ACTIVITY SHEETS
EXERCISE 1

Review on Definitions of a D.E., Elimination of Arbituary constants and family of curves.

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Year and section: BSME-2C
Date submitted:

For numbers 1 to 9, for the following differential equations, determine (a) order, (b) degree (c) linearity, (d) unknown function, and (e) independent variables

1. $x\frac{d^3y}{dx^3} + 7\frac{dy}{dx} - 7y = 6$

(a) = 3
(b) = 1
(c) = linear
(d) = y
(e) = x

2. $x^2y'' - xy' = 1 + \cos y$

(a) = 2
(b) = 1
(c) = non-linear
(d) = y
(e) = x

3. $\left(\frac{d^2q}{dr^2}\right)^3 + \left(\frac{d^2q}{dr^2}\right)^2 + y\frac{dq}{dy} = 3$

(a) = 2
(b) = 3
(c) = non-linear
(d) = q
(e) = r, y

4. $\left(\frac{dy}{dx}\right)^4 - 3y^2 = e^y$

(a) = 1
(b) = 4
(c) = non-linear
(d) = y
(e) = x

5. $\frac{d^2x}{dt^2} - 2xt = \left(\frac{dx}{dt}\right)$

(a) = 2
(b) = 1
(c) = non-linear
(d) = x
(e) = t

6. $z^2\frac{d^3v}{dz^3} - z\frac{dv}{dt} = 1 - \cos z$

(a) = 3
(b) = 1
(c) = linear
(d) = v
(e) = z, t

7. $\left(\frac{db}{dp}\right)^7 = 3p$

(a) = 1
(b) = 7
(c) = non-linear
(d) = b
(e) = p

8. $xy''' - x^2y'' - x^3y' - y = x$

(a) = 3
(b) = 1
(c) = linear
(d) = y
(e) = x

9. $\frac{d^nx}{dy^n} = y^2 + 1$

(a) = n
(b) = 1
(c) = non-linear
(d) = x
(e) = y

For numbers 10 + 15, Elimination of Arbituary constants: Find the differential equation whose solutions correspond to the following equation.

10. $3x^2 - xy^2 = 0$

$6x - y^2 + x\,2yy' = 0$

$2xyy' + 6x - y^2 = 0$

11. $y = \emptyset \cos(x+\theta)$

$y' = \emptyset\left[-\sin(x+\theta)\right]$

$\emptyset = \dfrac{y'}{-\sin(x+\theta)}$

by substituting:

$y = \left[\dfrac{y'}{-\sin(x+\theta)}\right]\cos(x+\theta)$

$\boxed{y = -\cot(x+\theta)\,y'}$

12. (E1) $y = C_1e^{-x} + C_2e^{-2x}$

(E2) $y' = -C_1e^{-x} - 2C_2e^{-2x}$

(E3) $y'' = C_1e^{-x} + 4C_2e^{-2x}$

Eliminating C_1 in E_1 and E_2 by addition gives:

(E4) $y + y' = -C_2e^{-2x}$

Eliminating C_1 in E_2 and E_3 by addition, we obtained:

(E5) $y' + y'' = 2C_2e^{-2x}$

Eliminating C_2 in E_4 and E_5 by multiplying E_4 by 2, we have:

$2y + 2y' = -2C_2e^{-2x}$

$+ \quad y' + y'' = 2C_2e^{-2x}$

By addition we have the desired equation:

$2y + 2y' + y' + y'' = 0$

$\boxed{2y + 3y' + y'' = 0}$

13. $(x-r)^2 + y^2 = r^2$

$2(x-r)(1) + 2yy' = 0$

$2x - 2r + 2yy' = 0$

$x - r + yy' = 0$; $r = x + yy'$

by substituting:

$\left[x - (x+yy')\right]^2 + y^2 = (x+yy')^2$

$(yy')^2 + y^2 = x^2 + 2xyy' + (yy')^2$

$2xyy' - y^2 - x^2$

$\boxed{x^2 + 2xyy' - y^2 = 0}$

14. $Ax^3 + Bx^2 - y^4 = 0$ (E1)

$3Ax^2 + 2Bx - 4y^3y' = 0$ (E2)

$6Ax + 2B - 4y^3y' - 12y^2(y')^2 = 0$ (E3)

Eliminating A by E1 and E2:

$-3\left[Ax^3 + Bx^2 - y^4 = 0\right]$

$+ \quad x\left[3Ax^2 + 2Bx - 4y^3y' = 0\right]$

$-Bx^2 + 3y^4 - 4y^3y' = 0$ (E4)

Eliminating A by E2 and E3:

