Lecture 24 (Ch. 8)

We now have a procedure that starts with H_0, H_1, and leads to a p-value which measures the weight of evidence against H_0, in favor of H_1. Then comparison with α leads to a decision to reject or not.

In prev. example, we had $n = 64$, $\bar{x}_{obs} = 34.4$, $s = 1.1$, and asked "Does data provide evidence to support $\mu > 34$?" Then

$H_0: \mu \le 34$
$H_1: \mu > 34$

I always write these so that H_0 and H_1 have opposite directions, because it's logical. The book does not. The "equality" in H_0 just reminds us that it's sufficient to test $H_0: \mu = 34$. (The Blue note).

$$\therefore \text{p-value} = pr(\bar{X} > \bar{x}_{obs}) = prob(t > t_{obs}) = pr(t > 2.91) \overset{df=64-1}{=} 0.0025,$$

$$t = \frac{\bar{x}_{obs} - \mu_0}{s/\sqrt{n}} = \frac{34.4 - 34}{1.1/\sqrt{64}} = 2.91$$

Since p-value $< \alpha$, then there is evidence to support $\mu > 34$.

It is tempting to say the above "conclusion" (at $\alpha = .05$), that $\mu > 34$, is obvious and trivial. After all the sample gave $\bar{x}_{obs} = 34.4$, which is greater than 34 already.

It's NOT obvious! Suppose the sample/data gave $\bar{x}_{obs} = 34.1$, ie. still larger than 34 (μ_0). Then

$$t_{obs} = \frac{34.1 - 34}{1.1/\sqrt{64}} = .73 \Rightarrow \text{p-value} = prob(t > 0.73) = 0.24$$

This p-value is larger than any reasonable α.
So, we cannot reject H_0 in favor of H_1 even though the obs. sample mean is bigger than 34.
34.1 is larger than 34, but just not enough (in units of standard error, $\frac{s}{\sqrt{n}}$) to justify rejecting H_0 ($\mu < 34$) in favor of H_1 ($\mu > 34$).

A_priori !

There are many ways to rephrase the statement/question in a problem. Here are some of Them:

$\boxed{\alpha = .05}$ Data says: $n = 64$, $\bar{x} = 34.4$, $s = 1.1$

$\hookrightarrow t_{obs} = \dfrac{34.4 - 34}{1.1/\sqrt{64}} = 2.91$

Does data support $\mu > 34$? $\Longrightarrow$ "prior claim": $H_0: \mu \leq 34$

$H_0: \mu \leq 34$
$H_1: \mu > 34$

p-value $= \text{prob}(\bar{x} > \bar{x}_{obs}) = \text{prob}(t > t_{obs})$

$= \text{prob}(t > 2.91) = .0025 < \alpha$

$\therefore$ Reject Ho $(\mu \leq 34)$ in favor of H_1 $(\mu > 34)$.

$\therefore$ Data does support The claim $\mu > 34$.

Does data support $\mu < 34$?

$H_0: \mu \geq 34$
$H_1: \mu < 34$

p-value $= \text{prob}(\bar{x} < \bar{x}_{obs}) = \text{prob}(t < t_{obs})$

$= \text{prob}(t < 2.91) = 0.998 > \alpha$

$\therefore$ Cannot Reject Ho $(\mu \geq 34)$ in favor of H_1 $(\mu < 34)$.

$\therefore$ Data does not support The claim $\mu < 34$.

Does data contradict $\mu \geq 34$? $\leftarrow$ prior claim: $H_0: \mu \geq 34$

$H_0: \mu \geq 34$
$H_1: \mu < 34$

p-value $= \text{prob}(\bar{x} < \bar{x}_{obs}) = \text{prob}(t < t_{obs})$

$= \text{prob}(t < 2.91) = 0.998 > \alpha$

$\therefore$ Cannot Reject Ho $(\mu \geq 34)$ in favor of H_1 $(\mu < 34)$.

$\therefore$ Data does not contradict The claim $\mu \geq 34$.

Does data contradict $\mu \leq 34$?

$H_0: \mu \leq 34$
$H_1: \mu > 34$

p-value $= \text{prob}(\bar{x} > \bar{x}_{obs}) = \text{prob}(t > t_{obs})$

$= \text{prob}(t > 2.91) = .0025 < \alpha$

$\therefore$ Reject Ho $(\mu \leq 34)$ in favor of H_1 $(\mu > 34)$.

$\therefore$ Data does contradict The claim $\mu \leq 34$.

