Computational Method in Finance Project 1

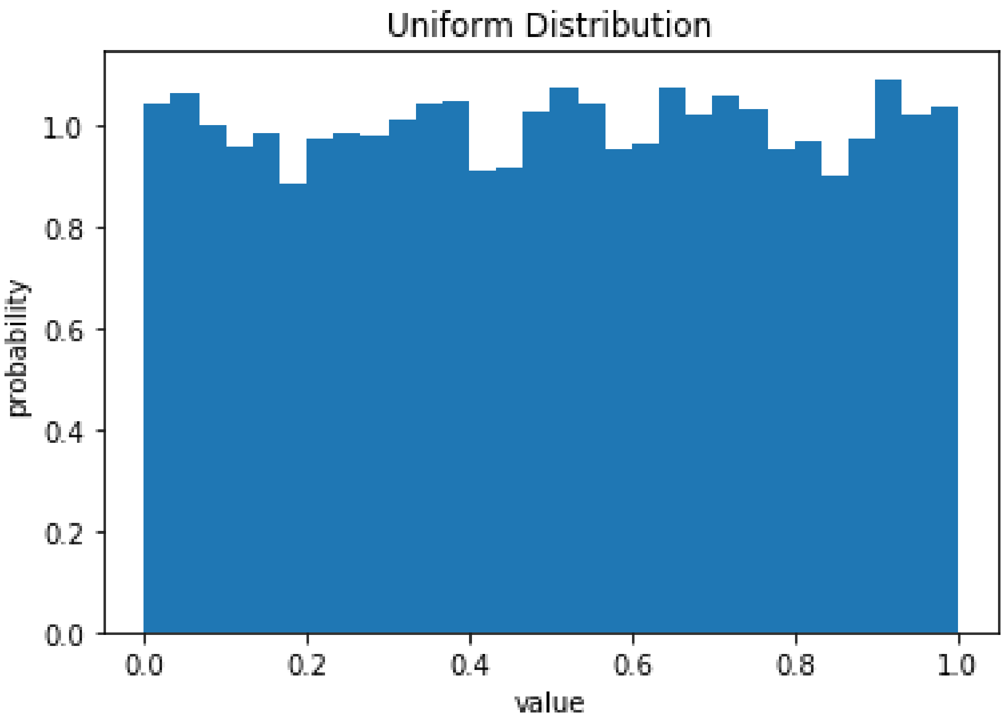
January 17, 2019

1. (a)The following code generate 10,000 Uniform distributed random number:

```
In [4]: import numpy as np
        import matplotlib.pyplot as plt
        import math
        import random
        import time

        #1--------------------------------
        #Set random number generators
        a = 7**5
        b = 0
        m = 2**31-1

        #Creat a list to store random numbers with U[0,1]
        unif = [None] * 10001
        unif[0] = 1

        #This loop is for generating random numbers with U[0,1]
        i = 1
        while i < 10001:
            unif[i] = np.mod(a*unif[i-1]+b,m)
            i = i+1
        unif = [x/m for x in unif]

        #Delete the first observation x_0 and keep x_1 to x_10000
        del unif[0]

        #Draw the histogram of the sample uniform distribution
        plt.figure()
        ax1 = plt.hist(unif, normed = True, bins = 30)
        plt.title("Uniform Distribution")
        plt.ylabel("probability")
        plt.xlabel("value")

        print("mean: ", np.mean(unif))
        print("std: ", np.std(unif))
```

```
mean:  0.5015954347987638
std:  0.28936371860979276
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:20: RuntimeWarning: c
```



(b)built in function results:

```
In [3]: #Use built-in function to generate uniform distribution
        build = np.random.uniform(0,1,10000)
        print("build in function mean: ", np.mean(build))
        print("build in function std: ",np.std(build))
```

```
build in function mean:  0.5020738374619756
build in function std:  0.28693499755277346
```

