Li_Jiaqi_Project7

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```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import scipy.stats as si
```

1 Problem 1

Following is the Value given by Black Scholes

```
In [4]: #Problem 1-----------------------------------------------------
        S0 = 10
        sigma = 0.2
        r = 0.04
        dt = 0.002
        dX = sigma*np.array([np.sqrt(dt),np.sqrt(3*dt),np.sqrt(4*dt)])
        K = 10
        T = 0.5
        steps = int(T/dt)
        S = np.arange(4,17,1)

        ###############B-S######################
        def f_BS(S0,type):
            K=10.0
            T=0.5
            sigma=0.2
            r=0.04
            d_1=(np.log(S0/K)+(r+0.5*sigma**2)*T)/(sigma * np.sqrt(T))
            d_2=d_1-sigma*np.sqrt(T)
            if type=="call":
                option=(S0*si.norm.cdf(d_1,0.0,1.0)-K*np.exp(-r*T)*si.norm.cdf(d_2,0.0,1.0))
            if type=="put":
                option=(K*np.exp(-r*T)*si.norm.cdf(-d_2, 0.0, 1.0)-S0*si.norm.cdf(-d_1,0.0,1.0
            return option

        BS_EFD = np.zeros(13)
        for i in range(13):
            BS_EFD[i] = f_BS(S[i],"put")
```

```
In [8]: print("Balck Scholes: ",BS_EFD,)
```

```
Balck Scholes:  [5.80198673e+00 4.80198692e+00 3.80205782e+00 2.80535743e+00
 1.84426859e+00 1.02442812e+00 4.64694534e-01 1.71536863e-01
 5.24596231e-02 1.36511383e-02 3.10746681e-03 6.34561801e-04
 1.18785901e-04]
```

```
In [9]: ###############EFD######################
        def f_EFD(S0,dX,sigma,dt,r,K,steps):
            Pu = dt*(sigma**2/(2*dX**2)+(r-sigma**2/2)/(2*dX))
            Pm = 1-dt*sigma**2/(dX**2) - r*dt
            Pd = dt*(sigma**2/(2*dX**2)-(r-sigma**2/2)/(2*dX))

            TerP = np.arange(np.log(100)+dX,np.log(1)-dX,-dX)
            index = np.where(np.exp(TerP)<S0)[0][0]
            N = len(TerP)
            A = np.zeros(shape = (N,N))
            move = np.array([0,1,2])
            A[0,move] = [Pu,Pm,Pd]
            A[1,move] = [Pu,Pm,Pd]
            for i in range(1,(N-3)):
                A[i+1,move+i] = [Pu,Pm,Pd]
            A[N-2,move+N-3] = [Pu,Pm,Pd]
            A[N-1,move+N-3] = [Pu,Pm,Pd]

            F = np.maximum(K - np.exp(TerP),0)

            B = np.zeros(N)
            B[-1] = np.exp(TerP[-2])-np.exp(TerP[-1])

            for i in range(steps):
                F = np.dot(A,F)+B
            option = np.mean([F[index-1],F[index]])
            return option

        optionE = np.zeros((3,13))
        for i in range(3):
            for j in range(13):
                optionE[i,j] = f_EFD(S[j],dX[i],sigma,dt,r,K,steps)

        Err_EFD = BS_EFD - optionE
```

Following is the option prices calculated by EXPLICIT METHOD with column represents different S0 values and row represents different dX values:

```
In [12]: print(optionE)
```

```
[[5.78827438e+00 4.78252372e+00 3.79936205e+00 2.81673938e+00
  1.85171073e+00 1.04077220e+00 4.67558599e-01 1.66762471e-01
```

```
 5.58697956e-02 1.45709312e-02 2.90395697e-03 6.80252673e-04
 1.04023936e-04]
[5.78462035e+00 4.81160099e+00 3.79211478e+00 2.78859638e+00
 1.89704928e+00 1.03172067e+00 4.57218418e-01 1.72958472e-01
 4.85085971e-02 1.30618534e-02 2.75245387e-03 6.58877253e-04
 1.33288592e-04]
[5.77016098e+00 4.80476841e+00 3.82596355e+00 2.78566460e+00
 1.88464056e+00 1.01294878e+00 4.49636834e-01 1.76474418e-01
 5.25783055e-02 1.60492028e-02 2.72412706e-03 8.00402203e-04
 1.28975428e-04]]
[[ 1.37123580e-02  1.94632023e-02  2.69577780e-03 -1.13819449e-02
  -7.44214180e-03 -1.63440741e-02 -2.86406454e-03  4.77439259e-03
  -3.41017247e-03 -9.19792970e-04  2.03509834e-04 -4.56908719e-05
   1.47619650e-05]
 [ 1.73663872e-02 -9.61406677e-03  9.94304568e-03  1.67610516e-02
  -5.27806852e-02 -7.29254735e-03  7.47611606e-03 -1.42160894e-03
   3.95102604e-03  5.89284820e-04  3.55012943e-04 -2.43154515e-05
  -1.45026905e-05]
 [ 3.18257570e-02 -2.78148964e-03 -2.39057252e-02  1.96928364e-02
  -4.03719637e-02  1.14793477e-02  1.50577001e-02 -4.93755484e-03
  -1.18682373e-04 -2.39806455e-03  3.83339752e-04 -1.65840402e-04
  -1.01895264e-05]]
```

Following is the option prices ERRORS with EXPLICIT METHOD compared with Black Scholes with column represents different S0 values and row represents different dX values:

```
In [13]: print(Err_EFD)
```


```
[[ 1.37123580e-02  1.94632023e-02  2.69577780e-03 -1.13819449e-02
  -7.44214180e-03 -1.63440741e-02 -2.86406454e-03  4.77439259e-03
  -3.41017247e-03 -9.19792970e-04  2.03509834e-04 -4.56908719e-05
   1.47619650e-05]
 [ 1.73663872e-02 -9.61406677e-03  9.94304568e-03  1.67610516e-02
  -5.27806852e-02 -7.29254735e-03  7.47611606e-03 -1.42160894e-03
   3.95102604e-03  5.89284820e-04  3.55012943e-04 -2.43154515e-05
  -1.45026905e-05]
 [ 3.18257570e-02 -2.78148964e-03 -2.39057252e-02  1.96928364e-02
  -4.03719637e-02  1.14793477e-02  1.50577001e-02 -4.93755484e-03
  -1.18682373e-04 -2.39806455e-03  3.83339752e-04 -1.65840402e-04
  -1.01895264e-05]]
```


