Name: _____

ID: _____

Quiz section or time: _____

7 + 14

Stat/Math 390, Spring, Test 2, Feb. 22, 2013; Marzban

CLOSED everything. ONLY a half-size "cheat sheet" is allowed. Check front page and back page.

Multiple-choice: mark answers on these pages. DO NOT EXPLAIN. There is wrong-answer penalty.

The rest: SHOW answer & WORK on these pages; NO CREDIT FOR CORRECT ANSWER WITHOUT EXPLANATION

Points

3.[illegible] **1.** Which of the following is/are true. In general, in a regression model $y = \alpha + \beta x$,

a) larger correlation implies larger β c) larger β implies a large R^2.

b) larger correlation implies larger R^2. d) larger R^2 implies smaller s_e (std dev of errors).

3.[illegible] **2.** Which of the following is/are true. In general, in a regression model $y = \alpha+\beta_1 x_1+\beta_2 x_2+\beta_3 x_1 * x_2$,

a) R^2 cannot be used to assess goodness of fit

b) collinearity between x_1 and x_2 is implied by the model

c) β_1 cannot be interpreted as a rate

d) unique estimates for α, β_i cannot be obtained even when there is no collinearity.

ac, bc, dc → 0.5

1 5.45 **3.** For which of the following quantities a sampling distribution does NOT exist?

a) Population mean b) Sample variance c) Sample minimum d) All of the above.

5.16 **4.** Which of the following is/are true.

a) If two events with nonzero probs are independent, then they cannot be mutually exclusive.

b) If two events with nonzero probs are mutually exclusive, then they cannot be independent.

c) If two events are independent, then they cannot be drawn on a Venn diagram at all.

d) If $P(A|B) = P(A \cap B)$, then A and B are independent.

ac, ad, ..., bc, bd → 0

abc → 0.5

7.50 **5.** Which of the following is/are true?

a) There is a 95% prob that the upper limit of a random 2-sided, 95% CI exceeds the true value.

b) We are 95% confident that the upper limit of an observed 2-sided, 95% CI exceeds the true value.

c) There is a 95% prob that an observed sample mean will be within the observed 2-sided, 95% CI.

d) None of the above.

ad, bd, cd → 0.5

1 7.[illegible] **6.** Two different bridge designs are under consideration, with μ_A, μ_B denoting their true life-time, respectively. To see if A has a longer life time than B, the most appropriate quantity to compute is

a) 2-sided CI for $\mu_A - \mu_B$ c) upper confidence bound for $\mu_A - \mu_B$

b) lower confidence bound for $\mu_A - \mu_B$ d) lower confidence bound for $\mu_B - \mu_A$

1 7.[illegible] **7.** A 2-sided, 99% CI for the difference between two means is found to be (-1,10). Is there a difference in the two means? a) Yes b) No c) We cannot tell.

~2 left hw5 **8.** Out of 110 boys, 100 answered Yes to a question, while 20 out of 120 girls answered Yes to the same question. **a)** Write the contingency table that summarizes the association between gender (Boy/Girl) and the answer (Yes/No). **b)** What kind of association is this data suggesting?

a)

Gender	Answer: Y	Answer: N	
Boy	100	10	110
Girl	20	100	120

b) Boys Tend to say Yes

Girls " " " No

OR, There is an association between gender, and the answer.

~ 3 (left 1) 2.5

9. Show that $SS_{explained}$, defined as $\sum(\hat{y}_i - \bar{y})^2$, is equal to $\hat{\beta}\, S_{xy}$, where $\hat{y}_i = \hat{\alpha} + \hat{\beta}\, x_i$, and $\hat{\alpha}, \hat{\beta}$ are OLS estimates, and $S_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y})$.

$SS_{expl.} = \sum (\hat{y}_i - \bar{y})^2$ ← Defn.

$= \sum (\hat{\alpha} + \hat{\beta} x_i - \bar{y})^2$ ← $\hat{y} = \hat{\alpha} + \hat{\beta} x_i$

$= \sum (\bar{y} - \hat{\beta}\bar{x} + \hat{\beta} x_i - \bar{y})^2$ ← $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$ (Normal eqn)

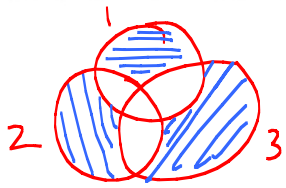
$= (\hat{\beta})^2 \underbrace{\sum (x_i - \bar{x})^2}_{S_{xx}} = \hat{\beta} \cdot \frac{S_{xy}}{S_{xx}} \cdot S_{xx} = \hat{\beta}\, S_{xy}$

