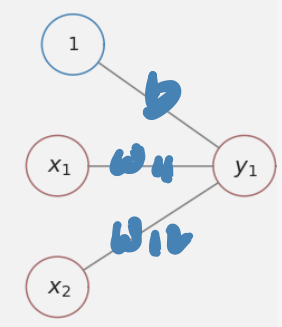
Neural Networks Part II

Feed-Forward Neural Networks

Previously on CSCI 4622

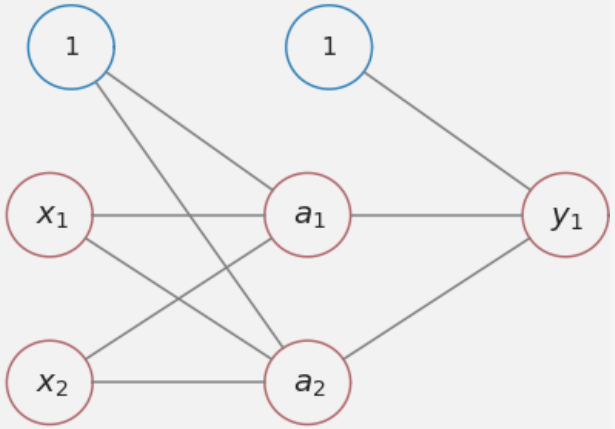
The Perceptron as a NN: Can learn linearly separable things like OR, AND, NAND, etc



$$\hat{y} = \begin{cases} 0 & \text{if } \mathbf{w}^T\mathbf{x} + b \leq 0 \\ 1 & \text{if } \mathbf{w}^T\mathbf{x} + b > 0 \end{cases}$$

x_1	0	1	0	1
x_2	0	0	1	1
x_1 OR x_2	0	1	1	1

The Multilevel Perceptron: Can learn non-linearly separable things like XOR



Layer 1 to 2:

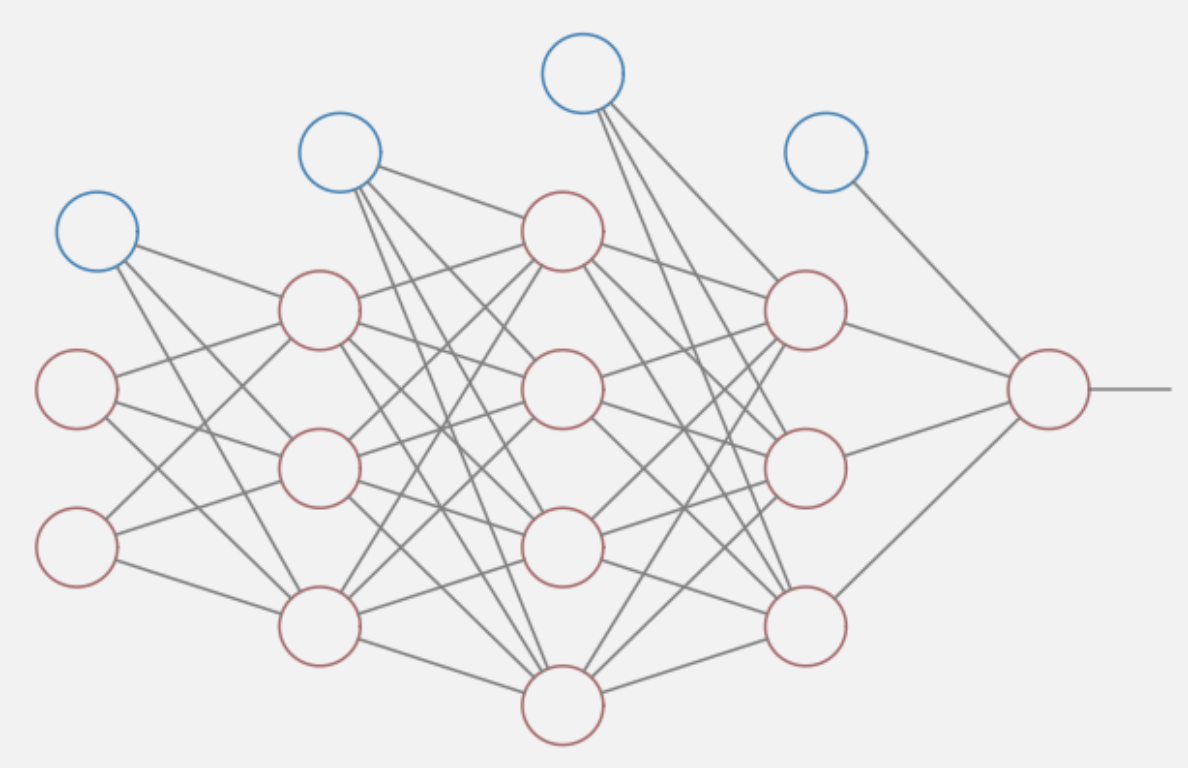
$$\begin{aligned} \mathbf{z}^2 &= W^1\mathbf{x}^1 + \mathbf{b}^1 \\ \mathbf{a}^2 &= \mathrm{I}(\mathbf{z}^2 > 0) = \mathrm{I}(W^1\mathbf{x}^1 + \mathbf{b}^1 > 0) \end{aligned}$$

Layer 2 to 3:

$$\begin{aligned} \mathbf{z}^3 &= W^2\mathbf{a}^2 + \mathbf{b}^2 \\ \hat{y} &= \mathrm{I}(\mathbf{z}^3 > 0) = \mathrm{I}(W^2\mathbf{a}^2 + \mathbf{b}^2 > 0) \end{aligned}$$

The Multilayer Perceptron

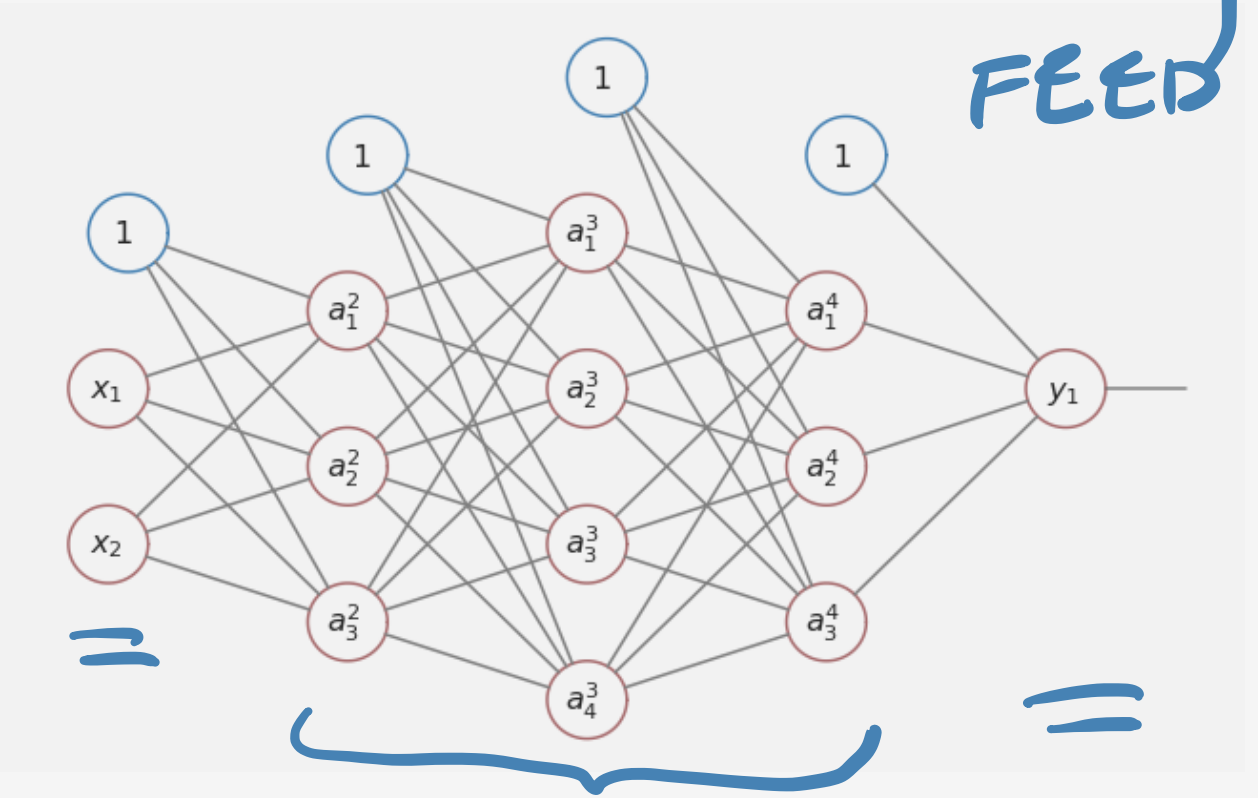
String together lots of perceptron units



The Multilayer Perceptron

String together lots of perceptron units

FULLY-CONNECT
FEED-FORWARD
MLP

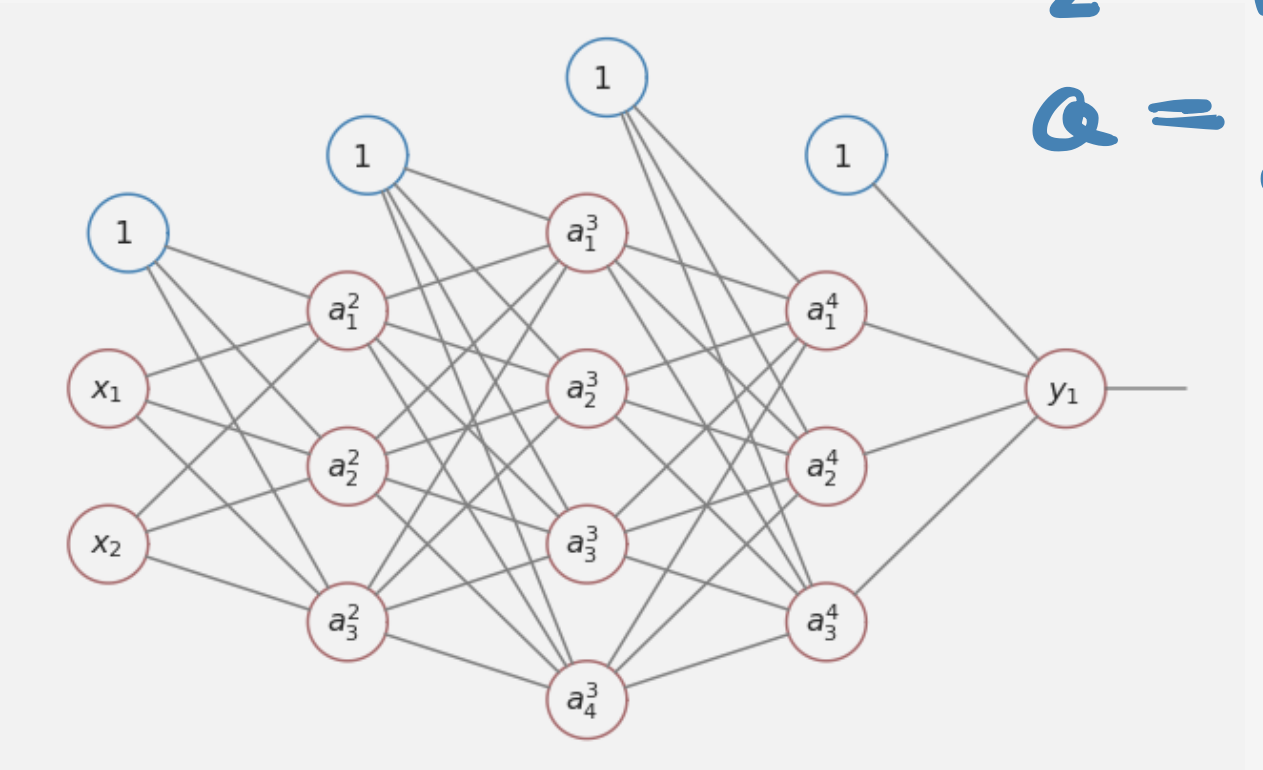


Input Layer Hidden Layers Output Layer

The Multilayer Perceptron

String together lots of perceptron units

$$z = Wa + b$$
$$a = g(z)$$



Inputs, activities, activations, and outputs all live on nodes (neurons)