```
In [14]: ##############IFD#####################
         def f_IFD(S0,dX,sigma,dt,r,K,steps):
             Pu = -0.5*dt*(sigma**2/(dX**2)+(r-sigma**2/2)/(dX))
             Pm = 1+dt*sigma**2/(dX**2) + r*dt
             Pd = -0.5*dt*(sigma**2/(dX**2)-(r-sigma**2/2)/(dX))
```

```python
        TerP = np.arange(np.log(20)+dX,np.log(1)-dX,-dX)
        index = np.where(np.exp(TerP)<S0)[0][0]
        N = len(TerP)
        A = np.zeros(shape = (N,N))
        move = np.array([0,1,2])
        A[0,[0,1]] = [1,-1]
        A[1,move] = [Pu,Pm,Pd]
        for i in range(N-3):
            A[i+1,move+i] = [Pu,Pm,Pd]
        A[-2,[-3,-2,-1]] = [Pu,Pm,Pd]
        A[-1,[-2,-1]] = [1,-1]

        F = np.maximum(K - np.exp(TerP),0)

        B = np.zeros(N)
        B[1:-1]= F[1:-1]
        B[-1] = np.exp(TerP[-2])-np.exp(TerP[-1])

        for i in range(steps):
            F = np.dot(np.linalg.inv(A),B)
            B = np.zeros(N)
            B[1:-1]= F[1:-1]
            B[-1] = np.exp(TerP[-2])-np.exp(TerP[-1])
        option = np.mean([F[index-1],F[index]])
        return option

    optionI = np.zeros((3,13))
    for i in range(3):
        for j in range(13):
            optionI[i,j] = f_IFD(S[j],dX[i],sigma,dt,r,K,steps)

    Err_IFD = BS_EFD - optionI
```

Following is the option prices calculated by IMPLICIT METHOD with column represents different S0 values and row represents different dX values:

```
In [18]: print(optionI)
```

```
[[5.78615809e+00 4.77987335e+00 3.79620443e+00 2.81324436e+00
  1.84818180e+00 1.03731054e+00 4.64793689e-01 1.65396135e-01
  5.54744034e-02 1.45954925e-02 2.97585487e-03 7.21040535e-04
  1.17320123e-04]
 [5.77770751e+00 4.80301019e+00 3.78178412e+00 2.77681125e+00
  1.88475334e+00 1.02060432e+00 4.49471776e-01 1.69001398e-01
  4.72665970e-02 1.27939425e-02 3.80624496e-03 6.76593934e-04
  1.43367723e-04]
 [5.76803507e+00 4.80212986e+00 3.82281916e+00 2.78216134e+00
  1.88109112e+00 1.00944414e+00 4.46860453e-01 1.74992002e-01
```

```
 5.21952606e-02 1.60484797e-02 2.79061044e-03 8.42000965e-04
 1.43323902e-04]]
```

Following is the option prices ERRORS with IMPLICIT METHOD compared with Black Scholes with column represents different S0 values and row represents different dX values:

```
In [17]: print(Err_IFD)
```

```
[[ 1.58286438e-02  2.21135696e-02  5.85339343e-03 -7.88692764e-03
  -3.91320875e-03 -1.28824154e-02 -9.91545740e-05  6.14072864e-03
  -3.01478027e-03 -9.44354280e-04  1.31611936e-04 -8.64787335e-05
   1.46577856e-06]
 [ 2.42792216e-02 -1.02326877e-03  2.02737010e-02  2.85461830e-02
  -4.04847472e-02  3.82379965e-03  1.52227583e-02  2.53546515e-03
   5.19302609e-03  8.57195783e-04 -6.98778148e-04 -4.20321323e-05
  -2.45818214e-05]
 [ 3.39516641e-02 -1.42944369e-04 -2.07613364e-02  2.31960938e-02
  -3.68225278e-02  1.49839850e-02  1.78340819e-02 -3.45513839e-03
   2.64362551e-04 -2.39734147e-03  3.16856365e-04 -2.07439163e-04
  -2.45380008e-05]]
```

```
In [19]: ##############CNFD######################
         def f_CNFD(S0,dX,sigma,dt,r,K,steps):
             Pu = -1/4*dt*(sigma**2/(dX**2)+(r-sigma**2/2)/(dX))
             Pm = 1+dt*sigma**2/(2*dX**2) + r*dt/2
             Pd = -1/4*dt*(sigma**2/(dX**2)-(r-sigma**2/2)/(dX))

             TerP = np.arange(np.log(20)+dX,np.log(1)-dX,-dX)
             F = np.maximum(K - np.exp(TerP),0)

             index = np.where(np.exp(TerP)<S0)[0][0]
             N = len(TerP)
             move = np.array([0,1,2])
             X = np.zeros(shape = (N,N))
             X[0,[0,1]] = 0
             X[1,move] = [-Pu,-(Pm-2),-Pd]
             for i in range(1,(N-3)):
                 X[i+1,move+i] = [-Pu,-(Pm-2),-Pd]
             X[-2,move+N-3] = [-Pu,-(Pm-2),-Pd]

             Y = np.zeros(N)
             Y[-1] = np.exp(TerP[-2])-np.exp(TerP[-1])

             A = np.zeros(shape = (N,N))
             A[0,[0,1]] = [1,-1]
             A[1,move] = [Pu,Pm,Pd]
             for i in range(1,(N-3)):
```

```
            A[i+1,move+i] = [Pu,Pm,Pd]
        A[-2,move+N-3] = [Pu,Pm,Pd]
        A[-1,[-2,-1]] = [1,-1]

        B = np.dot(np.linalg.inv(A),X)
        D = np.dot(np.linalg.inv(A),Y)

        for i in range(steps):
            F = np.dot(B,F)+D

        option = np.mean([F[index],F[index-1]])
        return option

    optionCN = np.zeros((3,13))
    for i in range(3):
        for j in range(13):
            optionCN[i,j] = f_CNFD(S[j],dX[i],sigma,dt,r,K,steps)

