# Robotic HW3 Chen Hao Cheng

① ultrasound sensor measures distance $x = \frac{c\Delta t}{2}$, c is speed of sound,
$\Delta t$ = difference btw emitting and receiving a signal
variance time meaurement $\Delta t$ be $\sigma^2$

What can you say about $x$, when c is assumed to be constant?
Hint: How does a change in $\Delta t$ affect $x$?

If c is constant, and denominator is also constant (2),
the only thing that can change $x$ is $\Delta t$.
If the different btw emitting and receiving a signal ($\Delta t$) is getting bigger, then $x$ will be bigger to. In other word, if the ultrasound sensor will measure the distance that's smaller, the $x$ will be smaller too.

② a unicycle turning w/ angular velocity $\dot{\phi}$ w/ radius $r$.

speed: $v = f(\dot{\phi}, r) = r\dot{\phi}$ — $x$ $\qquad \frac{df}{d\dot{\varphi}} = r$

$rx \rightarrow f' = r$

use the error propagation law to calculate the resulting variance of your speed estimate $\boxed{\sigma_v^2}$

傳播,繁殖

Error Propagation law:

$$\sigma_y^2 = \frac{\partial df}{\partial x}^2 \sigma_x^2 \quad \text{Page 133 (8.1)}$$

base on book, let the standard deviation of $x$ be given by $\sigma_x$, then we can calculate $\sigma_y^2$

Consider $y = f(x)$ w/ $v = f(\dot{\phi}, r)$ based on book, then $\dot{\phi}$ has a standard deviation $\sigma^2_\phi$. Consider the speed equation, we can think $v$ has a standard deviation of $\sigma_v^2 = \frac{\partial df}{\partial \phi}^2 \sigma_\phi^2$

Since f function equals to $r\dot{\phi}$, then we can say $\sigma^2 v = \frac{\partial d(r\dot{\phi})}{\partial \phi}^2 \sigma \phi$

if we take derivative for f function such that $\qquad = r^2 \sigma_\phi^2$

$f(\dot{\phi}, r) = r\dot{\phi}$, $\frac{df}{d\dot{\phi}} \Rightarrow r$

so, we will get $\sigma^2 v = \frac{\partial r^2}{\partial \phi} \sigma \phi$

Thus, $\sigma^2 v = r^2 \sigma^2_\phi$

③ $P(\text{marker} \mid \text{reading})$

Derive $P(\text{marker} \mid \text{reading})$ < marker $P(\text{reading} \mid \text{marker})$, $P(\text{marker})$

Use the Bayes Law:

$$P(B \mid A) = \frac{P(A \mid B)\,P(B)}{P(A)}$$

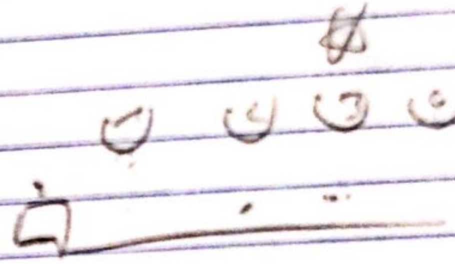
$$P(\text{marker} \mid \text{reading}) = \frac{P(\text{reading} \mid \text{marker}) \cdot P(\text{marker})}{P(\text{reading})}$$

Since $P(\text{marker})$ & $P(\text{reading} \mid \text{marker})$ are given, then we can derive $P(\text{reading})$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

③ ⓑ Based on previous answer, we get Bayes Law

$$P(\text{marker} \mid \text{reading}) = \frac{P(\text{reading} \mid \text{marker})}{P(\text{reading})} P(\text{marker})$$



Reading a marker correctly = 0.9
" marker wrong = 0.1

Do not see a marker = 0.2
Do see a marker = 0.8

Total : 4 markers

calculate the Prob. to be indeed underneath marker 3.

$P_1 = \ldots\ldots$
$P_2 =$
$P_3 =$
$P_4 =$

$$\boxed{P_3} = \frac{P_3}{P_1+P_3+P_2+P_1}$$

Since there are 4 of markers so each one is 25% = 0.25 for P(marker)

$$P(\text{marker} \mid \text{reading}) = \frac{(0.8)(0.25)}{0.9} = 0.222$$

We need to consider that it reads first two and not see a marker, which is 20% (0.2), and then it sees the marker 3 which is 80% (0.8), and consider it reading correctly which is 90% (0.9)

marker 1   marker 2   marker 3

$0.2 \times 0.2 \times 0.8 \times 0.9 = 0.0288$
$= 2.88\,\%$

(3b)

suppose the robot reads the first one? But it's wrong?

$(0.1 \times 0.8) = 0.08 = 8\%$

suppose the robot reads the second one and it's wrong?

$0.2 \times (0.8 \times 0.1) = 0.016 = 1.6\%$

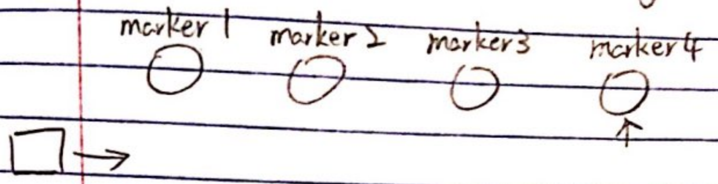
suppose the robot reads the fourth marker and it's wrong?

$0.2 \times 0.2 \times (0.8 \times 0.1) \times (0.8 \times 0.9) = 0.002304 = 0.23\%$

Normalize

$$P_3 = \frac{P_3}{P_1 + P_2 + P_3 + P_4} = \frac{0.0288}{0.08 + 0.016 + 0.0288 + 0.002304} = 0.3537$$

© Could the robot also possibly be underneath maker 4?




Reading a marker correctly $= 0.9$
" marker wrong $= 0.1$

Do not see a marker $= 0.2$
Do see a marker $= 0.8$

We need to consider that there has been 3 that's passed which means not see markers (0.2), we know it's the fourth one, then skip 80%

Thus, $0.2 \times 0.2 \times 0.2 \times 0.9 = 0.0072$

We also need to consider what if the robot read marker 3 that the robot thinks it's the fourth marker?

Then, it will be

$$0.2 \times 0.2 \times \overset{\text{marker 3}}{\underset{\text{see a marker}}{0.8}} \times \underset{\text{see marker wrong}}{0.1} = 0.0032$$

~~and multiply the total probability~~ of reading a marker as marker 3

$$0.0032 \times \frac{1}{3} = 0.001067 = 0.1067\%$$

next page.

(c)

$P_1 = P(m=1 | r=3)$ = under marker 1, reading as marker 3

$P_2 = P(m=2 | r=3)$

$P_3 = P(m=3 | r=3)$

$P_4 = P(m=4 | r=3)$

Normalize

$$\frac{P_4}{P_1+P_2+P_3+P_4} = \frac{P(m=4|r=3)}{P(m=1|r=3)+P(m=2|r=3)+P(m=3|r=3)+P(m=4|r=3)}$$

$$= \frac{0.002}{0.266+0.053+0.096+0.002}$$

$$= 0.004796 \approx 0.5\%$$