The Multilayer Perceptron

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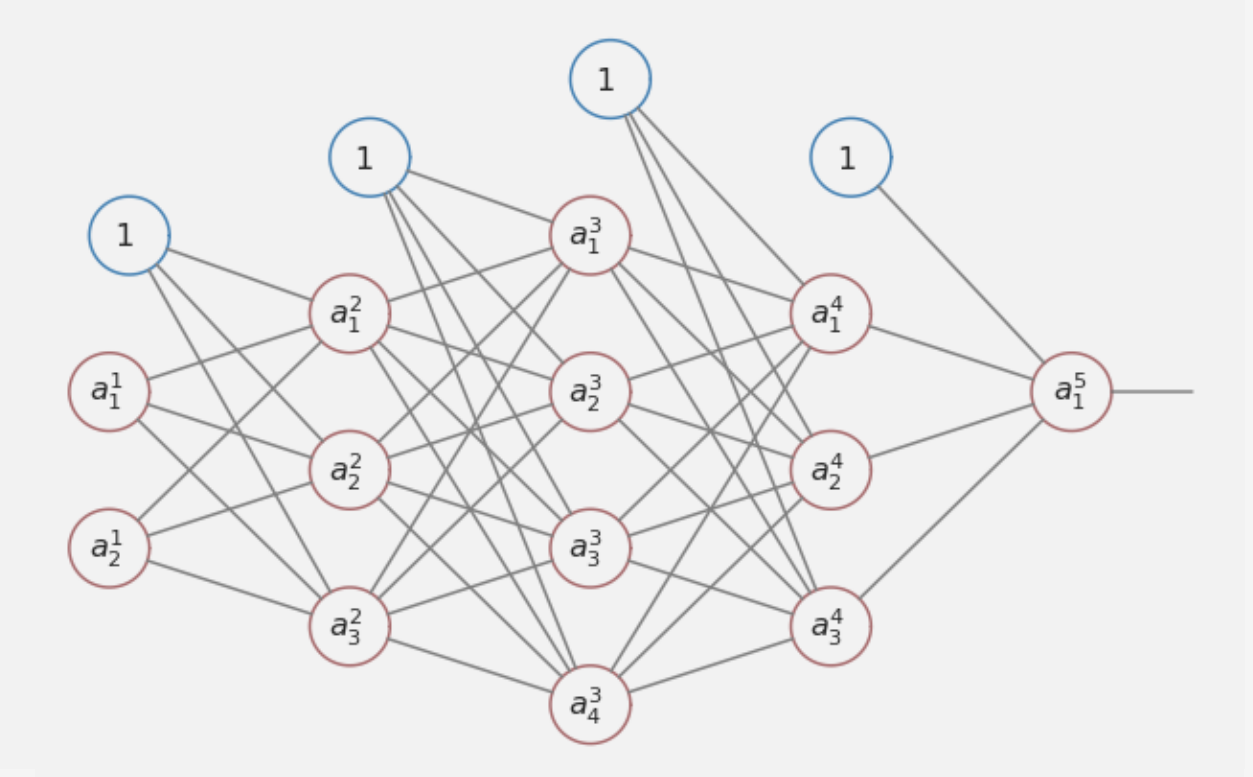

For simplicity: Think of inputs and outputs as activations too

The Multilayer Perceptron

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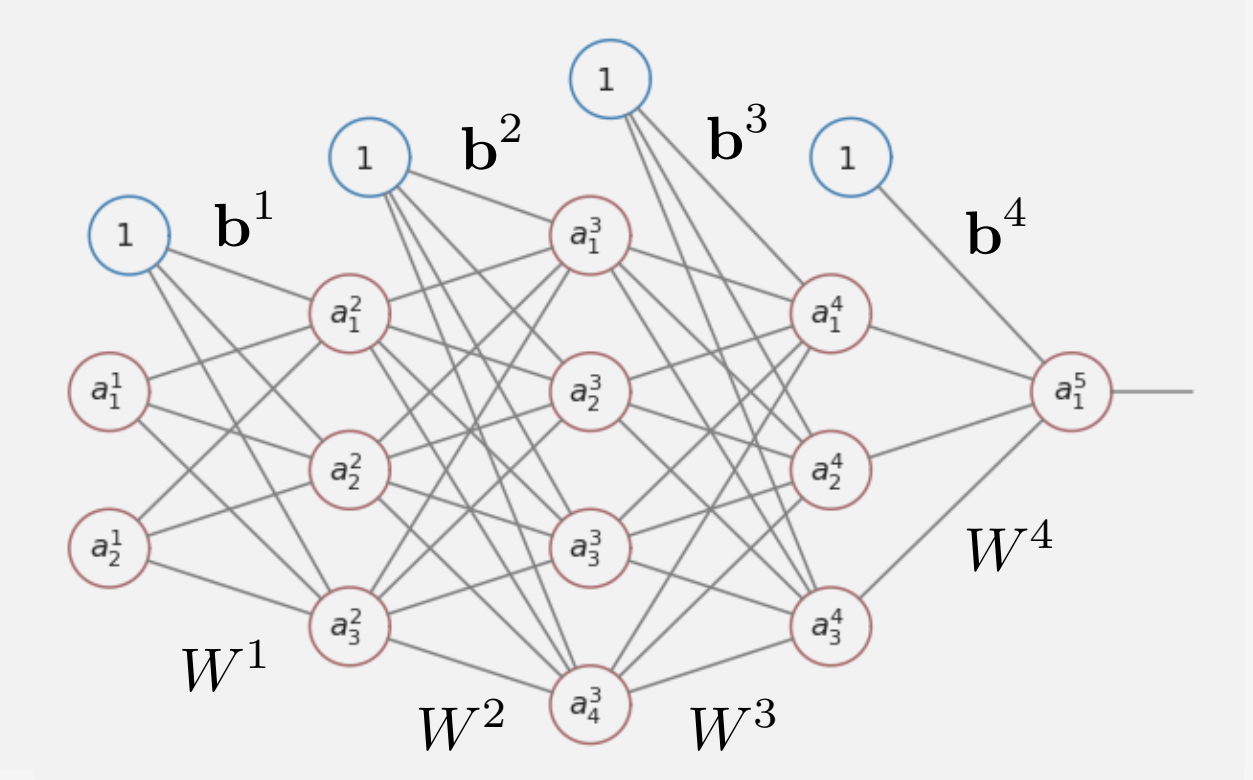
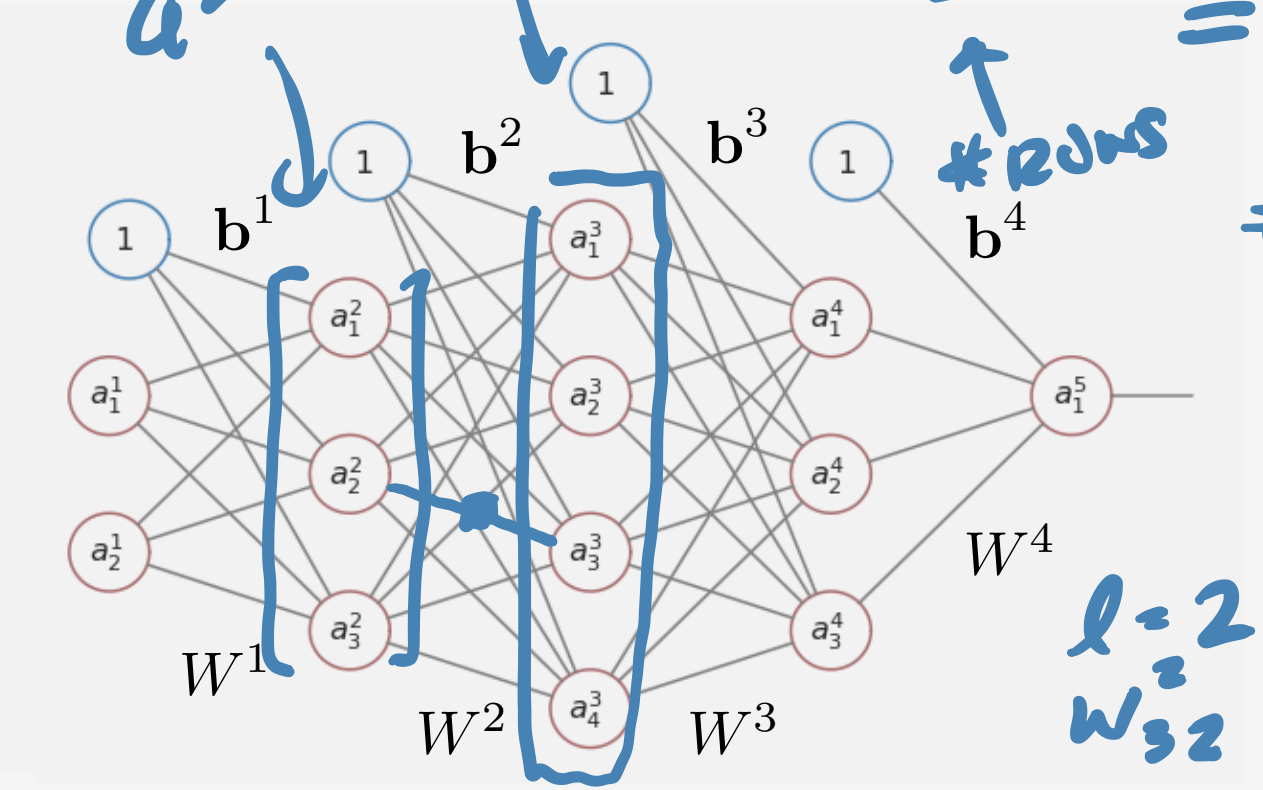


Each transition between layers has associated set of weights an biases

The Multilayer Perceptron

String together lots of perceptron units



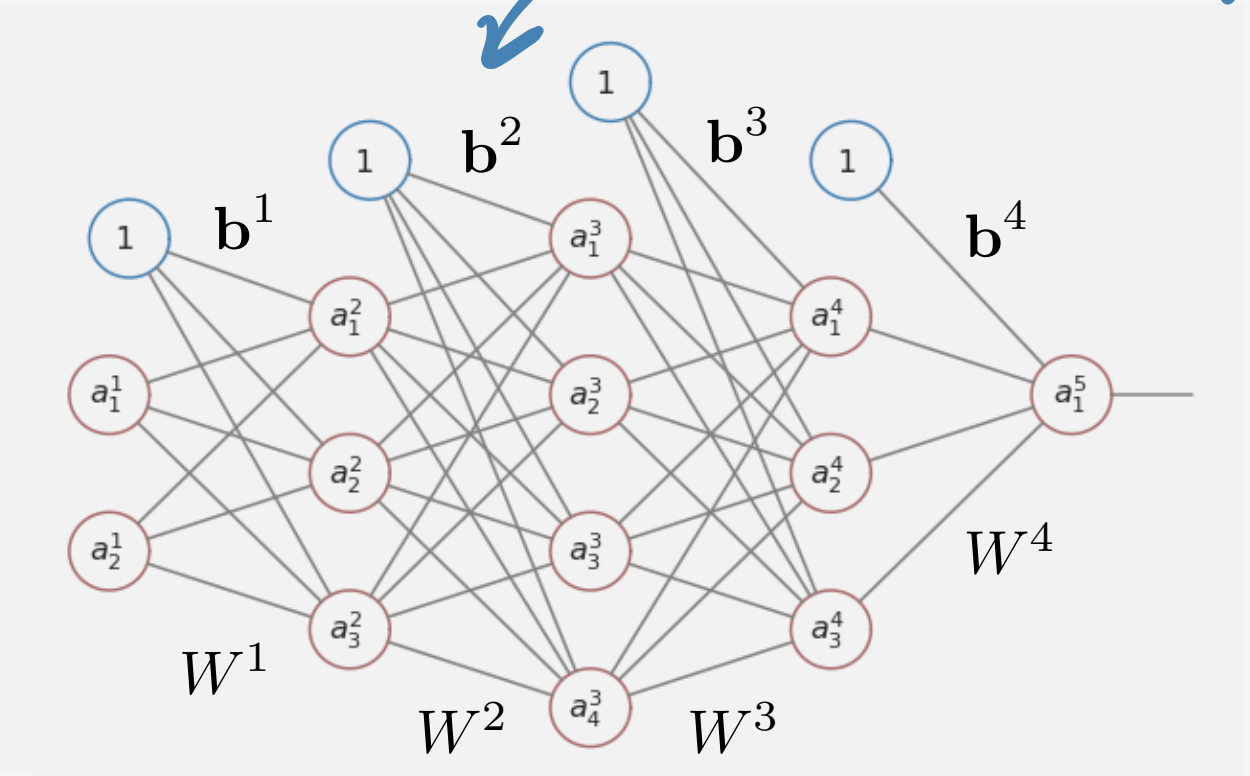
Store the weights connecting layers ℓ and $\ell+1$ in the matrix W^ℓ

