hw1 Jiaqi

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Problem 1

1.

Let $Y = \delta_t B_t$, then we can derive that:

$$\begin{aligned}
P(Y \geq y) &= P(\delta_t B_t \geq y) \\
&= P(B_t = 1)P(\delta_t B_t \geq y | B_t = 1) + P(B_t = 0)P(\delta_t B_t \geq y | B_t = 0) \\
&= pP(\delta_t \geq y)
\end{aligned}$$

Since $\delta_t \sim N(0, 1)$, $Y = \delta_t B_t \sim N(0, p)$.

We also have:

$$\begin{gathered}
E(\delta_t^3) = E(\epsilon_t^3) = 0 \\
E(\delta_t^4) = E(\epsilon_t^4) = 3 \\
E((\delta_t B_t)^3) = 0, E((\delta_t B_t)^4) = 3p
\end{gathered}$$

Calculate mean, vairnace, skewness, and kurtosis:

$$\begin{aligned}
r_t &= \mu + \sigma\epsilon_t + B_t(\mu_J + \sigma_J \delta_t) \\
&= \mu + \sigma\epsilon_t + B_t\mu_J + B_t\sigma_J\delta_t
\end{aligned}$$

$$\begin{aligned}
E(r_t) &= E(\mu + \sigma\epsilon_t + B_t\mu_J + B_t\sigma_J\delta_t) \\
&= \mu + 0 + \mu_J p + 0 = \mu + \mu_J p
\end{aligned}$$

$$\begin{aligned}
Var(r_t) &= Var(\mu + \sigma\epsilon_t + B_t\mu_J + B_t\sigma_J\delta_t) \\
&= \sigma^2 + \mu_J^2 p(1-p) + \sigma_J^2 p^2
\end{aligned}$$

$$\begin{aligned}
S(r_t) &= S(\mu + \sigma\epsilon_t + B_t\mu_J + B_t\sigma_J\delta_t) \\
&= 0 + 0 + \mu_J^3 \frac{1-2p}{p(1-p)} + 0 \text{ where } E((\delta_t B_t)^3) = 0 \\
&= \mu_J^3 \frac{1-2p}{p(1-p)}
\end{aligned}$$

$$\begin{aligned}
K(r_t) - 3 &= K(\mu + \sigma\epsilon_t + B_t\mu_J + B_t\sigma_J\delta_t) - 3 \\
&= 3\sigma^2 + \mu_J^4 \frac{3p^2 - 3p + 1}{p(1-p)} + 3p\sigma_j^4 - 3 \text{ where } E((\delta_t B_t)^4) = 3p
\end{aligned}$$

2.

For daily log-returns, data is skewed and has high kurtosis. Heterodasticity is also a problem. Thus, a simple log normal model is not a good model in this situation because of violation of assumptions.

In the Bernoulli-normal model, a jump is introduced into the model, which helps improve the model.

3.

```
sim = rnorm(600,0.008,0.063)

mu = 0.012; sigma = 0.05; p = 0.15; mu_j = -0.03; sigma_j = 0.1
epsilon = rnorm(600,0,1)
delta = rnorm(600,0,1)
bernoulli = rbinom(600,1,p)
r = mu + sigma*epsilon + bernoulli*(mu_j+sigma_j*delta)
mu_r = mean(r)
std = sd(r)
skew = sum((r-mu_r)^3/(std^3))/(length(r)-1)
kurt = sum((r-mu_r)^4/(std^4))/(length(r)-1)
print(paste("unconditional mean is ", round(mu_r,2)))
```

```
## [1] "unconditional mean is  0"
```

```
print(paste("unconditional standard deviation is ", round(std,2)))
```

```
## [1] "unconditional standard deviation is  0.07"
```

