STAT 391 Homework 1

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1. Problem 1- Practice with Probability

a Estimate $\theta = (\theta_0 \; \theta_1 \; \ldots \; \theta_4)$

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observations = 0
counter = {0:0, 1:0, 2:0, 3:0, 4:0}
for line in open(r'C:\Users\johnn\Documents\UW\SchoolWorks
\\2018Spring\STAT391\HW1\hw2-little-amazon.dat').readlines():
    line = int(line.rstrip())
    counter[line] = counter[line] + 1
    observations = observations + 1

theta = [counter[0]/observations, counter[1]/observations, \
          counter[2]/observations, counter[3]/observations,
             counter[4]/observations]
print(theta)
```

As the result, I got my θ to be

$$\theta = (0.149, 0.396, 0.049, 0.255, 0.151)$$

And the sufficient statistics are the counts for each title, which is Table1

b A customer buys 3 books. What is the probability that he buys "War and Peace", "Harry Potter", "Probability" in this order? Assign the event that a customer buys the i^{th} book as E_i, then we are looking for

Table 1: Sufficient Statistics for Books

Book ID	Book Title	Count
0	War and Peace	149
1	Harry Potter & the Deathly Hallows	396
2	Winnie the Pooh	49
3	Get rich NOW	255
4	Probability	151

the probability that $P(E_0) \cdot P(E_1) \cdot P(E_4)$ since the book that every time that customer gets is an independent random

$$P(E_0) \cdot P(E_1) \cdot P(E_4) = 0.149 * 0.396 * 0.151 \approx 0.008910$$

And getting these three books has $2 * 3 = 6$ combinations, and we are only looking for one of those, thus

$$P = \binom{6}{1} \cdot (P(E_0) \cdot P(E_1) \cdot P(E_4)) \approx 0.001485$$

Therefore, the probability that the customer buys "War and Peace", "Harry Potter", "Probability" in this order is 0.001484934.

c A customer buys 4 books. What is the probability that she buys only non-fiction, that is, $N = 3, 4$? Denote the event that the customer buys 4 books and she buys only non-fiction as E. Then

$$P(E) = (P(E_3) + P(E_4))^4 = (0.255 + 0.151)^4 \approx 0.02717$$

d A customer buys 2 "Probability" books and 3

fiction (i.e 0 or 1 or 2) books. What is the probability of this event? Denote the event that the customer buys 2 "Probability" books and 3

fiction (i.e 0 or 1 or 2) books as E. Then

$$P(E) = P(E_4)^2 * (P(E_0) + P(E_1) + P(E_2))^3 = 0.151^2 * (0.149 + 0.396 + 0.049)^3 \approx 0.004779$$

e A customer buys n books. What is the probability that he buys at least one "Probability"? Denote the event that the customer buys at least one "Probability" among n books he bought

$$P(E) = 1 - P(\neg E) = 1 - (1 - 0.151)^n$$

Table 2: Possible Results of Prediction

Outcome	Prediction	Result
H	H	1
H	T	0
T	H	0
T	T	1

2. Problem 2 Practice with probability

A man claims to have extrasensory perception. As a test, a fair coin is fipped 10 times, and the man is asked to predict the outcome in advance. He gets 7 out of 10 correct.

a Let Y refer to the number of correct tests, and denote the outcomes of the 10 individual tests with the random variables $X_1, X_2, \cdots X_10$.What are the distributions of each X_i? What is the relationship between $X_{1:10}$ and Y? For each random varibale X_i, it has a Bernoulli distribution to be either 1 (for correct test) or 0 (for incorrect test). The relationship between $X_i,\ 1 \leq i \leq 10$ and Y is

$$Y = \sum_{i=1}^{10} X_i$$

b What is the probability that he would have done at least this well if he had no ESP, i.e. if his guesses were essentially random? The event he would have done at least this well can be rewrite as $Y \geq 7$, denoting Y as the number of correct tests. And since Y is the sum of Bernoulli random variables, it will also be a Bernoulli random variable. From the result table above, we can conclude that the probability to get a head in a single trial is $p = 0.5$.

$$\begin{aligned} P(Y \geq 7) &= P(Y = 7) + P(Y = 8) + P(Y = 9) + P(Y = 10) \\ &= \binom{10}{7} p^7 (1-p)^3 + \binom{10}{8} p^8 (1-p)^2 + \binom{10}{9} p^9 (1-p)^1 + \binom{10}{10} p^1 0 (1-p)^0 \\ &\approx 0.1719 \end{aligned}$$

c Suppose the test is changed - now, the coin is flipped until the man makes an incorrect guess. He guesses the fi rst two correctly, but guesses the third wrong. What is the probability of this experimental outcome (again, assuming no ESP)? Denote the event that guessing first 2 correct and the third wrong to be E. Then,

$$P(E) = 0.5^2(1 - 0.5)^1 = 0.125$$

d Assume both tests were planned beforehand. What is the probability that both of these tests turned out the way they did? In other words, the plan was to

first flip the coin 10 times and count how many times the man is correct (Y) then to continue flipping until the man makes the next mistake, at flip 10 + Z. You are asked the probability that Y = 7 and Z = 3.

$$\begin{aligned} P(\{Y = 7\} \cup \{Z = 3\}) &= P(Y = 7) \cdot P(Z = 3) \\ &= \binom{10}{7} p^7 (1 - p)^3 \cdot p^2 (1 - p)^1 \\ &\approx 0.01465 \end{aligned}$$