**MASTER OF FINANCIAL ENGINEERING**
**UCLA Anderson School**
**Credit Risk**
**Prof. Holger Kraft**
**Problem Set 2**
**Due: Oct 10, 3pm (before TA session)**

**Problem 2 (Term Structure)** Suppose that the following prices of Treasury bonds are given ($N$: notional, $K$: coupon, $T$: time to maturity):

| | $T$ | Price | $K$ | $N$ |
|---|---|---|---|---|
| 1st Bond | 1 | 99.00 | 2.0 | 100 |
| 2nd Bond | 2 | 98.00 | 2.5 | 100 |
| 3rd Bond | 3 | 97.50 | 3.0 | 100 |

(a) Calculate the spot rates, $r(1)$, $r(2)$, and $r(3)$.

(b) What kind of term structure do you get (flat, normal, inverse)?

(c) A fourth bond is given by

| | $T$ | Price | $K$ | $N$ |
|---|---|---|---|---|
| 4th Bond | 3 | ??? | 4.0 | 100 |

Calculate the fair price of this bond.

**Problem 3 (Bond Spread Components)**

A firm has issued several bonds. Some of them are more liquid, others are less liquid. The following bond prices are given (N: notional, K: coupon, T: time to maturity).

| T | $\text{price}_{liquid}$ | $\text{price}_{illiquid}$ | K | N |
|---|---|---|---|---|
| 1 | 99 | 98 | 1 | 100 |
| 2 | 98 | 97 | 2 | 100 |
| 3 | 98.5 | 96.5 | 3 | 100 |

The column $\text{price}_{liquid}$ contains the prices of the liquid bonds. We assume that the liquid bonds are as liquid as Treasury bonds. The column $\text{price}_{illiquid}$ contains the prices of the bonds that are less liquid than Treasury bonds. For simplicity, assume that coupons are paid annually.

(a) Compute the zero rates (aka spot rates), $r_0(1)$ and $r_0(2)$ and $r_0(3)$, of the liquid and illiquid bonds.

(b) Which kind of term structure is generated by the liquid bonds?

(c) Calculate the prices of the following bonds:

| T | $\text{price}_{liquid}$ | $\text{price}_{illiquid}$ | K | N |
|---|---|---|---|---|
| 3 | ? | ? | 4 | 100 |

(d) The term structure of Treasury bonds is given by $r_0(1) = 1\%$, $r_0(2) = 2\%$, and $r_0(3) = 2.5\%$. Decompose the spreads of the illiquid bonds into a credit component and a liquidity component and express values of the components in basis points (bps).

**Problem 4 (Firm Value Model)**

We consider a firm that has only issued equity in the form of common stock and debt in the form of a zero-coupon bond. The zero-coupon bond has a face value of 100 million dollars. The firm value is currently 140 million dollars and the asset volatility $\sigma$ is fixed at 20%. For simplicity, the default-free interest rate is assumed to be 0%. Finally, assume that the firm value follows a geometric Brownian motion (Merton model).

(a) Calculate the value of the bond if the bond matures in 9 years.

(b) What is the spread of the bond?

(c) Calculate the value of equity.

(d) Apply Ito's lemma and derive the formula for the volatility of equity. Compute the volatility for setting above. Is the volatility of equity bigger or smaller than the volatility of the assets? Why?

(e) What is the spread of the bond if the bond's time to maturity goes to zero? Explain your answer by using the pricing formula for debt.

(f) Why is this an issue in practice?

**Hints.** Here are some values of the cdf of the standard-normal distribution

| $d_1$ | $d_2$ | $N(-d_1)$ | $N(d_2)$ |
|---|---|---|---|
| 2.127 | 1.927 | 0.017 | 0.973 |
| 1.575 | 1.292 | 0.058 | 0.902 |
| 1.344 | 0.997 | 0.090 | 0.841 |
| 1.214 | 0.814 | 0.112 | 0.792 |
| 1.130 | 0.683 | 0.129 | 0.753 |
| 1.073 | 0.583 | 0.142 | 0.720 |
| 1.031 | 0.502 | 0.151 | 0.692 |
| 1.000 | 0.434 | 0.159 | 0.668 |
| 0.976 | 0.376 | 0.165 | 0.646 |
| 0.957 | 0.325 | 0.169 | 0.627 |

**Problem 5 (Equity and Debt)**

We assume that a firm is financed by equity (stocks) and debt (one zero-coupon bond with maturity 1y and notional 100 million dollars). Equity consists of 1 million shares of stock that are currently selling at 18 dollars each. In one year, two scenarios can occur:

1. The firm redeems the zero-coupon bond ("no default"): Then the value of equity is assumed to be 20 million dollars.
2. The firm defaults on its debt ("default"): Then equity is worthless and the recovery rate on debt is assumed to be 60%.

Additionally, a (default-free) Treasury bond with maturity 1y and notional 100 dollars is currently selling at 95 dollars.

(a) Calculate the risk-neutral probabilities of survival and default.

(b) What is the current debt value?

(c) Compute the current firm value and the firm values in both future scenarios.

(d) What is the default-free interest rate for one year?

(e) How big is the current spread between the Treasury bond and the corporate bond?

**Problem 6 (Default Risk)**

Assume that there are three traded securities in the market: A risk-free treasury zero-coupon bond with a notional of 1 dollar and a current price of 1 dollar, a risky corporate zero-coupon bond with a notional of 1 dollar and a current price of 96 cents, and stocks with a current price of 100 dollars. The stocks and corporate bonds are issued by the same firm. In one year, there are three possible scenarios (boom, stagnation, recession). In the first and second scenario, the company does not default and repays the bond. The stock price is assumed to be 120 dollars in the first scenario and 80 dollars in the second scenario. In the third scenario, the company defaults and 50% of the notional of the corporate bond is recovered. The stock price is zero.

(a) What is the risk-free interest rate?

(b) Calculate the yield and the spread of the corporate zero-coupon bond?

(c) What are the risk-neutral probabilities for the three scenarios?

(d) Assume that you hold the corporate bond. What credit derivative can eliminate the default risk from your position in the corporate bond? What is the payoff structure of this product in our one-period model?

(e) Calculate the price of this security, i.e. the fair upfront payment. Assume that there are no running payments.

(f) Compare the yield of the corporate bond with the upfront payment from (e). What do you realize?

**Problem 7 (Default and Duration)**

We consider a corporate zero-coupon bond that matures in one year. Assume for simplicity that the risk-free interest rate $r$, the default intensity and the recovery rate of the company are constant. Besides, suppose that we can write the default intensity as

$$\lambda = a + br,$$

where $a$ and $b$ are constants.

(a) Which signs of $a$ and $b$ do you expect? Explain your answer.

(b) Write down the corporate bond price assuming

   (i) recovery of market value and

   (ii) recovery of Treasury (aka recovery of par at maturity).

The sensitivity of a Treasury bond w.r.t. interest rate changes is defined as

$$D = -\frac{\frac{\partial p(0,1)}{\partial r_0}}{p(0,1)}$$

where $r_0$ is the initial value of the short rate.

(c) Calculate the corresponding sensitivity of the corporate bond for the models in (b).

(d) Is the sensitivity of a corporate bond bigger or smaller than the sensitivity of a Treasury bond with the same maturity? Why? Which parameter drives the results?

(e) Relate your findings to the interpretation of the classical duration (Macauley duration).