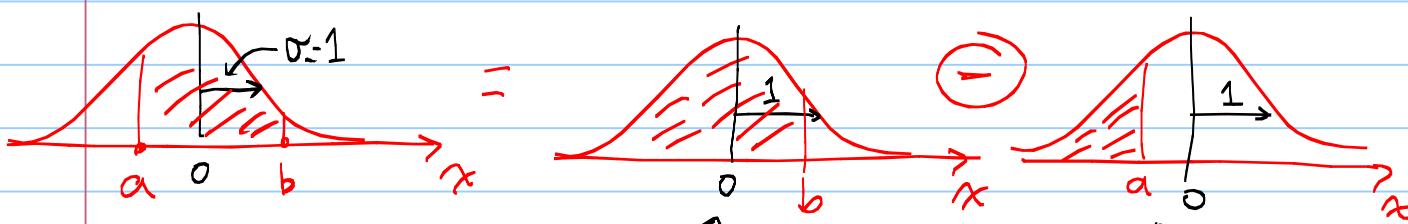
Lecture 6 (Ch. 1).

Last time we learned how to find area to the left of $x=a$, when x follows the std. Normal. i.e. the proportion of times (Think prob.) that $x<a$, if $x \sim N(0,1)$. So, we have to be tricky/smart about finding areas between 2 numbers:

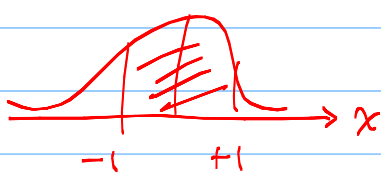
prop$(a<x<b)$ = area between a & b =



= prop ($x < b$) − prop($x < a$)

Both of These can be obtained from Table I.

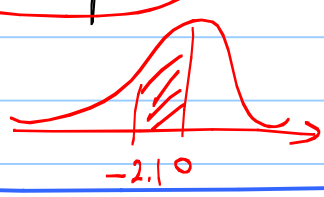
Example: What is the area under the std. Normal between −1 and +1 ?



$= .8413 - .1587 = \boxed{0.68}$

"famous" 68%

Example: How about between −2.1 and 0 ?



$= 0.5 - .0179 = \boxed{0.4821}$

Q1: what is the area to the right of (−2.1) ?

a) − 0.0179 b) + 0.0179 c) 0.9821

$1-(0.0179)$ or $0.4821+0.5$

Normal ($\mu=0, \sigma=1$)

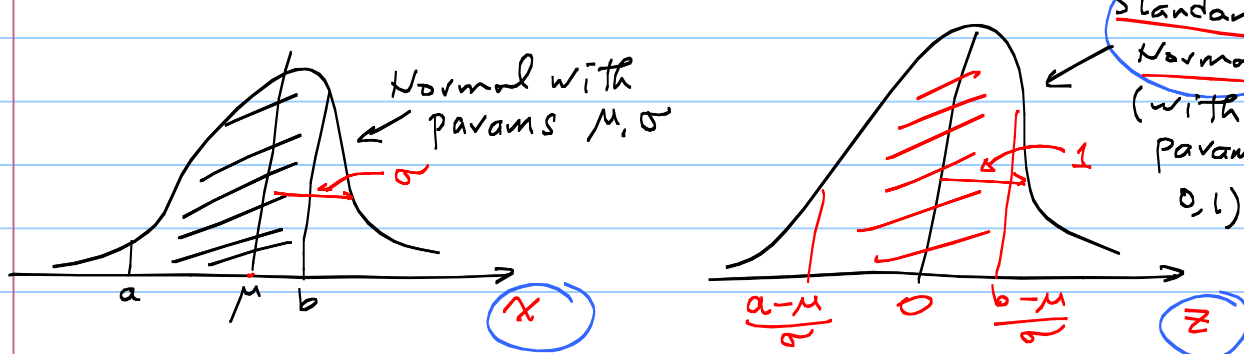
Now, we know how to find area/prop. under std. normal.

How do we handle $N(\mu, \sigma)$?

It would be impossible to tabulate values for every value of the 2 parameters, μ, σ. Need one more trick!

The trick is to "standardize" (ie. change variables):

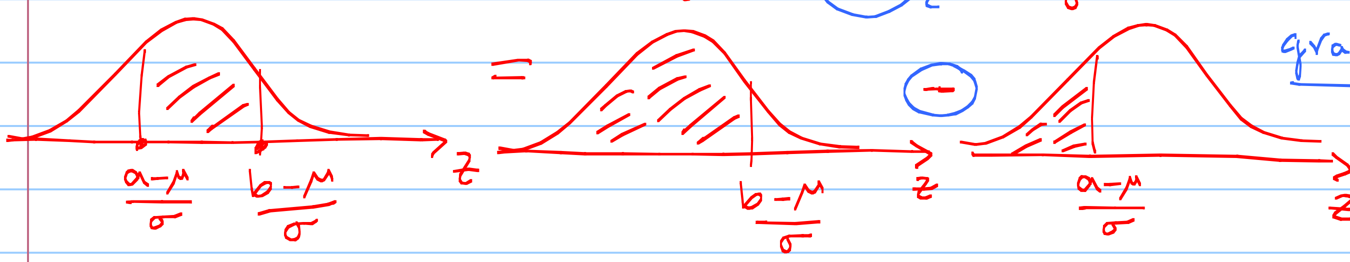
$$x \longrightarrow \boxed{z = \frac{x-\mu}{\sigma}} \quad (z\text{-score})$$



See hw in last set.