w_{ij}^ℓ is the weight from node j in layer ℓ to node i in layer $\ell+1$

The Multilayer Perceptron

String together lots of perceptron units

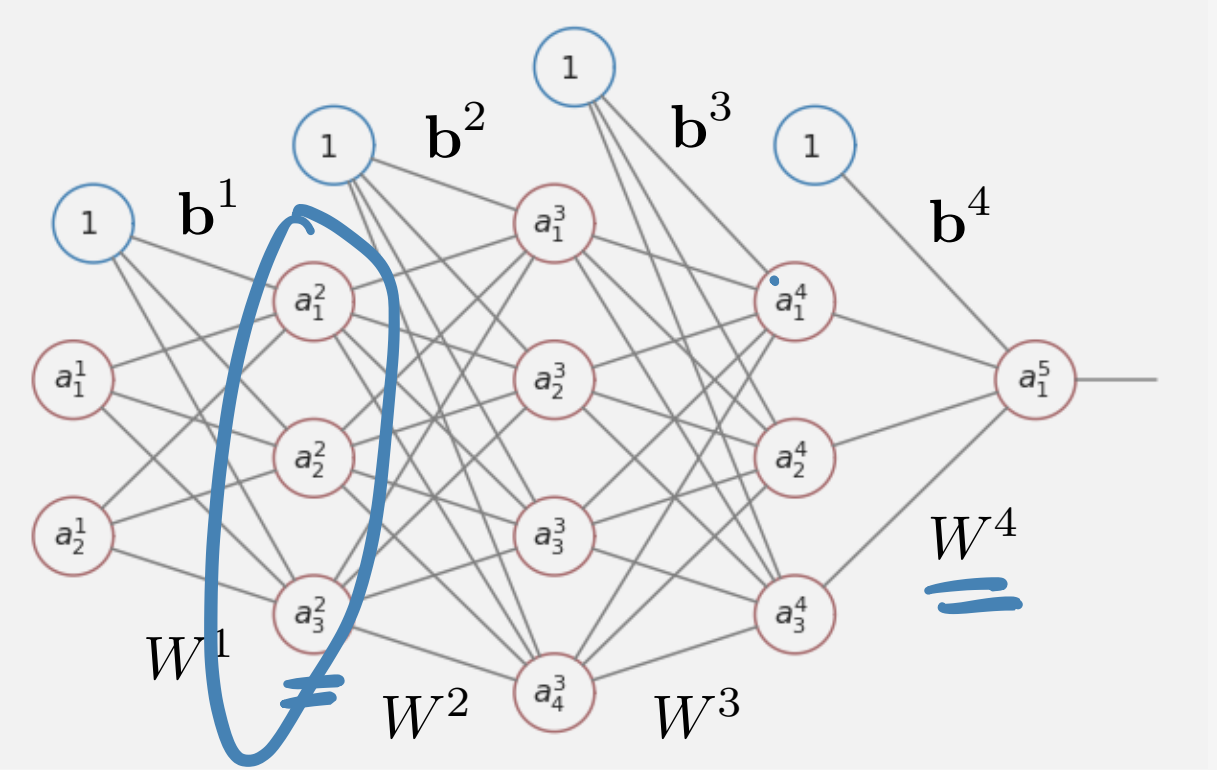
biases in layer 3



Store the biases for layer $\ell + 1$ in the vector

$\mathbf{b}^\ell$ is the bias feeding into node i in layer $\ell + 1$

The Multilayer Perceptron



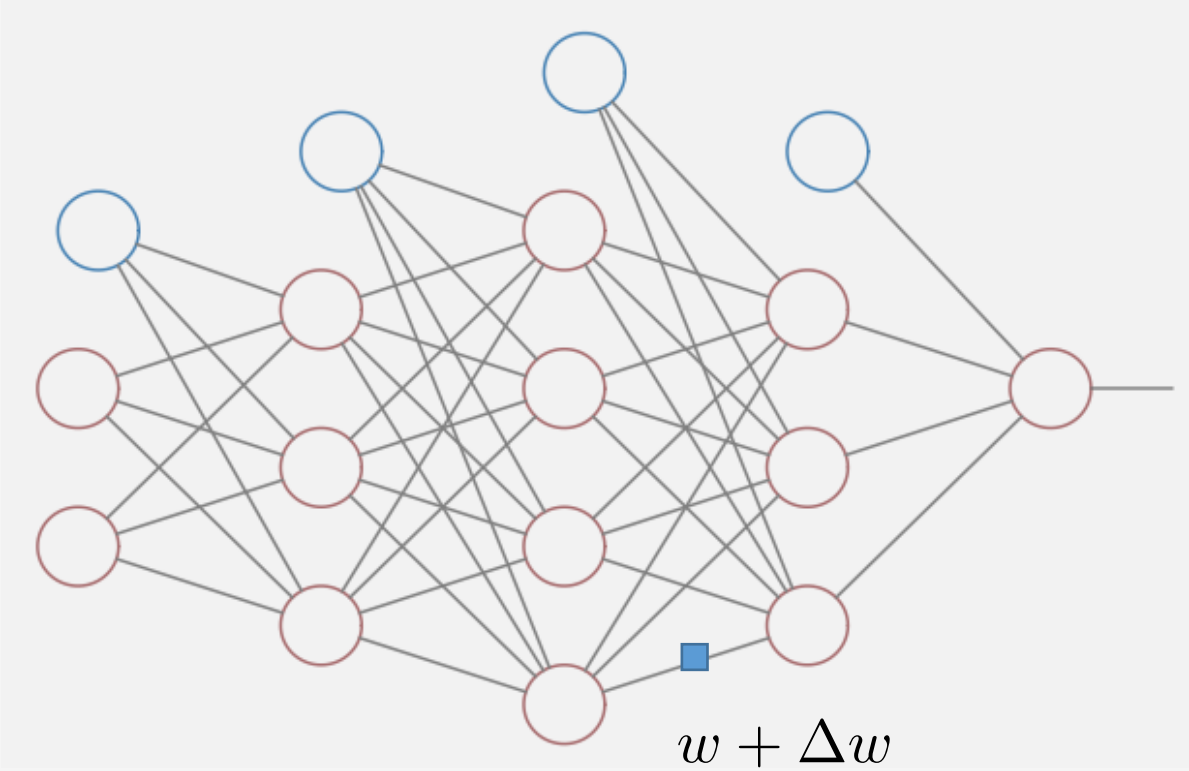
Question: What are the dimensions of each of the following?

W^2: 4×3, $\mathbf{b}^3$: 3, $\mathbf{z}^2$: 3, W^4: 1×3

Sigmoid Neurons

Next week we'll see how we can learn the weights and the biases in a NN using SGD

This poses a problem with standard perceptron neurons with the Indicator activation function



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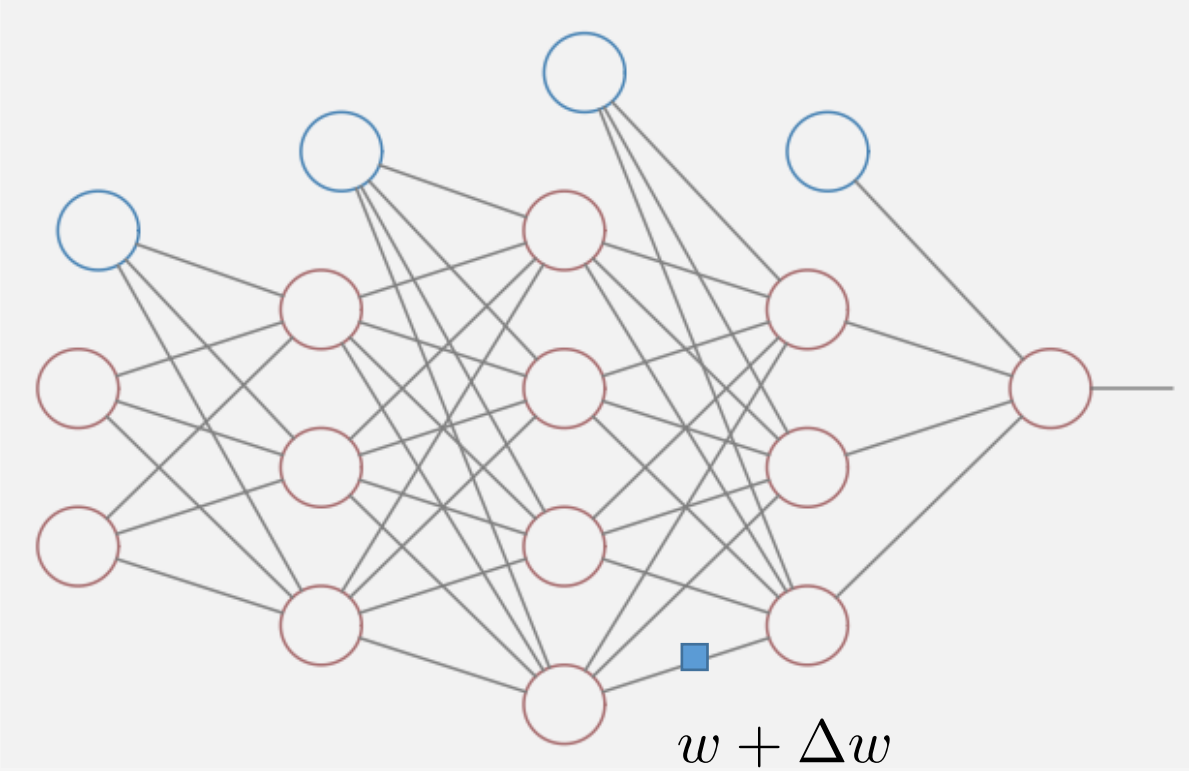

Q: What activation could we use instead?

$$\text{output} + \Delta\text{output}$$

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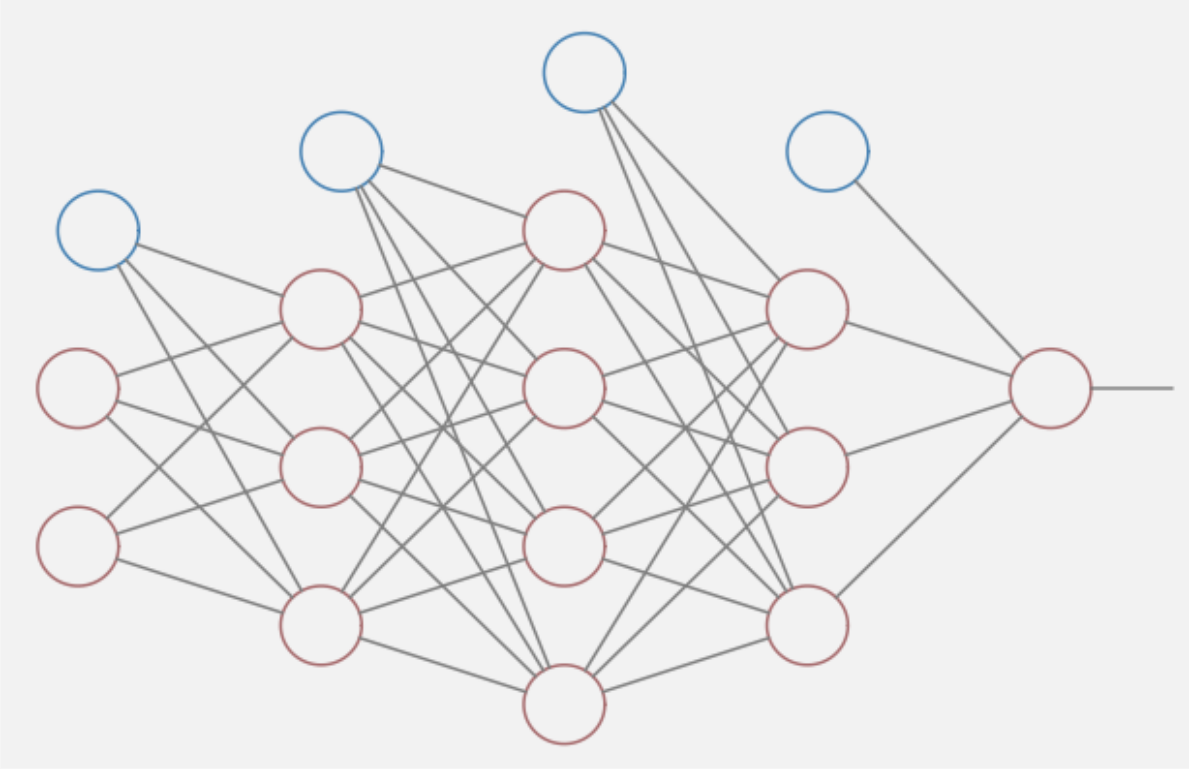
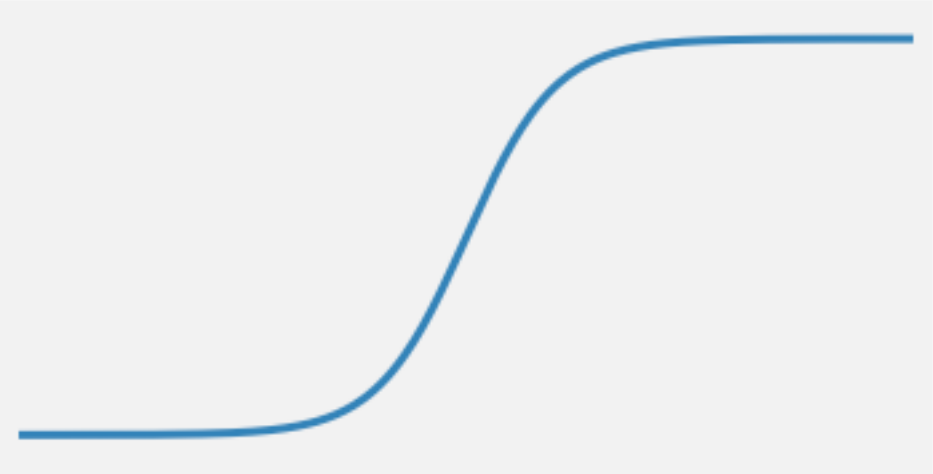


