Name: ____________________

ID: ____________________

Quiz section or time: ____________________

Stat/Math 390, Spring, Test 3, June 7, 2013; Marzban

7 + 17.5

Same instructions as before ...

Points

1 8.2 **1.** To decide whether the pipe welds in a nuclear power plant meet specifications, a random sample of welds is to be selected and the strength of each weld (force required to break the weld) determined. Suppose a population mean strength of $100 lb/in^2$ is the dividing line between welds meeting specification or not doing so. The most appropriate H_0, H_1 pair is

a) $H_0 : \mu \leq 100$; $H_1 : \mu > 100$ b) $H_0 : \mu \geq 100$; $H_1 : \mu < 100$ c) $H_0 : \mu = 100$; $H_1 : \mu \neq 100$

α = prob(Reject H_0 ... | H_0 = T) = prob(Bad error) Bad error: ($\mu > 100$ | $\mu < 100$) => $H_0 : \mu < 100$

1 8.? **2.** In testing a pair of hypotheses, H_0 and H_1, on a population mean, at significance level α, which statement is NOT true

a) α is some area under the sampling distribution of $\overline{x}$, when H_0 is true.

b) p-value is some area under the sampling distribution of $\overline{x}$, when H_0 is true.

c) β is some area under the sampling distribution of $\overline{x}$, when H_1 is true.

d) p-value is some area under the sampling distribution of $\overline{x}$, when H_1 is true.

1 8.36 7.? **3.** An article presented the results of an experiment to compare the yield (kg/ha) of Sundance winter wheat and Manitou spring wheat. Data are collected from nine test plots on which each of the two types of wheat is planted. To test whether the true average yield for the winter wheat is more than 500 kg/ha higher than for spring wheat, the most appropriate test is

a) 1-sample t-test b) 2-sample (indep. t-test) c) 2-sample paired t-test d) 1-way ANOVA F-test

1 9.78 **4.** Suppose there is a difference between two population means. If data are in fact paired, then an unpaired test will generally yield a _____ p-value than that of a paired test.

a) Higher b) Lower c) zero d) Cannot tell in general.

1 9.14 **5.** In an experiment five groups of rats consisting of six rats per group are put on diets with differing amounts of carbohydrates. Then, the DNA content (in mg/g) of the liver of each rat is determined. We want to see if the diets have an effect on the mean DNA content in the liver. Which test is appropriate?

a) t-test b) chi-squared test c) 1-Way ANOVA test d) None of the above.

2 Last Lect Labs **6.** Mark all correct answers. Resampling methods like bootstrap or cross-validation are useful for

a) building confidence intervals b) model selection c) hypothesis testing d) avoiding overfitting

0.5 each

all over **7.** Under each of the following H_0, write **all** the appropriate test statistics for any H_1. You don't have to specify H_1 in your answers; just choose from z, t, chi-sqd, and F.

$\mu = 0$	$\pi = 0.5$	$\mu_1 = \mu_2$	$\pi_1 = \pi_2$
z OR t	z (NOT t) AND chi-sqd	z OR t AND F	z OR chi-sqd
$\pi_1 = 0.2, \pi_2 = 0.8$	$\mu_1 = \mu_2 = \mu_3$	$\pi_1 = 0.2, \pi_2 = 0.3, \pi_3 = 0.5$	slope $\beta_i = 0$ (fixed i)
(NOT t) z AND chi-sqd	F	chi-sqd	z OR t
intercept $\alpha = 0$	$\beta_1 = \beta_2 = 0$	mean $y(x) = 1.5$	single $y(x) = 1.5$
z OR t	F	z OR t	z OR t

because chi-sqd comes up in homog. which we skipped This Qtr

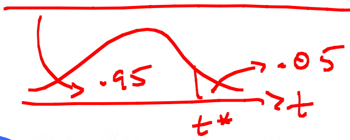
$\frac{12}{4} = 3$

i.e. 0.25 each box

~ 2 7.40 **8.** A sample of 19 joints give a sample mean limit stress of 8.0 MPa, and a sample standard deviation of 1 MPa. Write down (do not derive) an appropriate interval estimate for the true average limit stress of all such joints. Work at 95% confidence level. Note that a high value of stress limit means a stronger/better joint. Show work, but you don't have to do arithmetic.

1-sided, right $t^* \Rightarrow 2$
2-sided, $t^* = 2.1 \Rightarrow 1$
2-sided, $t^* = 2.9, 1.7 \Rightarrow 0.5$

Lower conf. Bound : $\bar{x} - t^* \frac{s}{\sqrt{n}}$



$\Rightarrow t^* = 1.75$ Table VI

df $= n-1 = 18$

$$\Rightarrow \boxed{8 - 1.75 \frac{1}{\sqrt{19}}}$$

8.42 **9.** Consider our study center. It is staffed with the expectation that 40% of its clients are from the business school, 30% from engineering, 20% from social science, and the other 10% from agriculture. A random sample of 10 clients reveals 3, 4, 2, and 1 from the four departments. Does this data suggest that the percentages on which staffing is based are not correct?

~ ~~3~~ 2.5 **a)** To that end, perform an appropriate hypothesis test to compute the p-value (or its range).

H_0: $\pi_1 = 0.4$, $\pi_2 = 0.3$, $\pi_3 = 0.2$, $\pi_4 = 0.1$ $\quad$ π_i = true prop. of clients from dept. i

H_1: At least 1 of These is wrong.

