Lecture 8 (Ch. 1)

Last time we derived The binomial mass function

$$p(X=x) = \frac{n!}{x!\,(n-x)!}\,\pi^x (1-\pi)^{n-x}$$

(Table II)

$x = 0, 1, 2, \ldots$

X = number of 1's out of n 0/1 Things

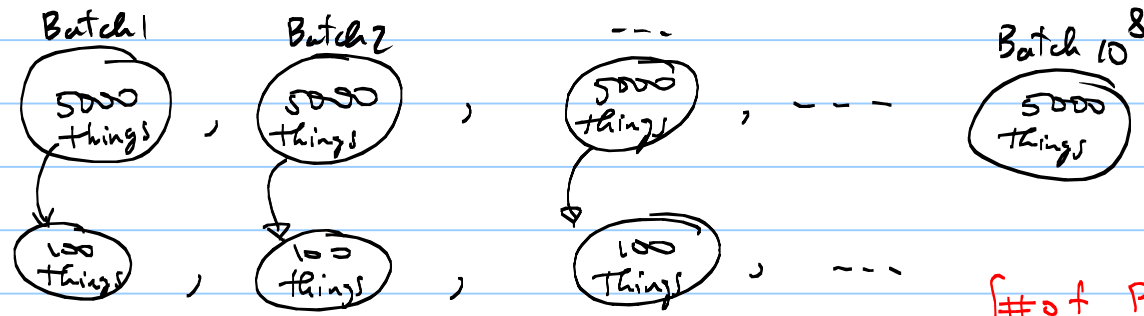
π = prop. of 1's in The population

$p(X=x)$ = prop. of times we get x 1's out of n 0/1 Things

= prob. of $X=x$.

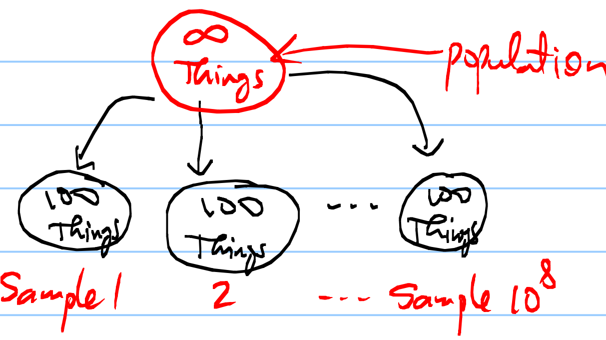
$\sum_{x=0}^{n} p(x) = 1$ because $p(x)$ = proportion.

Example 1.22 (p.53)



$X =$ 3 (e.g.) , 5 (e.g.) , 0 (e.g.) , --- $=$ { # of Bads in sample of size 100.

Assume the lots are identical, i.e. the company manufacturing the 5000 things is extremely consistent. Then, the picture looks like this:



Q What proportion of these 10^8 lots will have $X = 0, 1, ---, 100$?

G = Good
B = Bad

Sample = $\{\underbrace{G, G, ---, G}_{100}\}$

Sample = $\{\underbrace{B, B, ---, B}_{100}\}$

Suppose we know the prop. of Bads, period, in the pop. = 0.5% = .005 = π

Then $P(X = x) = \binom{100}{x} \pi^x (1-\pi)^{100-x}$

prop. of lots with $X = 0$: $\binom{100}{0} \pi^0 (1-\pi)^{100}$ = .6058

$= 1$: $\binom{100}{1} \pi^1 (1-\pi)^{100-1}$ = .3044

$= 2$: = .0757

$= 3$: Etc. = .0124

⋮

Lab.

Important Interpretation

In the long-run we expect
- ~ 60% of the lots to be all good.
- ~ 30% " " " to have 1 bad out of 100.
- ~ 7% " " " 2 bads " " ".
  (i.e. 7% of the lots to be 2% defective)

Q1: A sample is taken from a population of boys & girls. The binomial mass function provides the proportion of ______ with certain characteristics.