(c) Comparing part(a) and part(b), we can tell that the simulation gets a very similar mean and standard deviation as the biult in function does

2. (a)Generate 10,000 random numbers with following distribution

$$X = \begin{cases} -1 & \text{with probability } 0.30 \\ 0 & \text{with probability } 0.35 \\ 1 & \text{with probability } 0.20 \\ 2 & \text{with probability } 0.15 \end{cases} \tag{1}$$

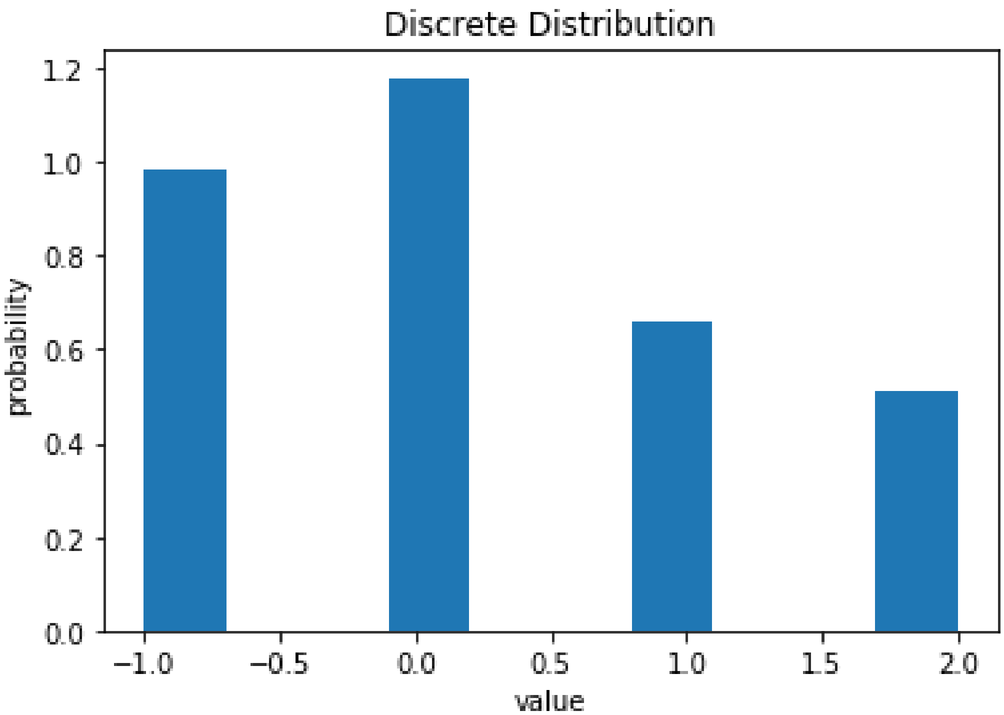
```
In [5]: #2-----------------------------------
        #Creat a list to store random numbers from the discrete distribution
        dist = [None] * 10000

        #Set probabilities for different outcomes from the distribution
        p1 = 0.3; p2 = 0.35; p3 = 0.2; p4 = 0.15

        #This loop is for generating the discrete distribution
        i = 0
        while i < 10000:
            if unif[i] <= p1:
                dist[i] = -1
            elif p1 < unif[i] <= p1+p2:
                dist[i] = 0
            elif p1+p2 < unif[i] <= p1+p2+p3:
                dist[i] = 1
            elif p1+p2+p3 < unif[i] <= p1+p2+p3+p4:
                dist[i] = 2
            i = i + 1
```

(b)Draw the histogram and calculate mean and standard deviation:

```
In [6]: print("mean: ", np.mean(dist))
        print("std: ", np.std(dist))

        #Draw the histogram
        plt.figure()
        ax2 = plt.hist(dist, normed = True)
        plt.title("Discrete Distribution")
        plt.ylabel("probability")
        plt.xlabel("value")
```

```
mean:  0.2085
std:  1.0309353762481916
```

```
Out[6]: Text(0.5, 0, 'value')
```



