

CSCI 112

Introduction to computer Science -I

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Unsigned and Signed Integers

Standard Number Lengths

Storage Type	#bits	Capacity
Byte	8	256
Word	16	65536
Doubleword	32	4294967296
Quadword	64	18446744073709551616

Negative numbers

How to represent negative numbers?

- Signed magnitude
- One's complement
- Two's complement
- Excess (biased)

Signed magnitude

- +ve
 - All positions can be used with respective place values
 - 00110110 = ?
- Signed Representation
 - Most significant bit is reserved, 1 for -ve and 0 for +ve)
 - All other bit positions can be used with respective place values
 - 10110110 = ?

One's complement form

- Negate by flipping each bit
- Example (8-bits):

$$\begin{array}{l} 37 = 0010 \ 0101 \\ -37 = 1101 \ 1010 \end{array} \begin{array}{c} \updownarrow \\ \text{FLIP} \end{array}$$

Two's complement form

- Negate by flipping each bit *and adding 1*
- Example (8-bits):

$$\begin{array}{r} 37 = 0010\ 0101 \\ 1101\ 1010 \end{array} \begin{array}{c} \updownarrow \\ \text{FLIP} \end{array}$$

$$\begin{array}{r} + 1 \\ \hline \end{array}$$

$$-37 = 1101\ 1011$$

Excess(biased) form

- Add bias to two's complement
- Example (8-bit excess 128):

$$\begin{array}{r} 37 = 0010\ 0101 \\ \quad 1101\ 1010 \end{array} \begin{array}{c} \uparrow \\ \downarrow \end{array} \text{FLIP}$$
$$\begin{array}{r} \quad + \quad \quad \quad 1 \\ \hline \quad 1101\ 1011 \\ +1000\ 0000 \\ \hline 0101\ 1011 = -37 \end{array}$$

Convert and Compare format

- Signed Magnitude

$$123_{10} = 64 + 32 + 16 + 8 + 2 + 1 = 0111\ 1011_2$$

$$-123_{10} \Rightarrow 1111\ 1011_2$$

- One's Complement (flip the bits)

$$-123_{10} \Rightarrow 1000\ 0100_2$$

- Two's Complement (add 1 to one's complement)

$$-123_{10} \Rightarrow 1000\ 0101_2$$

- Excess 128 (add 128 to two's complement)

$$-123_{10} \Rightarrow 0000\ 0101_2$$

Picking a format

Points for considerations

- Check for negative numbers?
- Test if a number is zero?
- Add & subtract positive & negative numbers?
- Determine if an overflow has occurred?
- Check if one number is larger than another?

Implemented in hardware: simpler => better

Representing 3 bit signed integer

Decimal	Unsigned	Signed Mag	1's Comp	2's Comp	Excess 4
7	111				
6	110				
5	101				
4	100				
3	011	011	011	011	111
2	010	010	010	010	110
1	001	001	001	001	101
0	000	000/100	000/111	000	100
-1		101	110	111	011
-2		110	101	110	010
-3		111	100	101	001
-4				100	000

Binary addition

- Simple, just a few rules
 - $0 + 0 = 0$
 - $0 + 1 = 1$
 - $1 + 0 = 1$
 - $1 + 1 = 10$

Binary addition (Signed Mag)

- Add two positives: 75 , 46

$$\begin{array}{r}
 111 \\
 0 \quad 1001011 \\
 +0 \quad 0101110 \\
 \hline
 0 \quad 1111001
 \end{array}$$

Binary addition (Signed Mag)

- Add two positives : 107 , 46

$$\begin{array}{r} 1 111 \\ 0 1101011 \\ + 0 0101110 \\ \hline 0 0011001 \end{array} \rightarrow 25 \quad (?)$$

- No room for carry => overflow

Binary addition (Signed Mag)

- Add two negatives :-46, -25

$$\begin{array}{r} \\ 111 \\ 1 \ 0101110 \\ +1 \ 0011001 \\ \hline 1 \ 1000111 \end{array}$$

- Signs are same, just add the numbers

Binary addition (Signed Mag)

- Add mixed numbers: 46, -25

$$\begin{array}{r} 02 \quad 02 \\ 0 \quad 0101110 \\ +1 \quad 0011001 \\ \hline 0 \quad 0010101 \end{array}$$

- Sign of the larger number becomes the sign of the result. Subtraction.

Binary addition (Signed Mag)

- Observations
- Signed magnitude representation is easy for people to understand, but it requires complicated computer hardware.
- Another disadvantage of signed magnitude is that it allows two different representations for zero: positive zero and negative zero.
- For these reasons (among others) computers systems employ complement systems for numeric value representation.

X86 -Unsigned Representation

- Just binary in one of the standard lengths
- E47A is the word-length unsigned representation for the decimal number 58490.

X86-Signed Representation

- 2's complement representation used in 80x86
- One of the standard lengths
- High-order (leading) bit gives sign
 - 0 for positive
 - 1 for negative
- For a negative number, you must perform the 2's complement operation to find the corresponding positive number.

2's Complement Operation

- +/- button on many calculators
- Manually by subtracting from 100...0
- E47A represents a negative word-length signed number since E = $\underbrace{11}_{\text{minus}}10$
- $10000 - E47A = 1B86 = 7046_{10}$
so E47A is the 2's complement signed representation for -7046

Multiple Interpretations

- One pattern of bits can have many different interpretations.
- The word FE89 can be interpreted as
 - An unsigned number whose decimal value is 65161
 - A signed number whose decimal value is -375