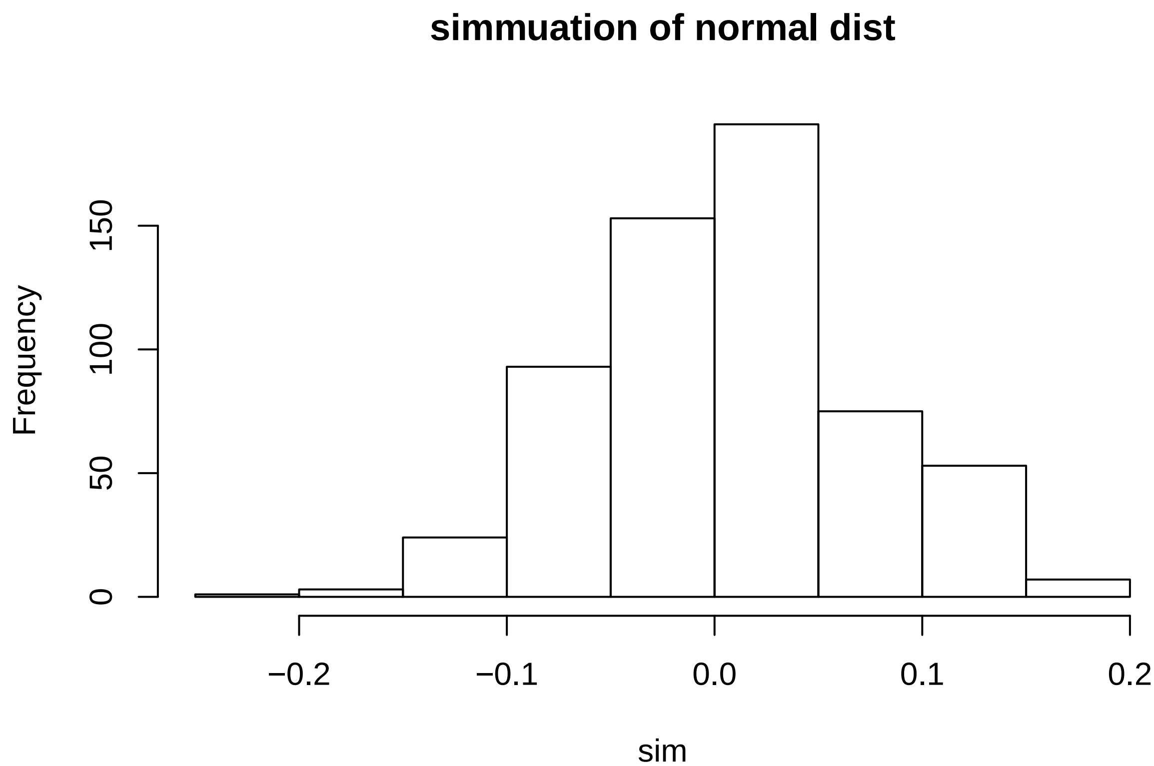
```
print(paste("unconditional skewness is ", round(skew,2)))
```

```
## [1] "unconditional skewness is  -0.3"
```

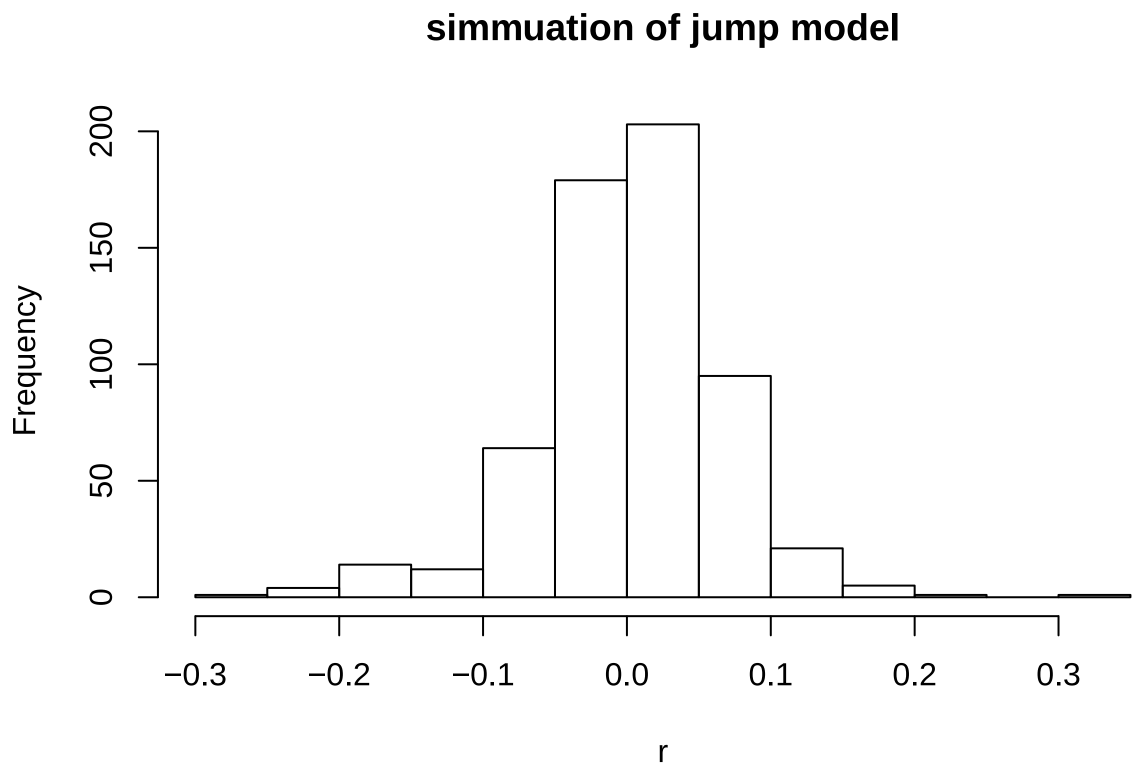
```
print(paste("unconditional kurtosis is ", round(kurt,2)))
```

```
## [1] "unconditional kurtosis is  5.05"
```