($\hat{\beta} = \frac{S_{xy}}{S_{xx}}$)

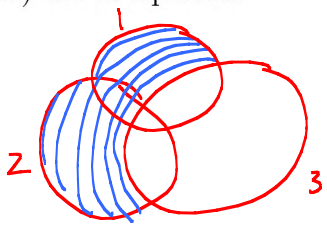
~ 2 (3.2)

10. An engineering firm is constructing power plants at 3 different sites. Let the events $E_i, (i = 1, 2, 3)$ denote the event that the plant at site i is completed. Draw a Venn diagram that depicts the 3 events as intersecting circles, and shade the region corresponding to **a)** exactly one of the plants is completed. **b)** Only the plant at site 1 or site 2 (or both) are completed.

a)



b)

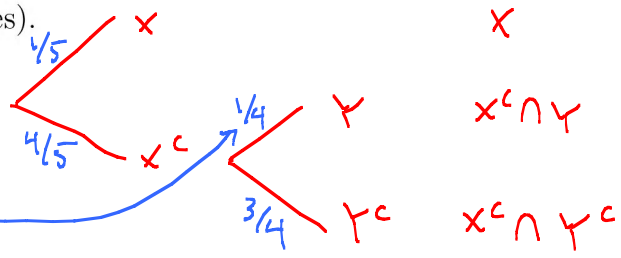


(3.8)

11. A method is proposed for testing a shipment of five items (call them A, B, C, D, and E). Ethan randomly samples one item and tests it; the remaining four items are sent to Kelly who randomly samples one item and tests it. Suppose that only item A is defective in the shipment. We want to know the prob that item A will be found in this method, BUT, you must answer as follows:

a) Let X = "the event when Ethan draws A", and Y = "the event when Kelly draws A." Draw the appropriate tree diagram showing ALL the events (in terms of X, Y, X^c, Y^c), and the corresponding probabilities (on the branches).

~ 2 / 2.5



$P(Y \mid X^c)$

After Ethan draws 1 item, there are 4 left.

~ 1

b) Write the **event** "A is found" in terms of X, Y, X^c, Y^c: $X \cup Y$ [or $X \cup (Y \cap X^c)$]

~ 2 / 1.5

c) Compute the probability of the event in part b, explaining every step.

Class "Trick" or...

$\text{prob}(X \cup Y) = P(X) + P(Y) - \underbrace{P(X \cap Y)}_{0} = P(X) + P(Y \mid X)\,P(X) + P(Y \mid X^c)\,P(X^c)$

$= \frac{1}{5} + 0 + \frac{1}{4} \cdot \frac{4}{5} = \frac{2}{5}$

Add Rule

2.5

If X occurs, Y cannot occur.

$\therefore$ X, Y are mut. excl. $\Rightarrow P(X \cap Y) = 0$ (Defn.)

$P(Y \mid X)\,P(X) = 0$

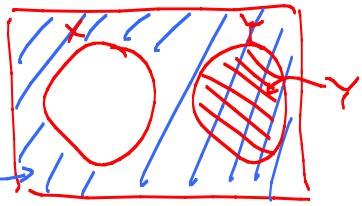
$P(X \cup (Y \cap X^c)) = P(X) + P(Y \cap X^c) - P(X \cap (Y \cap X^c))$

Add Rule

$= \frac{1}{5} + \frac{4}{5} \cdot \frac{1}{4}$ (By Tree property)

Y

mut. excl. X^c



7.17 **12.** **a)** What is the confidence level for the upper confidence bound $\bar{x}+0.84\,s/\sqrt{n}$? SHOW WORK.

~~2~~ 1

$\therefore \mu < \bar{x} + .84\,\frac{s}{\sqrt{n}}$

$\therefore -.84\frac{s}{\sqrt{n}} < \bar{x} - \mu$

$-.84 < \frac{\bar{x}-\mu}{s/\sqrt{n}}$

Conf. level = prob $(-.84 < z)$ = $1 - \text{prob}(z < -.84) = 1 - .2005 =$ **.7995**

b) Now, for a general upper confidence bound $\bar{x} + z^*\,s/\sqrt{n}$, derive a probabilistic expression that gives the confidence level. Hint: what is the event whose probability is the confidence level?

~~2~~ 1.5

$\mu < \bar{x} + z^* s/\sqrt{n}$

$\therefore -z^* < \frac{\bar{x}-\mu}{s/\sqrt{n}}$

Conf. level = prob $(z > -z^*)$
