Project 3 Jiaqi Li

January 31, 2019

Followings are functions from previous project:

```
In [62]: import numpy as np
         import math
         import matplotlib.pyplot as plt
         import scipy.stats as bs

         a = 7**5
         b = 0
         m = 2**31-1

         #This function generates uniform distribution
         def f_unif(n,x_0):
             U = [None] * (n+1)
             U[0] = x_0
             for i in range(1,(n+1)):
                 U[i] = np.mod(a*U[i-1]+b,m)
             del U[0]
             U = [x/m for x in U]
             return U

         #This function generates normal distribution
         def f_norm(n,U):
             Z_1 = [None]*n
             Z_2 = [None]*n
             for i in range(n):
                 Z_1[i] = np.sqrt(-2*np.log(U[2*i]))*np.cos(2*math.pi*U[2*i+1])
                 Z_2[i] = np.sqrt(-2*np.log(U[2*i]))*np.sin(2*math.pi*U[2*i+1])
                 i = i + 1
             return Z_1+Z_2

         #This function generates a brownian motion
         def f_w(T,Z):
             W = [np.sqrt(T)*x for x in Z]
             return W
```

1 Problem 1

The following functions are used for computing probabilities:

```
In [65]: n = 1000
         x = 500
         Y0 = 3/4
         X0 = 1

         def f_YP(n,x,dt,Y0,k):
             count = 0
             All = [None]*100
             for j in range(100):
                 U = f_unif(n,j+10)
                 Z = f_norm(x,U)
                 W = f_w(dt,Z)
                 Y = [None]*(n+1)
                 Y[0] = Y0
                 t = dt
                 for i in range(n):
                     Y[i+1] = Y[i] + (2/(1+t)*Y[i] + (1+t**3)/3)*dt+(1+t**3)/3*(W[i])
                     t = t + dt
                 if Y[n] > k:
                     count = count + 1
                 All[j] = Y[n]
             out = [All,count]
             return out

         def f_XV(n,x,dt,X0,k):
             All = [None]*100
             for j in range(100):
                 U = f_unif(n,j+100)
                 Z = f_norm(x,U)
                 W = f_w(dt,Z)
                 X = [None]*(n+1)
                 X[0] = X0
                 for i in range(n):
                     X[i+1] = X[i] + (1/5 - 1/2*X[i])*dt+2/3*(W[i])
                 All[j] = np.sign(X[n])*np.absolute(X[n])**k
             return All
```

Each of the simulatoins runs 10^5 calculations, thus, it takes a litile bit time to run each simulation

```
In [70]: dt2 = 2/n
         P_Y2 = f_YP(n,x,dt2,Y0,5)[1]/100
         print(P_Y2)
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:15: RuntimeWarning: o
  from ipykernel import kernelapp as app
```

```
0.98
```

$P(Y_2 > 5) \approx 0.98$

```
In [67]: dt3 = 3/n
         E_Y3 = np.mean(f_YP(n,x,dt3,Y0,10000)[0])
         print(E_Y3)
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:15: RuntimeWarning: o
  from ipykernel import kernelapp as app
```

```
26.38584746299788
```

$E(Y_3) \approx 26.38$

```
In [68]: E_X2_13 = np.mean(f_XV(n,x,dt2,X0,1/3))
         print(E_X2_13)
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:15: RuntimeWarning: o
  from ipykernel import kernelapp as app
```

```
0.6555430986101816
```

$E(X_2^{\frac{1}{3}}) \approx 0.65$

```
In [69]: X2 = f_XV(n,x,dt2,X0,1)
         Y2 = f_YP(n,x,dt2,Y0,10000)[0]
         X2g1 = [None]*100
         for i in range(100):
             if X2[i] > 1:
                 X2g1[i] = 1
             else:
                 X2g1[i] = 0

         E = np.mean([a*b*c for a,b,c in zip(X2,Y2,X2g1)])
         print(E)
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:15: RuntimeWarning: o
  from ipykernel import kernelapp as app
```

```
3.9432667688269656
```