A) people in the sample (Boy or Girl) B) people in the pop. (Boy or Girl) C) $\boxed{\text{Samples}}$ D) None of the above

For large n, all the factorials get nasty.

Q How did the French handle this problem?

A Poisson noted that if

$n \to$ large.

$\pi \to$ small [rare event]

$\boxed{n\pi = \text{const} = \lambda}$, then

$$\binom{n}{x} \pi^x (1-\pi)^{n-x} \xrightarrow[\text{approx.}]{} \frac{e^{-\lambda}\lambda^x}{x!} = \text{poisson mass function with param } \lambda$$

(Table III)

Recall $x = \#$ of 1's. = discrete

No $n!$. No π. Just λ (= avg. rate of occurrence)

In the example, since we know n & π, we can compute λ:

$$\lambda = n \cdot \pi = 100(.005) = 0.5$$

Then prop. $(X=0) \simeq \dfrac{e^{-\lambda}\lambda^0}{0!} = e^{-.5} = \boxed{0.6065}$

Similarly for prop $(X = 1, 2, 3), \ldots \approx$ exact answers from binomial.

As mentioned previously, and proved later, λ of the poisson dist. can be interpreted as the "average" of x.

Although I derived Poisson as a large-n limit of binomial, it turns out that some problems can be solved with Poisson, quite independantly of Binomial, e.g. when you have λ (average rate) but not n or π.

Examples of data that follow the poisson distr:

- # of bombs dropped over London per block.
- # of knots per unit length of wood.
- # of crashes (cars, planes, buildings) per year.
- # of people arriving at a teller per unit time. $= X$

Eg. An avg. of 4 people arrive at a teller per hour. What's the proportion (prob) of 3 people arriving in any hour?

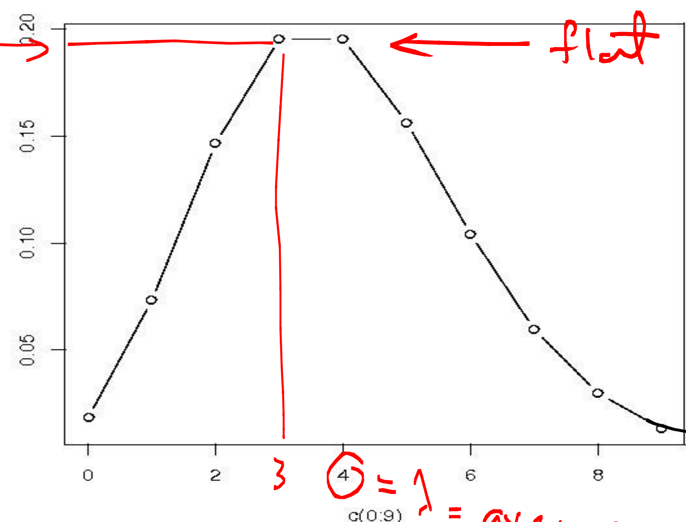
Assume $X =$ poisson with $\lambda = 4$ people/hr.

$$P(X=3) = e^{-4} 4^3 / 3! = 0.19$$

flat (see hw).

Lab

poisson with $\lambda = 4$ for $x = 0, 1, \cdots 9$



3 $\quad$ 4 $= \lambda =$ average arrival rate

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The poisson mass function in The Teller example is "flat" at The top, ie. $p(x)$ has the same value at $x=3$ and $x=4$. Show That, quite generally, The poisson mass function has the same value at $x=\lambda$ (ie. at The average) and at $x=(\lambda-1)$.

hw-lect8-2

Consider The examples of Poisson in lecture.

a) Find another example (google, books, ...) That qualifies as a Poisson variable. Call it X, and define it clearly.

b) Assume, or even guess, what The value of The λ parameter may be for your example. Remember, λ is The average x. State That value, with The correct units.

c) plot The poisson dist. with That value of λ (by R or by hand)

d) Compute $p(x=0)$, and interpret it.