ACTIVITY SHEETS

EXERCISE 4

Review on Integrating Factor (linear equation, bernoulli equation, substitution method)

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(1.) $y' + y\cot x = 5e^{\cos x}$

$\frac{dy}{dx} + y\cot x = 5e^{\cos x}$; linear in y

$P(x) = \cot x$, $Q(x) = 5e^{\cos x}$

$I(x) = e^{\int P(x)dx}$

$I(x) = e^{\int \cot x\, dx}$

$I(x) = e^{\ln \sin x}$

$I(x) = \sin x$

Then multiplying I(x) to the equation:

$\sin x \frac{dy}{dx} + y\cot x \sin x = 5\sin x\, e^{\cos x}$

$\int d(y\sin x) = \int 5\sin x\, e^{\cos x} dx$

$y\sin x = -5e^{\cos x} + c$

$y = \left(-5e^{\cos x} + c\right)\left(\frac{1}{\sin x}\right)$

$\boxed{y = \left(-5e^{\cos x} + c\right)\csc x}$

(2.) $xy' = y + x^3 + 3x^2 - 2x$

$\frac{1}{x}\left[xy' = y + x^3 + x^2 - 2x\right]$

$y' = \frac{y}{x} + x^2 + x - 2$

$\frac{dy}{dx} = \frac{y}{x} + x^2 + x - 2$; linear in y

$P(x) = -\frac{1}{x}$, $Q(x) = x^2 + 3x - 2$

$I(x) = e^{\int P(x)dx}$

$I(x) = e^{\int (-\frac{1}{x})dx}$

$I(x) = e^{-\ln x} \rightarrow I(x) = -x$

then multiplying I(x) to the equation:

$-x\frac{dy}{dx} + y = -x^3 - 3x^2 - 2x$

$-\int d(xy) = \int(-x^3 - 3x^2 - 2x)dx$

$-xy = -\frac{x^4}{4} - x^3 - x^2 + c$

$\boxed{y = \frac{x^3}{4} + x^2 + x + c}$

(3.) $y' - 2y\cot 2x = 1 - 2x\cot 2x - 2\csc 2x$

$\frac{dy}{dx} - 2y\cot 2x = 1 - 2x\cot 2x - 2\csc 2x$; linear in y

$P(x) = -2\cot 2x$, $Q(x) = 1 - 2x\cot 2x - 2\csc 2x$

$I(x) = e^{\int P(x)dx}$

$I(x) = e^{\int(-2\cot 2x)dx}$

$I(x) = e^{-\ln 2x}$

$I(x) = -2x$

Then multiplying I(x) in the equation:

$-2x\frac{dy}{dx} + 4xy\cot 2x = -2x + 4x^2\cot 2x + 4x\csc 2x$

$-\int(2xy)d = \int(-2x + 4x^2\cot 2x + 4x\csc 2x)dx$

$-2xy = -x^2 + \frac{2x^3}{3}\ln 2x - 2(\ln|\csc x + \cot x|) + c$

$-\frac{1}{2x}\left[-2xy = -x^2 + \frac{2x^3}{3}\ln 2x - 2(\ln|\csc x + \cot x|) + c\right]$

$\boxed{y = \frac{x}{2} - \frac{x^2}{3}\ln 2x - \ln(\csc x + \cot x) + c}$

(4.) $\frac{dy}{dx} + 2xy = 4x$; linear in y

$P(x) = 2x$, $Q(x) = 4x$

$I(x) = e^{\int P(x)dx}$

$I(x) = e^{\int 2x\, dx}$

$I(x) = e^{x^2}$

Then multiplying I(x) to the equation:

$e^{x^2}\frac{dy}{dx} + e^{x^2}xy = 4xe^{x^2}$

$\int d(e^{x^2}y) = \int 4xe^{x^2}dx$

$ye^{x^2} = 2e^{x^2} + c$

$\left[ye^{x^2} = 2e^{x^2} + c\right]\frac{1}{e^{x^2}}$

$y = 2 + \frac{c}{e^{x^2}}$

$\boxed{y = 2 + ce^{-x^2}}$

5. $y' + y = \sin x$; linear in y

$$\frac{dy}{dx} + y = \sin x$$

$P(x) = 1$, $Q(x) = \sin x$

$$I(x) = e^{\int P(x)dx}$$

$$I(x) = e^{\int 1 dx}$$

$$I(x) = e^{x}$$

Then multiplying $I(x)$ to the equation:

$$e^x \frac{dy}{dx} + e^x y = e^x \sin x$$

$$\int d(e^x y) = \int e^x \sin x \, dx$$

$$e^x y = e^x(-\cos x) - \int -\cos x \cdot (e^x) dx$$

$$e^x y = -e^x \cos x + \int e^x \cos x$$

$$e^x y = -e^x \cos x + e^x \sin x - \int \sin x (e^x) dx$$

$$e^x y = \frac{1}{2}\left[e^x \sin x - e^x \cos x\right] + C$$

$$e^x y = \frac{e^x}{2}\sin x - \frac{e^x}{2}\cos x + C$$

$$\frac{1}{e^x}\left[e^x y = \frac{e^x}{2}\sin x - \frac{e^x}{2}\cos x + C\right]$$

$$\boxed{y = \frac{\sin x}{2} - \frac{\cos x}{2} + C}$$

For numbers 16-20 :