    Err_CNFD = BS_EFD - optionCN
```

Following is the option prices calculated by CRANK-NICOLSON METHOD with column represents different S0 values and row represents different dX values:

```
In [20]: print(optionCN)
```

```
[[5.78615025e+00 4.77986543e+00 3.79618944e+00 2.81315496e+00
  1.84800517e+00 1.03735518e+00 4.65072477e-01 1.65546083e-01
  5.54490323e-02 1.45137172e-02 2.92332848e-03 6.96397803e-04
  1.09897568e-04]
 [5.77769967e+00 4.80300227e+00 3.78176861e+00 2.77671722e+00
  1.88457606e+00 1.02065681e+00 4.49751356e-01 1.69154845e-01
  4.72259555e-02 1.27133178e-02 3.74858670e-03 6.53347323e-04
  1.35037185e-04]
 [5.76802723e+00 4.80212194e+00 3.82280456e+00 2.78206810e+00
  1.88091432e+00 1.00950193e+00 4.47139902e-01 1.75151977e-01
  5.21634270e-02 1.59674806e-02 2.74066139e-03 8.15729714e-04
  1.35100459e-04]]
```

Following is the option prices ERRORS with CRANK-NICOLSON METHOD compared with Black Scholes with column represents different S0 values and row represents different dX values:

```
In [22]: print(Err_CNFD)
```

```
[[ 1.58364851e-02  2.21214876e-02  5.86838001e-03 -7.79753206e-03
  -3.73657477e-03 -1.29270547e-02 -3.77942894e-04  5.99078022e-03
  -2.98940920e-03 -8.62578907e-04  1.84138327e-04 -6.18360017e-05
   8.88833323e-06]
```

```
[ 2.42870629e-02 -1.01535244e-03  2.02892182e-02  2.86402128e-02
 -4.03074669e-02  3.77131384e-03  1.49431781e-02  2.38201803e-03
  5.23366758e-03  9.37820477e-04 -6.41119890e-04 -1.87855218e-05
 -1.62512836e-05]
[ 3.39595054e-02 -1.35023367e-04 -2.07467338e-02  2.32893290e-02
 -3.66457267e-02  1.49261921e-02  1.75546327e-02 -3.61511374e-03
  2.96196123e-04 -2.31634239e-03  3.66805423e-04 -1.81167913e-04
 -1.63145582e-05]]
```

2 Problem 2

```
In [23]: #Problem 2---------------------------------------------------------
         S0 = 10
         sigma = 0.2
         r = 0.04
         dt = 0.002
         dS = 0.25
         K = 10
         T = 0.5
         steps = int(T/dt)
```

Following is the function that calculates the option prices by using GENERALIZED FINITE-DIFFERENCE method

```
In [24]: ##############Generalized Finite-Difference#####################
         def f_A_PDE(a,dS,r,sigma,dt,S0,K,steps,name):
             TerP = np.arange(0,20+dS,dS)
             index = np.abs(TerP-S0).argmin()
             n = len(TerP)

             j = np.arange(1,n-1,1)
             alpha = a
             a1 = 0.5*((sigma**2)*(j**2)-r*j)*(1-alpha)
             a2 = -1/dt-((sigma**2)*(j**2)+r)*(1-alpha)
             a3 = 0.5*((sigma**2)*(j**2)+r*j)*(1-alpha)
             b1 = 0.5*((sigma**2)*(j**2)-r*j)*alpha
             b2 = 1/dt-((sigma**2)*(j**2)+r)*alpha
             b3 = 0.5*((sigma**2)*(j**2)+r*j)*alpha

             P = np.maximum(np.round(K - TerP,4),0)
             C = np.maximum(np.round(TerP - K,4),0)

             A = np.zeros(shape = (n,n))
             move = np.array([0,1,2])
             A[0,[0,1]] = [1,-1]
             for i in range(n-2):
                 A[i+1,move+i] = [a1[i],a2[i],a3[i]]
```

```
        A[-1,[-2,-1]] = [1,-1]

        B = np.zeros(shape = (n,n))
        for i in range(n-2):
            B[i+1,move+i] = [-b1[i],-b2[i],-b3[i]]

        eC = np.zeros(n)
        eC[0] = TerP[0]-TerP[1]
        eP = np.zeros(n)
        eP[0] = TerP[-2]-TerP[-1]

        for i in range(steps):
            C = np.dot(np.linalg.inv(A),np.dot(B,C)+eC)
            C = np.maximum(C,np.maximum(np.round(TerP - K,4),0))
            P = np.dot(np.linalg.inv(A),np.dot(B,P)+eP)
            P = np.maximum(P,np.maximum(np.round(K - TerP,4),0))

        if name == "C":
            return C[index]
        else:
            return P[index]
```

The following generates the options prices for American Put/Call Option

```
In [25]: optionC = np.zeros((9,13))
         optionP = np.zeros((9,13))
         S = np.arange(4,17,1)
         dS = [0.25,1,1.25]
         a = [1,0,0.5]

         pS = [0,0,0,1,1,1,2,2,2]
         pa = [0,1,2,0,1,2,0,1,2]
         for i in range(9):
             for j in range(len(S)):
                 optionC[i,j] = f_A_PDE(a[pa[i]],dS[pS[i]],r,sigma,dt,S[j],K,steps,"C")
         for i in range(9):
             for j in range(len(S)):
                 optionP[i,j] = f_A_PDE(a[pa[i]],dS[pS[i]],r,sigma,dt,S[j],K,steps,"P")
```

```
In [26]: print(optionC)
```

```
[[3.23696901e-10 4.73061853e-07 8.95673120e-05 3.48932677e-03
  4.20739932e-02 2.21037817e-01 6.60797846e-01 1.36832742e+00
  2.25001819e+00 3.21149181e+00 4.20060288e+00 5.19630645e+00
  6.18995908e+00]
 [7.15081363e-10 6.94829322e-07 1.04309683e-04 3.65166541e-03
  4.24139964e-02 2.20923454e-01 6.60217199e-01 1.36799609e+00
  2.25004803e+00 3.21150804e+00 4.20007668e+00 5.19418020e+00
  6.18404856e+00]
```

