Math 327 Homework 5

We did not cover the Integral Test (Corollary 9.11 and Corollary 9.13) so do not use it below.

1. Determine if the following series converge. Explain.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n-2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{n^2+1}\right)^3$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$. Here you need $\displaystyle\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e > 1$.

   (e) $\displaystyle 1 + \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7} + \ldots$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

2. Redo your exam questions. Even if you got full points on a question, you can improve your presentation of your proof, maybe taking off unnecessary details or filling in some steps, words or explanations.

   (a) Find the infimum and supremum of $S = \left\{\dfrac{2n+5}{3n-1} : n \in \mathbf{N}\right\}$ and prove your claims.

   (b) Determine if the following are true of false. If true, briefly explain why. If false, give a counter-example.

      1. An increasing bounded sequence converges.
      2. The set $\mathbf{Q}$ of rational numbers is closed.
      3. A sequence (a_n) converges if and only if $(|a_n|)$ converges.
      4. Every bounded set in $\mathbf{R}$ has a least upper bound.
      5. If $a_n > 0$ for all n and $a_n \to a$, then $a > 0$.

   (c) Define a sequence (a_n) recursively by $a_1 = 1$ $a_{n+1} = \dfrac{1+a_n}{2+a_n} = 1 - \dfrac{1}{2+a_n}$.

      1. Prove by induction on n that $\dfrac{-1+\sqrt{5}}{2} < a_n$ for all n.
      2. Prove that (a_n) is monotone.
      3. What is the limit of (a_n)?

   (d) Let $S = \{s_1, s_2, ..., s_k\}$ be a finite set of real numbers. Prove that S is closed.