# Sensor and actuator placement for contaminant containment in water distribution networks

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#### Introduction

- ► Public water distribution networks are vulnerable to contamination, particularly from bio-terrorism [1, 2]
- Solution: use sensors to detect contaminants, and valves (actuators) to stop further contamination.
- ► Review of containment strategies, using optimization [3].
- ► Formulated as multi-objective linear program on a graph theoretic abstraction of network to simultaneously get optimum placement in terms of number of sensors, actuators used.

### Legend and definitions

- Node is a point in the abstract graph of the distribution network. Total number:
- Vulnerable nodes  $\bullet$  are the only attack-able nodes.  $i^{th}$  node is vulnerable:  $\mathbf{y_i} = \mathbf{0}$
- ► Demand nodes  $\bigcirc$  are points of exit from the network.  $i^{th}$  node is demand:  $\mathbf{y_i} = \mathbf{1}$
- Sensors, assumed capable of instantly and accurately detecting contamination of that node, placed at  $i^{th}$  node:  $\mathbf{x_i} = \mathbf{1}$
- Actuators  $\mathbf{z}$  cut off flow from one end to the other. Placed on  $i^{th}$  edge  $\mathbf{z_i} = \mathbf{1}$
- ► Goal: No demands node must stay contaminated, with minimum total number of sensors + actuators (cardinality).  $min \sum \mathbf{x_i} + \sum \mathbf{y_i}$

## Sensor Placement

▶ Given the specification of a network, vulnerable nodes and demand nodes, we can come up with a sensor placement strategy to detect any contaminant

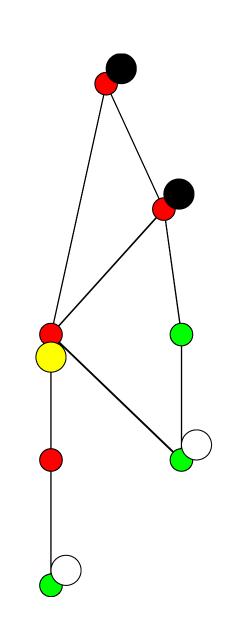


Fig. 1: Example network; basic sensor placement

Constraints:  $\mathbf{A}\mathbf{x} \leq [\mathbf{0}]$  where  $A_i = 1$  if  $i^{th}$  node is affected by the vulnerable node.

- ➤ We can also add constraints to make sure contaminant is detected before any demand is compromised.
- Once we detect contaminants, we can shut off source (partition the network) to prevent further contamination.

#### **Actuator Placement**

► This actuator placement is independent of sensor placement - global optima at optima of sub-problems.

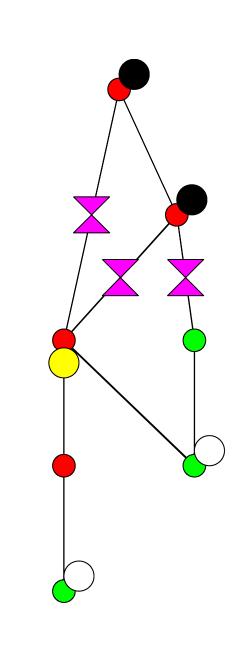


Fig. 2: Independent sensor and actuator placement on example

Adding the requirement of detection before contamination of any vulnerable node

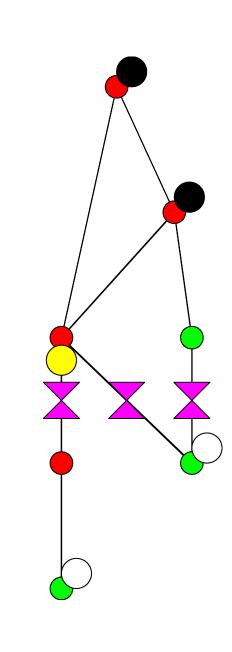


Fig. 3: Trying to solve for containment after sensor placement

► We can extend this and later descriptions to identifiability – knowing which set of sensors were attacked.