Exp. counts: 4, 3, 2, 1 $\quad (10\,\pi_i)$

obs counts: 3, 4, 2, 1

$\chi^2_{obs} = \left(\frac{1}{4} + \frac{1}{3} + 0 + 0\right) = \frac{7}{12} \approx 0.7$

$\Rightarrow$ p-value $= \text{prob}(\chi^2 > \chi^2_{obs})$

$= \text{prob}(\chi^2 > 0.7)$

$\boxed{\text{p-value} > 0.1}$ $\quad$ Table VII

df $= 4-1 = 3$

~ ~~2~~ 1 **b)** State the conclusion "In English", i.e. **regarding the percentages**, at $\alpha = 0.05$.

Because p-value $> \alpha \Rightarrow$ There is insufficient evidence to reject The expected percentages assigned to The departments.

9.45 9.41 **10.** Consider a 2-sided, 2-sample hypothesis test of two means, i.e., $H_0 : \mu_1 - \mu_2 = 0$, vs. $H_1 : \mu_1 - \mu_2 \neq 0$. Also, suppose the two samples are independent and with equal size, i.e., $n_1 = n_2 = n/2$.

~ 2 **a)** Write down (do not derive) the expression of the t-statistic, for testing H_0 and H_1, in terms of $n, \overline{y_1}, \overline{y_2}, s_1^2, s_2^2$, where $\overline{y_i}$ and s_i are the sample mean and standard deviation of the two samples.

$$t = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{\frac{s_1^2}{n/2} + \frac{s_2^2}{n/2}}} = \sqrt{\frac{n}{2}} \cdot \frac{\bar{Y}_1 - \bar{Y}_2}{\sqrt{s_1^2 + s_2^2}}$$

~ 3 **b)** Now, suppose we want to test the above H_0 and H_1 with an F-test. To that end, start from the following definition of F, and show that it is equal to the square of the t-statistic found in part a.

$$F = \frac{SS_{between}/(k-1)}{SS_{within}/(n-k)} \quad , \quad SS_{between} = \sum_i^k n_i\,(\overline{y}_i - \overline{\overline{y}})^2 \quad , \quad SS_{within} = \sum_i^k (n_i - 1)\, s_i^2 \quad , \quad \overline{\overline{y}} = \sum_i^k \frac{n_i}{n} \overline{y}_i$$

$$\bar{\bar{Y}} = \frac{n_1}{n}\bar{Y}_1 + \frac{n_2}{n}\bar{Y}_2 = \frac{1}{2}\bar{Y}_1 + \frac{1}{2}\bar{Y}_2$$

$$SS_{between} = n_1(\bar{Y}_1 - \bar{\bar{Y}})^2 + n_2(\bar{Y}_2 - \bar{\bar{Y}})^2 = \frac{n}{2}\left[\left(\bar{Y}_1 - \tfrac{1}{2}\bar{Y}_1 - \tfrac{1}{2}\bar{Y}_2\right)^2 + \left(\bar{Y}_2 - \tfrac{1}{2}\bar{Y}_1 - \tfrac{1}{2}\bar{Y}_2\right)^2\right]$$

$$= \frac{n}{2}\left[\frac{1}{4}(\bar{Y}_1 - \bar{Y}_2)^2 + \frac{1}{4}(\bar{Y}_2 - \bar{Y}_1)^2\right] = \boxed{\frac{n}{4}(\bar{Y}_1 - \bar{Y}_2)^2}$$

$$SS_{within} = (n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 = \left(\frac{n}{2} - 1\right)s_1^2 + \left(\frac{n}{2} - 1\right)s_2^2 = \boxed{\frac{1}{2}(n-2)(s_1^2 + s_2^2)}$$

$$F = \frac{\frac{n}{\cancel{4}\,2}(\bar{Y}_1 - \bar{Y}_2)^2/(2-1)}{\cancel{\frac{1}{2}(n-2)}\,(s_1^2 + s_2^2)/\cancel{(n-2)}} = \frac{n}{2}\frac{(\bar{Y}_1 - \bar{Y}_2)^2}{s_1^2 + s_2^2} = t^2$$

11.? 5.51

11. In a simple regression problem with $n = 16$, $s_x = 1/\sqrt{8}$, and $s_e = 4$, we are making a single prediction at $x = \overline{x} + 1$.

$\sim \cancel{2}$ 1 **a)** Use the following (approximate) formula for $s_{\hat{y}} = s_e\sqrt{\frac{1}{n} + \frac{(x-\overline{x})^2}{ns_x^2}}$ to show that $s_{\hat{y}} = 3$.

$$S_{\hat{y}} = 4\sqrt{\frac{1}{16} + \frac{(\cancel{\overline{x}}+1-\cancel{\overline{x}})^2}{16\cdot\frac{1}{8}}} = 4\sqrt{\frac{1}{16}+\frac{1}{2}} = \cancel{4}\sqrt{\frac{9}{\cancel{16}}} = 3$$

~ 3 **b)** What is the value of T_0 such that Prob (prediction error $> T_0$) $= 0.01$? Note: Do NOT compute a prediction interval.

Standardize.

$$0.01 = \text{prob}(\text{pred. err.} > T_0) = \text{Prob}\Big(\underbrace{\frac{\text{pred. err}}{S_{\text{pred. err.}}}}_{t} > \frac{T_0}{S_{\text{pred. err}}}\Big)$$

$$S_{\text{pred. err.}} = \sqrt{S_{\hat{y}}^2 + S_e^2} = \sqrt{9+16} = 5$$

$$0.01 = \text{prob}\Big(t > \underbrace{\frac{T_0}{5}}_{2.6}\Big) \implies \boxed{T_0 = 5(2.6)}$$

2.6 Table VI

df = n − 2 = 14