3. (a)Generate 1,000 random numbers with Binomail distribution with n = 44 and p = 0.64

In [7]:
```
#3-----------------------------------
#Set seed so that each time we can gerenate the same sequence
#of random numbers for each loop that is used to generate
#binomial random numbers
#By setting such a seed, our result will be consistent and easy
#to study with
random.seed(9)

#Creat a list to store random numbers from the bernoulli distribution
ber = [None] * 44
p = 0.64

#Creat a list to store random numbers from the binomail distribution
B = [None]*1000

#This loop will generate 1000 binomially distributed random numbers
x = 1
while x < 1001:
    u = [None] * 45
    #generate random number for x_0 in each loop
    u[0] = random.randint(1,100)
    #generate a set of 44 uniformly distributed random numbers
```

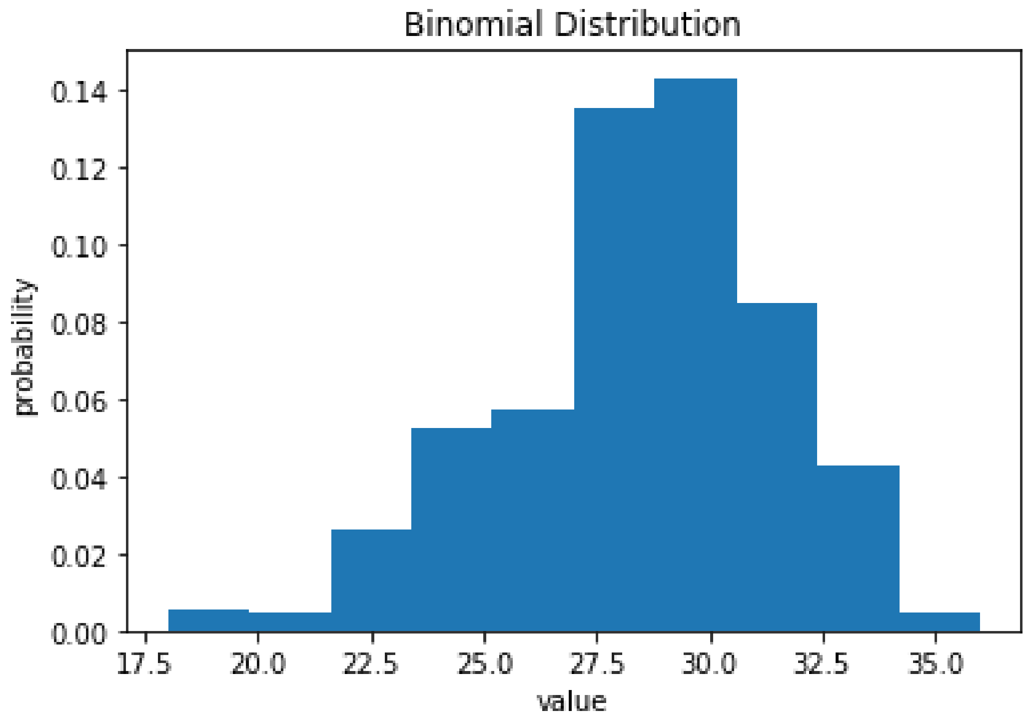
```
#each set of these numbers will generate 1 random number
#with Binomial(44,0.64)
i = 1
while i < 45:
    u[i] = np.mod(a*u[i-1]+b,m)
    i = i+1
del u[0]
j = 0
while j < 44:
    if u[j]/m <= p:
        ber[j] = 1
    else:
        ber[j] = 0
    j = j + 1
B[x-1] = sum(ber)
x = x + 1
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:27: RuntimeWarning: o
```

(b)Draw histogram

```
In [9]: #Draw the histogram
        plt.figure()
        ax3 = plt.hist(B, normed = True)
        plt.title("Binomial Distribution")
        plt.ylabel("probability")
        plt.xlabel("value")
```

```
Out[9]: Text(0.5, 0, 'value')
```