$E(X_2 Y_2 1(X_2 > 1)) \approx 3.94$

2 Problem 2

In [9]:
```
#2---------------------------------------------------
def f_XV2(n,x,dt,X0,k):
    All = [None]*100
    for j in range(100):
        U1 = f_unif(n,j+1000)
        U2 = f_unif(n,j+2000)
        Z1 = f_norm(x,U1)
        Z2 = f_norm(x,U2)
        W1 = f_w(dt,Z1)
        W2 = f_w(dt,Z2)
        X = [None]*(n+1)
        X[0] = X0
        for i in range(n):
            X[i+1] = X[i] + 1/4*X[i]*dt + 1/3*X[i]*W1[i] - 3/4*X[i]*W2[i]
        All[j] = np.sign(1+X[n])*np.absolute(1+X[n])**k
    return All

EX3_2 = np.mean(f_XV2(n,x,dt3,X0,1/3))
print(EX3_2)
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:16: RuntimeWarning: c
  app.launch_new_instance()


1.3574080066094518
```

$E(1+X_3)^{\frac{1}{3}} \approx 1.36$

In [11]:
```
U1 = f_unif(n,1)
U2 = f_unif(n,2)
Z1 = f_norm(x,U1)
Z2 = f_norm(x,U2)
W1 = f_w(3,Z1)
W2 = f_w(3,Z2)
Y3 = [np.exp(-0.08*3+1/3*a+3/4*b) for a,b in zip(W1,W2)]
EY3_2 = [(1+y)**(1/3) for y in Y3]
EY3_2 = np.mean(EY3_2)
print(EY3_2)
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:16: RuntimeWarning: c
  app.launch_new_instance()


1.3421793474167587
```

$E(1+Y_3)^{\frac{1}{3}} \approx 1.34$

3 Problem 3

(a) pricing function for European Option

In [14]:
```
#3-------------------------------------------
#a
def Euro_Call(S0,T,X,r,sigma):
    n = 1000
    u = f_unif(n,1234)
    z1 = f_norm(500,u)
    z2 = [-x for x in z1]
    w1 = f_w(T,z1)
    w2 = f_w(T,z2)
    x1 = S0*np.exp(sigma*np.array(w1)+(r-(sigma**2/2))*T)
    x2 = S0*np.exp(sigma*np.array(w2)+(r-(sigma**2/2))*T)
    ST_red1 = [None]*n
    ST_red2 = [None]*n
    for i in range(n):
        if (x1[i]-X) > 0:
            ST_red1[i] = x1[i]-X
        else:
            ST_red1[i] = 0
        if (x2[i]-X) > 0:
            ST_red2[i] = x2[i]-X
        else:
            ST_red2[i] = 0
    c_red1 = np.array(ST_red1)*np.exp(-r*T)
    c_red2 = np.array(ST_red2)*np.exp(-r*T)
    c_red = np.mean((np.array(c_red1)+np.array(c_red2))/2)
    return c_red
```

(b) Black-Scholes formula pricing function

In [15]:
```
#b
def f_N(x):
    d1 = 0.0498673470
    d2 = 0.0211410061
    d3 = 0.0032776263
    d4 = 0.0000380036
    d5 = 0.0000488906
    d6 = 0.0000053830
    if x >= 0:
        N = 1 - 1/2*(1+d1*x+d2*x**2+d3*x**3 \
                          +d4*x**4+d5*x**5+d6*x**6)**(-16)
    else:
        N = 1 - (1 - 1/2*(1+d1*(-x)+d2*(-x)**2 \
                               +d3*(-x)**3+d4*(-x)**4+d5*(-x)**5+d6*(-x)**6)**(-16))
    return N
```

```python
def B_S(S0,T,X,r,sigma):
    d1 = (np.log(S0/X) + (r+(sigma**2)/2)*T)/(sigma*np.sqrt(T))
    d2 = d1 - sigma*np.sqrt(T)
    C = S0*f_N(d1) - np.exp(-r * T)*X*f_N(d2)
    return C
```