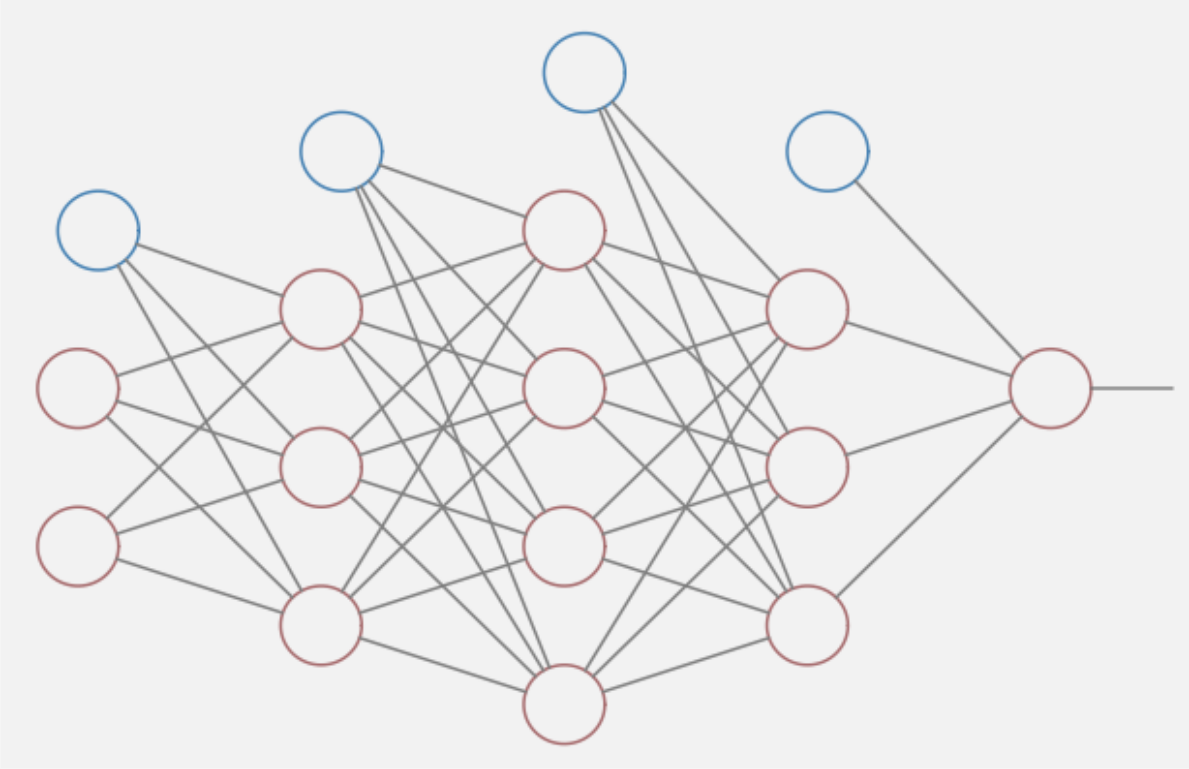
Q: What activation could we use instead?

A: How about a sigmoid activation?



Sigmoid Neurons

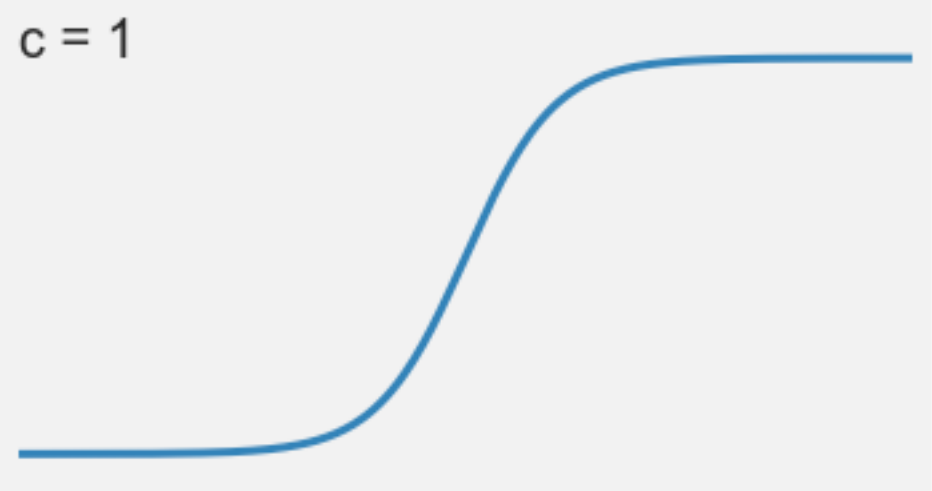
Is a sigmoid neuron all that different from a perceptron?



For a fixed input. $\mathbf{x}$,

multiply weights and bias by a factor c

$$\text{sigm}[c(\mathbf{w}^T x + b)]$$



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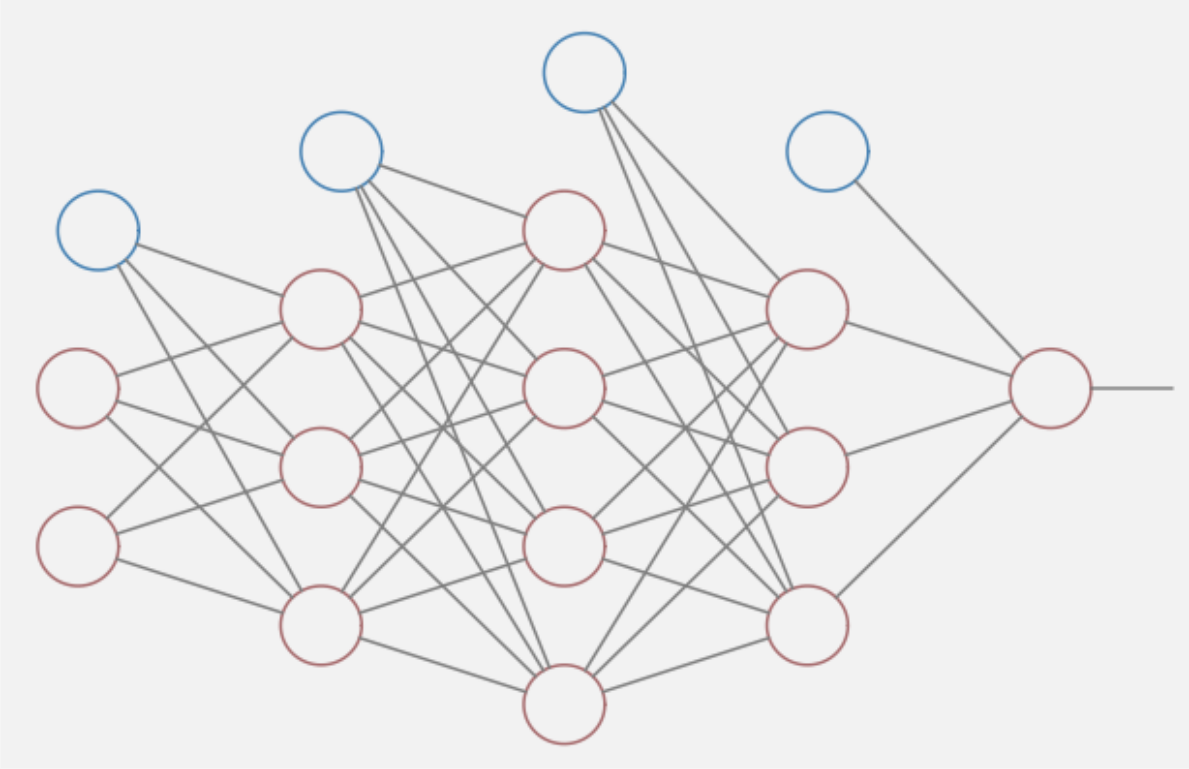

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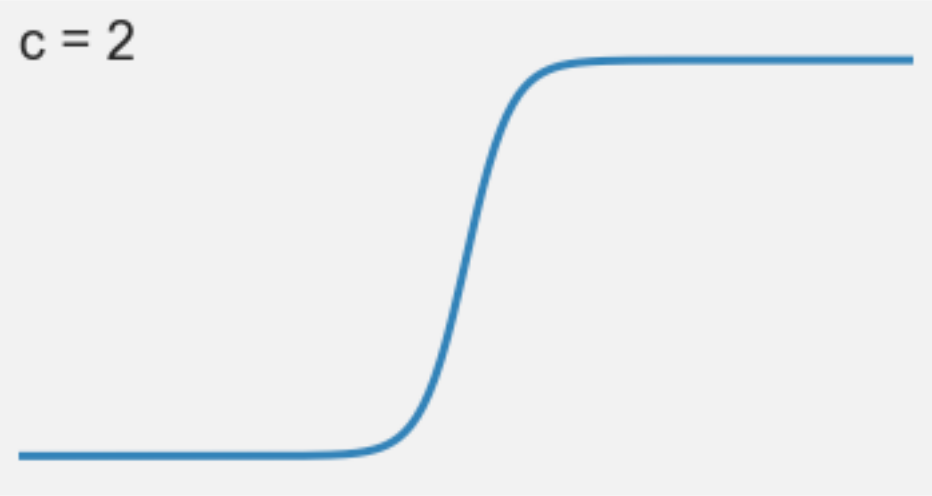

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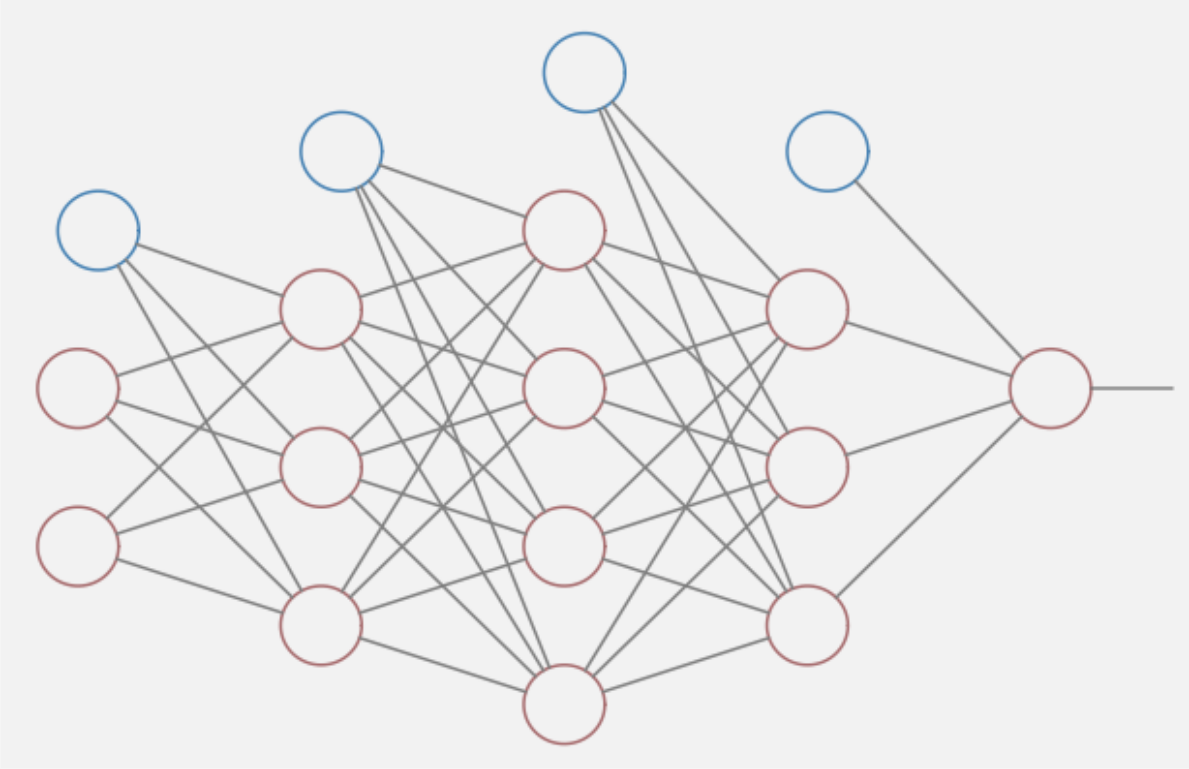

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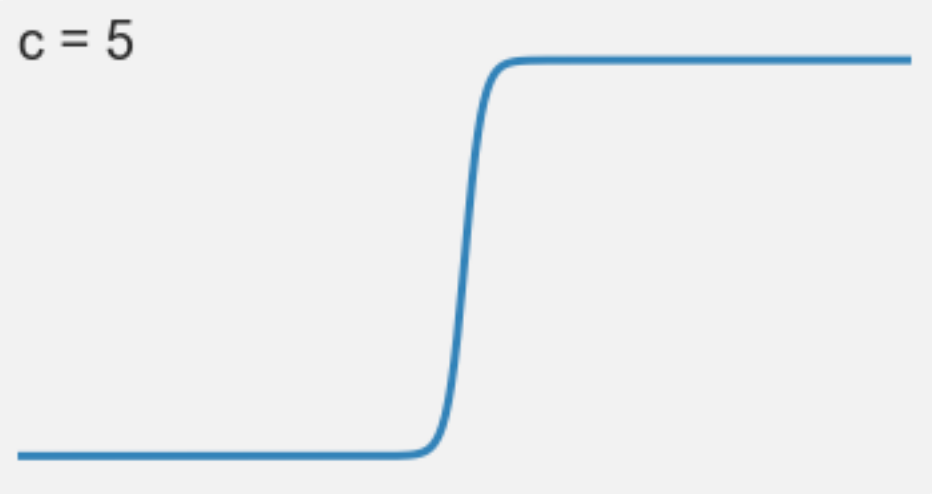