```
[4.91185022e-10 5.77781039e-07 9.68547354e-05 3.57066850e-03
 4.22441497e-02 2.20980311e-01 6.60507750e-01 1.36816144e+00
 2.25003115e+00 3.21148329e+00 4.20024434e+00 5.19487489e+00
 6.18595847e+00]
[7.92163922e-07 2.64510236e-05 5.06920716e-04 5.88940615e-03
 4.29164858e-02 2.01044471e-01 6.23047544e-01 1.34812264e+00
 2.24495750e+00 3.21142689e+00 4.20098941e+00 5.19616136e+00
 6.18932658e+00]
[8.98402156e-07 2.87121514e-05 5.31679021e-04 6.02573911e-03
 4.32324386e-02 2.01054174e-01 6.22287807e-01 1.34785662e+00
 2.24494162e+00 3.21103546e+00 4.19910596e+00 5.19019725e+00
 6.17409466e+00]
[8.44309397e-07 2.75709614e-05 5.19263256e-04 5.95763041e-03
 4.30747307e-02 2.01048932e-01 6.22667641e-01 1.34798610e+00
 2.24492747e+00 3.21111016e+00 4.19952831e+00 5.19140032e+00
 6.17673664e+00]
[1.18170076e-06 5.71238451e-05 1.38619739e-03 1.83159179e-02
 1.83159179e-02 1.37262478e-01 5.96307245e-01 1.55398273e+00
 2.72281218e+00 2.72281218e+00 3.95284590e+00 5.19604680e+00
 6.43617980e+00]
[1.30190422e-06 6.06167536e-05 1.42978419e-03 1.85328723e-02
 1.85328723e-02 1.37463642e-01 5.95522972e-01 1.55385564e+00
 2.72254696e+00 2.72254696e+00 3.95088809e+00 5.18808734e+00
 6.41198505e+00]
[1.24099393e-06 5.88597004e-05 1.40796747e-03 1.84245399e-02
 1.84245399e-02 1.37362970e-01 5.95913751e-01 1.55390480e+00
 2.72256656e+00 2.72256656e+00 3.95121063e+00 5.18923776e+00
 6.41480464e+00]]
```

dS = 0.25	S0=4	S0=5	S0=6	S0=7	S0=8	S0=9	S0=10	S0=11	S0=12	S0=13	S0=14	S0=15	S0=16
Implicit	3.23697e-10	4.73062e-07	8.95673e-05	0.00348934	0.0420742	0.221038	0.660798	1.36833	2.25002	3.21149	4.2006	5.19631	6.18996
Explitcit	7.15086e-10	6.94829e-07	0.000104301	0.00365106	0.0424104	0.22092	0.660217	1.368	2.25005	3.21151	4.20008	5.19418	6.18405
Crank-Nicolson	4.91185e-10	5.77781e-07	9.68547e-05	0.00357067	0.0422441	0.22098	0.660508	1.36816	2.25003	3.21148	4.20024	5.19487	6.18596

dS = 1	S0=4	S0=5	S0=6	S0=7	S0=8	S0=9	S0=10	S0=11	S0=12	S0=13	S0=14	S0=15	S0=16
Implicit	7.92164e-07	2.64510e-05	0.000506921	0.00588941	0.0429165	0.201044	0.623048	1.34812	2.24496	3.21143	4.20099	5.19616	6.18933
Explitcit	8.98402e-07	2.87122e-05	0.000531679	0.00602574	0.0432324	0.201054	0.622288	1.34786	2.24494	3.21104	4.19911	5.1902	6.17409

dS = 1	S0=4	S0=5	S0=6	S0=7	S0=8	S0=9	S0=10	S0=11	S0=12	S0=13	S0=14	S0=15	S0=16
Crank-Nicolson	8.44309e-07	2.75716e-05	[illegible]	[illegible]	[illegible]	[illegible]	0.622668	1.34799	2.24493	3.21111	4.19953	5.1914	6.17674

dS = 1.25	S0=4	S0=5	S0=6	S0=7	S0=8	S0=9	S0=10	S0=11	S0=12	S0=13	S0=14	S0=15	S0=16
Implicit	1.18175e-06	5.71238e-05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.55398	2.72281	2.72281	3.95285	5.19605	6.43618
Explplitcit	1.30196e-06	6.06168e-05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.55386	2.72255	2.72255	3.95089	5.18809	6.41199
Crank-Nicolson	1.24095e-06	5.88597e-05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.5539	2.72257	2.72257	3.95121	5.18924	6.4148

```
In [27]: print(optionP)
```

```
[[6.00000000e+00 5.00000000e+00 4.00000000e+00 3.00000000e+00
  2.00000000e+00 1.08128231e+00 4.80663846e-01 1.75062833e-01
  5.31157712e-02 1.37847278e-02 3.14472884e-03 6.46276053e-04
  1.22144542e-04]
 [6.00000000e+00 5.00000000e+00 4.00000000e+00 3.00000000e+00
  2.00000000e+00 1.08057257e+00 4.79729532e-01 1.74591758e-01
  5.31327553e-02 1.39374913e-02 3.25050207e-03 6.92775642e-04
  1.38052364e-04]
 [6.00000000e+00 5.00000000e+00 4.00000000e+00 3.00000000e+00
  2.00000000e+00 1.08092228e+00 4.80191555e-01 1.74823182e-01
  5.31222962e-02 1.38607313e-02 3.19766197e-03 6.69528843e-04
  1.30056887e-04]
 [6.00000000e+00 5.00000000e+00 4.00000000e+00 3.00000000e+00
  2.00000000e+00 1.05162709e+00 4.39978944e-01 1.54452305e-01
  4.81521757e-02 1.38759316e-02 3.79614482e-03 1.00389879e-03
  2.59842593e-04]
 [6.00000000e+00 5.00000000e+00 4.00000000e+00 3.00000000e+00
  2.00000000e+00 1.05125183e+00 4.39081179e-01 1.54173471e-01
  4.82546415e-02 1.40218530e-02 3.88624541e-03 1.04610273e-03
  2.76910435e-04]
 [6.00000000e+00 5.00000000e+00 4.00000000e+00 3.00000000e+00
  2.00000000e+00 1.05143853e+00 4.39529996e-01 1.54312272e-01
  4.82033850e-02 1.39490939e-02 3.84130557e-03 1.02501380e-03
  2.68354863e-04]
 [6.25000000e+00 5.00000000e+00 3.75000000e+00 2.50000000e+00
  2.50000000e+00 1.25337401e+00 4.15327729e-01 1.09703667e-01
  2.56281147e-02 2.56281147e-02 5.59366486e-03 1.17666331e-03
  2.43108776e-04]
 [6.25000000e+00 5.00000000e+00 3.75000000e+00 2.50000000e+00
  2.50000000e+00 1.25331072e+00 4.14427377e-01 1.09625147e-01
  2.57729041e-02 2.57729041e-02 5.69067855e-03 1.21736388e-03
```

