ACTIVITY SHEETS
EXERCISE 3

Review on Integrating Factor (Exact Equations, Integrating Factor by Inspection, Integrating Factor by Formula

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Subject: MATH213ME

Date Submitted:

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for no. 1 - 5:

1.) $2xydx + (y^2+x^2)dy = 0$

Test for exactness:

$M = 2xy \qquad N = y^2 + x^2$

$\frac{\partial M}{\partial y} = 2xy \qquad \frac{\partial N}{\partial x} = y^2 + x^2$

$= 2x \qquad = 2x$

∴ IT IS AN EXACT EQUATION

Finding Solution:

$\frac{\partial F}{\partial x} = M(x,y) \quad \frac{\partial F}{\partial y} = N(x,y)$

$\frac{\partial F}{\partial x} = 2xy$

$\partial F = (2xy)\partial x$

$\int \partial F = \int 2xy \, \partial x$

$F = yx^2 + T(y)$

$\frac{\partial F}{\partial y} = x^2 + T'(y) \; ; \; \frac{\partial F}{\partial y} = N$

$x^2 + T'(y) = x^2 + y^2$

$T'(y) = y^2$

$\int T'(y) = \int y^2 dy$

$T(y) = \frac{y^3}{3}$

then substitute $\frac{y^3}{3}$ in $T(y)$ to obtain soln.

$F = yx^2 + T(y) \; ; \; F = C$

$= yx^2 + \frac{y^3}{3}$

$\therefore \boxed{yx^2 + \frac{y^3}{3} = C}$

2.) $(xy + x^2)dx + \frac{1}{2}x^2 dy = 0$

Test for exactness:

$M = xy + x^2 \qquad N = \frac{1}{2}x^2$

$\frac{\partial M}{\partial y} = xy + x^2 \qquad \frac{\partial N}{\partial x} = \frac{1}{2}x^2$

$= x \qquad = x$

∴ It IS AN EXACT EQUATION

Finding Solution:

$\frac{\partial F}{\partial x} = M(x,y) \quad \frac{\partial F}{\partial y} = N(x,y)$

$\frac{\partial F}{\partial x} = xy + x^2$

$\partial F = (xy + x^2)\partial x$

$\int \partial F = \int (xy + x^2)\partial x$

$F = \frac{yx^2}{2} + \frac{x^3}{3} + T(y)$

$\frac{\partial F}{\partial y} = \frac{1}{2}x^2 + T'(y) \; ; \; \frac{\partial F}{\partial y} = N$

$\frac{1}{2}x^2 + T'(y) = \frac{1}{2}x^2$

$T'(y) = 0$

$\int T'(y) = 0$

$T(y) = 0$

then substitute 0 in $T(y)$ to obtain soln:

$F = \frac{yx^2}{2} + \frac{x^3}{3} + T(y) \; ; \; F = C$

$F = \frac{yx^2}{2} + \frac{x^3}{3} + 0$

$\therefore \boxed{\frac{yx^2}{2} + \frac{x^3}{3} = C}$

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(3) $(2xy-3x^2)dx + (x^2+2y)dy = 0$

Test for exactness:

$M = 2xy-3x^2 \qquad N = x^2+2y$

$$\frac{\partial M}{\partial y} = 2xy-3x^2 \qquad \frac{\partial N}{\partial x} = x^2+2y$$

$$= 2x \qquad\qquad = 2x$$

∴ IT IS AN EXACT EQUATION

Finding Solution:

$$\frac{\partial F}{\partial x} = M(x,y) \qquad \frac{\partial F}{\partial y} = N(x,y)$$

$$\frac{\partial F}{\partial x} = 2xy-3x^2$$

$$\partial F = (2xy-3x^2)\partial x$$

$$\int \partial F = \int (2xy-3x^2)\partial x$$

$$F = yx^2 - x^3 + T(y)$$

thus:

$$\frac{\partial F}{\partial y} = x^2 + T'(y)\ ; \quad \frac{\partial F}{\partial y} = N$$

$$x^2 + T'(y) = x^2 + 2y$$

$$T'(y) = 2y$$

$$\int T'(y) = \int 2y\,dy$$

$$T(y) = y^2$$

then substitute y^2 in $T(y)$ to obtain soln:

$$F = yx^2 - x^3 + T(y) \quad ; \quad F = C$$

$$F = yx^2 - x^3 + y^2$$

$$\therefore \boxed{yx^2 - x^3 + y^2 = c}$$

(4) $(2xy - \tan y)dx + (x^2 - x\sec^2 y)dy = 0$

Test for Exactness:

$M = 2xy - \tan y \qquad N = x^2 - x\sec^2 y$

$$\frac{\partial M}{\partial y} = 2xy - \tan y \qquad \frac{\partial N}{\partial x} = x^2 - x\sec^2 y$$

$$= 2x - \sec^2 y \qquad\qquad = 2x - \sec^2 y$$

∴ IT IS AN EXACT EQUATION

Finding Solution:

$$\frac{\partial F}{\partial x} = M(x,y) \qquad \frac{\partial F}{\partial y} = N(x,y)$$

$$\frac{\partial F}{\partial x} = 2xy - \tan y$$

$$\partial F = (2xy - \tan y)\partial x$$

$$\int \partial F = \int (2xy - \tan y)\partial x$$

$$F = yx^2 - x\tan y + T(y)$$

thus:

$$\frac{\partial F}{\partial y} = x^2 - x\sec^2 y + T'(y)\ ; \quad \frac{\partial F}{\partial y} = N$$

$$x^2 - x\sec^2 y + T'(y) = x^2 - x\sec^2 y$$

$$T'(y) = 0$$

$$\int T'(y) = 0$$

$$T(y) = 0$$

Then substitute 0 in $T(y)$ to obtain soln:

$$F = yx^2 - x\tan y + T(y)\ ; \quad F = C$$

$$F = yx^2 - x\tan y + 0$$

$$\therefore \boxed{yx^2 - x\tan y = c}$$

(5) $(\cos x\cos y - \cot x)dx - \sin x\sin y\,dy = 0$

Test for exactness:

$M = \cos x\cos y - \cot x \qquad N = -\sin x\sin y$

$$\frac{\partial M}{\partial y} = \cos x\cos y - \cot x \qquad \frac{\partial N}{\partial x} = -\sin x\sin y$$

$$= -\cos x\sin y \qquad\qquad = -\cos x\sin y$$

∴ IT IS AN EXACT EQUATION

Finding solution:

$$\frac{\partial F}{\partial x} = \cos x\cos y - \cot x$$

$$\partial F = (\cos x\cos y - \cot x)\partial x$$

$$\int \partial F = \int (\cos x\cos y - \cot x)\partial x$$

$$F = \cos y\sin x - \ln(\sin x) + Ty$$

Thus:

$$\frac{\partial F}{\partial y} = -\sin x\sin y + T'(y)\ ; \quad \frac{\partial F}{\partial y} = N$$

$$-\sin x\sin y + T'(y) = -\sin x\sin y$$

$$T'(y) = 0$$

$$\int T'(y) = 0$$

$$T(y) = 0$$

Then substitute 0 in $T(y)$ to obtain soln:

$$F = \cos y\sin x - \ln(\sin x) + Ty \quad ; \quad F = C$$

$$F = \cos y\sin x - \ln(\sin x) + 0$$

$$\therefore \boxed{\cos y\sin x - \ln(\sin x) = C}$$

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For no. 16 - 20 :

