- ASSUMPTIONS: ϵ IS INDEPENDENT FROM EVERYTHING
- COMPUTING EXPECTED TEST MSE AT POINT x_0
- NOTATION: $f = f(x_0)$, $\hat{f} = \hat{f}(x_0)$
- ASSUMPTION: $y = f + \epsilon$

$$E[(y-\hat{f})^2] = E[(f+\epsilon-\hat{f})^2] = E[((f-\hat{f})+\epsilon)^2]$$

$$= E[(f-\hat{f})^2 + 2\epsilon(f-\hat{f}) + \epsilon^2] = E[(f-\hat{f})^2] + E[2\epsilon(f-\hat{f})] + E[\epsilon^2]$$

$$= E[(f-\hat{f})^2] + \underbrace{2E[\epsilon]E[f-\hat{f}]}_{\epsilon \text{ IS INDEP.}} + E[\epsilon^2] = E[(f-\hat{f})^2] + E[\epsilon^2]$$

$$= \underbrace{E[(f-\hat{f})^2]}_{\text{REDUCIBLE}} + \underbrace{VAR(\epsilon)}_{\text{IRREDUCIBLE}}$$

REDUCIBLE ERROR DECOMP

$$E[(f-\hat{f})^2] = E[(f-E[\hat{f}]+E[\hat{f}]-\hat{f})^2]$$

$$= E[(f-E[\hat{f}])^2 + 2(E[\hat{f}]-\hat{f})(f-E[\hat{f}]) + (E[\hat{f}]-\hat{f})^2]$$

$$= E[(f-E[\hat{f}])^2] + \underbrace{2(f-E[\hat{f}])E[E[\hat{f}]-\hat{f}]}_{\text{B/C } (f-E[\hat{f}]) \text{ IS DETERMINISTIC}} + E[(E[\hat{f}]-\hat{f})^2]$$

$$= E[(f-E[\hat{f}])^2] + \underbrace{2(f-E[\hat{f}])\cdot 0}_{\text{B/C } E[\hat{f}]-E[\hat{f}]} + E[(E[\hat{f}]-\hat{f})^2]$$

$$= \underbrace{(f-E[\hat{f}])^2}_{\text{B/C DETERMINISTIC}} + E[(E[\hat{f}]-\hat{f})^2]$$

$$= [BIAS(\hat{f})]^2 + VAR(\hat{f})$$