16. $y' + y = y^2$

$$\frac{dy}{dx} + y = y^2$$

$$\left[\frac{dy}{dx} + y = y^2\right]\frac{1}{y^2}$$

$$y^{-2}\frac{dy}{dx} + y^{-1} = 1$$

Let $v = y^{-1}$

$$dv = -y^{-2}dy$$

$$\frac{dv}{-1} = y^{-2}dy$$

then substituting:

$$\frac{dv}{-dx} + v = 1 \text{ ; linear in } v$$

$$-1\left[\frac{dv}{-dx} + v = 1\right]$$

$$\frac{dv}{dx} - v = -1$$

$$v \cdot e^{\int P(x)dx} = \int Q(x) e^{\int P(x)dx} dx$$

$$v \cdot e^{\int 1 dx} = \int -1 e^{\int 1 dx} dx$$

$$v \cdot e^x = -\int e^x dx$$

$$v \cdot e^x = -e^x + C ; \ v = y^{-1}$$

$$y^{-1}e^x = -e^x + C$$

$$\frac{e^x}{y} = -e^x + C$$

$$y\left[\frac{e^x}{y} = -e^x + C\right]$$

$$\boxed{e^x = -ye^x + Cy}$$

17. $\frac{dx}{dy} + y^2 x = x^2 y^2$

$$\left[\frac{dx}{dy} + y^2 x = x^2 y^2\right]\frac{1}{x^2}$$

$$x^{-2}\frac{dx}{dy} + y^2 x^{-1} = y^2 \text{ ; Let } v = x^{-1}$$

$$dv = -x^{-2}dx$$

$$\frac{dv}{-1} = x^{-2}dx$$

then substituting:

$$\left[\frac{dv}{-dy} + y^2 v = y^2\right] - 1$$

$$\frac{dv}{dy} - y^2 v = -y^2 \text{ ; linear in } v$$

$$v \cdot e^{\int P(y)dy} = \int Q(y) e^{\int P(y)dy} dy$$

$$v \cdot e^{\int -y^2 dy} = \int -y^2 e^{\int -y^2 dy} dy$$

$$v \cdot e^{\frac{-y^3}{3}} = \int -y^2 e^{-\frac{y^3}{3}} dy$$

$$v \cdot e^{-y^3/3} = e^{-y^3/3} + C \text{ ; } v = x^{-1}$$

$$x^{-1} e^{-y^3/3} = e^{-y^3/3} + C$$

$$x\left(\frac{e^{-y^3/3}}{x}\right) = \left(e^{-y^3/3} + C\right)x$$

$$\boxed{e^{-y^3/3} = xe^{-y^3/3} + Cx}$$

18. $6y^2dx - x(2x^3+y)dy = 0$

$$\left[6y^2dx - x(2x^3+y)dy = 0\right]\frac{1}{6y^2dy}$$

$$\frac{dx}{dy} + \left[\frac{-2x^4-xy}{6y^2}\right] = 0$$

$$\frac{dx}{dy} + \left[\frac{-2x^4}{6y^2} - \frac{xy}{6y^2}\right] = 0$$

$$\frac{dx}{dy} + \left[\frac{-x^4}{3y^2} - \frac{x}{6y}\right] = 0$$

$$\frac{dx}{dy} - \frac{x}{6y} = \frac{x^4}{3y^2}$$

$$\frac{dx}{dy} - \frac{1}{6y}(x) = \frac{1}{3y^2}(x^4)$$

$$\left[\frac{dx}{dy} - \frac{1}{6y}(x) = (x^4)\frac{1}{3y^2}\right]\frac{1}{x^4}$$

$$x^{-4}\frac{dx}{dy} - \frac{1}{6y}x^{-3} = \frac{1}{3y^2}$$

Let $v = x^{-3}$

$dv = -3x^{-4}dx$

$\frac{dv}{-3} = x^{-4}dx$

then substituting:

$$\frac{dv}{-3dy} - \frac{1}{6y}v = \frac{1}{3y^2}$$

$$-3\left[\frac{dv}{-3dy} - \frac{1}{6y}v = \frac{1}{3y^2}\right]$$

$$\frac{dv}{dy} + \frac{1}{2y}v = -\frac{1}{y^2}$$; linear in v

$$v \cdot e^{\int P(y)dy} = \int Q(y)e^{\int P(y)dy}dy$$

$$v \cdot e^{\int \frac{1}{2y}dy} = \int -\frac{1}{y^2}e^{\int \frac{1}{2y}dy}dy$$

