Name: _________________________

ID: ______________________________

Quiz section or time: __________ 12.5 + 15.5

Stat/Math 390, Summer, Test 3, Aug. 21, 2014; Marzban

Same as before ...

Points

1 **1.** Which of following statements is true? If significance level α is 0.01, then (reject H_0 | H_0 = F)

a) About 1% of the time we will incorrectly reject H_0. Defn. of α

b) About 99% (i.e., 1-0.01) of the time we will correctly reject H_0.

c) About 1% of the time we will be unable to reject H_0, when it is false. ← related to β or power

d) About 99% (i.e., 1-0.01) of the time we will be unable to reject H_0 when it is false. ← Not α.

1.5 Lect 19, p.6 **2.** Which of following statements is/are true? 0.5 each

a) Data provide evidence for H_1, against H_0.

b) A small p-value implies more evidence for H_1, against H_0.

c) Beliefs based on no data should be placed under H_0.

d) H_0 and H_1 are statements regarding sample statistics.

1 8.46 **3.** Three different design configurations are being considered for a particular component. There are four possible failure modes for the component. An engineer obtains data on the number of failures in each mode for each of the configurations, and wants to see if the configurations appear to have an effect on type of failure. What is the best test?

a) A chi-squared test on 1 population with multiple categories

b) A chi-squared test of homogeniety

c) A 1-way ANOVA F-test.

d) An F-test of model utility

2 8.6 **4.** Joe is expected to manufacture a concrete whose mean breaking point μ is at least $1300 KN/m^2$. Although larger values of μ are desirable, increasing μ is an expensive process. But smaller values of μ can have disasterous consequences. Joe should set up the problem so that $\alpha = prob($ $\mu > 1300$ $|$ $\mu < 1300$ $)$. Fill in the blanks with statements regarding μ.

Technically, "reject $\mu<1300$ in favor of $\mu>1300$."

1 hw-AN **5.** The chi-squared test is a generalization of the z-test to more than two

a) populations b) categories c) cases d) none of the above.

1 Lect 22, p.8 **6.** The ANOVA F-test is a generalization of the t-test to more than two

a) populations b) categories c) cases d) none of the above.

1 9.8 **7.** The area to the right of F = 3.86 at df=(3,9) is 0.05. If your data have those df values, but you observe F to be 4.0, the appropriate conclusion is to ______ H_0 at $\alpha = 0.05$.

a) Reject bigger F, lower p. b) Not reject c) Insufficient information.

1 Lect 27 **8.** Circle the correct statement(s). A multivariate (joint) distribution

a) for two variables x and y must be symmetric in x and y exchange.

b) is defined only for continuous variables.

c) is defined only for discrete variables.

d) cannot be defined for a mixture of continuous and discrete variables.

e) none of the above.

$f(x,y) \geq 0$, $\sum\sum f(x,y) = 1$ } all that is necessary.

1 Lect 24, p.6 **9.** In regression, if the standard deviation of errors σ_ϵ is **known**, then the distribution of the quantity (estimation error)/σ_ϵ is $\sigma_{est.\,err.}$

a) N(0,1) b) t with df = n-(k+1) c) insufficient information.

1 **10.** We want to see if there is a useful relationship between two continuous variables x and y. So we fit the model $y = \alpha_1 + \beta_1 x$, and test $\beta_1 = 0$. If we fit the model $x = \alpha_2 + \beta_2 y$ and test $\beta_2 = 0$, then the p-value for the second test will be

11.12

a) the same   b) smaller   c) larger   d) insufficient information.

ρ is symmetric. Test of β = Test of ρ.

1 **11.** Suppose you fit $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$ to a data set. Which statement is NOT true?

3.36 11.2

a) F-test can test model utility.
b) t-test can test whether the interaction term is statistically significant.
c) The confidence interval for β_1 estimates the average change in y associated with a 1-unit change in x_1, in a way that conveys information about precision and reliability.
d) The prediction interval for y predcits a single future value of y in a way that conveys information about precision and reliability.

12. The following is summary data on daily caffeine consumption for a (small) sample of n=16 adult women: $\bar{x} = 212mg, s = 24mg$. Suppose it had previously been believed that population mean consumption was at most 200 mg. Does the given data contradict prior belief?

