

Three more easier (maybe) induction problems:

1. Suppose $x \in \mathbb{R}$, $x \neq 1$. Use induction on n to show that

$$\sum_{i=0}^{n-1} x^i = \frac{x^n - 1}{x - 1}.$$

Notes: 1a. This is an extremely useful formula, for example in the theory of power series. It is used to prove convergence of the so-called “geometric series”, which is studied in Math 327 (although you’ve probably already seen this in first-year calculus).

1b. To be honest, it’s actually easier to prove this formula by direct algebraic manipulation: just multiply both sides by $x - 1$ and simplify. But I want you do it by induction, just for the practice.

2. Use induction on n to show that for all $n \in \mathbb{N}$, $3^{2n} - 1$ is divisible by 8.

(Maybe the inductive step on this one doesn’t qualify as “easy”. But don’t throw up your hands in despair; play around with the formulas to find a relation between the $n + 1$ case and the n case. If you don’t see how, don’t beat yourself up over it! Ask for a hint.)

3. Use induction on n to show that $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$.