Compute P(X $\geq$ 40)

In [10]:
```
#Compute P(X>=40)
P = sum(1 for i in B if i >= 40)

#Compute theoretical value of P(X>=40) to compare with the empirical result
k = 40
n = 44
P_T = 0
while k < 45:
    P_true = math.factorial(n)/(math.factorial(k)*math.factorial(n-k))* \
             (p**k)*((1-p)**(n-k))
    P_T = P_T + P_true
    k = k + 1
print("Empirical probability =", P)
print("Theoretical Probability =", P_T,"is approximately 0")
```

```
Empirical probability = 0
Theoretical Probability = 4.8236638455676785e-05 is approximately 0
```

From above comparision, we can tell that P(X $\geq$ 40) computed from sample and from theoretical formula are approximately the same

4. (a)Generate 10,000 Exponentially distributed random numbers with $\lambda = 1.5$

```
In [12]: lam = 1.5

         #Use the uniform distributed random numbers created in question 1
         #to generate a exponential distribution
         U_4 = [1-x for x in unif]
         X_4 = -1/lam*np.log(U_4)
```

(b)Compute $P(X \geq 1)$ and $P(X \geq 4)$

```
In [14]: #Compute P(X>=1) and P(X>=4)
         P_1 = sum(1 for i in X_4 if i >= 1)/10000
         P_4 = sum(1 for i in X_4 if i >= 4)/10000
         print("P(X >= 1) =", P_1)
         print("P(X >= 4) =", P_4)
```

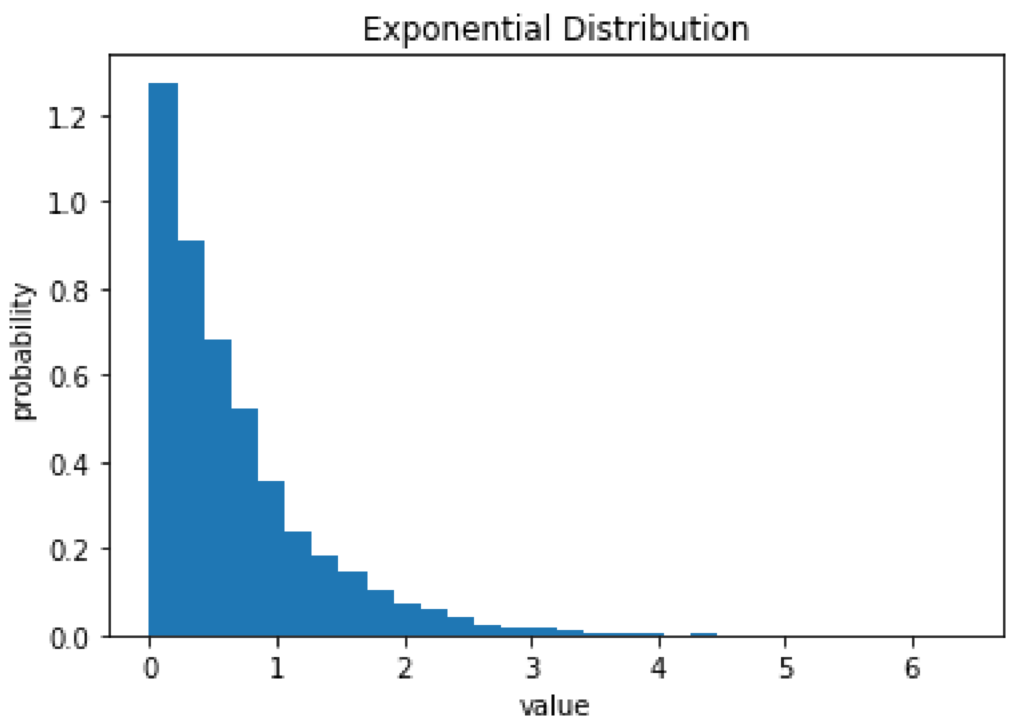
```
P(X >= 1) = 0.2223
P(X >= 4) = 0.002
```

