(t > -t^*) = \text{prob}(t > -2)$$

$$= \text{prob}(t < +2) = 1 - \text{prob}(t > +2) = 1 - .025 = \boxed{.975}$$

See last page for hyp-test approach

$df = 61 - 1 = 60$

~ 3 8.44 **11.** We toss a die 6 $=n$ times and get 6, 3, 4, 3, 5, 6. Is there evidence that the die is unfair? State the hypotheses and compute the appropriate statistic. No need for p-value.

H_0: fair die $(\pi_1 = \frac{1}{6}, \pi_2 = \frac{1}{6}, \cdots \pi_6 = \frac{1}{6})$, $\pi_{0i} = \frac{1}{6}$.

H_1: unfair die (At least 1 of these is wrong)

where π_i = prob of getting side i of die.

0 1's

2 3's

2 6's

Expected counts : $n\pi_{0i} = 6(\frac{1}{6}, \cdots) = (1, 1, 1, 1, 1, 1)$

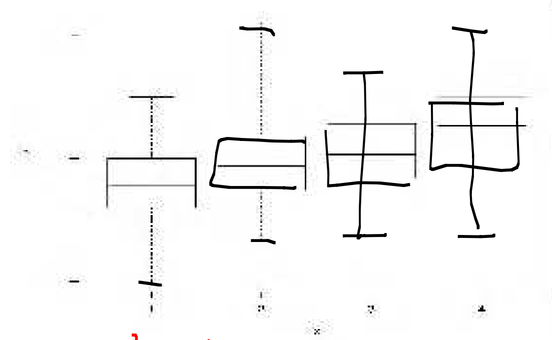
observed counts : $(0, 0, 2, 1, 1, 2)$

$$\chi^2 = \sum \frac{(obs - exp)^2}{exp} = 1[(0-1)^2 + (0-1)^2 + (2-1)^2 + (1-1)^2 + (1-1)^2 + (2-1)^2] = \boxed{4}$$

If continued $df = k - 1 = 5 \Rightarrow$ p-value $> 0.1 \Rightarrow$ cannot reject fairness.

CRG Religion example

~3 **12.** Consider the data on height (of something) y for 4 different levels of a categorical variable x (adjacent plot). The sample size at each level of x is 11, and the sample variance of y at each level of x is $1m^2$. Suppose the mean of y at each level of x is not known, but the sample variance of the 4 means is $1/11\, m^2$. We want to see if x has an effect on y. State the hypotheses and compute the appropriate statistic. No need for p-values.



→ 1-way ANOVA, $k=4$, $n_i=11$, $s_W^2=1$, $s_B^2=\frac{1}{11}$

$H_0: \mu_1=\mu_2=\mu_3=\mu_4$ μ_i = True mean of y when $x=i$

H_1: At least 2 μ_i's are different.

$$SS_B=\sum_i^k n_i(\bar{y}_i-\bar{\bar{y}})^2 = 11\sum_i^k(\bar{y}_i-\bar{\bar{y}})^2 = 11\,(k-1)s_B^2 = 11(4-1)\frac{1}{11}=3$$

$$SS_W=\sum_i^k(n_i-1)s_W^2 = (11-1)[1+1+1+1]=(11-1)4=40$$

$$F=\frac{SS_B/(k-1)}{SS_W/(n-k)}=\frac{3/(4-1)}{40/(4\times 11-4)}=\frac{3/3}{40/40}=\boxed{1}$$

$4(11)=\sum^k n_i$

If continued, df $=(3,40)$ ⇒ p-value >0.1 ⇒ No evidence that x has effect on y.

~3 PI version of hw-AJ **13.** A regression fit to 4 points has yielded the equation $y=2+x$, with $s_e=1$. From the data, we have $\bar{x}=1$, $S_{xx}=100/251$. What is the prob that the prediction error at $x=6$ will exceed 16?

$k=1$

$$? = \text{prob}(\text{pred err.} > 16)$$

$$= \text{prob}\left(\frac{\text{pred err}}{s_{\text{pred err}}} > \frac{16}{s_{\text{pred. err}}}\right) = \text{prob}\left(t > \frac{16}{s_{\text{pred err}}}\right)$$

$$s^2_{\text{pred err}} = s_e^2+s_{\hat{y}}^2 = s_e^2+s_e^2\left[\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}\right] = s_e^2\left[1+\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}\right]$$

$$=1\left[1+\frac{1}{4}+\frac{(6-1)^2}{100/251}\right] = 1+\frac{1}{4}+\cancel{25}\left(\frac{251}{\cancel{100}_4}\right) = 1+\frac{252}{4} = 1+63=64$$

$$s_{\text{pred err}}=\sqrt{64}=8$$

df $= n-(k+1) = 4-(1+1)=2$

$$P = \text{prob}\left(t>\frac{16}{8}\right) = \text{prob}(t>2) = \boxed{.092}$$

Hypothesis-testing procedure for #10.

Appropriate H_0, H_1:

$H_0: \mu_d \geq 7$ $\qquad \mu_d$ = mean of differences across pairs

$H_1: \mu_d < 7$

compare with $t^* = 2$

$t_{obs} = \dfrac{\bar{d} - \Delta}{s_d/\sqrt{n}} = \dfrac{1-7}{3\sqrt{61}/\sqrt{61}} = -2$

Table df = 60

p-value = prob$(t < t_{obs})$ = prob$(t < -2) = .025$

If $\alpha < .025$, then we cannot reject $H_0 (\mu_d \geq 7)$.

$\underset{\text{"}}{\alpha}$ = 1 − Conf. level $\Longrightarrow$ Conf. level > 1 − .025 = $\boxed{0.975}$