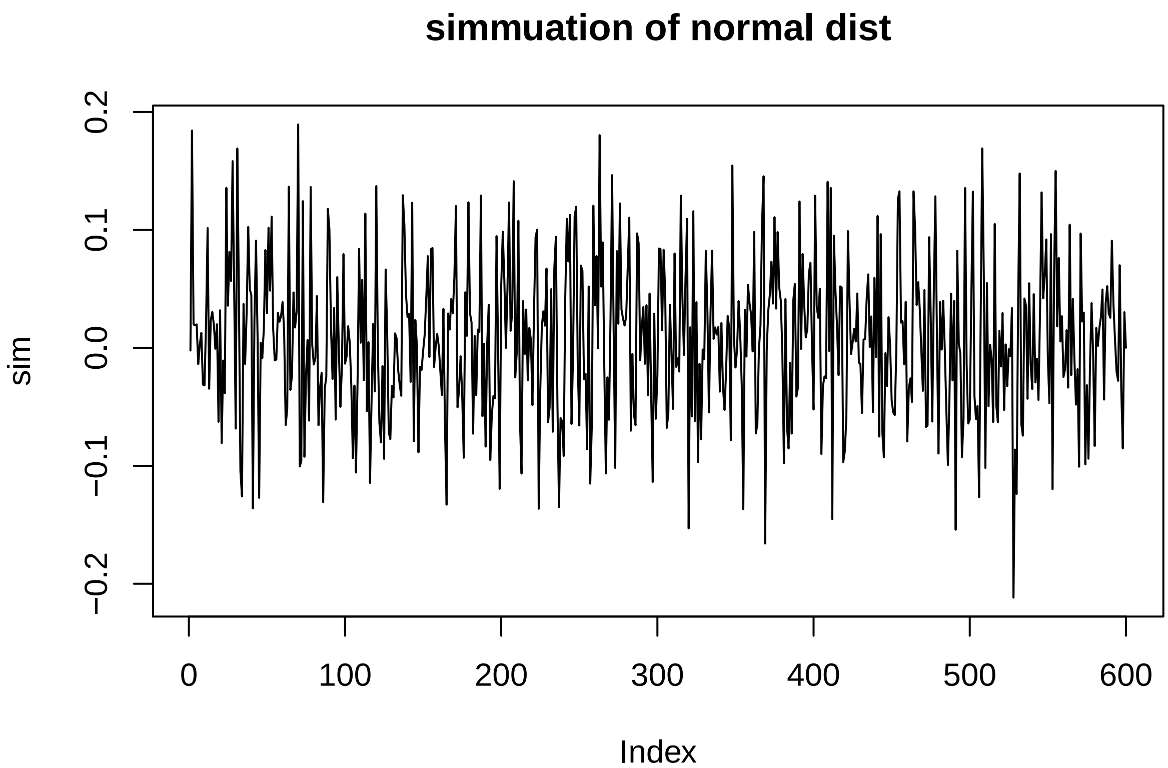
```
hist(sim, main = "simmuation of normal dist")
```



