Lecture 5 (Ch. 1)

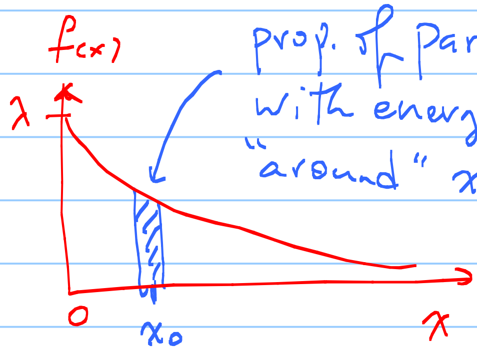
Std. Normal

Last time we learned about a few special dists (Bernoulli, Binom).
There are a few (more) special distributions which arise frequently either because they have desirable mathematical properties, or because there are lots of data in the real world whose histograms look like these distributions.

1) Exponential (family), x = cont.

E.g. Radiated Heat (ie. energy of particles)

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{else} \end{cases}$$



prop. of particles with energy "around" x_0.

Note the parameter/meaning

$\lambda > 0$

Later we will see that $\frac{1}{\lambda}$ can be interpreted as a "mean."

e.g. energy of particles coming out of uranium.

Another example: (inter) arrival time between indep. events.

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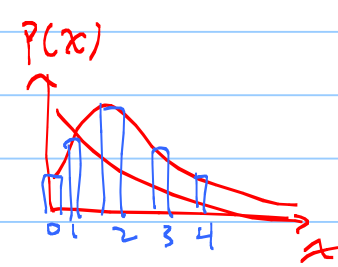
2) Poisson, x = discrete

$$p(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$



parameter.

e.g. # of bombs dropped over London per block.

λ = avg. rate " " " " "

$\lambda > 0$

3) Binomial, x = discrete

We'll derive its mass function, next time, but it's :

$$p(x) = \underbrace{\boxed{\frac{n!}{x!\,(n-x)!}}}_{\binom{n}{x}} \cdot \pi^{x} (1-\pi)^{n-x}, \quad x = 0, 1, \cdots n$$

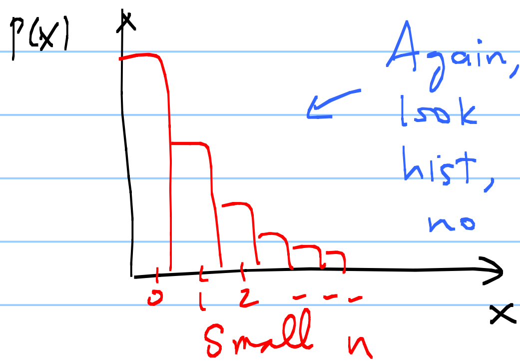
# of Hs out of n tosses (x)

prob. of H on a single toss (π)

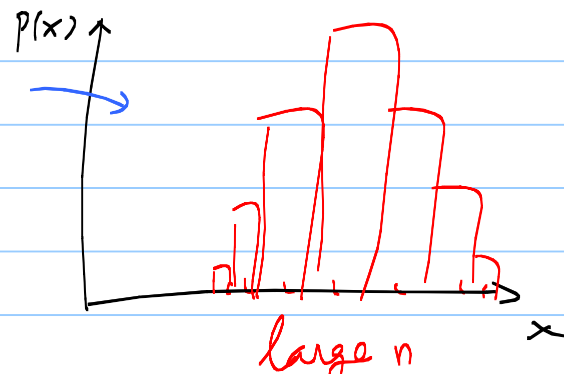
Note it is a mass function ; $p(x) \geq 0$, $\sum_{x=0}^{n} p(x) = 1$

" it has parameters/meaning : n, π. [n = integers , $0 < \pi < 1$]

Depending on the value of the params, it can look like



Again, note: look like hist, but no data.



In Lab you'll see how these look for different π values.

E.g. # of Heads out of n tosses.

# of defective gates on a chip with n gates

# of girls in a sample of size n

Etc.

4) Normal/Gaussian, x = cont.

