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DOF: how many variables are required to determine position of a mechanism in space.

① DOF of standard, four-wheel, hand-pushed lawnmower?

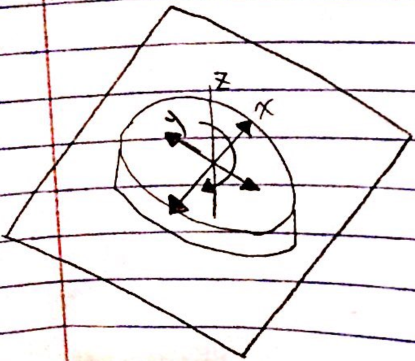
Google definition: A direction in which independent motion can occur.

A lawn mower can be pushed <u>forward</u> or pulled <u>backward</u>.
so, it's 1 degree of freedom

But how people mow their whole lawns?
People remove a constraint by lifting up the front or back wheels so it allows to turn.
The rest of degrees of freedom depending on human's hand of degrees of freedom. If we are just talking the lawnmower, it's just one.

② What are the maximum degrees of freedom for objects driving on the plane?

If the objects' body is rigid, it can only has 3 DOF.
2 translational (x, y)
1 rotational (θ_z)



③ ⓐ Calculate the angle btw vectors $(\overset{\frac{\sqrt{2}}{2}}{\cos 45°}, \underset{a}{\overset{-\frac{\sqrt{2}}{2}}{-\sin 45°}}, 0)^T$ & $(\overset{\frac{\sqrt{2}}{2}}{\sin 45°}, \underset{b}{\overset{\frac{\sqrt{2}}{2}}{\cos 45°}}, 0)^T$

$$\cos\theta = \frac{\overline{a}\cdot\overline{b}}{|\overline{a}||\overline{b}|}$$

$$\overline{a}\cdot\overline{b} = a_x\cdot b_x + a_y\cdot b_y + a_z\cdot b_z$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + {}^{-}\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + 0\cdot 0 = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{-\sqrt{2}}{2}\right)^2 + 0 = \frac{1}{2} + -\frac{1}{2} + 0 = 0$$

$$|\overline{a}| = \sqrt{a_x^2 + b_y^2 + C_z^2} = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + 0} = \sqrt{\frac{1}{2} + \frac{1}{2} + 0} = 1$$

$$|\overline{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2} = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + 0} = \sqrt{\frac{1}{2} + \frac{1}{2} + 0} = 1$$

$$\cos\theta = \frac{0}{1\cdot 1} = 0 \Rightarrow \cos^{-1}(0) = 90$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

ⓑ

$$\begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \end{bmatrix} = \begin{bmatrix} i & j & K \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix} =$$

$$= i(0-0) - j(0-0) + K\left[\left(\frac{\sqrt{2}}{2}\right)^2 - \left(-\left(\frac{\sqrt{2}}{2}\right)^2\right)\right]$$
$$= K\left(\frac{1}{2} + \frac{1}{2}\right) = K(1)$$

$$[0, 0, 1]$$

In order to form coordinate system, all vector orthogonal (cross product)

dot product is scalar

Basis: $\begin{bmatrix}1 & 0 & 0\end{bmatrix}$
$\begin{bmatrix}0 & 1 & 0\end{bmatrix}$
$\begin{bmatrix}0 & 0 & 1\end{bmatrix}$

4a) Write out the entries of a rotation matrix ${}^A_BR$ assuming basis vectors x_A, y_A, z_A and x_B, y_B, z_B

$${}^A_BR = \begin{bmatrix} {}^A\hat{x}_B & {}^A\hat{Y}_B & {}^A\hat{Z}_B \end{bmatrix} = \quad {}^A\hat{x}_B = \begin{bmatrix} \hat{x}_B \cdot \hat{x}_A \\ \hat{x}_B \cdot \hat{Y}_A \\ \hat{x}_B \cdot \hat{Z}_A \end{bmatrix} \quad {}^A\hat{Y}_B = \begin{bmatrix} \hat{Y}_B \cdot \hat{x}_A \\ \hat{Y}_B \cdot \hat{Y}_A \\ \hat{Y}_B \cdot \hat{Z}_A \end{bmatrix} \quad {}^A\hat{Z}_B = \begin{bmatrix} \hat{Z}_B \cdot \hat{x}_A \\ \hat{Z}_B \cdot \hat{Y}_A \\ \hat{Z}_B \cdot \hat{Z}_A \end{bmatrix}$$

b) $\hat{x}_B = [0\,1\,0]^T$ in frame $\{A\}$ the inverse of ${}^A_BR = {}^A_BR^T$

$$\hat{x}_B = [0\,1\,0]^T = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$${}^A\hat{x}_B = {}^A_BR \; {}^B x_B$$

$$= \begin{bmatrix} {}^A\hat{x}_B & {}^A\hat{Y}_B & {}^A\hat{Z}_B \end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix} 0 + {}^A\hat{Y}_B + 0 \end{bmatrix} = {}^A\hat{Y}_B$$