Now, given the similarity between C.I. and the hypothesis testing approach (ie. the p-value way) guess what the hypotheses for a 2-sample test are:

$H_0: \mu_2 \square \mu_1$ $\qquad$ $H_1: \mu_2 \square \mu_1$ $\quad$ (ie. $\mu_2 - \mu_1 \square 0$)

It turns out we can solve a more general problem:

$H_0: \mu_2 - \mu_1 \square \Delta$ $\qquad$ $H_1: \mu_2 - \mu_1 \square \Delta$

I.e. Instead of zero, use Δ, the null parameter.
You can always set it to zero, if desired.

⇒ $\therefore$ If 2-samples are indep., then assuming $H_0 = T$,

$$z = \frac{(\bar{x}_2 - \bar{x}_1) - \Delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

$$t = \frac{(\bar{x}_2 - \bar{x}_1) - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t\text{-dist. with } df = \text{Welch.}$$

Then, p-values are computed just as before:

$$\text{p-value} = \begin{cases} \text{prob}(t > t_{obs}) & \text{if } H_1: \mu_2 - \mu_1 > \Delta \\ \text{prob}(t < t_{obs}) & \text{if } H_1: \mu_2 - \mu_1 < \Delta \\ \text{twice "tail"} & \text{if } H_1: \mu_2 - \mu_1 \neq \Delta \end{cases}$$

(Table VI)

⇒ If the two samples are paired: Make a new column:

x_1	x_2	$d = x_1 - x_2$
-	.	⋮
-	.	
-	.	
		$\bar{d}, s_d$

$$t = \frac{\bar{d} - \Delta}{s_d / \sqrt{n}} \sim t\text{-dist.} \quad df = n - 1$$

p-value computed as before.

Reconsider this example:

Example: 82 students have picked-up their test, but 30 have not, even 1 week after the test was returned.

Call these 2 groups "Attenders" and "Non-attenders".

		n	$\bar{x}$	s
1	Non-attend	30	11.8	3.32
2	Attend	82	13.25	3.04

Sample suggests that mean of Attend is higher than Non-attend.
Is this true for the population (ie. all 390 courses)?

95% Conf. level

With μ_1 = mean of test 1 for Non-attend students

μ_2 = " " Attend students.

1-sided interval

We built the LOWER conf. bound for $\mu_2-\mu_1$:

$.31 \xrightarrow{\mu_2-\mu_1}$

$$(\bar{x}_2-\bar{x}_1) - 1.645\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = 1.45 - 1.645(0.693) = 0.31$$

Interpretation: We are 95% confident that $\mu_2 > \mu_1 + 0.31$.

Corollary: zero not in the interval $\Rightarrow$ There is evidence that $\mu_2>\mu_1$.

Q1! What are H_0, H_1?

A) $H_0: \mu_2-\mu_1 < 0$, $H_1: \mu_2-\mu_1 > 0$

B) $H_0: \mu_2-\mu_1 > 0$, $H_1: \mu_2-\mu_1 < 0$

C) $H_0: \mu_2-\mu_1 = 0$, $H_1: \mu_2-\mu_1 \neq 0$

1-sided test

Above question $\Rightarrow$ "Does data provide evidence for $\mu_2>\mu_1$?"

$H_0: \mu_2-\mu_1 \leq 0$

$H_1: \mu_2-\mu_1 > 0$

$$t_{obs} = \frac{1.45-0}{0.693} = 2.1$$

$$\text{p-value} = \text{prob}(t > 2.1) \overset{\text{Table VI}}{\doteq} 0.0205 \Rightarrow \text{At } \boxed{\alpha = .05}, \text{ p-value} < \alpha.$$

$\therefore$ Reject H_0 in favor of H_1

$\mu_2 < \mu_1$ $\qquad$ $\mu_2 > \mu_1$

$$df = \frac{\left[\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right]^2}{\frac{1}{n_1-1}\left[\frac{s_1^2}{n_1}\right]^2 + \frac{1}{n_2-1}\left[\frac{s_2^2}{n_2}\right]^2} = 47.91$$

$\therefore$ There is evidence for $\mu_2>\mu_1$.

hw_lect24-1

Above, we have discussed tests of means. Later, we will talk about tests of proportions. But you should already be able to do this problem.

A hw problem asked does it appear that π_x (The true proportion of defective screws) exceeds 2.5%. There, the appropriate interval is the upper conf. Bound for π_x. Which of the following is appropriate pair of hypotheses?

A)

$H_0: \pi_x \le 2.5\%$

$H_1: \pi_x > 2.5\%$

B)

$H_0: \pi_x \geq 2.5\%$

$H_1: \pi_x < 2.5\%$

C)

$H_0: \pi_x = 2.5\%$

$H_1: \pi_x < 2.5\%$

Do not do This. It's all wrong!

See hw_lect25-1 instead.