8.62

~ 1 **a)** Clearly state the hypotheses in terms of well-defined parameters.
~ 2 **b)** Compute the p-value, and
~ 1 **c)** State the answer in "English" at $\alpha = 0.05$.

a) μ = True/pop. mean caffeine consumption $H_0: \mu \leq 200$ $H_1: \mu > 200$

b) $t_{obs} = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{212 - 200}{24/\sqrt{16}} = \dfrac{12}{6} = 2$ $df = n-1 = 15$

p-value = $prob(\bar{X} > \bar{X}_{obs}) = prob(t > t_{obs}) = prob(t > 2) = 0.032$

c) p-value $< \alpha \Rightarrow$ Reject H_0 in favor of $H_1 \Rightarrow$ Data contradicts prior belief $\mu < 200$

13. Consider the pixels on an image. In natural images it is expected that 20% of the pixels are Red, 30% are Green, and 50% are Blue. A random image with a total of 200 pixels is selected, and it is found that the proprotion of Red, Green, and Blue pixels is 10%, 40%, and 40%, respectively. Is this image a natural image? Perform an appropriate test

8.42

Don't add to 1. But go ahead.

$\sim$ ~~2~~ 1 **a)** Clearly stating H_0, H_1 in terms of well-defined quantities,
~ 2 **b)** Computing the appropriate statistic (i.e., no p-value necessary).

a) $H_0: \pi_1 = .2,\ \pi_2 = .3,\ \pi_3 = .5$, π_i = pop. prop. of pixels of color i.
H_1: At least one of These is wrong

b) Expected counts: $200(.2),\ 200(.3),\ 200(.5)$
observed counts: $200(.1),\ 200(.4),\ 200(.4)$

$$\chi^2 = \sum \frac{(exp - obs)^2}{exp} = \frac{[200(.2) - 200(.1)]^2}{200(.2)} + \frac{[200(.3) - 200(.4)]^2}{200(.3)} + \frac{[200(.5) - 200(.4)]^2}{200(.5)}$$

$$= 200\left[\frac{(.1)^2}{.2} + \frac{(.1)^2}{.3} + \frac{(.1)^2}{.5}\right] = 200(.1)^2\left[\frac{10}{2} + \frac{10}{3} + \frac{10}{5}\right]$$

$$= 200(.1)^2 10\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5}\right) = 200(.1)^2 10\left(\frac{15+10+6}{30}\right) = 200(.1)^2 10\,\frac{31}{30} = \frac{62}{3} \sim 20$$

3

~~2~~ 9.11 **14.** In an experiment to investigate the lifetime of 4 different brands of light bulbs, five light bulbs of each brand are randomly selected and tested. The resulting ANOVA table is produced. Fill in the missing elements, and show your work.

Source	df	SS	MS	F
Brand	3	54	18	9
Error	16	32	2	X
Total	19	86	X	X

k-1

n-k

$20-1$

$= 86-32$

$\frac{32}{16} = 2$

$\frac{54}{3}$

$k=4, \; n_i = 5 \rightarrow n = \sum_i^4 n_i = 20$

$F = \frac{MS}{MS} = \frac{18}{2}$

[0.5 for each of df, df, df, $MS = \frac{SS}{df}$, $SST = SSE + SS$, $F = \frac{MS}{MS}$

Lct27

~~2~~ 2.5 **15.** In simple linear regression, in the limit $n \to \infty$, what happens to the CI and the PI for $y(x)$? Present a mathematical argument.

$CI: \hat{y} \pm t^* s_e \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{S_{xx}}}$, $S_{xx} = (n-1)s_x^2$; $\frac{1}{n} \to 0$, $\frac{(x-\bar{x})^2}{S_{xx}} \to 0$

$\therefore CI \to \hat{y} \pm 0$

$PI: \hat{y} \pm t^* s_e \sqrt{1 + \underbrace{\frac{1}{n} + \frac{(x-\bar{x})^2}{S_{xx}}}_{\to 0}}$

$\therefore PI \to \hat{y} \pm t^* s_e$

3 PI version of hw-AK

~~2~~ **16.** In a simple regression problem with $n = 16$ cases, and $s_e = \frac{1}{\sqrt{17}}$, a prediction of 2.5 is made at the mean of x. At what prediction level does a lower prediction bound for an individual's prediction exceed 2.0? Recall that a lower prediction (and confidence) bound is constructed from a self-evident fact of the form $prob(t < t^*) =$ confidence level.

Lower prediction bound : $\hat{y} - t^* s_e \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{S_{xx}}}$

At $x = \bar{x}$: $2.5 - t^* \frac{1}{\sqrt{17}} \sqrt{1 + \frac{1}{16} + 0} = 2.5 - t^* \frac{1}{\sqrt{17}} \sqrt{\frac{17}{16}} = 2.5 - \frac{1}{4} t^*$

Exceed 2 : $2.5 - \frac{1}{4} t^* > 2 \Rightarrow t^* < 4(2.5 - 2) = 4\left(\frac{1}{2}\right)$

$\underline{t^* < 2}$

prediction level $= prob(t < t^*)$

$= prob(t < 2) = 1 - .032 = \underline{0.968}$.

$df = n - (k+1) = n - 2 = 14$