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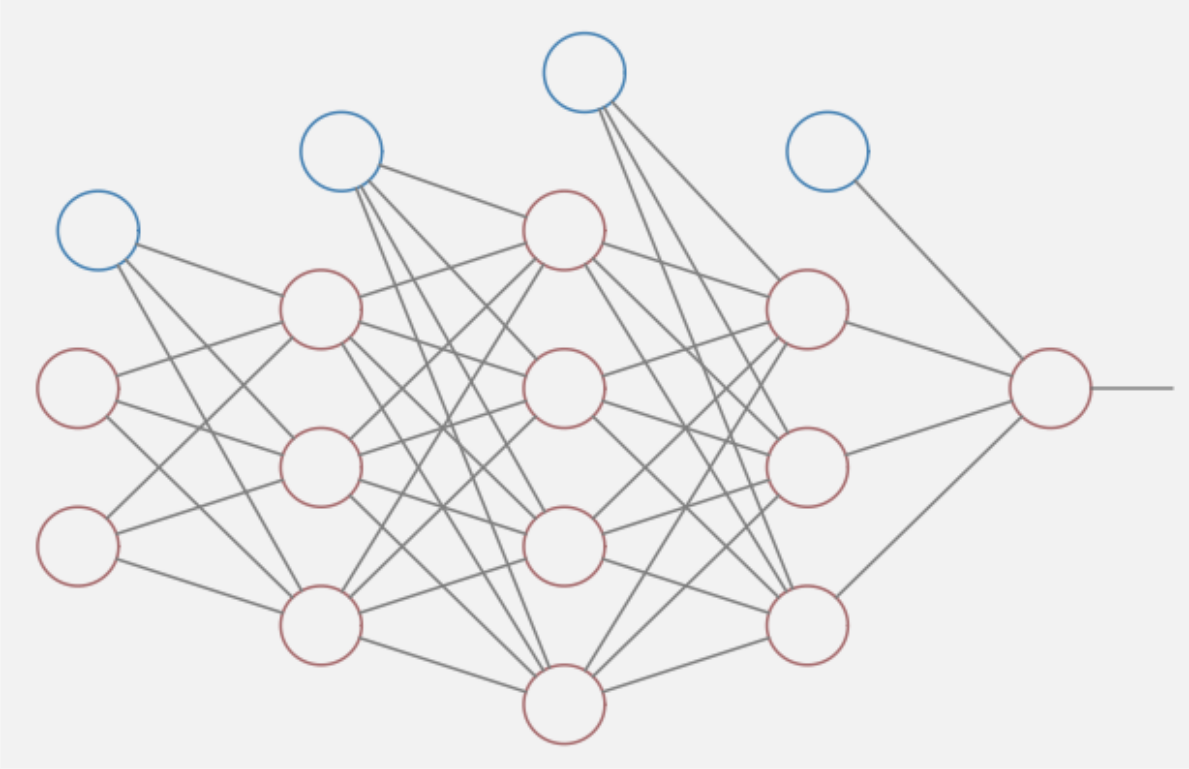

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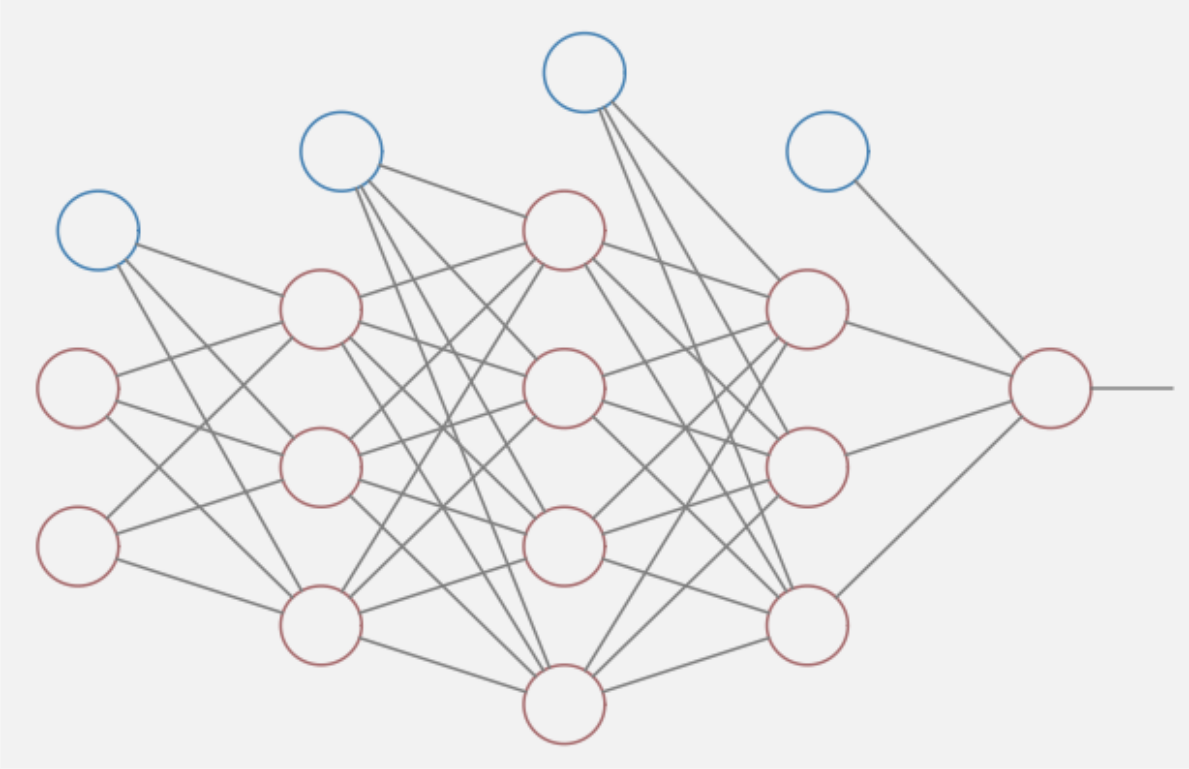

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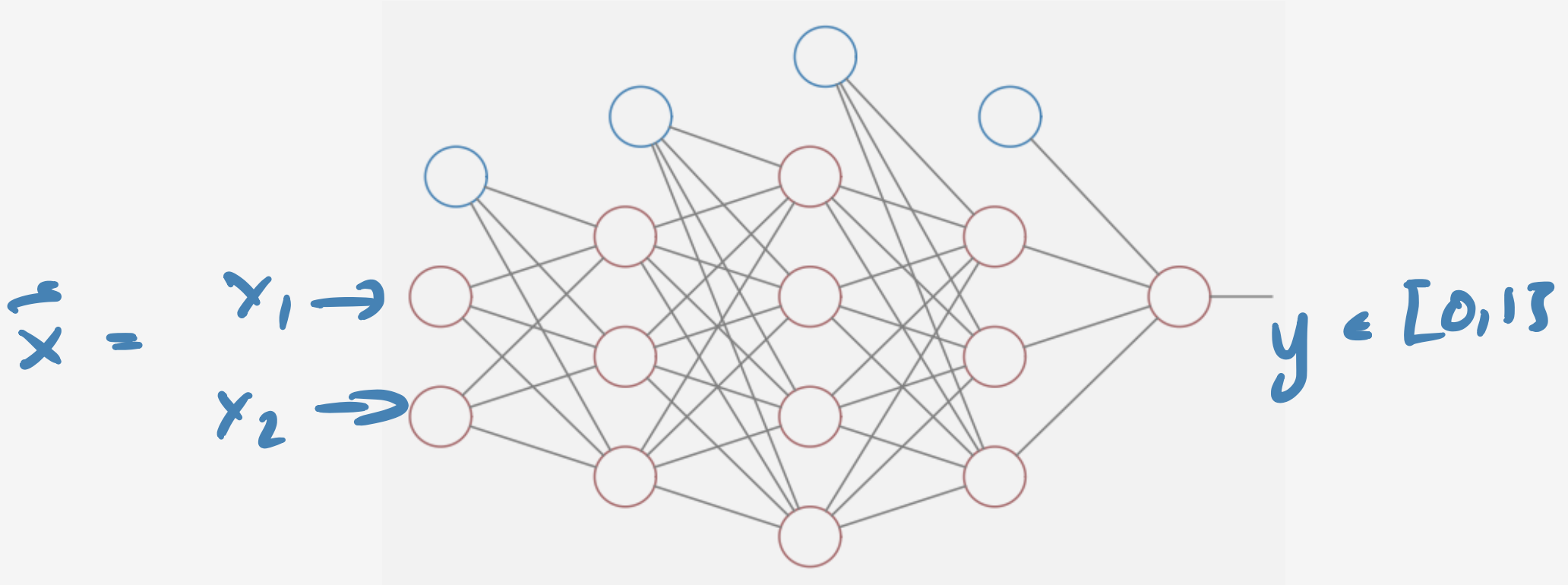
Predictions: Forward Propagation

Suppose we've learned the weights in a NN with sigmoid neurons.

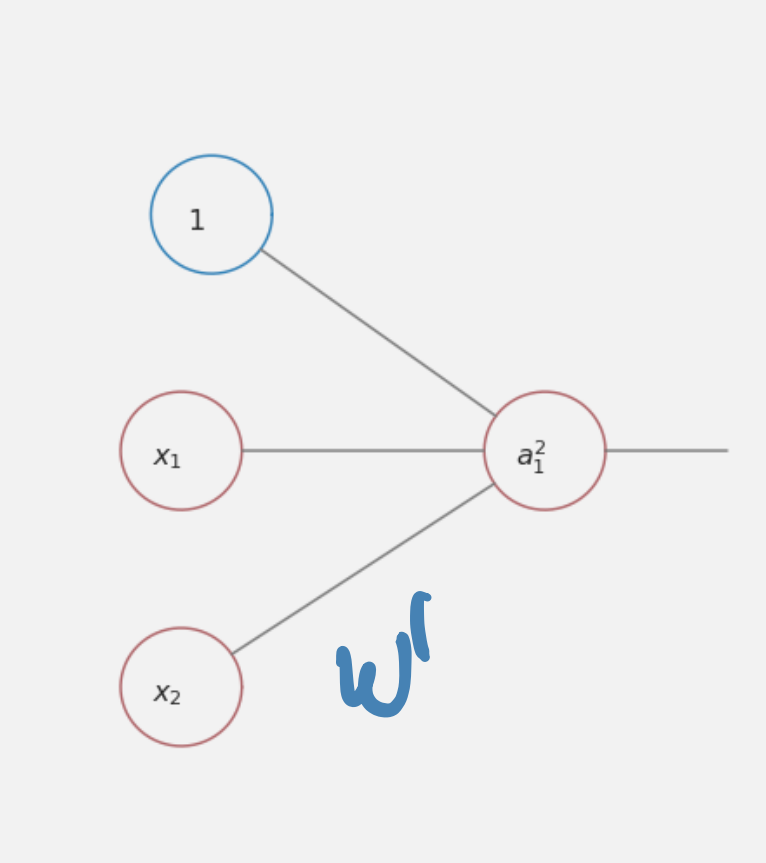
Given an input $\mathbf{x}$, how do we make a prediction?



Predictions: Forward Propagation

Suppose we've learned the weights in a NN with sigmoid neurons.

