Stat 462 HW4 Jiaqi Li

1. $\hat{\varepsilon} = Y - \hat{Y}$

$= Y - HY$

$= (I - H)Y$

$Var(\hat{\varepsilon}) = Var[(I-H)Y]$

$= (I-H) Var(\hat{\varepsilon}) (I-H)^T$

$= (I-H) \sigma^2 I (I-H)^T$

Since H is symmetric and idempotent, $H = H^T$ and $HH = H$ is true

Since I is symmetric, $(I-H)$ is also symmetric and idempotent, which means $(I-H) = (I-H)^T$ and $(I-H)(I-H) = (I-H)$

then, we have:

$Var(\hat{\varepsilon}) = (I-H)\sigma^2 I (I-H)$

$= \sigma^2 (I-H)(I-H)$

$= \sigma^2 (I-H)$

2. $X^T X \hat{\beta} = X^T y$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & -1 \\ 2 & 4 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & 0 & 0 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & -1 \\ 2 & 4 & 0 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 2 & 4 \\ 0 & 6 & 12 \\ 0 & 12 & 24 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -4 \end{bmatrix} \Rightarrow \begin{cases} 4\beta_0 + 2\beta_1 + 4\beta_2 = -2 \\ 6\beta_1 + 12\beta_2 = -2 \\ 12\beta_1 + 24\beta_2 = -4 \end{cases}$$

$$\Rightarrow \begin{cases} \beta_0 = -\frac{1}{3} \\ \beta_1 = -\frac{1}{3} - 2\beta_2 \\ \beta_2 \text{ unbounded} \end{cases}$$

$\Rightarrow$ Since β_2 is unbounded here, we will have infinitely many solutions.