```
 2.57124201e-04]
[6.25000000e+00 5.00000000e+00 3.75000000e+00 2.50000000e+00
 2.50000000e+00 1.25334221e+00 4.14877237e-01 1.09664129e-01
 2.57006742e-02 2.57006742e-02 5.64228210e-03 1.19702451e-03
 2.50097916e-04]]
```

dS = 0.25	S0=4	S0=5	S0=6	S0=7	S0=8	S0=9	S0=10	S0=11	S0=12	S0=13	S0=14	S0=15	S0=16
Implicit	6	5	4	3	2	1.08128	0.480664	0.175063	0.0531158	0.0137847	0.00314473	0.000646276	0.000122145
Explitcit	6	5	4	3	2	1.08057	0.47973	0.174592	0.0531328	0.0139375	0.0032505	0.000692776	0.000138052
Crank-Nicolson	6	5	4	3	2	1.08092	0.480192	0.174823	0.0531223		0.0138607	0.00319766	0.000669529

dS = 1	S0=4	S0=5	S0=6	S0=7	S0=8	S0=9	S0=10	S0=11	S0=12	S0=13	S0=14	S0=15	S0=16
Implicit	6	5	4	3	2	1.05163	0.439979	0.154452	0.0481522	0.0138759	0.00379614	0.00100039	0.000259843
Explitcit	6	5	4	3	2	1.05125	0.439081	0.154173	0.0482546	0.0140219	0.00388625	0.0010461	0.00027691
Crank-Nicolson	6	5	4	3		2	1.05144	0.43953	0.154312	0.0482034	0.0139491	0.00384031	0.00102501

