Homework #1

Game Theory and Its Applications

Contents

- Simulating the execution of one graph game based on your student ID

Student ID mod 8	Game to simulate
0	Multi-Domination Game
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Multi-Domination Game [YC14]

- Players: node set $\{p_1, p_2, \ldots, p_n\}$
- Strategies: $c_i \in \{0 \text{ (OUT)}, 1 \text{ (IN)}\}$ for all p_i
- Utility functions (C: strategy profile)

$$u_i(C) = \begin{cases} (\sum_{p_j \in M_i} g_j(C)) - \beta & \text{if } c_i = 1 \\ 0 & \text{otherwise,} \end{cases}$$

$\beta > 0$: constant
M_i: closed neighbors of p_i

where

$$g_j(C) = \begin{cases} \alpha, & \text{if } v_j(C) \le k_j \\ 0, & \text{otherwise} \end{cases}$$

$\alpha > \beta$: constant

where

$$v_j(C) = \sum_{p_k \in M_j} c_k$$

The number of nodes that dominate p_j

k-Domination Game [YC14]

- Players: node set $\{p_1, p_2, \ldots, p_n\}$
- Strategies: {0 (OUT), 1 (IN)}

If $|N_i| < k$, c_i must be 1

$$u_i(C) = \begin{cases} \boxed{\alpha \qquad \text{if } |N_i| < k \text{ and } c_i = 1} \\ \sum_{p_j \in N_i} g_j(C) - \beta & \text{if } |N_i| \geq k \text{ and } c_i = 1 \\ 0 & \text{otherwise} \end{cases}$$

where

$$g_i(C) = \begin{cases} \alpha, & \text{if } c_i = 0 \text{ and } v_i(C) \leq k \\ 0, & \text{otherwise} \end{cases}$$

$\boxed{\alpha > \beta > 0}$

where

$$v_i(C) = \sum_{p_j \in N_i} c_j$$

N_i (not M_i): p_i's open neighbors (p_i excluded)

Maximal Independent Set (MIS) Game (Symmetric) [YHT16]

- Players: nodes p_i's
- Strategies: $c_i \in \{1 \text{ (IN)}, 0 \text{ (OUT)}\}$
- Utility functions:

$$u_i(C) = \sum_{p_j \in N_i} \omega(c_i, c_j) + c_i$$

N_i: p_i's open neighbors

where

$$\omega(c_i, c_j) = -\alpha c_i c_j$$

$\alpha > 1$: constant

$$BR_i(c_{-i}) = \begin{cases} 0, & \text{if } \exists p_j \in N_i, c_j = 1 \\ 1, & \text{otherwise.} \end{cases}$$

Asymmetric MIS Game [YHT16]

- Player's utility

$$u_i(C) = \sum_{p_j \in L_i} \omega(c_i, c_j) + c_i$$

L_i: p_i's neighbors that have equal or higher priority

where

$$\omega(c_i, c_j) = -\alpha c_i c_j$$

$\alpha > 1$: constant

$$BR_i(c_{-i}) = \begin{cases} 0, & \text{if } \exists p_j \in L_i, c_j = 1 \\ 1, & \text{otherwise.} \end{cases}$$

Players only care neighbors that have priority equal to or higher than theirs

Weighted MIS Game [YHT16]

- Each node has a weight and we want to maximize the total weight in the MIS
- One approach: using priority
- Possible priority functions:

$$\frac{W(p_i)}{\deg(p_i)+1} \qquad\qquad \frac{W(p_i)}{\mathrm{W}(p_i)+\sum_{p_j \in N_i} W(p_j)}$$

MIS-based IDS Game [YS18]

- p_i's utility:

$$\alpha > 1$$

$$u_i(C) = c_i \left(1 - \alpha \sum_{p_j \in L_i} c_j \right)$$

L_i: set of p_i's neighboring node p_j with $\deg(p_j) \geq \deg(p_i)$.

prefer nodes with higher node degrees

- Best response of p_i

$$BR_i(c_{-i}) = \begin{cases} 0, & \text{if } \exists p_j \in L_i, c_j = 1 \\ 1, & \text{otherwise.} \end{cases}$$

Symmetric MDS-based IDS Game [YS18]

- Let $M_i = N_i \cup \{p_i\}$. Define $$v_i(C) = \sum_{p_j \in M_i} c_j$$
- Let $\alpha > 1$ be a constant. Define $g_i(C)$ as $$g_i(C) = \begin{cases} \alpha & \text{if } v_i(C) = 1 \\ 0 & \text{otherwise,} \end{cases}$$
- Let $\gamma > n\alpha$ be a constant. Define $$w_i(C) = \sum_{p_j \in N_i} c_i c_j \gamma,$$
- Let $0 < \beta < \alpha$. p_i's utility: $$u_i(C) = \begin{cases} \boxed{\left(\sum_{p_j \in M_i} g_j(C)\right)} - \beta - \boxed{w_i(C)} & \text{if } c_i = 1 \\ 0 & \text{otherwise,} \end{cases}$$

gain of dominance

penalty of violating independence

Asymmetric MDS-based IDS Game [YS18]

- Let $M_i = N_i \cup \{p_i\}$. Define

$$v_i(C) = \sum_{p_j \in M_i} c_j$$

- Let $\alpha > 1$ be a constant. Define $g_i(C)$ as

$$g_i(C) = \begin{cases} \alpha & \text{if } v_i(C) = 1 \\ 0 & \text{otherwise,} \end{cases}$$