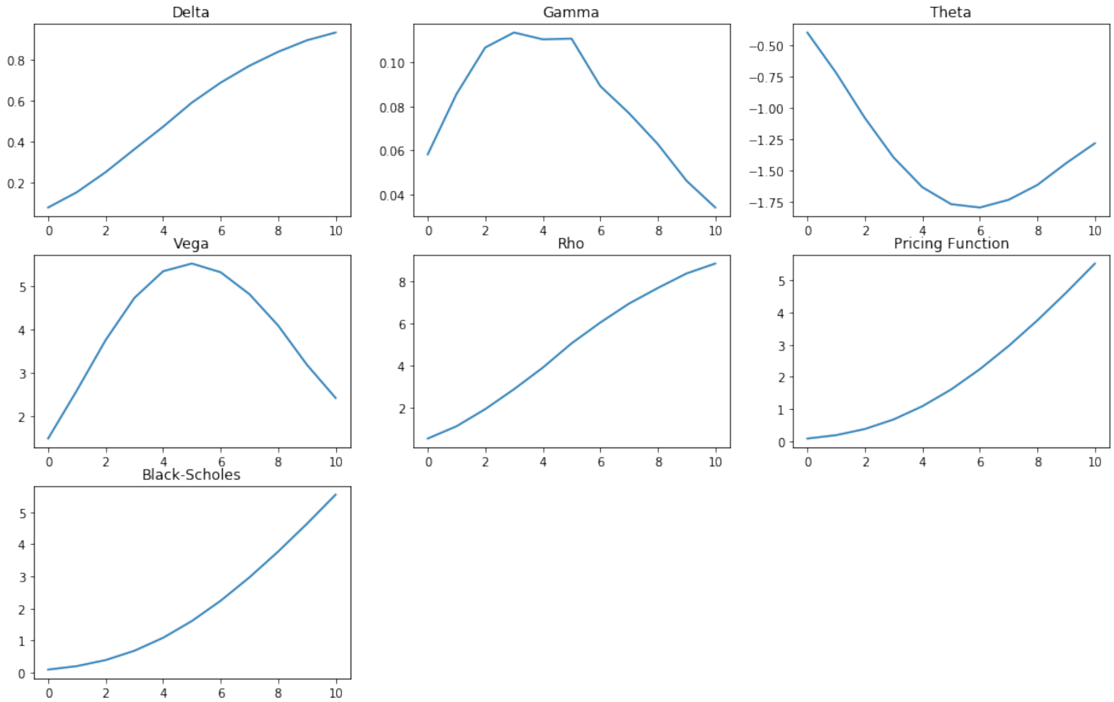
(c) five greeks for prices from 15 to 25

In [24]:
```python
#c
S0 = np.array([i for i in range(15,26)])
T = 0.5; X = 20; r = 0.04; sigma = 0.25
d1 = (np.log(S0/X) + (r+(sigma**2)/2)*T)/(sigma*np.sqrt(T))
d2 = d1 - sigma*np.sqrt(T)

Delta = [None]*11; Gamma = [None]*11; Theta = [None]*11; Vega = [None]*11;
Rho = [None]*11; ECp = [None]*11; ECn = [None]*11; EC = [None]*11;
C1 = [None]*11; C2 = [None]*11
for i in range(11):
    C1[i] = Euro_Call(S0[i],T,X,r,sigma)
    C2[i] = B_S(S0[i],T,X,r,sigma)
    Delta[i] = (Euro_Call(S0[i]+0.1,T,X,r,sigma) - Euro_Call(S0[i],T,X,r,sigma))/0.1
    ECp[i] = Euro_Call(S0[i]+1,T,X,r,sigma)
    ECn[i] = Euro_Call(S0[i]-1,T,X,r,sigma)
    Gamma[i] = (ECp[i] + ECn[i] - 2*np.array(C1[i]))/(1**2)
    Theta[i] = (Euro_Call(S0[i],T,X,r,sigma)-Euro_Call(S0[i],T+0.01,X,r,sigma))/0.01
    Vega[i] = (Euro_Call(S0[i],T,X,r,sigma+0.01)-Euro_Call(S0[i],T,X,r,sigma))/0.01
    Rho[i] = (Euro_Call(S0[i],T,X,r+0.001,sigma)-Euro_Call(S0[i],T,X,r,sigma))/0.001
plt.figure(1, figsize=(16, 10))
plt.subplot(331)
ax1 = plt.plot(Delta)
plt.title("Delta")
plt.subplot(332)
ax2 = plt.plot(Gamma)
plt.title("Gamma")
plt.subplot(333)
ax3 = plt.plot(Theta)
plt.title("Theta")
plt.subplot(334)
ax4 = plt.plot(Vega)
plt.title("Vega")
plt.subplot(335)
ax5 = plt.plot(Rho)
plt.title("Rho")
plt.subplot(336)
ax6 = plt.plot(C1)
plt.title("Pricing Function")
plt.subplot(337)
ax7 = plt.plot(C2)
plt.title("Black-Scholes")
```

```
D:\anaconda_distribution\Anaconda\lib\site-packages\ipykernel_launcher.py:16: RuntimeWarning: c
  app.launch_new_instance()
```