## Simultaneous Placement

- The globally optimal solution can only be obtained by iterating through the process, a non-linear optimization problem.
- A simultaneous linear program formulation for both sensor and actuator placement makes this problem tractable.
- ➤ Within the linear program, we store the minimum distance to sensing for each vulnerable node, and require that each actuator be beyond that distance:

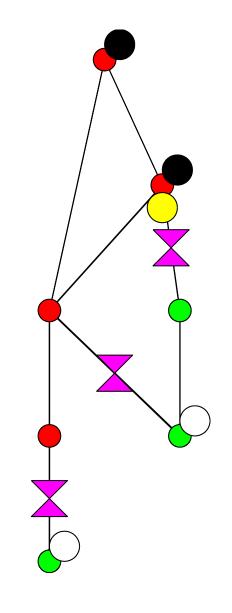


Fig. 4: Result of simultaneous optimization on example network

#### Results

► The formulation was applied to solving the case study used in [4]. Cardinality of 26 was obtained, for allowing no compromised demands:

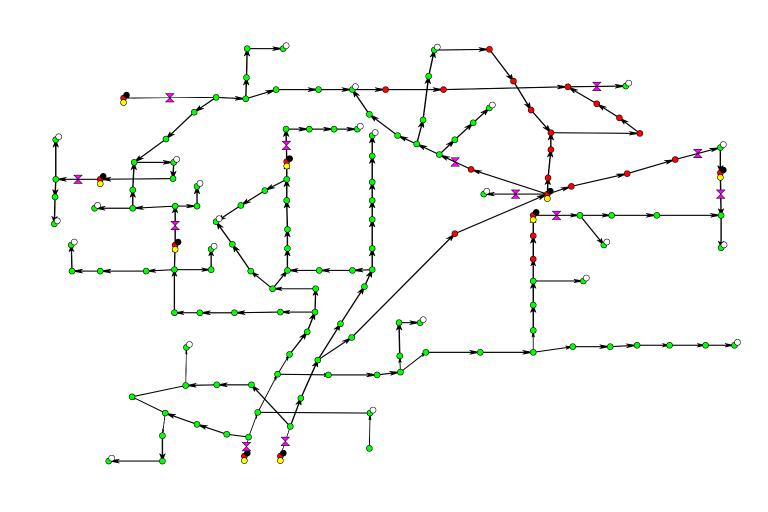


Fig. 5: Results on Bangalore network, with vulnerable nodes [1 2 3 19 32 37 39 53 66]

► The results for allowing initial contamination of demand nodes from the work was reproduced (cardinality of 21):

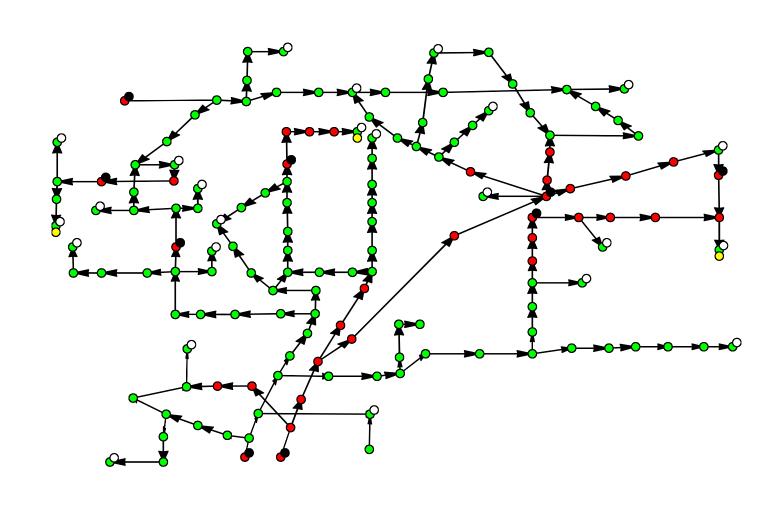


Fig. 6: Reproduction of results from [4]

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#### References

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- [4] Venkata Reddy Palleti, Varghese, Shankar Narasimhan and Raghunathan Rengasamy: Actuator network design to mitigate contamination effects in water distribution networks