E.g.  x = weight, height, temperature

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

π ∟ 3.1415

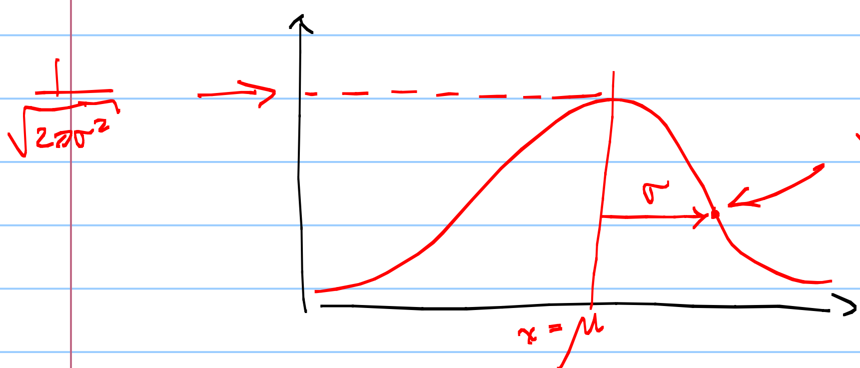
parameters meaning: μ, σ.

μ: measure of "location" or centrality.

σ: measure of "spread"

Unimportant

"inflection point"



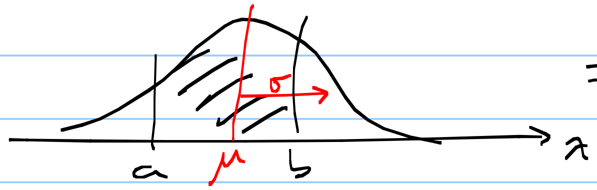
Important: Resist the temptation to call μ and σ "mean" and "standard deviation", at least for now. Otherwise you'll get very confused. They are simply parameters of the distribution.

Recall given a distr., we can compute the proportion of times x will be between 2 values:

(And also remember that proportions are important because they are measurable things)

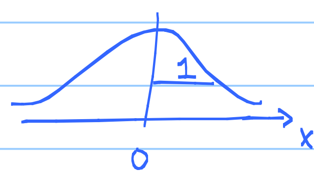
→ For binomial (n, π): $\displaystyle \sum_{x=a}^{b} \boxed{\overbrace{\binom{n}{x} \pi^x (1-\pi)^{n-x}}^{p(x)}}$

→ For Normal (μ, σ):



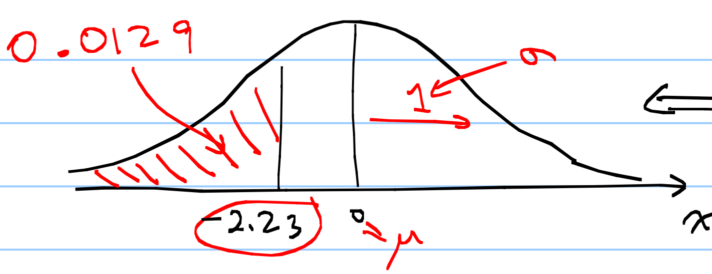
$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_a^b e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

→ For std. Normal: $\displaystyle \frac{1}{\sqrt{2\pi}} \int_a^b e^{-\frac{1}{2}x^2} dx$



Note: std. Normal = Normal$(\mu=0, \sigma=1)$.

Unfortunately, integrals of this type can be done only numerically. Their values are tabulated in Table I. E.g.



$$\Longleftrightarrow \int_{-\infty}^{-2.23} f(x)\, dx = 0.0129$$

Note: Std. Normal is symmetric.

In 390, use $\boxed{\text{Table I}}$, $\boxed{\text{NOT}}$ computers / calculators, except for problems that say "By R".

hw_lect 5-1

Show that

a) $\int_0^\infty \lambda e^{-\lambda x}\, dx = 1$

b) $\sum_{x=0}^\infty \frac{e^{-\lambda}\lambda^x}{x!} = 1$ [Hint: use the Taylor series expansion for $e^{+\lambda}$]

c) $\int_{-\infty}^\infty \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1$ [use $\int_{-\infty}^\infty e^{-\frac{1}{2}x^2} dx = \sqrt{2\pi}$]

hw_lect 5-2

Suppose the density function for x is given by the Normal dist. with parameters μ, σ. I.e.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

a) Compute the density function, $f(z)$, for $z = \frac{x-\mu}{\sigma}$.

Hint: $f(z)$ must satisfy $\int_{-\infty}^\infty f(z)\, dz = 1$.

So, start with $\int_{-\infty}^\infty f(x)\, dx = 1$, with $f(x)$ = above, and massage the expression until it becomes $\int_{-\infty}^\infty [\cdots] dz = 1$.

Then $f(z) = [\cdots]$.

Note: It is not necessary to perform any integrals.

b) From the form of $f(z)$, read off its μ and σ parameters. I.e. what are their values?