Out[24]: `Text(0.5, 1.0, 'Black-Scholes')`



In [29]:
```
print("Prices of options with underlying stocks from $15 to $25 computed by pricing f
print(np.round(C1,4))
```

```
Prices of options with underlying stocks from $15 to $25 computed by pricing function are:
[0.0738 0.1796 0.3711 0.6693 1.0813 1.6037 2.2371 2.9596 3.7591 4.6215
 5.53  ]
```

In [30]:
```
print("Prices of options with underlying stocks from $15 to $25 computed by Black-Sch
print(np.round(C2,4))
```

```
Prices of options with underlying stocks from $15 to $25 computed by Black-Scholes are:
[0.0858 0.1943 0.3828 0.6732 1.0787 1.6016 2.2345 2.963  3.7695 4.6362
 5.5471]
```

In [32]:
```
print("Deltas of options with underlying stocks from $15 to $25 are:")
print(np.round(Delta,4))
```

```
Deltas of options with underlying stocks from $15 to $25 are:
[0.0762 0.151  0.2487 0.3605 0.4706 0.5883 0.6856 0.7679 0.8361 0.8922
 0.9304]
```

```
In [33]: print("Gamma of options with underlying stocks from $15 to $25 are:")
         print(np.round(Gamma,4))
```

```
Gamma of options with underlying stocks from $15 to $25 are:
[0.0581 0.0856 0.1068 0.1137 0.1105 0.1109 0.0893 0.0769 0.0629 0.0462
 0.034 ]
```

```
In [34]: print("Theta of options with underlying stocks from $15 to $25 are:")
         print(np.round(Theta,4))
```

```
Theta of options with underlying stocks from $15 to $25 are:
[-0.4    -0.7225 -1.0832 -1.3994 -1.6372 -1.7727 -1.7995 -1.7377 -1.619
 -1.445  -1.2856]
```

```
In [35]: print("Vega of options with underlying stocks from $15 to $25 are:")
         print(np.round(Vega,4))
```

```
Vega of options with underlying stocks from $15 to $25 are:
[1.4969 2.6006 3.755  4.7215 5.3338 5.5103 5.3101 4.8077 4.0878 3.189
 2.4262]
```

```
In [36]: print("Rho of options with underlying stocks from $15 to $25 are:")
         print(np.round(Rho,4))
```

```
Rho of options with underlying stocks from $15 to $25 are:
[0.4998 1.0852 1.9036 2.8523 3.8806 5.0375 6.0316 6.9364 7.679  8.3736
 8.8472]
```

