ACTIVITY SHEETS
EXERCISE 2

Review on Methods for solving differential Equations of Order one.

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For numbers, 1 to 7, solve the following separable differential equations.

1. $y' = y^2x^3$

$$\frac{dy}{dx} = y^2x^3$$

$$\frac{dy}{y^2} = x^3dx$$

$$\int \frac{dy}{y^2} = \int x^3dx$$

$$\int y^{-2}dy = \int x^3dx$$

$$\frac{y^{-1}}{-1} = \frac{x^4}{4} + C$$

$$-\frac{1}{y} = \frac{x^4+4C}{4}$$

$$-4 = y(x^4+4C)$$

$$y = -\frac{4}{x^4+4C}$$

$$\boxed{y = \frac{4}{x^4+C}}$$

2. $y' = \frac{x+1}{y^4+1}$

$$\frac{dy}{dx} = \frac{x+1}{y^4+1}$$

$$dy(y^4+1) = dx(x+1)$$

$$\int y^4dy + \int dy = \int xdx + \int dx$$

$$\boxed{\frac{y^5}{5} + y = \frac{x^2}{2} + x + C}$$

3. $(x-1)dx + xy^2dy = 0$

$$\frac{(x-1)dx}{x} + y^2dy = 0$$

$$\int dx - \int\frac{dx}{x} + \int y^2dy = 0$$

$$x - \ln(x) + \frac{y^3}{3} + C = 0$$

$$3x - 3\ln(x) + y^3 + 3C = 0$$

$$\boxed{y^3 = 3\ln(x) - 3x - C}$$

4. $\tan^2 y\,dx = \sec^2 x\,dy$

$$\frac{dx}{\sec^2 x} = \frac{dy}{\tan^2 y}$$

$$\cos^2 x\,dx = \cot^2 y\,dy$$

$$\left(\frac{1+\cos 2x}{2}\right)dx = (\csc^2 y - 1)dy$$

$$\int\left(\frac{1+\cos 2x}{2}\right)dx = \int(\csc^2 y - 1)dy$$

$$\frac{1}{2}\int \cos 2x\,dx + \frac{1}{2}\int dx = \int \csc^2 dy - \int dy$$

$$\boxed{\frac{\sin 2x}{2} + \frac{x}{2} = -\cot y - y + C}$$

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For numbers 8 to 10, solve the following separable differential equations and determine the constant of integration when initial conditions are indicated.

8) $xyy' + (1-y^2) = 0$, when $x = 1$, $y = 4$

$$xy\left(\frac{dy}{dx}\right) + (1-y^2) = 0$$

$$xy\,dy = (y^2-1)\,dx$$

$$\frac{y\,dy}{y^2-1} = \frac{dx}{x}$$

$$\int \frac{y\,dy}{y^2-1} = \int \frac{dx}{x}$$

Let $u = y^2 - 1$

$du = 2y\,dy$

$$\int \frac{u}{a}\left(\frac{dy}{2y}\right) = \ln(x) + \ln(c)$$

$$\frac{1}{2}\ln(y^2-1) = \ln(x) + \ln(c)$$

$$e^{\cancel{\ln}(y^2-1)^{1/2}} = e^{\ln(x)(c)}$$

$$\left(\sqrt{y^2-1}\right)^2 = (xc)^2$$

$$y^2 - 1 = x^2c^2$$

$$y = \sqrt{cx^2+1}$$

then when $x = 1$, $y = 4$

thus: $y = \sqrt{cx^2+1}$

$$4 = \sqrt{c(1)^2+1}$$

$$(4)^2 = \left(\sqrt{c+1}\right)^2$$

$$16 = c + 1$$

$$16 - 1 = c$$

$\therefore \boxed{c = 15}$

Equation:

$$\boxed{y = \sqrt{15x^2+1}}$$

9) $x \ln y \ln x\,dy + dx = 0$ when $x = e$, $y = 1$

$$\int \ln y\,dy + \int \frac{dx}{x\ln x} = 0$$

Let $u = \ln y$, $v = y$

$du = \frac{dy}{y}$, $du = dy$

Let $u = \ln x$

$du = \frac{1}{2}dx$

$$y\ln y - \int y\left(\frac{1}{y}\right)dy = \int \frac{x\,du}{xu}$$

$$y\ln y - y = \ln(u) + C$$

$$y(\ln y - 1) = \ln(\ln(x)) + C$$

$$\ln(1) - 1 = \ln(\ln(e)) + C$$

$$-1 = 0 + C$$

$\therefore \boxed{C = -1}$

$$\boxed{y(\ln y - 1) = \ln[\ln(x)] - 1}$$

10) $(y^2 - y + 1)y' - y = y^3$, when $x = \frac{\pi}{4}$, $y = 1$

$$(y^2-y+1)\frac{dy}{dx} - y = y^3$$

$$(y^2-y+1)\,dy = (y^3+y)\,dx$$

$$\left(\frac{y^2-y+1}{y^3-y}\right)dy = dx$$

$$\int\left(\frac{y^2-y+1}{y^3-y}\right)dy = \int dx$$

$$\frac{y^2-y+1}{y(y^2-1)} = \frac{A}{y} + \frac{B}{y^2+1}$$

$$y^2 - y + 1 = A(y^2+1) + By$$

$A = 1$, $B = 1$

$$y^2 - y + 1 = Ay^2 + A + By$$

$$\int \frac{1}{y}dy - \int \frac{1}{y^2+1}dy = x + C\ ;\ \ln y = \arctan y = x + C$$

$$\boxed{\ln y = \arctan(y) = x + \ln(1) - \pi/4}$$

when $x = \frac{\pi}{4}$, $y = 1$

$$\therefore \ln(1) = \arctan(1) = \frac{\pi}{4} + C$$

$$\boxed{C = \arctan 1 - \frac{\pi}{4},\ C = \ln(1) - \frac{\pi}{4}}$$

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(5.) $y' = \cos^2 x \cos y$

$$\frac{dy}{dx} = \cos^2 x \cos y$$

$$\frac{dy}{\cos y} = \cos^2 x \, dx$$

$$\sec y \, dy = \left(\frac{1 + \cos 2x}{2}\right) dx$$

$$\int \sec y \, dy = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x \, dx$$

Let $u = 2x$

$du = 2dx$

$$\ln(\sec y + \tan y) = \frac{x}{2} + \frac{1}{2}\int \cos u \left(\frac{du}{2}\right)$$

$$\boxed{\ln(\sec y + \tan y) = \frac{x}{2} + \frac{\sin 2x}{4} + c}$$

(6.) $dr = b(\cos\theta \, dr + r \sin\theta \, d\theta)$

$$dr = b\cos\theta \, dr + br\sin\theta \, d\theta$$

$$dr - b\cos\theta \, dr = br\sin\theta \, d\theta$$

$$\frac{dr\,(1 - \cancel{b\cos\theta})}{r\,(1 - \cancel{b\cos\theta})} = \frac{b\cancel{r}\sin\theta \, d\theta}{\cancel{r}\,(1 - b\cos\theta)}$$

$$\int \frac{dr}{r} = \int \frac{b\sin\theta \, d\theta}{(1 - b\cos\theta)}$$

Let $u = 1 - b\cos\theta$

$du = b\sin\theta \, d\theta$

$$\ln(r) = \int \frac{b\sin\theta \, d\theta}{u}\left(\frac{du}{b\sin\theta \, d\theta}\right)$$

$$\ln(r) = \int \frac{du}{u}$$

$$e^{\ln(r)} = \ln(1 - b\cos\theta) + e^{\ln C}$$

$$\boxed{r = c(1 - b\cos\theta)}$$

(7.) $x\,dy - 2y\,dx = y^2 dx$

$$x\,dy = y^2 dx + 2y\,dx$$

$$x\,dy = dx(y^2 + 2y)$$

$$\frac{dy}{(y^2 + 2y)} = \frac{dx}{x}$$

$$\int \frac{dy}{y^2 + 2y} = \int \frac{dx}{x}$$

$$\int \frac{dy}{y(y+2)} = \int \frac{dx}{x}$$

$$\frac{1}{y(y+2)} = \frac{A}{y} + \frac{B}{y+2} = \ln(x) + C$$

$$1 = A(y+2) + By$$

Let $y = -2$

$1 = A(-2+2) + B(-2)$

$B = -\frac{1}{2}$

Let $y = 0$

$1 = A(0+2) + B(0)$

$A = \frac{1}{2}$

$$\int \frac{\frac{1}{2}}{y} dy + \int \frac{\left(-\frac{1}{2}\right) dy}{y+2} = \ln(x) + c$$

$$\frac{1}{2}\ln(y) - \frac{1}{2}\ln(y+2) = \ln(x) + \ln C$$

$$\boxed{\ln(y) - \ln(y+2) = 2\ln(x) + C}$$

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