Given an input $\mathbf{x}$, how do we make a prediction? First, think about a simple network:



$$W^1 = [w^1_{11} \; w^1_{12}] \qquad b^1 = [b^1_1]$$

$$z^2 = W^1 x + b$$

$$a^2 = g(W^1 x + b)$$

$$\downarrow$$

$$y$$

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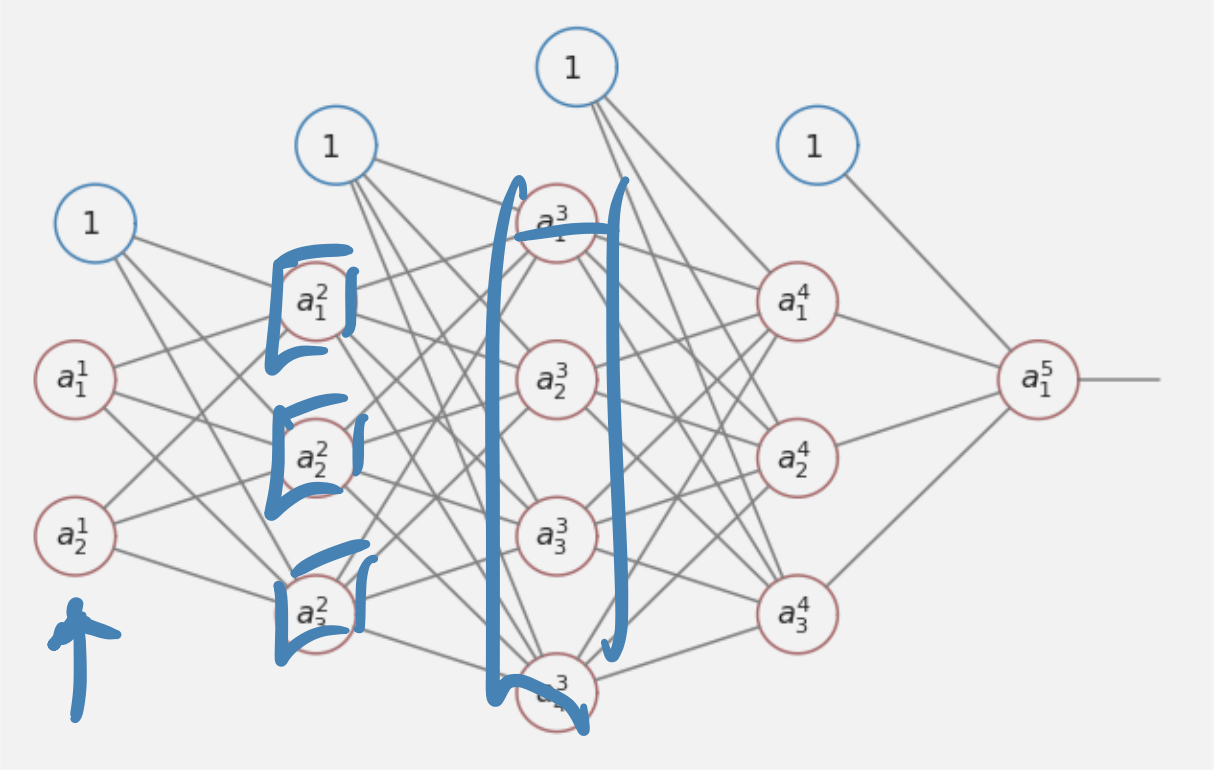

$z^2 = W^1 a^1 + b^1$

$a^2 = g(z^2)$

$z^3 = W^2 a^2 + b^2$

$a^3 = g(z^3)$

Re: $\mathbf{a}^{\ell+1} = g(\mathbf{z}^{\ell+1})$ where $\mathbf{z}^{\ell+1} = W^\ell \mathbf{a}^\ell + \mathbf{b}^\ell$

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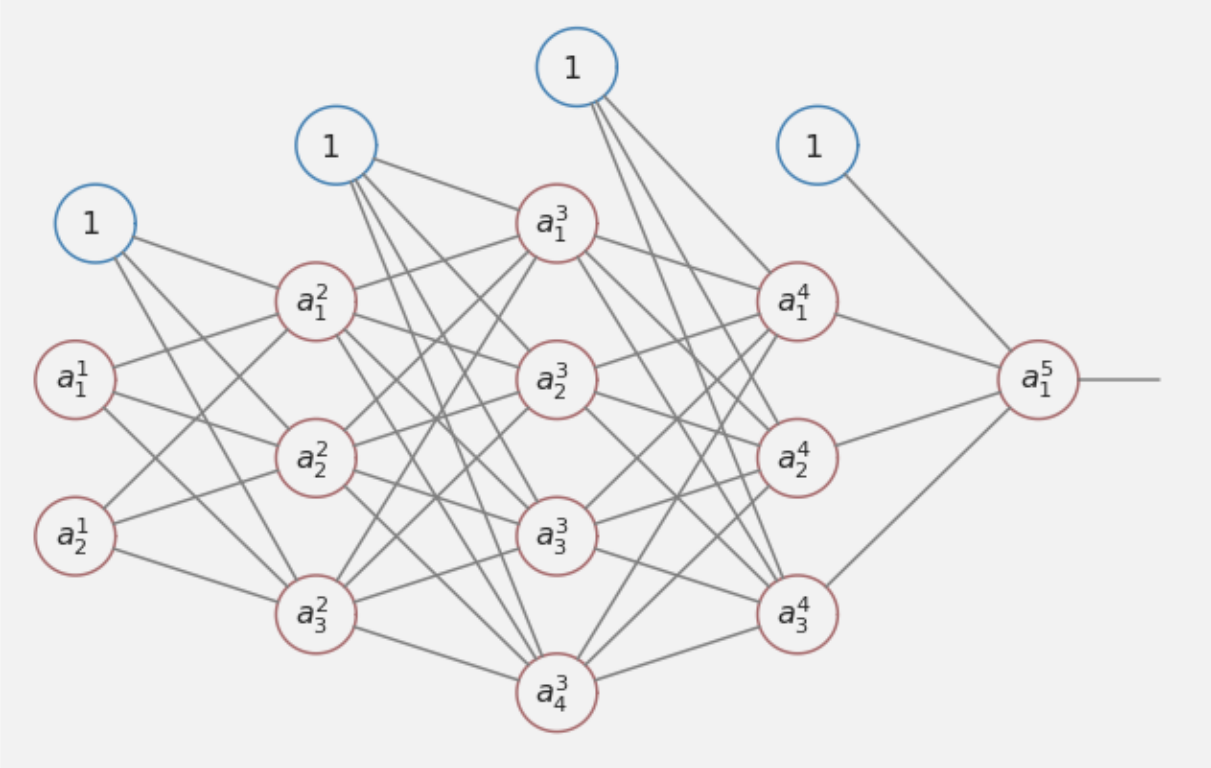

Suppose the network has L layers

Intialize $\mathbf{a}^1 = \mathbf{x}$

for $\ell = 1, \ldots, L-1$

$$\mathbf{z}^{\ell+1} = W^\ell \mathbf{a}^\ell + \mathbf{b}^\ell$$

$$\mathbf{a}^{\ell+1} = g(\mathbf{z}^{\ell+1})$$

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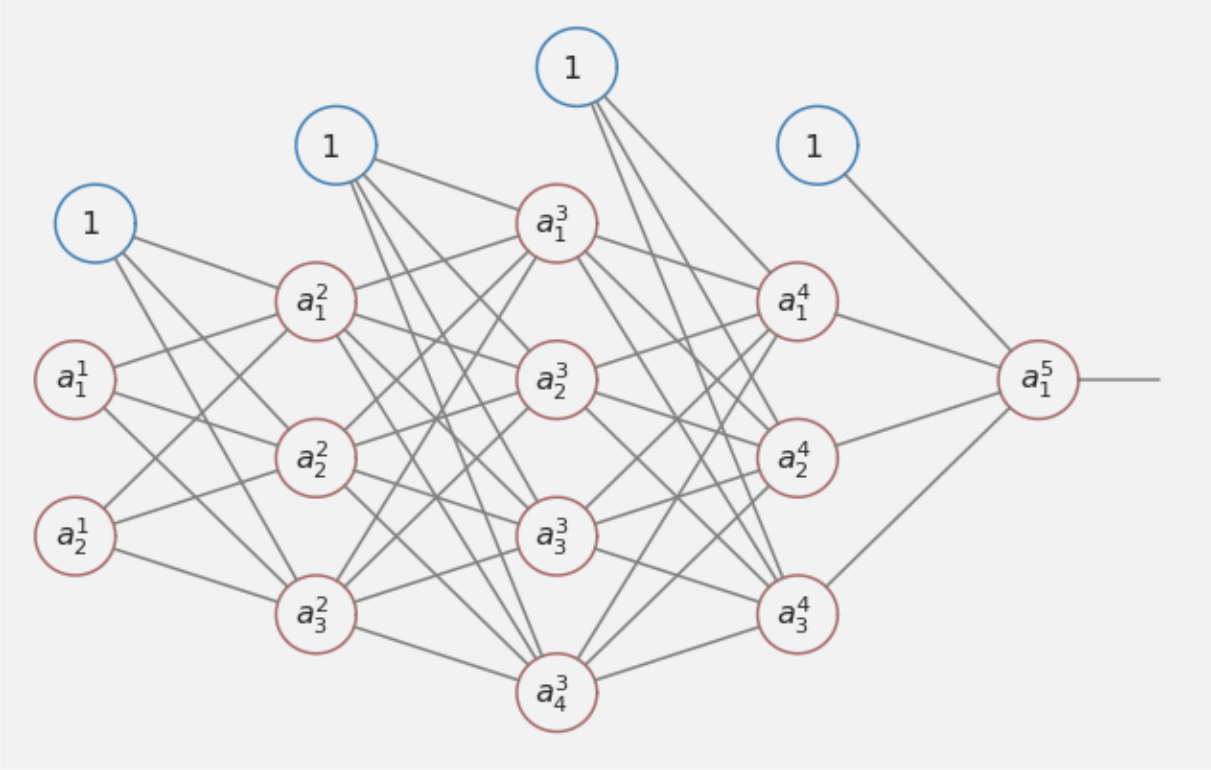

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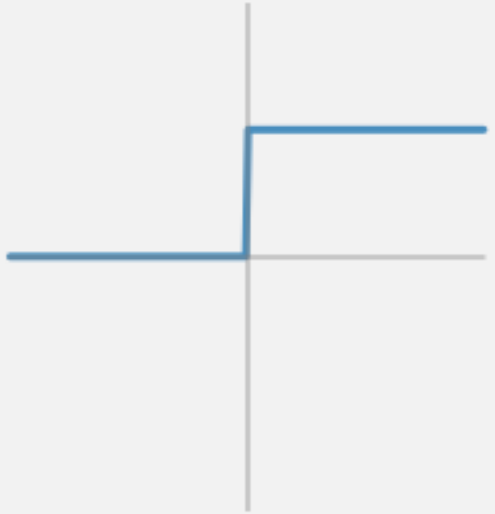
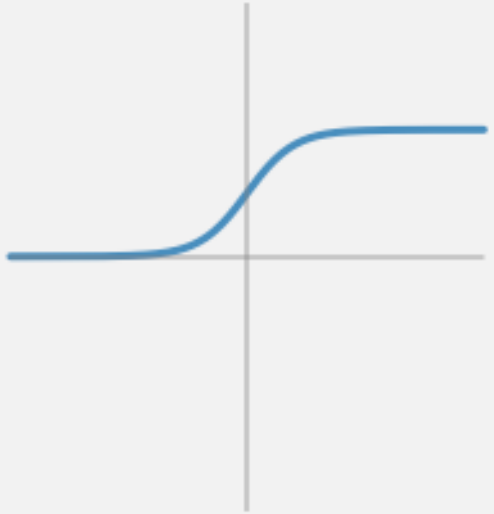
If classification and sigmoid neuron output

then predict based on $\mathbf{a}^L$

Other Popular Activation Functions

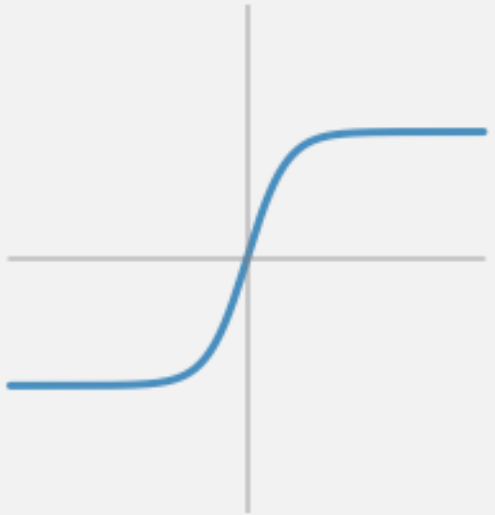
perceptron





sigmoid

tanh



ReLu

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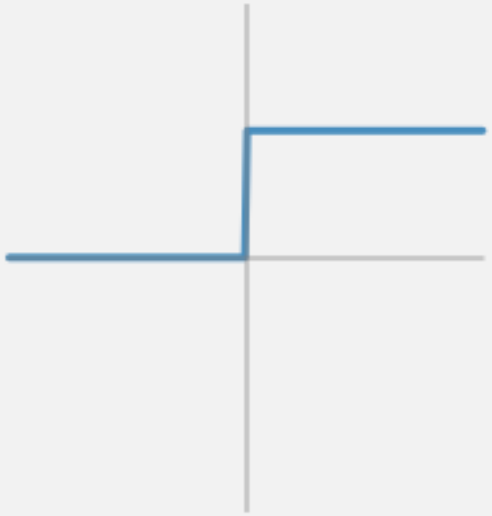