4 Problem 4

```
In [41]: #4----------------------------------
         def f_corr2W(n,x,var,rho):
             covM = np.array([[1,rho],[rho,1]])
             L = np.linalg.cholesky(covM)
             N1 = np.random.normal(0,1,n)
             N2 = np.random.normal(0,1,n)
             dWt1 = np.sqrt(var[0])*L[0,0]*N1
             dWt2 = np.sqrt(var[1])*(L[1,0]*N1 + L[1,1]*N2)
             r = [dWt1,dWt2]
             return r
```

```python
def f_max(x):
    if x > 0:
        r = x
    else:
        r = 0
    return r

def f_C(n,x,T,V0,S0,K,rho,scheme = None):
    dt = T/n
    C = [None]*1000
    var = [dt,dt]
    for j in range(1000):
        W = f_corr2W(n,x,var,rho)
        Wt1 = W[0]
        Wt2 = W[1]
        v = [None]*(n+1)
        v[0] = V0
        s = [None]*(n+1)
        s[0] = S0
        if scheme == "Reflection":
            for i in range(n):
                s[i+1] = s[i] + r*s[i]*dt + np.sqrt(np.absolute(v[i]))*s[i]*Wt1[i]
                v[i+1] = np.absolute(v[i]) \
                        + a*(b-np.absolute(v[i]))*dt \
                        + sigma*np.sqrt(np.absolute(v[i]))*Wt2[i]
        elif scheme == "Partial":
            for i in range(n):
                s[i+1] = s[i] + r*s[i]*dt + np.sqrt(f_max(v[i]))*s[i]*Wt1[i]
                v[i+1] = v[i] + a*(b-v[i])*dt \
                        + sigma*np.sqrt(f_max(v[i]))*Wt2[i]
        elif scheme == "Full":
            for i in range(n):
                s[i+1] = s[i] + r*s[i]*dt + np.sqrt(f_max(v[i]))*s[i]*Wt1[i]
                v[i+1] = v[i] + a*(b-f_max(v[i]))*dt \
                        + sigma*np.sqrt(f_max(v[i]))*Wt2[i]
        else:
            for i in range(n):
                s[i+1] = s[i] + r*s[i]*dt + np.sqrt(v[i])*s[i]*Wt1[i]
                v[i+1] = v[i] + a*(b-v[i])*dt \
                        + sigma*np.sqrt(v[i])*Wt2[i]
                if v[i+1] < 0:
                    print("Warning: Encounter negative Vt value!")
                    print("         Use other scheme!")
                    return np.nan
        if (s[n] - K) > 0:
            C[j] = (s[n] - K)*np.exp(-r*T)
        else:
```

```
                C[j] = 0
        return C
```

Aboves are functions that can compute the price of an European Call Option with different methods (Reflection,Partial Truncation, Full Truncation)

```
In [56]: rho = -0.6; r = 0.03; S0 = 48; V0 = 0.05; sigma = 0.42
         a = 5.8; b = 0.0625; T = 0.5; K = 50
         n = 1000; x = 500
         C_ref = f_C(n,x,T,V0,S0,K,rho,scheme = "Reflection")
         C_part = f_C(n,x,T,V0,S0,K,rho,scheme = "Partial")
         C_full = f_C(n,x,T,V0,S0,K,rho,scheme = "Full")
         print(np.round([np.mean(C_ref),np.mean(C_part),np.mean(C_full)],3))
```

```
[2.584 2.635 2.762]
```

Since I used different seeds to compute the prices each time, the prices generated by each method are slightly different. Even though prices are different by using different methods, they are very similar compared to each other. This is because with T = 0.5, all dVt could be positive and therefore, each method will generate similar price for the option.

Method	Option Price
Reflection	2.584
Partial Truncation	2.635
Full Truncation	2.762

5 Problem 5

Following is the Halton Sequence generation function:

```
In [47]: def f_HaltonS(base,n):
             seq = np.zeros(n)
             bits = 1+math.ceil(np.log(n)/np.log(base))
             bs = np.array([i+1 for i in range(bits)])
             b = 1/(base**bs)
             d = np.zeros(bits)
             for i in range(n):
                 j = 0; ok = 0
                 while ok == 0:
                     d[j] = d[j]+1
                     if d[j] < base:
                         ok = 1
                     else:
                         d[j] = 0; j = j+1
                 seq[i] = np.dot(d,b)
             return  seq
```

(a) generate 100 2-dimensional vectors of Uniform [0,1] × [0,1]

```
In [48]: #5-------------------------------
         #a
         X = f_unif(100,1)
         Y = f_unif(100,10)
         unif2d = [X,Y]
```

(b) generate 100 points of the 2-dimensional Halton sequences, using bases 2 and 7

```
In [49]: x_h_2 = f_HaltonS(2,100)
         y_h_7 = f_HaltonS(7,100)
         halton27 = [x_h_2,y_h_7]
```