$-2\left[3Ax^2 + 2Bx - 4y^3y' = 0\right]$

$+ \quad x\left[6Ax + 2B - 4y^3y' - 12y^2(y')^2 = 0\right]$

$-2Bx + 8y^3y' - 4xy^3y'' - 12xy^2(y')^2 = 0$ (E5)

Eliminating B by E4 and E5:

$-2\left[-Bx^2 + 3y^4 - 4y^3y' = 0\right]$

$+ \quad x\left[-2Bx + 8y^3y' - 4xy^3y'' - 12xy^2(y')^2 = 0\right]$

$-6y^4 + 8xy^3y' + 8xy^3y' - 4xy^3y'' - 12xy^2(y')^2 = 0$

$\therefore \boxed{16xy^3y' - 4xy^3y' - 12xy^2(y')^2 - 6y^4 = 0}$

15. $(x-h)^2 + y^2 = r^2$

$2(x-h)(1) + 2yy' = 0$

$2x - 2h + 2yy' = 0$

$x - h + yy' = 0$; $h = x + yy'$

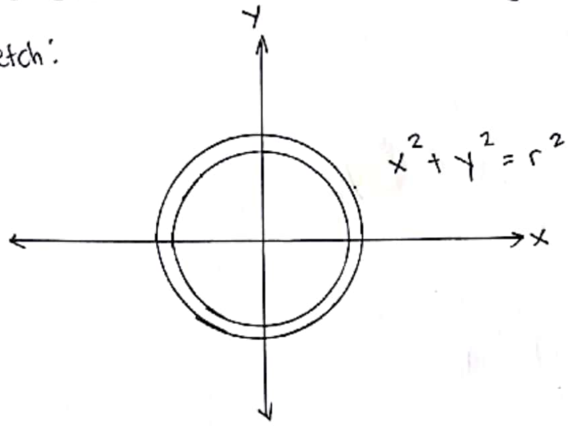
$0 = 1 + y'y' + yy''$

$\boxed{yy'' + (y')^2 + 1 = 0}$

For numbers 16 to 20, obtain the differential equation of the family of curves described and sketch some members of the family.

16. Circles with center at the origin.

Sketch:



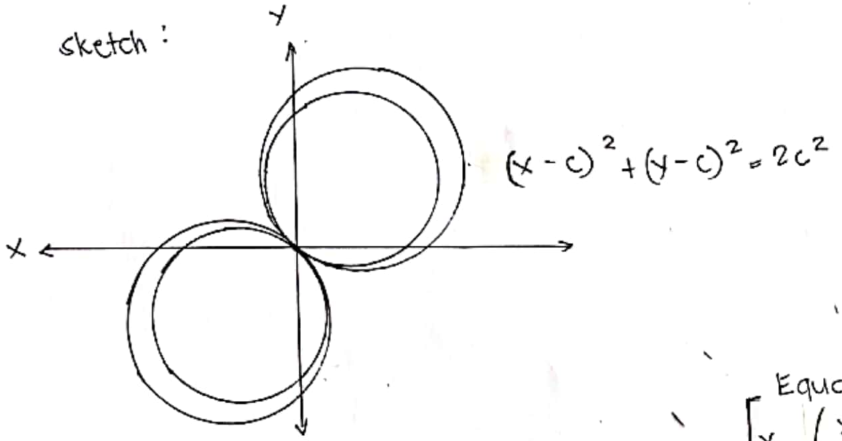
Solution: $x^2 + y^2 = r^2$

$$2x\,dx + 2y\,dy = 0$$

$$\boxed{x\,dx + y\,dy = 0}$$

17. Circles with centers on $y = x$ and passing through the origin.

Sketch:



Solution: $(x-c)^2 + (y-c)^2 = 2c^2$

$$2(x-c)(1) + 2(y-c)(y') = 0.$$

$$2x - 2c + 2yy' - 2cy' = 0$$

$$x - c + yy' - cy' = 0$$

$$x + yy' = c + cy'$$

$$x + yy' = c(1+y'); \; c = \frac{x+yy'}{1+y'}$$

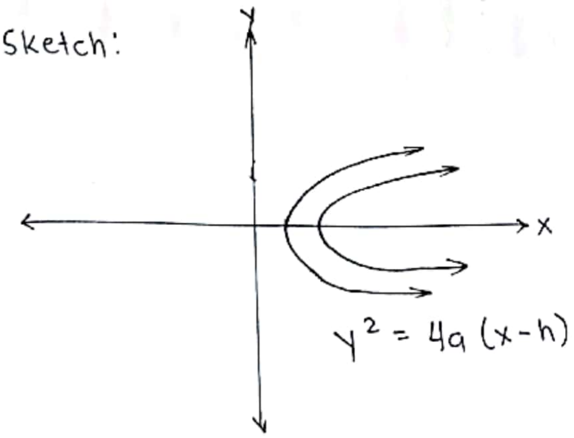
Equate c into E_1:

$$\left[x - \left(\frac{x+yy'}{1+y'}\right)\right]^2 + \left[y - \left(\frac{x+yy'}{1+y'}\right)\right]^2 = 2\left(\frac{x+yy'}{1+y'}\right)^2$$

$$\boxed{(xy')^2 - 2xy(y')^2 - yy' + y^2 - 2xy - x^2 - 4xyy' = 0}$$

18. Parabolas with vertex and focus on the x-axis opening in the right.

Sketch:



Solution: $y^2 = 4a(x-h)$; $y^2 = ax - ah$
(since $4a$ is constant)

$$2yy' - a$$

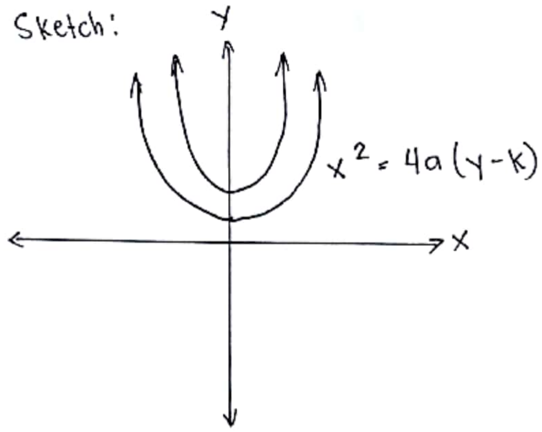
$$2(yy'' + y'y') = 0$$

$$2yy'' + 2(y')^2 = 0$$

$$\boxed{yy'' + (y')^2 = 0}$$

(19) Parabolas with vertex and focus on the y-axis opening upward.

Sketch:



Solution : $x^2 = 4a(y-k)$;
(since 4a is constant)

$x^2 = 4ay - 4ak$

$2x = 4ay'$

$2 = 4ay'' + 0(y')$

$2 = 4ay''$; $a = \frac{1}{2y''}$

by substituting to $2x = ay'$

→ $2x = \left(\frac{1}{2y''}\right) y'$

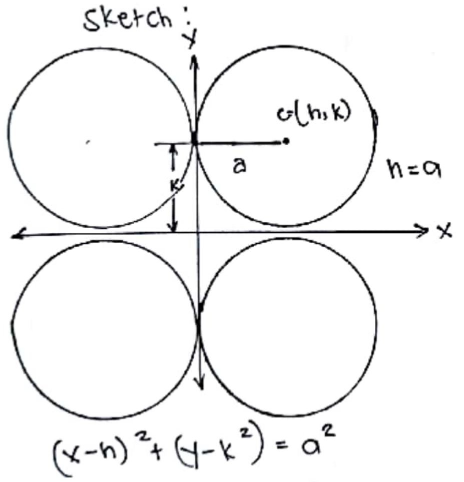
$2x = \frac{1}{2y'}$

$4xy' - 1$

$4xy' - 1 = 0$

(20) Circles tangent to the y-axis.

Sketch:



$(x-h)^2 + (y-k^2) = a^2$

E1 $(x-a)^2 + (y-k)^2 = a^2$; where $a = h$

since a is the parameter; there is only 1 constant in the equation so we are allowed to derive once.

Solution:

Derive: $2(x-a)(1) + 2(y-k)(y') = 0$

$(x-a) + (y-k)(y') = 0$

E2 $(y-k) = \frac{-(x-a)}{y'}$

Equate 2 into 1: $(x-a)^2 + \left[\frac{-(x-a)}{y'}\right]^2 = a^2$

$(x-a)^2 + \frac{(x-a)^2}{(y')^2} = a^2$

$(x-a)^2(y')^2 + (x-a)^2 = a^2(y')^2$

$(x-a)^2(y')^2 - a^2(y')^2 + (x-a)^2 = 0$

$(y')^2[(x-a)^2 - a^2] + (x-a)^2 = 0$

$(y')^2[x^2 - 2xa + a^2 - a^2] + (x-a)^2 = 0$

$(y')^2[x^2 - 2xa] + (x-a)^2 = 0$

$(y')^2(x^2 - 2xa) = -(x-a)^2$

$(y')^2 = \frac{-(x-a)^2}{-(2xa - x^2)}$

$$\boxed{(y')^2 = \frac{(x-a)^2}{2xa - x^2}}$$