$$ve^{1/2\ln y} = -\int \frac{1}{y^2}e^{1/2\ln y}dy$$

$$v\sqrt{y} = -\int \frac{1}{y^2}\sqrt{y}\,dy$$

$$v\sqrt{y} = -\int y^{-3/2}dy$$

$$v\sqrt{y} = \frac{2}{\sqrt{y}} + C$$

$$\left[v\sqrt{y} = \frac{2}{\sqrt{y}} + C\right]\sqrt{y}$$

$vy = 2 + C\sqrt{y}$; $v = x^{-3}$

$$x^{-3}y = 2 + C\sqrt{y}$$

$$x^3\left[\frac{y}{x^3} = 2 + C\sqrt{y}\right]$$

$$\boxed{y = 2x^3 + Cx^3\sqrt{y}}$$

19. $2x^3y' = y(y^2+3x^2)$

$$2x^3\frac{dy}{dx} = y(y^2+3x^2)$$

$$\left[2x^3\frac{dy}{dx} = y^3 + 3x^2y\right]\frac{1}{2x^3}$$

$$\frac{dy}{dx} = \frac{y^3}{2x^3} + \frac{3y}{2x}$$

$$\frac{dy}{dx} - \frac{3y}{2x} = \frac{y^3}{2x^3}$$

$$\frac{dy}{dx} - \frac{3}{2x}(y) = \frac{1}{2x^3}(y^3)$$

$$\left[\frac{dy}{dx} - \frac{3}{2x}(y) = \frac{1}{2x^3}(y^3)\right]\frac{1}{y^3}$$

$$y^{-3}\frac{dy}{dx} - \frac{3}{2x}(y^{-2}) = \frac{1}{2x^3}$$; Let $v = y^{-2}$

$dv = -2y^{-3}dy$

$\frac{dv}{-2} = y^{-3}dy$

then substituting:

$$-2\left[\frac{dv}{-2dx} - \frac{3}{2x}v = \frac{1}{2x^3}\right] \rightarrow \frac{dv}{dx} + \frac{3}{x}v = -\frac{1}{x^3}$$; linear in v

$$v \cdot e^{\int P(x)dx} = \int Q(x)e^{\int P(x)dx}dx$$

$$ve^{\int \frac{3}{x}dx} = \int -\frac{1}{x^3}e^{\int 3/x\,dx}dx$$

$$ve^{3\ln x} = \int \frac{1}{x^3}e^{3\ln x}dx$$

$$v\,3x = \int -\frac{3x}{x^3}dx$$

$$v\,3x = \int -3x^{-2}dx$$

$v\,3x = \frac{3}{x} + C$; $v = y^{-2}$

$$y^{-2}(3x) = \frac{3}{x} + C$$

$$y^2\left[\frac{3x}{y^2} = \frac{3}{x} + C\right]$$

$$\boxed{3x = \frac{3y^2}{x} + Cy^2}$$

(20) $y(6y^2 - x^{-1})dx + 2x\,dy = 0$

$$\left[(6y^3 - yx^{-1})dx + 2x\,dy = 0\right]\frac{1}{2x\,dx}$$

$$\frac{6y^3 - yx^{-1}}{2x} + \frac{dy}{dx} = 0$$

$$\frac{6y^3}{2x} - \frac{yx^{-1}}{2x} + \frac{dy}{dx} = 0$$

$$\frac{3y^3}{x} - \frac{y}{2x^2} + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} - \frac{y}{2x^2} = -\frac{3y^3}{x}$$

$$\left[\frac{dy}{dx} - \frac{y}{2x^2} = -\frac{3y^3}{x}\right]\frac{1}{y^3}$$

$$y^{-3}\frac{dy}{x} - \frac{1}{2x^2}(y^{-2}) = -\frac{3}{x} \; ; \text{ Let } v = y^{-2}$$

$$dv = -2y^{-3}dy$$

$$\frac{dv}{-2} = y^{-3}dy$$

Then substituting:

$$\frac{dv}{-2dx} - \frac{1}{2x^2}v = -\frac{3}{x}$$

$$\left[\frac{dv}{-2dx} - \frac{1}{2x^2}v = \frac{-3}{x}\right]-2$$

$$\frac{dv}{dx} + \frac{1}{x^2}v = \frac{6}{x} \; ; \text{ linear in } v$$

$$ve^{\int P(x)dx} = \int Q(x)\,e^{\int P(x)dx}dx$$

$$ve^{\int \frac{1}{x^2}dx} = \int \frac{6}{x}e^{\int \frac{1}{x^2}dx}dx$$

$$ve^{-\frac{1}{x}} = \int \frac{6}{x}e^{-1/x}dx$$

$$ve^{-\frac{1}{x}} = -6E_1\left(-\frac{1}{x}\right) \; ; \; v = y^{-2}$$

$$y^{-2}e^{-\frac{1}{x}} = -6E_1\left(-\frac{1}{x}\right) + C$$

$$\boxed{\frac{e^{-\frac{1}{x}}}{y^2} = -6E_1\left(-\frac{1}{x}\right) + C}$$