(c)Compute empirical mean and standard deviation

```
In [15]: mu_4 = np.mean(X_4)
         sigma_4 = np.std(X_4)
         print("mean =", mu_4, ", std =", sigma_4)
         #Draw the histogram
         plt.figure()
         ax4 = plt.hist(X_4, normed = True, bins = 30)
         plt.title("Exponential Distribution")
         plt.ylabel("probability")
         plt.xlabel("value")
```

```
mean = 0.670837071558907 , std = 0.668062748295854
```

```
Out[15]: Text(0.5, 0, 'value')
```



5. (a)Generate 5,000 Uniformly distributed random numbers

```
In [16]: #5---------------------------------------------------
         #Creat a function that can generate random number with U[0,1]
         #with sample size n and initial value x_0
         def f_unif(n,x_0):
             U = [None] * (n+1)
             U[0] = x_0
             i = 1
             while i < (n+1):
                 U[i] = np.mod(a*U[i-1]+b,m)
                 i = i+1
             del U[0]
             U = [x/m for x in U]
             return U

         #Generate a uniform distribution with 5000 observations and x_0 = 1
         U_5 = f_unif(n = 5000, x_0 = 1)
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:9: RuntimeWarning: o
  if __name__ == '__main__':
```

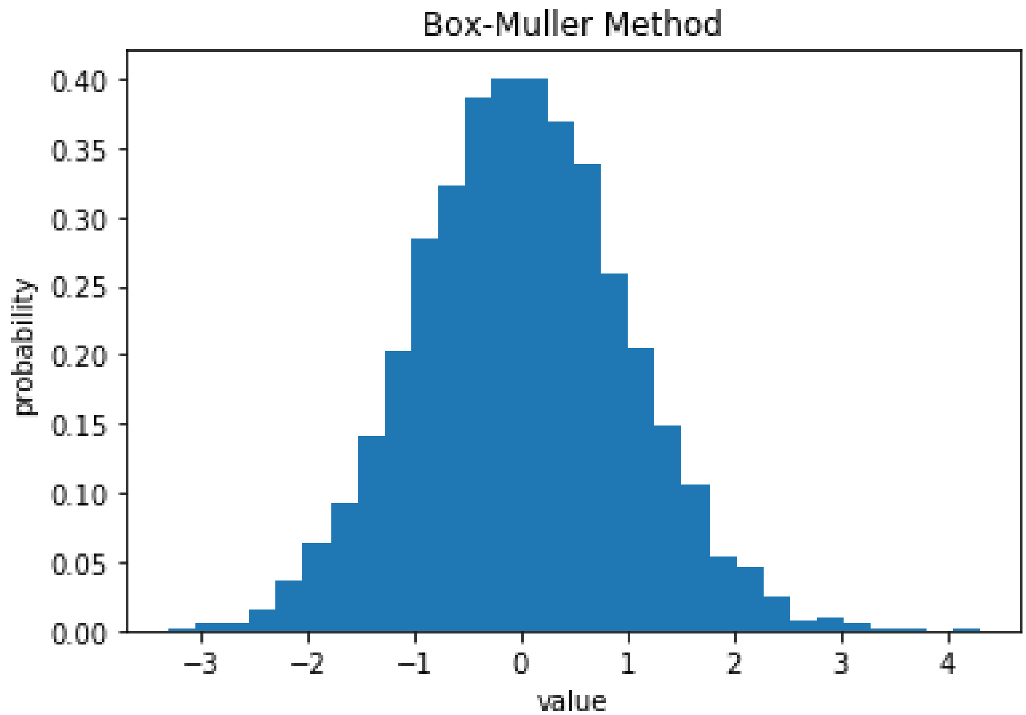
(b)Generate 5,000 Normally distributed numbers by Box-Muller Method

```
In [19]: #Creat 2 lists with length 2500 each
         #These 2 lists will be used to store normally distributed random numbers
         Z_1 = [None]*2500
         Z_2 = [None]*2500

         #Here we used Box-Muller Method
         for i in range(2500):
             Z_1[i] = np.sqrt(-2*np.log(U_5[2*i]))*np.cos(2*math.pi*U_5[2*i+1])
             Z_2[i] = np.sqrt(-2*np.log(U_5[2*i]))*np.sin(2*math.pi*U_5[2*i+1])
             i = i + 1

         #Combine 2 lists to get a normal distribution with 5000 observations
         Z_BM = Z_1+Z_2
```