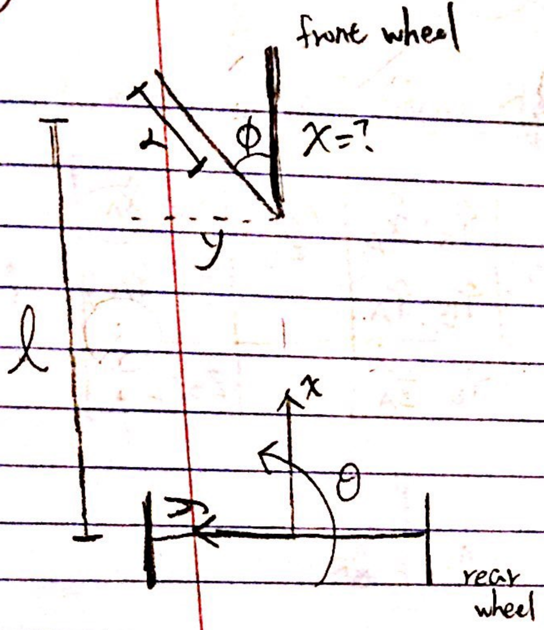
c) the inverse of ${}^A_BR$ is ${}^A_BR^T$
${}^B_AR$ is ${}^A_BR^T$

$${}^B_AR = \begin{bmatrix} {}^B\hat{x}_A & {}^B\hat{Y}_A & {}^B\hat{Z}_A \end{bmatrix}$$

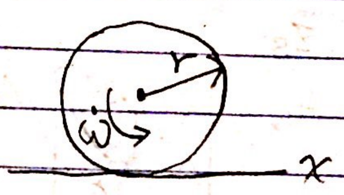
the entries of rotation matrix ${}^B_AR$ | ${}^B\hat{x}_A = \begin{bmatrix} \hat{x}_A \cdot \hat{x}_B \\ \hat{x}_A \cdot \hat{Y}_B \\ \hat{x}_A \cdot \hat{Z}_B \end{bmatrix}$ ${}^B\hat{Y}_A = \begin{bmatrix} \hat{Y}_A \cdot \hat{x}_B \\ \hat{Y}_A \cdot \hat{Y}_B \\ \hat{Y}_A \cdot \hat{Z}_B \end{bmatrix}$ ${}^B\hat{Z}_A = \begin{bmatrix} \hat{Z}_A \cdot \hat{x}_B \\ \hat{Z}_A \cdot \hat{Y}_B \\ \hat{Z}_A \cdot \hat{Z}_B \end{bmatrix}$

$$\begin{bmatrix} \hat{x}_A \cdot \hat{x}_B & \hat{Y}_A \cdot \hat{x}_B & \hat{Z}_A \cdot \hat{x}_B \\ \hat{x}_A \cdot \hat{Y}_B & \hat{Y}_A \cdot \hat{Y}_B & \hat{Z}_A \cdot \hat{Y}_B \\ \hat{x}_A \cdot \hat{Z}_B & \hat{Y}_A \cdot \hat{Z}_B & \hat{Z}_A \cdot \hat{Y}_B \end{bmatrix}^T \Rightarrow \begin{bmatrix} \hat{x}_A \cdot \hat{x}_B & \hat{x}_A \cdot \hat{Y}_B & \hat{x}_A \cdot \hat{Z}_B \\ \hat{Y}_A \cdot \hat{x}_B & \hat{Y}_A \cdot \hat{Y}_B & \hat{Y}_A \cdot \hat{Z}_B \\ \hat{Z}_A \cdot \hat{x}_B & \hat{Z}_A \cdot \hat{Y}_B & \hat{Z}_A \cdot \hat{Y}_B \end{bmatrix}$$

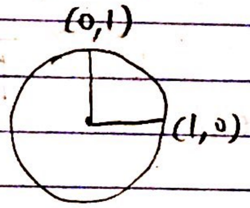
⑤



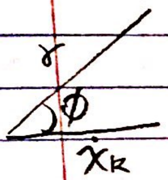
↑ = independent



$x = r\dot{\omega}$



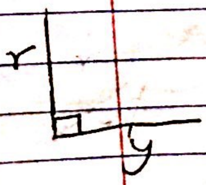
Kinematics of the mechanism



$\cos\phi = \frac{\dot{x}_R}{r} \Rightarrow \dot{x}_R = r\cos\phi$

But we need to muleiply speed $\dot{\omega}$

$\Rightarrow \dot{x}_R = r\dot{\omega}\cos\phi$



$\cos 90^\circ = \frac{\dot{y}_R}{r} \Rightarrow \dot{y}_R = r\dot{\omega}\overset{0}{\cancel{\cos 90^\circ}}$

↑

$= 0$ meaning that the tri-cycle is spining its own circle

θ: consider the whole tri-cycle

$= \frac{r\dot{\omega}\overset{对}{\sin}\phi}{l}$