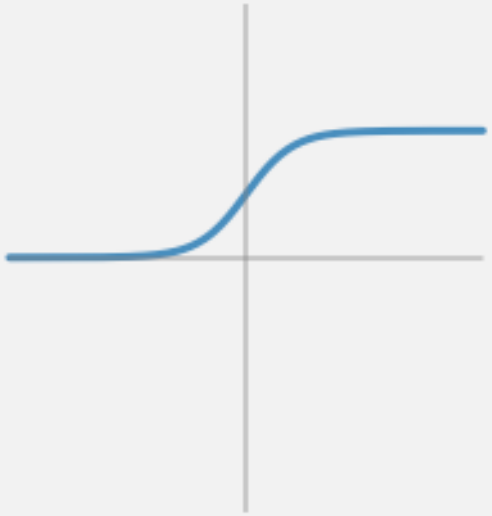

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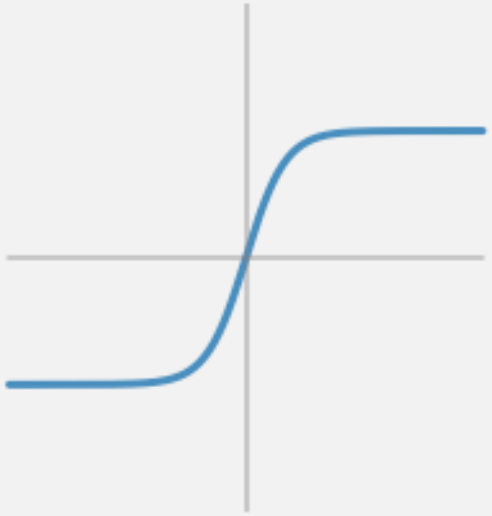
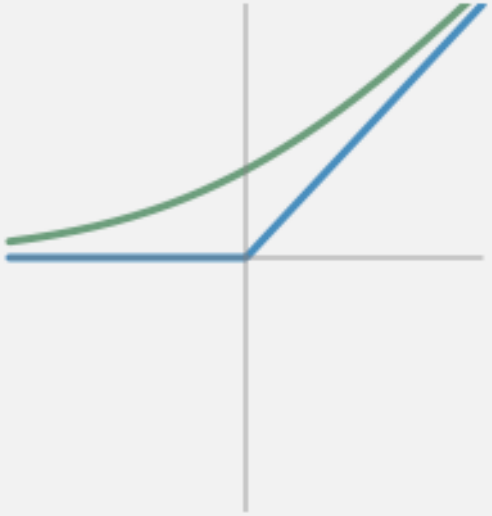
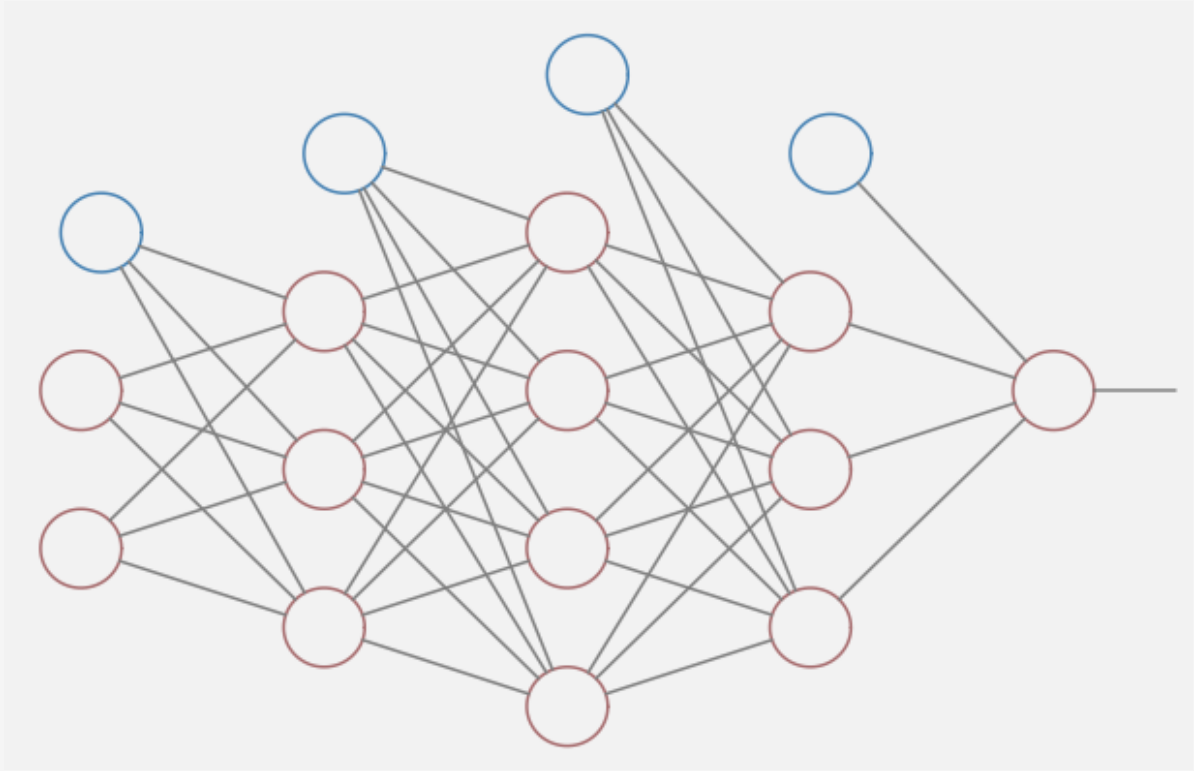




ReLu

Different Probs – Different Architectures

What if we want to do regression instead of classification?



Different Probs - Different Architectures

What if we want to do classification with more than two classes?

Pedestrian

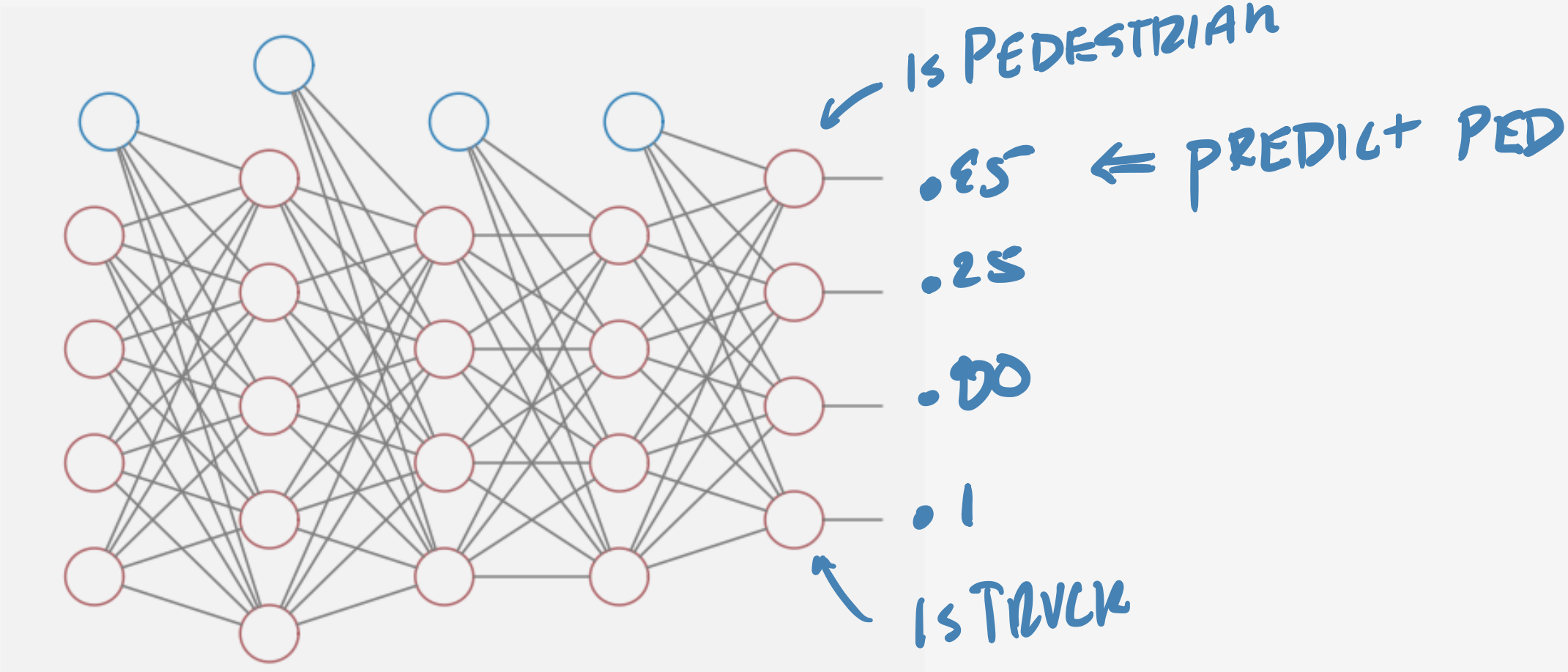
Car

Motorcycle

Truck

Different Probs – Different Architectures

What if we want to do classification with more than two classes?



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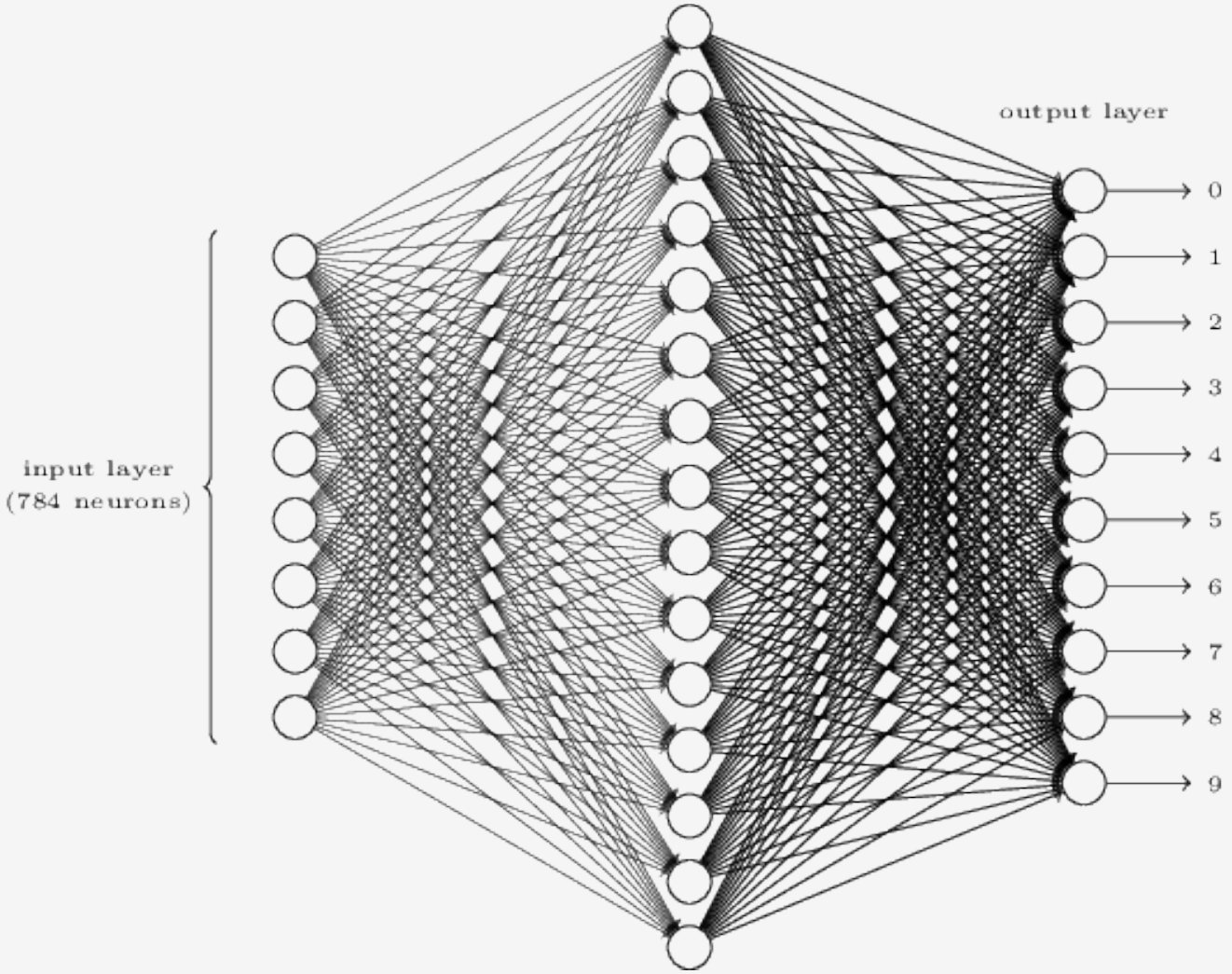
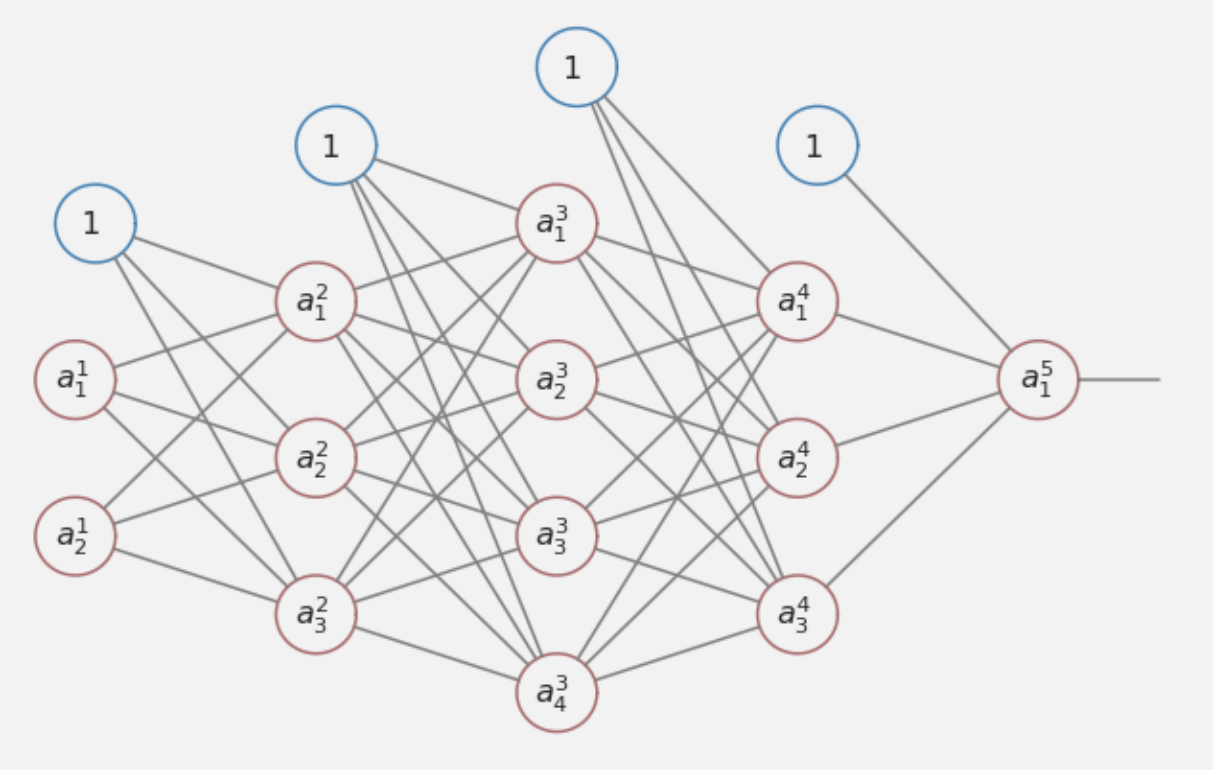


Loss Functions

How do we know if we're doing well?