(c)Compute empirical mean and standard deviation of above distribution, also generated histogram

```
In [20]: mu_BM = np.mean(Z_BM)
         std_BM = np.std(Z_BM)

         print("Mox-Muller method: mean =", mu_BM, ", std =", std_BM)

Mox-Muller method: mean = 0.015866199631777707 , std = 0.9904473293620408
```

```
In [21]: #Draw the histogram
         plt.figure()
         ax5 = plt.hist(Z_BM, normed = True, bins = 30)
         plt.title("Box-Muller Method")
         plt.ylabel("probability")
         plt.xlabel("value")

Out[21]: Text(0.5, 0, 'value')
```



(d)Generate 5,000 Normally distributed numbers by Polar-Marsaglia Method

```
In [22]: #Here we used Polar-Marsaglia Method
         Z_1_1 = [None]*2500
         Z_2_2 = [None]*2500
         for i in range(2500):
             V_1 = 2*U_5[2*i]-1
             V_2 = 2*U_5[2*i+1]-1
             W = V_1**2+V_2**2
             #drop V_1 and V_2 if W <= 1
             if W <= 1:
                 Z_1_1[i] = np.sqrt((-2*np.log(W))/W)*V_1
                 Z_2_2[i] = np.sqrt((-2*np.log(W))/W)*V_2
         Z_PM = Z_1_1 + Z_2_2
         Z_PM = [x for x in Z_PM if x != None]
         #Notice that this method will generate less than 5000 observations
```

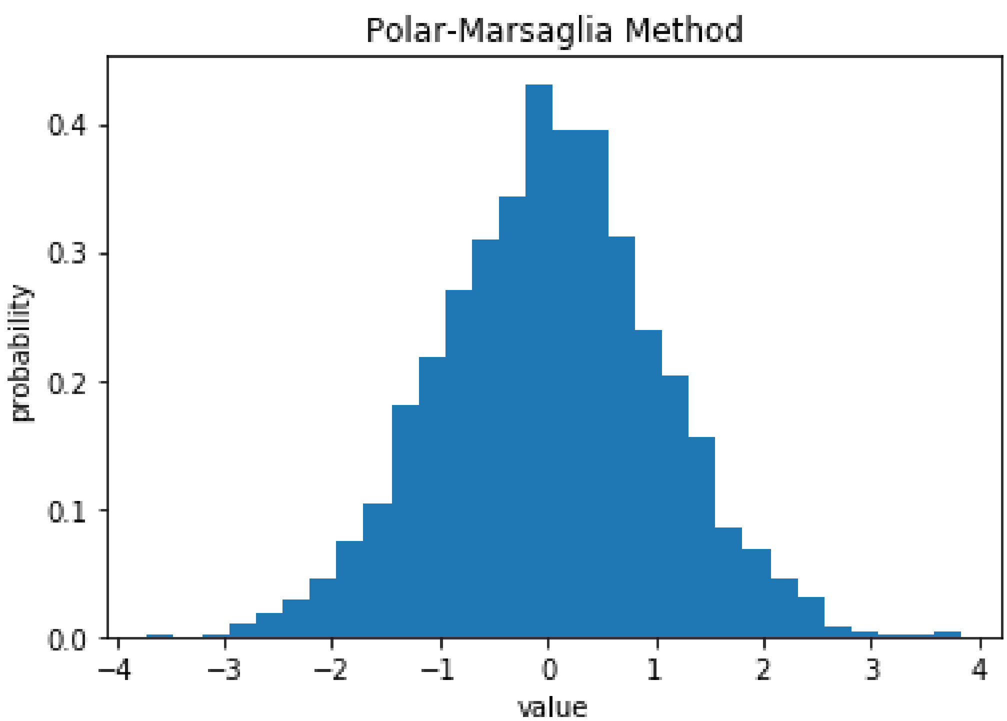
(e)Compute empirical mean and standard deviation of above distribution, also generated histogram

```
In [23]: mu_PM = np.mean(Z_PM)
         std_PM = np.std(Z_PM)

         print("Polar-Marsaglia method: mean =", mu_PM, ", std =", std_PM)
```

```
Polar-Marsaglia method: mean = 0.025241381616694033 , std = 1.0109944380282085
```

```
In [24]: #Draw the histogram
         plt.figure()
         ax6 = plt.hist(Z_PM, normed = True, bins = 30)
         plt.title("Polar-Marsaglia Method")
         plt.ylabel("probability")
         plt.xlabel("value")
```

```
Out[24]: Text(0.5, 0, 'value')
```