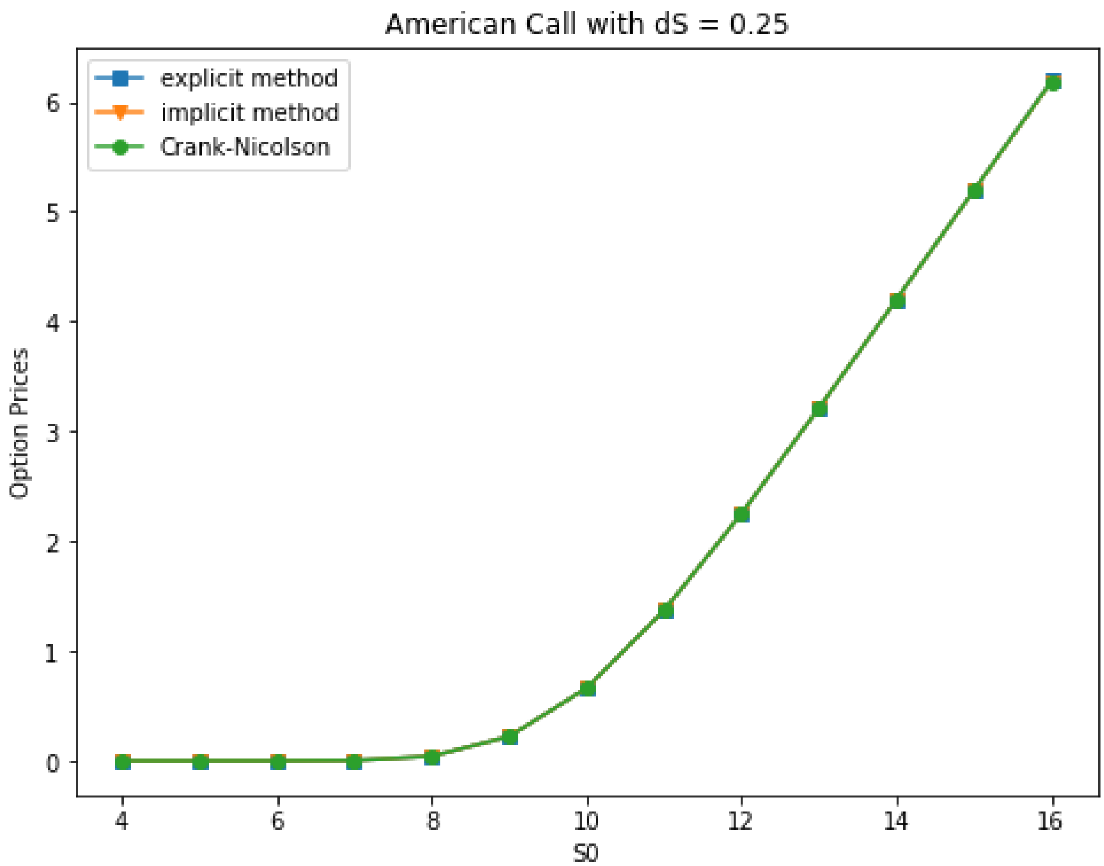
dS = 1.25	S0=4	S0=5	S0=6	S0=7	S0=8	S0=9	S0=10	S0=11	S0=12	S0=13	S0=14	S0=15	S0=16
Implicit	6.25	5	3.75	2.5	2.5	1.25337	0.415328	0.109704	0.0256281	0.0256281	0.00559366	0.00117666	0.000243109
Explitcit	6.25	5	3.75	2.5	2.5	1.25331	0.414427	0.109625	0.0257729	0.0257729	0.00569068	0.00121736	0.000257124
Crank-Nicolson	6.25	5	3.75	2.5	2.5	1.25334	0.414877	0.109664	0.0257007	0.0257007	0.00564228	0.00119702	0.000250098

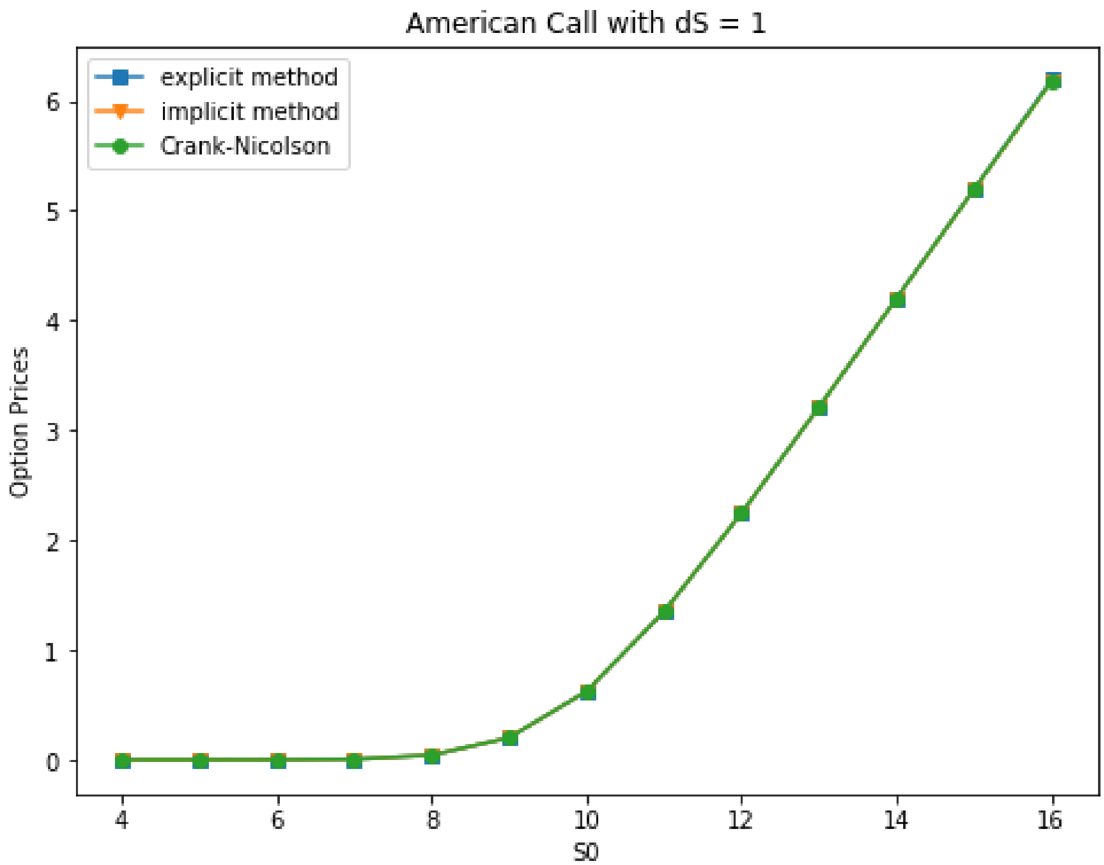
The followings are plots

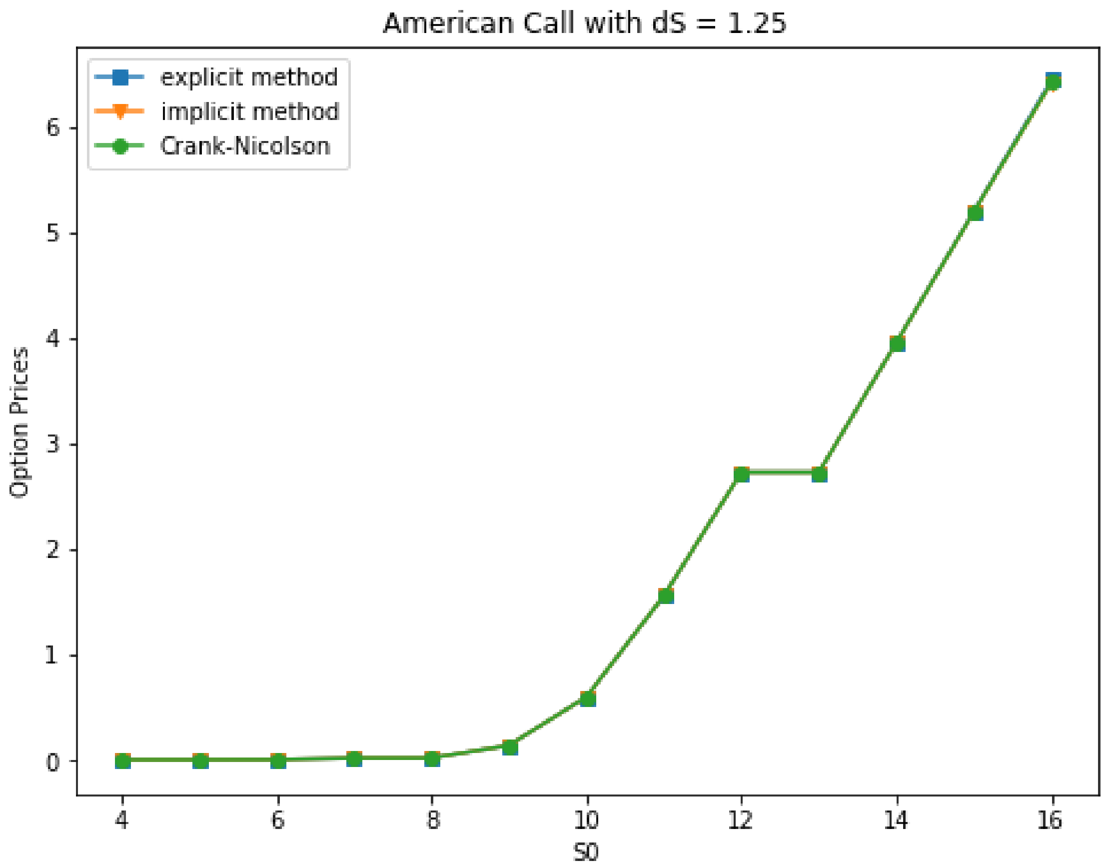
In [28]:
```
name = ["explicit method","implicit method","Crank-Nicolson"]
Shape = ["s","v","o"]
plt.figure(1,figsize = (8,6))
for i in range(3):
    plt.plot(np.arange(4,17,1),optionC[i,:],marker = Shape[i],label = name[i])
plt.legend()
plt.title("American Call with dS = 0.25")
plt.xlabel("S0")
plt.ylabel("Option Prices")