Single Output - Squared Loss

Regression or Classification

$$L(W, b, x, y) = \sum_{i=1}^{n} (y_i - a_i^5)^2$$

Loss Functions

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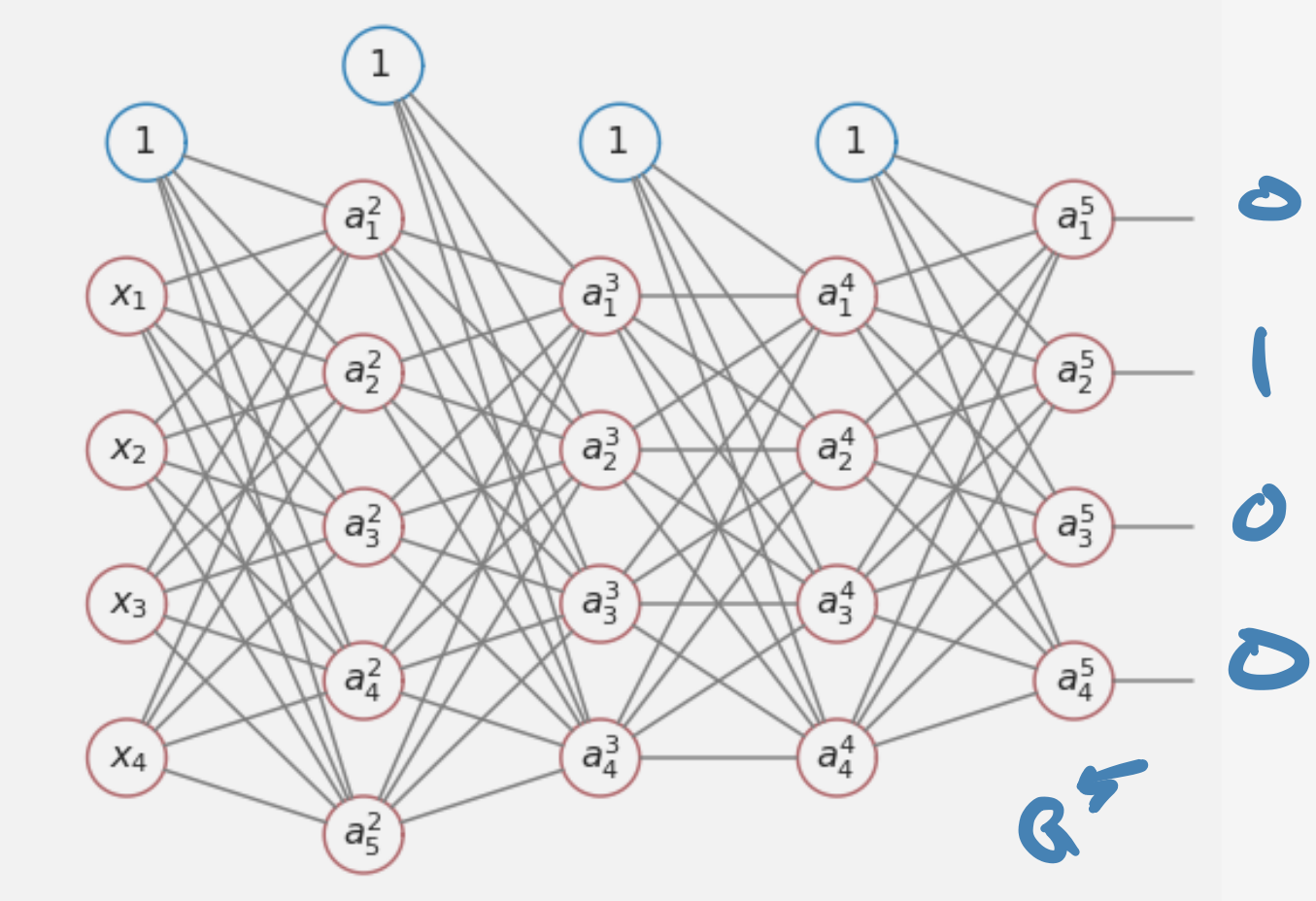


Multiple Outputs - Squared Loss

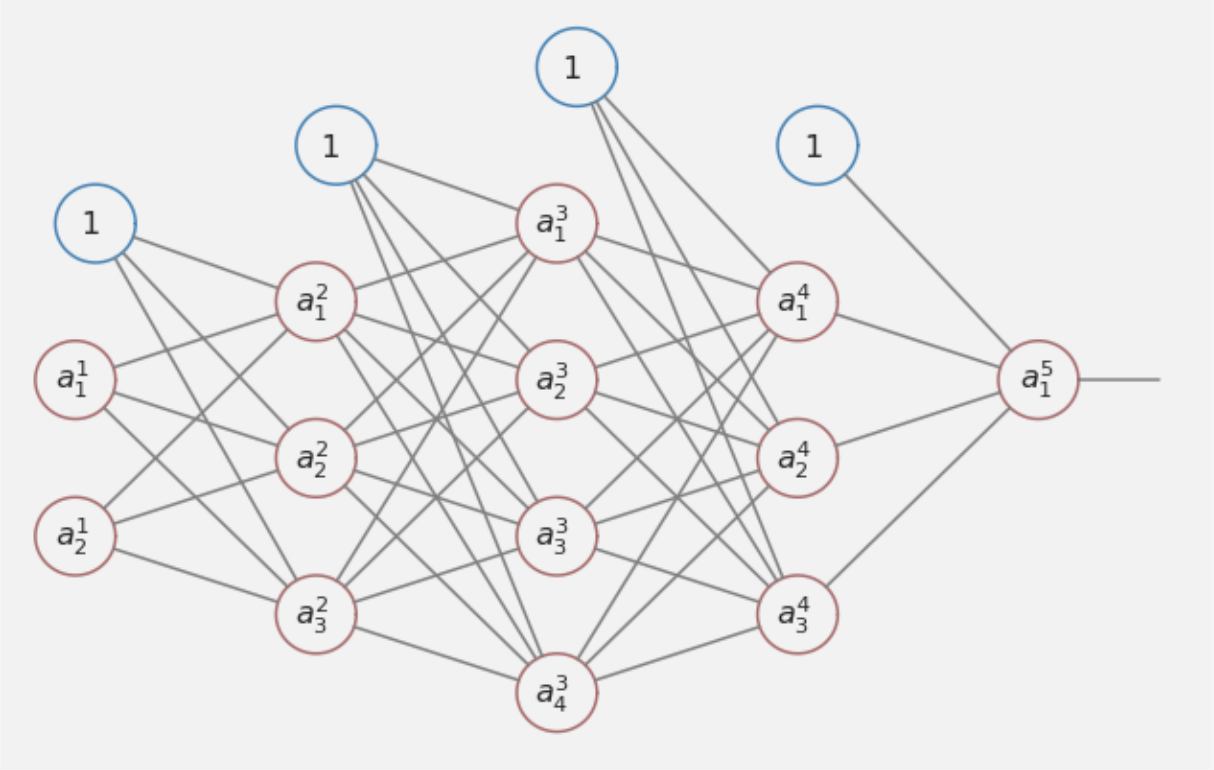
Classification

$$\sum_{i=1}^{n} \| \vec{y}(x) - a^L \|^2$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} .01 \\ .85 \\ .01 \\ .01 \end{bmatrix}$$

Loss Functions

How do we know if we're doing well?



Single Output - Cross-Entropy

~~Regression or~~ Classification

$$-\sum_{i=1}^{\tilde{n}} \{ y_i \cdot \log(a^L) + (1-y_i) \log(1-a^L) \}$$

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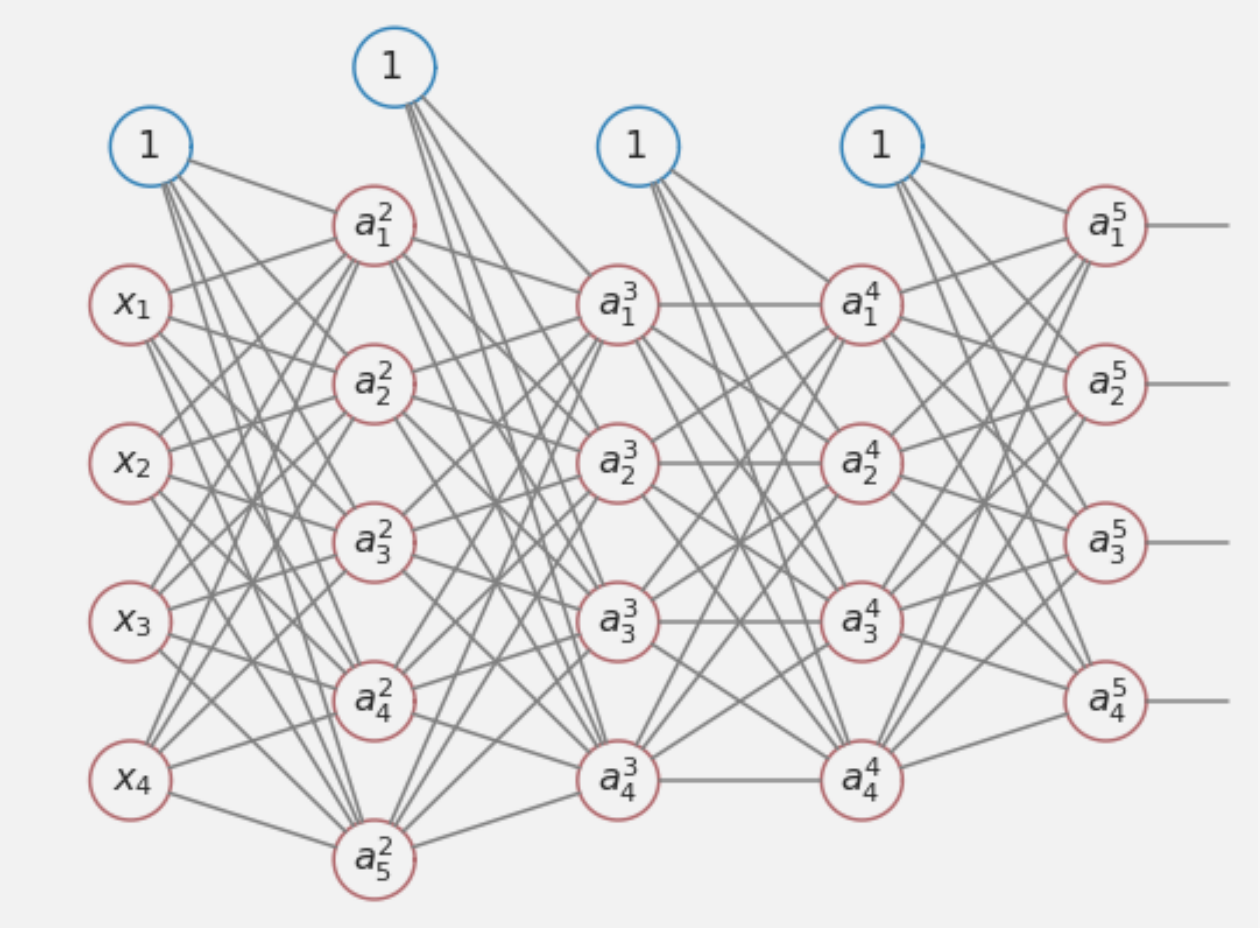

Multiple Outputs – SoftMax

Classification

Training a Feed-Forward NN

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Turns out those partial derivatives are not so easy to compute

This is where **Back Propagation** comes in. It will require:

Calculus **Linear Algebra** **An Iron Stomach**