(f)Compare execution time for using different methods

In order to compare execution time better, we will generate 2 normal distributions by using different methods with the same sample size.

```
In [73]: #Now we want to compare time efficiency of the two methods

         #The following uniform distribution is used to generate
         #2 normal distributions each with 5000 observations
         #by using different methods
         U_5_test = f_unif(10000,2)

         Z_1_test = [None]*2500
         Z_2_test = [None]*2500
```

```
#Box-Muller Method
#Record starting time
start_time1 = time.time()
for i in range(2500):
    Z_1_test[i] = np.sqrt(-2*np.log(U_5_test[2*i]))* \
                  np.cos(2*math.pi*U_5_test[2*i+1])
    Z_2_test[i] = np.sqrt(-2*np.log(U_5_test[2*i]))* \
                  np.sin(2*math.pi*U_5_test[2*i+1])
    i = i + 1

#Record ending time and compute time used
time_1 = (time.time() - start_time1)
print("--- %s seconds ---" % time_1)

Z_1_1_test = [None]*2500
Z_2_2_test = [None]*2500

#Record starting time
start_time2 = time.time()
x = 0
for i in range(10000):
    V_1 = 2*U_5_test[2*i]-1
    V_2 = 2*U_5_test[2*i+1]-1
    W = V_1**2+V_2**2
    if W <= 1:
        Z_1_1_test[x] = np.sqrt((-2*np.log(W))/W)*V_1
        Z_2_2_test[x] = np.sqrt((-2*np.log(W))/W)*V_2
        x = x + 1
    #when Z_1_1_test and Z_2_2_test both contain 2500 observations, exit loop
    if x > 2499:
        break

#Record ending time and compute time used
time_2 = (time.time() - start_time2)
print("--- %s seconds ---" % time_2)

print("Box-Muller takes", time_1, "seconds.")
print("Polar-Marsaglia takes", time_2, "seconds.")
```

```
--- 0.020933866500854492 seconds ---
--- 0.018950223922729492 seconds ---
Box-Muller takes 0.020933866500854492 seconds.
Polar-Marsaglia takes 0.018950223922729492 seconds.


D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:9: RuntimeWarning: o
  if __name__ == '__main__':
```

It looks like Box-Muller method takes longer time than Polar-Marsaglia method does. Thus, the Polar-Marsaglia method is more efficient. The reseaon is that the Box-Muller method uses trigonometirc functions to generate randon number while the Polar-Marsaglia method only need to do some simple calculation. Trigonometric functions are known to be time-consumming functions.