```
from random import *
 2
    import math
3
    import matplotlib.pyplot as plt # Import plot
    import numpy as np
5
    tries = 100000 # 10^5 tires
7
    needleLength = 0.5 # set the needle length
8
    ASpacing = 1 # set the spacing between the first set of parallel lines
9
    BSpacing = 3 # set the spacing between the second set of parallel lines
    topAngle = math.atan(ASpacing / BSpacing) # find the top angle in radians
10
    CSpacing = 0.3 * math.sqrt(10) # set the spacing between the thired set of parallel lines
11
12
    longrun = [] # for storing all long-run values
13
    n = list(range(0, tries))
14
15
    fig = plt.figure()
16
17
    for run in range(0, 10): # Ten runs
18
        estimation = [] # for storing the estimations
19
        hits = 0 # for keeping track of the number of needles that cross a line
20
        for needle in range(0,tries): # one needle per try
21
            # Initialize the (x, y) position of one end of the needle at random
22
            # Assuming the needle is put uniformly across the board
23
            # z is the diagnol-perspective position
24
            x = random() * ASpacing
            y = random() * BSpacing
25
26
            z = random() * CSpacing
27
28
            # Choose the angle that the needle makes with the horizontal in radians
29
            # we assume it is uniformly distributed in [0,2pi] radians
            angle = random() * 2 * math.pi
30
31
32
            # Use the generated angle and needle length to find (x1, y1) position of the other end
            x1 = x + math.cos(angle) * needleLength
33
34
            y1 = y + math.sin(angle) * needleLength
            z1 = z + math.cos(angle - topAngle) * needleLength
35
36
            # check if the needle cross lines in set A
37
            if x1 > ASpacing or x1 < 0:
38
                hits += 1
39
            # then check if the needle cross lines in set B
40
            elif y1 > BSpacing or y1 < 0:
                hits += 1
41
42
            # finally check if the needle cross lines in set C
            elif z1 > CSpacing or z1 < 0:
43
44
                hits += 1
45
46
            # For every try, record the estimated probablity
47
            estimation.append(hits / (needle + 1))
        plt.plot(n, estimation, label = "#" + str(run + 1) + " run")
48
49
        longrun.append(estimation[len(estimation) - 1])
    plt.legend(loc = 'best', prop={'size':8})
50
    plt.xlabel("Iterations")
51
52
    plt.ylabel("Estimated Probability")
53
    plt.title("Probability over iterations")
54
    fig.savefig('p1.png', dpi = fig.dpi)
    plt.xlim((50000, 100000))
55
56
    range_max = max(longrun)
57
    range_min = min(longrun)
58
    plt.ylim((range min, range max))
59
    plt.title("Probability over iterations after 50000 tries")
60
    fig.savefig('p1_variation.png', dpi = fig.dpi)
61
    print("Range of the long-run estimation is (" + str(range_min) + ", " + str(range_max) + ").")
62
63
64
65
```

```
 \# \ [Running] \ python \ "c:\Users\Johnnia\Desktop\46\Fall \ 2017\Math381\HW4\tempCodeRunnerFile.py" 
67
68
    # Range of the long-run estimation is (0.58559, 0.59012).
69
    # [Done] exited with code=0 in 5.58 seconds
70
71
72
    # Output for tries = 10^6
     \# \ [Running] \ python \ "c:\Users\Johnnia\Desktop\46\Fall \ 2017\Math381\HW4\tempCodeRunnerFile.py" \\
73
74
    # Range of the Long-run estimation is (0.586249, 0.587688).
75
76
    # [Done] exited with code=0 in 34.827 seconds
77
```

Output for tries = 10^5

66