1st Day	2nd Day
F,G | A	–
F,G | B	–
$\bar{F}$,G	F,G
	F,$\bar{G}$
	$\bar{F}$,G
	$\bar{F}$,$\bar{G}$

Table 1: Outcome Space

STAT 391 Homework 6

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1. Problem 1 - Statistical decision making

a Make a neatly labeled table of the outcome space for this problem
Table1 is the outcome space.

b What is the probability that Rob finds the graphics card on the second day?

$$
\begin{aligned}
P(F \text{ on the second day}) &= P(\bar{F}|B) * P(F|A) * P(A) + P(\bar{F}|A) * P(F|B) * P(B) \\
&= (1 - \frac{2}{5}) \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{5} \cdot \frac{2}{3} \\
&= \frac{1}{10} + \frac{2}{15} \\
&= \frac{7}{30}
\end{aligned}
$$

c What is the probability that he finds the graphics card?

$$\begin{aligned} P(F) &= P(F \text{ on the first day}) + P(F \text{ on the second day}) \\ &= P(F|B) * P(B) + P(F|A) * P(A) + \frac{7}{30} \\ &= \frac{4}{15} + \frac{1}{6} + \frac{7}{30} \\ &= \frac{2}{3} \end{aligned}$$

d What is the probability that he finds the card and the card is still good?

$$\begin{aligned} P(F,G) &= P(F|B) * P(B) + P(F|A) * P(A) + P(\bar{F}|B) * P(F|A) * P(A) * P(G|A) + P(\bar{F}|A) \\ &= \frac{4}{15} + \frac{1}{6} + \frac{1}{10} * \frac{4}{5} + \frac{2}{15} * \frac{3}{5} \\ &= \frac{89}{150} \end{aligned}$$

e What is the expected value of Rob's search policy? Consider the outcome space

2. Problem 3- Bayesian Inference

a Compute the probability that a customer buys all three books under $\bar{P}_{ABC}$.

$$\bar{P}_{ABC}(1,1,1) = 0.6 \cdot 0.3 \cdot 0.4 = 0.072$$

b Compute the probability that a customer buys all three books under $\tilde{P}$

$$\tilde{P}_{ABC}(1,1,1) = 0.1 \cdot 0.4 = 0.04$$

c Using $\bar{P}$ and $\tilde{P}$ from above, determine if the likelihood that Robin Hood is a man is higher than the likelihood that the (s)he's a woman.

$$\bar{P}_{ABC}(1,1,0) = 0.6 \cdot 0.3 \cdot (1 - 0.4) = 0.108 \tilde{P}_{ABC}(1,1,0) \quad = 0.1 \cdot (1 - 0.4) = 0.06$$

Therefore, the likelihood tells Robin Hood is more likely to be a woman.

d Determine the posterior probability that Robin Hood is a man.

$$\begin{aligned} P(man|A=1,B=1,C=0) &= \frac{P((1,1,0)|man)P(man)}{P(1,1,0)} \\ &= \frac{0.06 \cdot \frac{2}{3}}{0.06 \cdot \frac{2}{3} + 0.108 \cdot \frac{1}{3}} \\ &= \frac{0.04}{0.04 + 0.036} \\ &= \frac{10}{19} \end{aligned}$$

e Determine the posterior probability that RObin Hood is a man if Al doesn't recall whether Robin ordered Book C or not.

$$\begin{aligned} P((1,1)|woman) &= \bar{P}_{AB}(1,1) = 0.6 \cdot 0.3 = 0.18 \\ P((1,1)|man) &= \tilde{P}_{AB}(1,1) = 0.1 \\ P(man|A=1,B=1) &= \frac{P((1,1)|man)P(man)}{P(1,1)} \\ &= \frac{0.1 \cdot \frac{2}{3}}{0.18 \cdot \frac{1}{3} + 0.1 \cdot \frac{2}{3}} \\ &= \frac{10}{19} \end{aligned}$$

f Compute the value of Likelihood Ratio(LR) for the data $A = 1, B = 1, C = 0$

$$LR(A=1,B=1,C=0) = \frac{\bar{P}_{ABC}(1,1,0)}{\tilde{P}_{ABC}(1,1,0)} = \frac{0.108}{0.06} = 1.8$$

Compute the value of the LR if the data consists of 3 customers $A_1 = 1, B_1 = 0, C_1 = 0, A_2 = 0, B_2 = 1, C_2 = 0, A_3 = 1, B_3 = 0, C_3 = 1$.

$$\begin{aligned} LR(D) &= \frac{\bar{P}_{ABC}(1,0,0) \cdot \bar{P}_{ABC}(0,1,0) \cdot \bar{P}_{ABC}(1,0,1)}{\tilde{P}_{ABC}(1,0,0) \cdot \tilde{P}_{ABC}(0,1,0) \cdot \tilde{P}_{ABC}(1,0,1)} \\ &= \frac{0.6 * (1-0.3) * (1-0.4) * (1-0.6) * 0.3 * (1-0.4) * 0.6 * (1-0.3) * 0.4}{0.5 * (1-0.4) * 0.2 * 0.4 * 0.5 * 0.4} \\ &= 0.63504 \end{aligned}$$

g Give an example of a data set where $LR > 1$.

$$D : A_1 = 1, B_1 = 1, C_1 = 0, A_2 = 1, B_2 = 1, C_2 = 1$$

3. Problem 5 - Dirichlet/Beta Distribution

a Change the variables θ_j to $\xi_j = ln\theta_j$ and express L as a function of ξ.

$$L = P(\theta_1, \theta_2) = \theta_1^{n_1}\theta_2^{n_2} = n_1\xi_1 n_2\xi_2$$