plt.figure(2,figsize = (8,6))
for i in range(3):
    plt.plot(np.arange(4,17,1),optionC[i+3,:],marker = Shape[i],label = name[i])
plt.legend()
```

```
plt.title("American Call with dS = 1")
plt.xlabel("S0")
plt.ylabel("Option Prices")

plt.figure(3,figsize = (8,6))
for i in range(3):
    plt.plot(np.arange(4,17,1),optionC[i+6,:],marker = Shape[i],label = name[i])
plt.legend()
plt.title("American Call with dS = 1.25")
plt.xlabel("S0")
plt.ylabel("Option Prices")
```

Out[28]: Text(0, 0.5, 'Option Prices')




American Call with dS = 1
explicit method
implicit method
Crank-Nicolson
Option Prices
S0
0
1
2
3
4
5
6
4
6
8
10
12
14
16

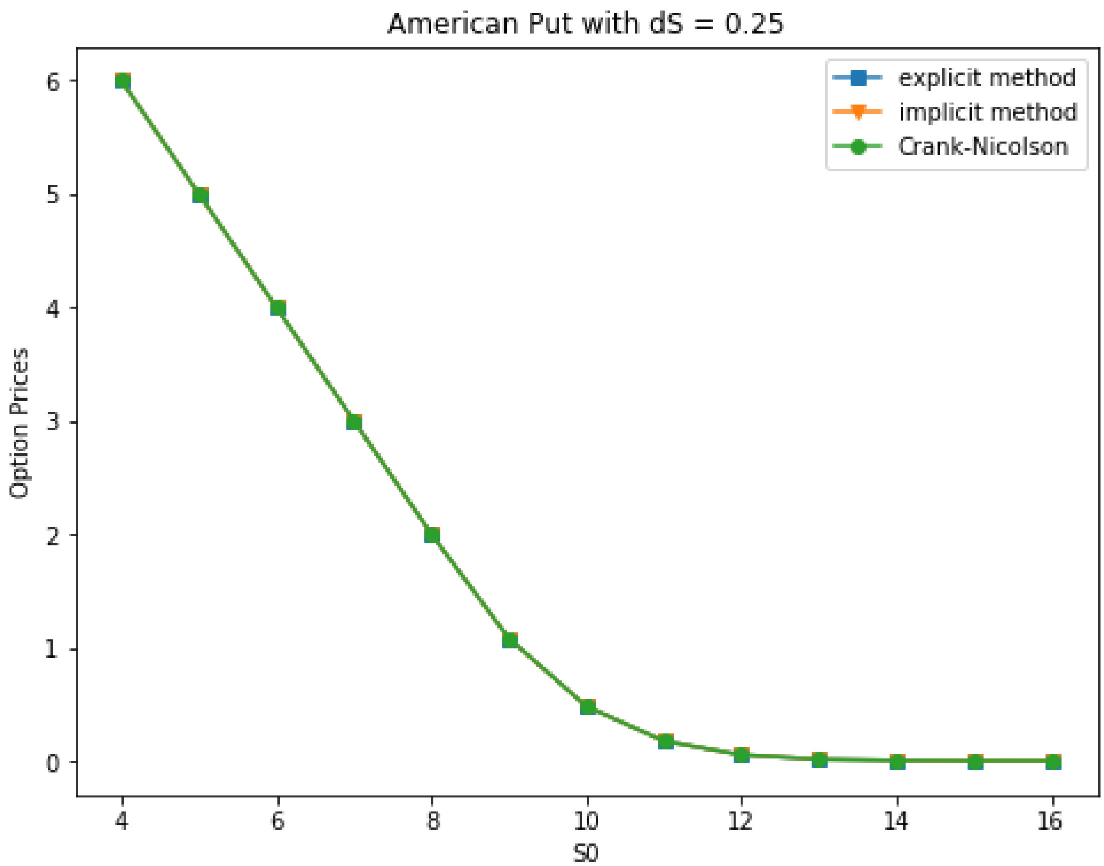


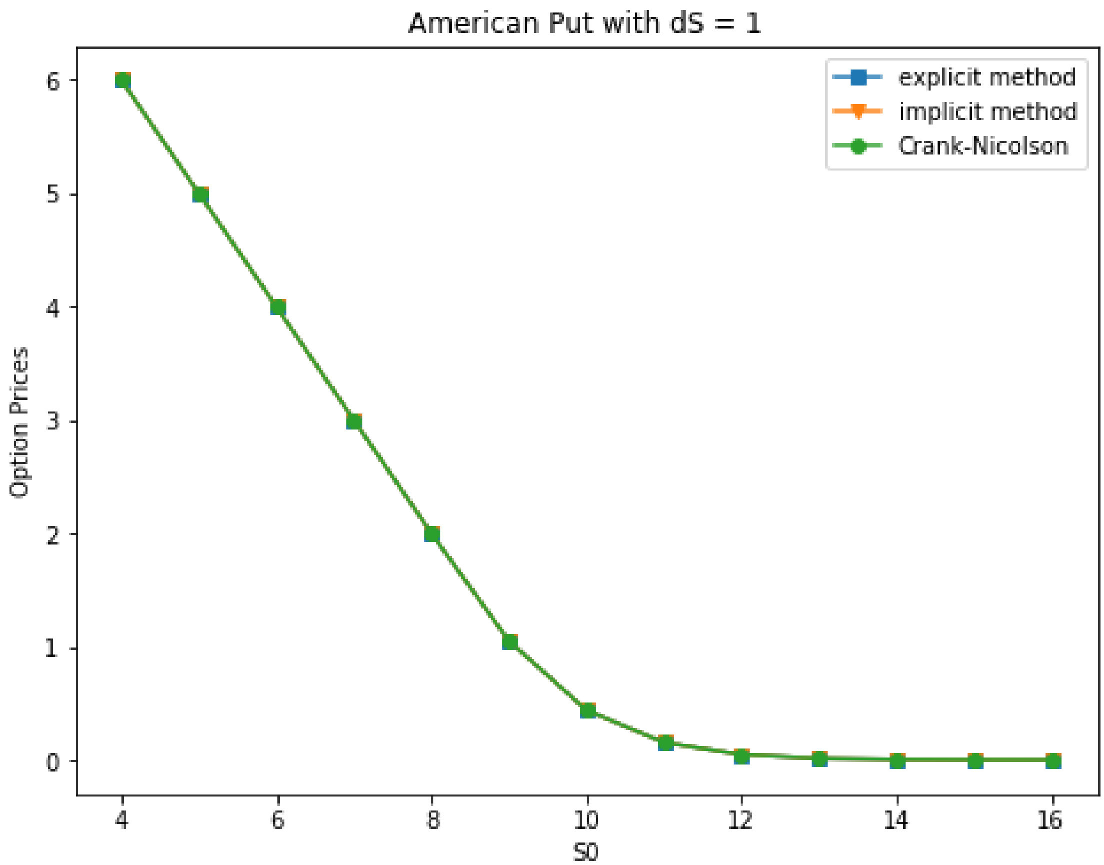
The plots show us that all the three methods are giving the very similar results since they are all overlaying on each other

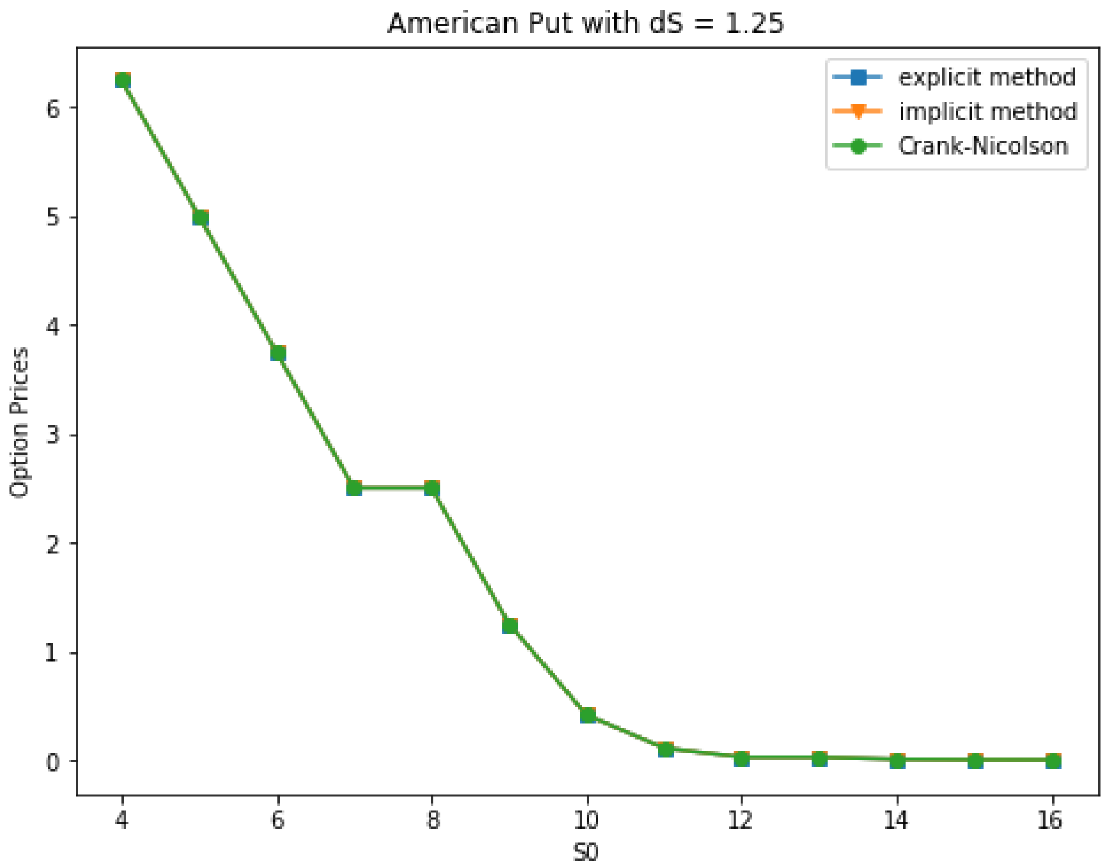
```
In [29]: plt.figure(4,figsize = (8,6))
         for i in range(3):
             plt.plot(np.arange(4,17,1),optionP[i,:],marker = Shape[i],label = name[i])
         plt.legend()
         plt.title("American Put with dS = 0.25")
         plt.xlabel("S0")
         plt.ylabel("Option Prices")

         plt.figure(5,figsize = (8,6))
         for i in range(3):
             plt.plot(np.arange(4,17,1),optionP[i+3,:],marker = Shape[i],label = name[i])
         plt.legend()
         plt.title("American Put with dS = 1")
         plt.xlabel("S0")
         plt.ylabel("Option Prices")

         plt.figure(6,figsize = (8,6))
         for i in range(3):
```

```
    plt.plot(np.arange(4,17,1),optionP[i+6,:],marker = Shape[i],label = name[i])
plt.legend()
plt.title("American Put with dS = 1.25")
plt.xlabel("S0")
plt.ylabel("Option Prices")
```

Out[29]: Text(0, 0.5, 'Option Prices')




American Put with dS = 1
explicit method
implicit method
Crank-Nicolson
Option Prices
S0
0
1
2
3
4
5
6
4
6
8
10
12
14
16



The plots show us that all the three methods are also giving the very similar results since they are all overlaying on each other