So, to compute area between 2 values:

$$\text{prop}(a < x < b) = \text{prop}\left(\frac{a-\mu}{\sigma} < \underbrace{\frac{x-\mu}{\sigma}}_{z} < \frac{b-\mu}{\sigma}\right)$$



graphically

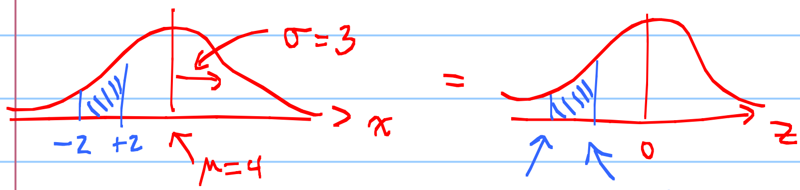
$$= \text{prop}\left(z < \frac{b-\mu}{\sigma}\right) - \text{prop}\left(z < \frac{a-\mu}{\sigma}\right)$$

Algebraically

Either way (algebraically or graphically) you can obtain the value of each term from Table 1.

Recall that Table 1 gives areas for z-scores.

Example: What's the area between -2 and $+2$ for a normal curve with $\mu = 4$, $\sigma = 3$

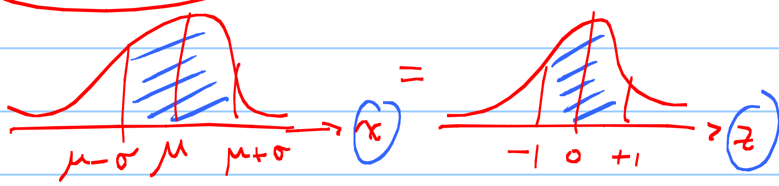


$= 0.2514 - 0.0228$

$= \boxed{0.2286}$

$-2 = \frac{-2-4}{3}$ $\quad \frac{2-4}{3} = -0.67$

Example: How about within σ of μ?



$= .8413 - .1587 = \boxed{0.6826}$

Famous 68%!

Q2: If $x \sim N(\mu, \sigma)$, what prop. of times will it be beyond $1\ \sigma$ of μ?

a) 0.6826 b) 0.3174 c) 1.8413

$1 - 0.6826$ or $2(.1587)$

Summary:

Given $f(x)$, and $x = a$ (and/or b), we can compute area.

If $f(x) =$ std. Normal, Then Table I. $z = \frac{x-\mu}{\sigma}$.

If $f(x) = \text{Normal}(\mu, \sigma)$, Then standardize first, and proceed ...

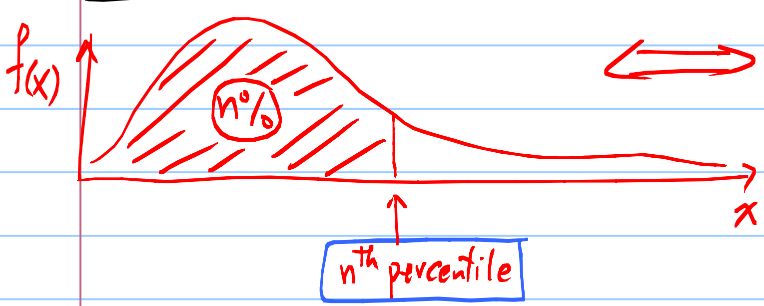
And always keep in mind That These areas translate to proportions (of times, of Things, ...)

Now, given $f(x)$, and area, compute x.

E.g. median: $$\int_{-\infty}^{\text{median}} f(x)\,dx = 1/2$$

Related to percentile, quantile, quartile

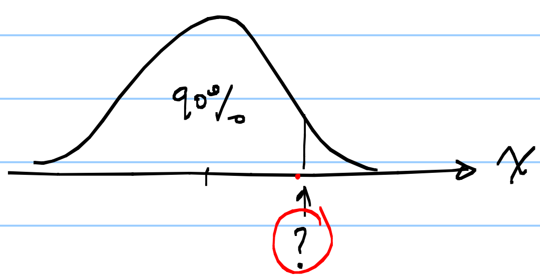
Defn:



$$\Longleftrightarrow \quad \int_{-\infty}^{n^{th}\text{ percentile}} f(x)\,dx = \frac{n}{100}$$

50^{th} percentile $= 0.5$ quantile $= 2^{nd}$ quartile
(median)

Example: What's the 90^{th} percentile of a normal distr. with params μ, σ?



$\Longrightarrow$

$$\frac{? - \mu}{\sigma} = 1.285$$

Table I

$$? = (\mu + 1.285\,\sigma)$$

Note: percentile is a number on the x-axis, not a percent.
I.e. a percentile of x has the same units as x.

By now, you should be able to (for hists AND dists)

1) compute the area to the left (or right) of $x=a$,
2) compute " " between $x=a$, $x=b$,
3) compute $x=a$, given the area to left (or right),

If the area is $n\%$, then $x=a$ is called the n^{th} percentile.

Left

Again, note that percentiles can be defined for dists AND hists

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hw-lect 6-1

What's the 10^{th} percentile of the uniform dist. between $-1, +1$?

Hint: for uniform dist. integration is trivial

hw-lect 6-2

Find the n^{th} percentile of an exponential dist. with param. λ.

Hint: The answer will depend on λ and n.