- Let $\gamma > n\alpha$ be a constant. Define

$$w_i(C) = \sum_{\boxed{p_j \in L_i}} c_i c_j \gamma.$$

only care neighbors with higher degrees

- Let $0 < \beta < \alpha$. p_i's utility:

$$u_i(C) = \begin{cases} \left(\sum_{p_j \in M_i} g_j(C)\right) - \beta - w_i(C) & \text{if } c_i = 1 \\ 0 & \text{otherwise,} \end{cases}$$

Pseudo Code for Game Simulation

```
randomize initial game state
move_count = 0
while the game does not reach NE
  randomly pick up one player who can improve its utility
  change this player's strategy to its best response
  move_count++
end while
verify that the game state is a valid solution
output game state and move_count
```

Performance Measurements of The Result

- Except weighted MIS game, the quality of the result can be measured by the number of elements in the set
  - We want to minimize the number of elements in a dominating set
  - We want to maximize the number of elements in an independent set
- We want to maximize the total weight in the weighted MIS game
- Besides, we want to minimize the number of player's movements (i.e., `move_count`)

Pseudo Code for Performance Evaluation

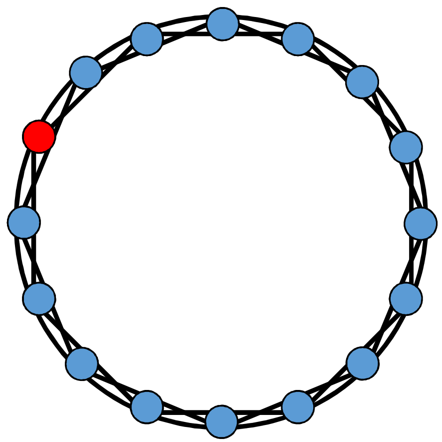
Topology: the WS model ($n = 30, k = 4$)
Adjustable parameter: p_r (0 to 0.8 step 0.2)

```
repeat every adjustable topology parameter
  repeat 100 times

      Code for Game Simulation

  calculate the averaged set cardinality and move_count
plot the results using x-y figures
```

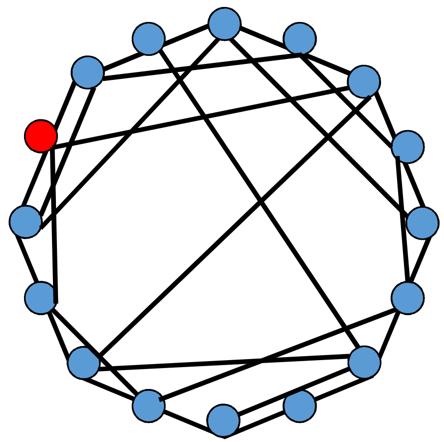
The WS Model [WS98]

- an n-node regular graph is first formed
  - each node has k edges connecting to its k nearest neighbors

$n = 16$
$k = 4$

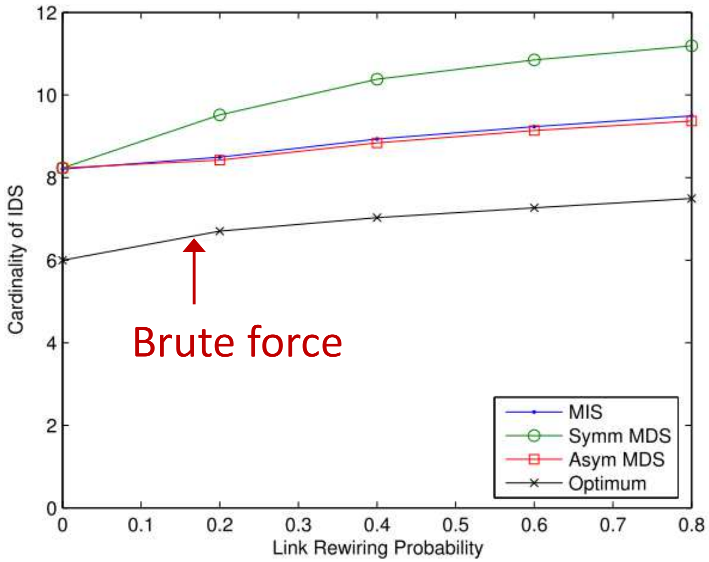


- rewire every edge to a randomly selected node with probability p_r



Sample Result (for IDS)

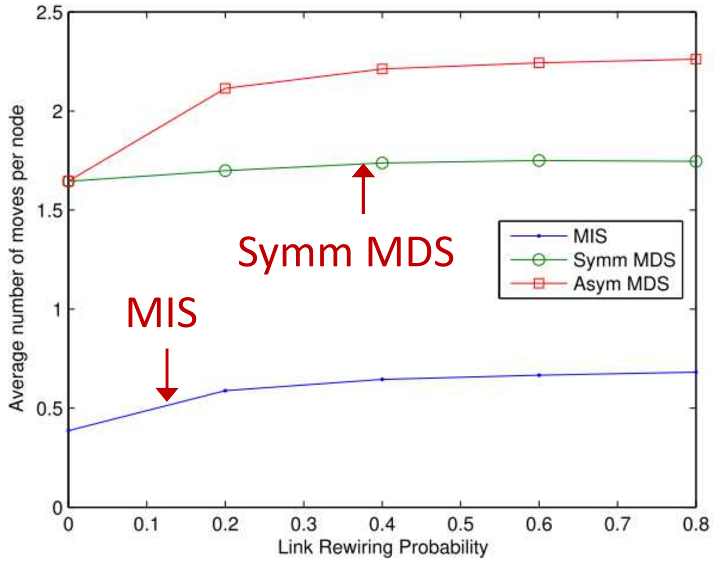
Avg. size of IDS



Brute force

Link rewiring prob.

Avg. #moves per node



Symm MDS

MIS

Link rewiring prob.

$n = 30, k = 4$

References

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