(16) $x\,dy - y\,dx = x^3y^2dx$

$\frac{1}{y^2}\left[x\,dy - y\,dx = x^3y^2dx\right]$

$\frac{x\,dy - y\,dx}{y^2} = x^3dx$

$\int d\left(-\frac{x}{y}\right) = \int x^3dx$

$-\frac{x}{y} = \frac{x^4}{4}$

$-\frac{x}{y} - \frac{x^4}{4} = C$

$\boxed{-4x - x^4y = 4yC}$

(17) $y^3 \sin 2x\,dx - 3y^2\cos^2 x\,dy = 0$

say: $y = y^3$ $\quad x = -\cos 2x + 1$

$dy = 3y^2$ $\quad dx = 2\sin(2x)dx$

then:

$2\left[y^3 \sin 2x\,dx - 3y^2\left(\frac{1+\cos 2x}{2}\right)dy = 0\right]$

$y^3 2\sin 2x\,dx - 3y^2(1+\cos 2x)\,dy = 0$

$y\,dx + x\,dy = d(xy)$

$y\,dx + x\,dy = y^3 2\sin 2x\,dx + (3y^2)\left[-(1+\cos 2x)\right]dy$

$= y^3 2\sin 2x\,dx - 3y^2(1+\cos 2x)\,dy$

$d(xy) = d(\cos 2x + 1)(y^3)$

$\int d(\cos 2x + 1)(y^3) = \int 0$

$\boxed{y^3(\cos 2x + 1) = C}$

(18) $y(x^2+y^2-1)dx + x(x^2+y^2+1)dy = 0$

$x^2y\,dx + y^3dx - y\,dx + x^3dy + xy^2dy + x\,dy = 0$

$(yx^2dx + x^3dy) + (y^3dx + xy^2dy) - (y\,dx + x\,dy) = 0$

$x^2(y\,dx + x\,dy) + y^2(y\,dx + xy\,dy) - (y\,dx + x\,dy) = 0$

$\left[(x^2+y^2)(y\,dx + x\,dy) - (y\,dx + x\,dy) = 0\right]\frac{1}{x^2+y^2}$

$(y\,dx + x\,dy) - \frac{y\,dx + x\,dy}{x^2+y^2} = 0$

$\int d(xy) - \int d\left(\arctan\frac{y}{x}\right) = 0$

$\boxed{xy - \arctan\frac{y}{x} = C}$

(19) $x\,dy - y\,dx = (xy)y^2dy$

$\left[x\,dy - y\,dx = xy^3dy\right]\frac{1}{xy}$

$\frac{x\,dy - y\,dx}{xy} = y^2dy$

$\int \frac{x\,dy - y\,dx}{xy} = \int y^2dy$

$\int d(\ln xy) = \frac{y^3}{3} + C$

$\ln xy = \frac{y^3}{3} + C$

$\boxed{\ln(xy) - \frac{y^3}{3} = C}$

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20. $x\,dy - y\,dx = (x^2 + xy - 2y^2)dx$

$$x\,dy - y\,dx = x^2dx + xy\,dx - 2y^2dx$$

$$\left[x\,dy - y\,dx = (x^2 + xy - 2y^2)dx\right]\frac{1}{x^2}$$

$$\frac{x\,dy - y\,dx}{x^2} = \frac{x^2 + xy + 2y^2}{x^2}dx$$

$$\frac{x\,dy - y\,dx}{x^2} = \left[1 + \frac{y}{x} - 2\left(\frac{y}{x}\right)^2\right]dx$$

$$\int d\left(\frac{y}{x}\right) = \int dx + \int \frac{y}{x}dx - \int \frac{2y^2}{x^2}dx$$

$$\frac{y}{x} = x + y\ln(x) - 2y^2\int\frac{1}{x^2}dx$$

$$\frac{y}{x} = x + y\ln(x) - 2y^2\left(-\frac{1}{x}\right)$$

$$\left[\frac{y}{x} = x + y\ln(x) + \frac{2y^2}{x} + c\right]x$$

$$y = x^2 + xy\ln(x) + 2y^2 + cx$$

$$\boxed{cx = y - x^2 - xy\ln(x) - 2y^2}$$

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