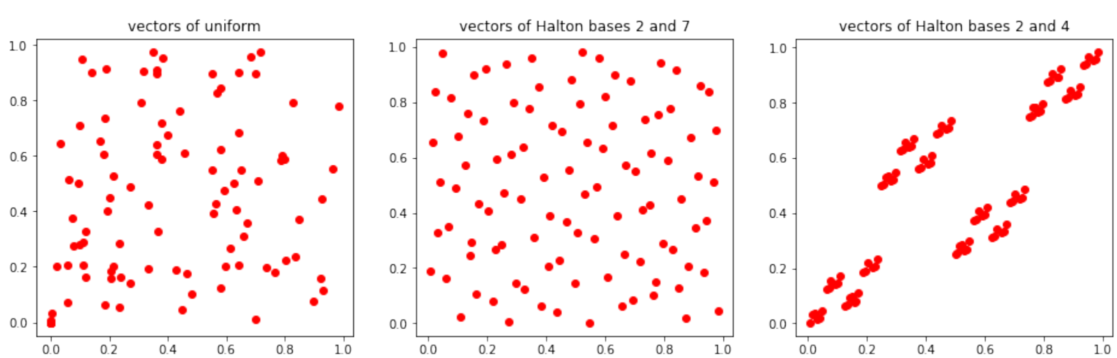
(c) generate 100 points of the 2-dimensional Halton sequences, using bases 2 and 4

```
In [50]: y_h_4 = f_HaltonS(4,100)
         halton24 = [x_h_2,y_h_4]
```

(d) Draw all 3 sequences generated above:

```
In [52]: plt.figure(2, figsize=(16, 10))
         plt.subplot(231)
         ax10 = plt.plot(unif2d[0],unif2d[1],"ro")
         plt.title("vectors of uniform")
         plt.subplot(232)
         ax11 = plt.plot(halton27[0],halton27[1],"ro")
         plt.title("vectors of Halton bases 2 and 7")
         plt.subplot(233)
         ax12 = plt.plot(halton24[0],halton24[1],"ro")
         plt.title("vectors of Halton bases 2 and 4")
```

```
Out[52]: Text(0.5, 1.0, 'vectors of Halton bases 2 and 4')
```



Comment: The first plot looks like there are some clusters of points, which makes the distribution of the points not very uniform. The second plot looks much more better than the first plot and its distributions looks uniform. The last plot looks terrible and points are barely uniformly distributed. The reason is that the Halton sequences generated are based on bases 2 and 4 where 4 is a non-prime number. This problem makes the points in the third plot cluster togehter since 2 and 4 are both multiples of 2.

(e) compute the following integral by using Halton sequences:

$$\int_0^1 \int_0^1 e^{-xy}(sin6\pi x + cos^{\frac{1}{3}}2\pi y)dxdy$$

```
In [54]: N = 10000
         x2 = f_HaltonS(2,N)
         y4 = f_HaltonS(4,N)
         y7 = f_HaltonS(7,N)
         x5 = f_HaltonS(5,N)
         integral24 = [None]*N
         integral27 = [None]*N
         integral57 = [None]*N
         for i in range(N):
             integral24[i] = np.exp(-x2*y4)*(np.sin(6*np.pi*x2) \
                             + np.sign(np.cos(2*np.pi*y4)) \
                             * np.absolute(np.cos(2*np.pi*y4))**(1/3))
             integral27[i] = np.exp(-x2*y7)*(np.sin(6*np.pi*x2) \
                             + np.sign(np.cos(2*np.pi*y7)) \
                             * np.absolute(np.cos(2*np.pi*y7))**(1/3))
             integral57[i] = np.exp(-x5*y7)*(np.sin(6*np.pi*x5) \
                             + np.sign(np.cos(2*np.pi*y7)) \
                             * np.absolute(np.cos(2*np.pi*y7))**(1/3))
         I24 = np.mean(integral24)
         I27 = np.mean(integral27)
         I57 = np.mean(integral57)
         print(np.round([I24,I27,I57],5))
```

```
[-0.00488  0.02611  0.02616]
```

Base	Integral value
(2,4)	-0.00488
(2,7)	0.02611
(5,7)	0.02616