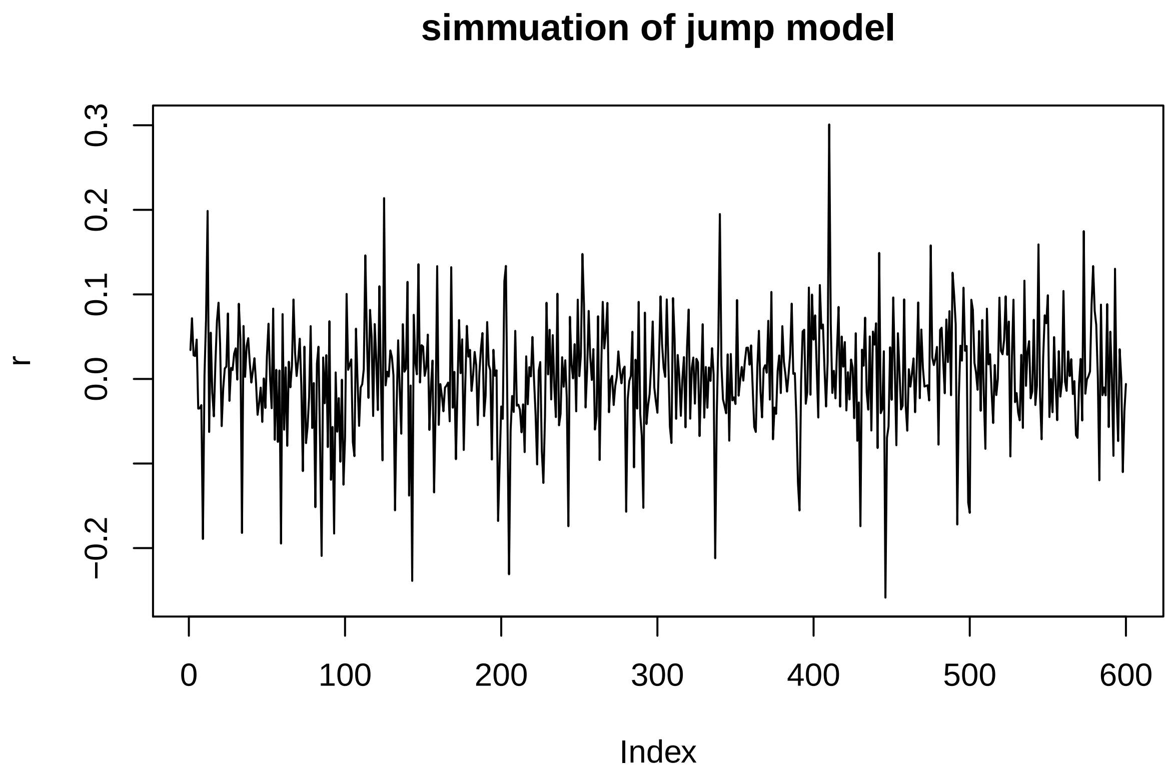

```
hist(r, main = "simmuation of jump model")
```




```
plot(sim, type = "l", main = "simmuation of normal dist")
```




```r
plot(r, type = "l", main = "simmuation of jump model")
```



Based on the graph, the jump model simulation looks more similarly to the market data.

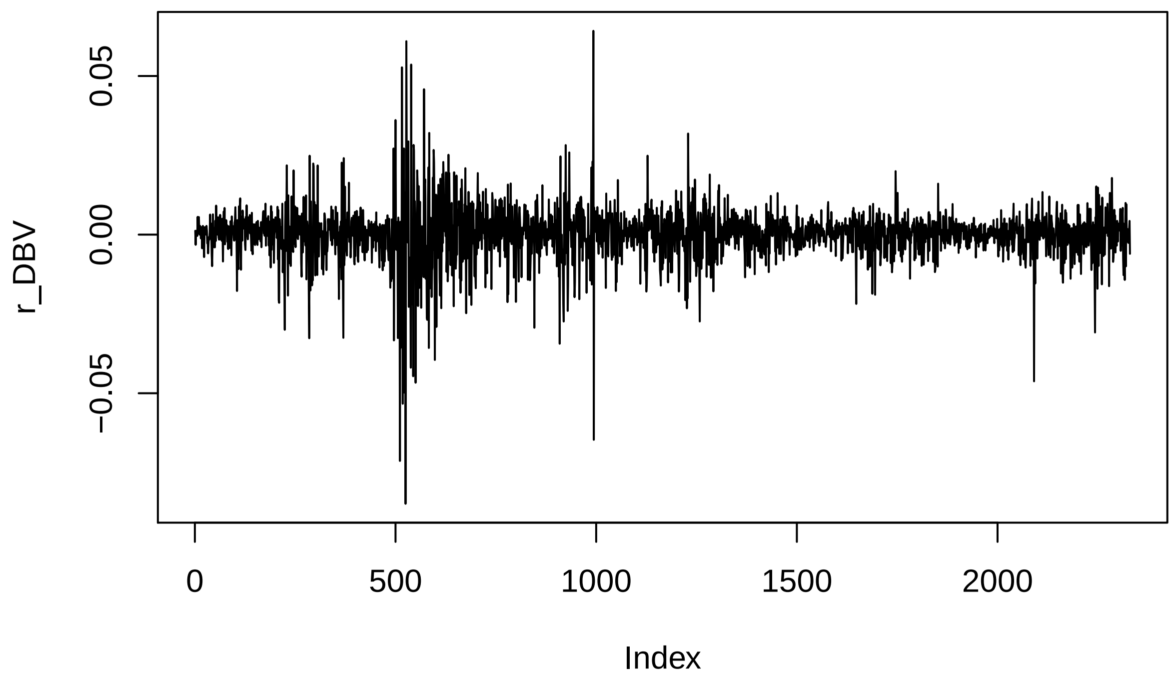
Problem 2

1.

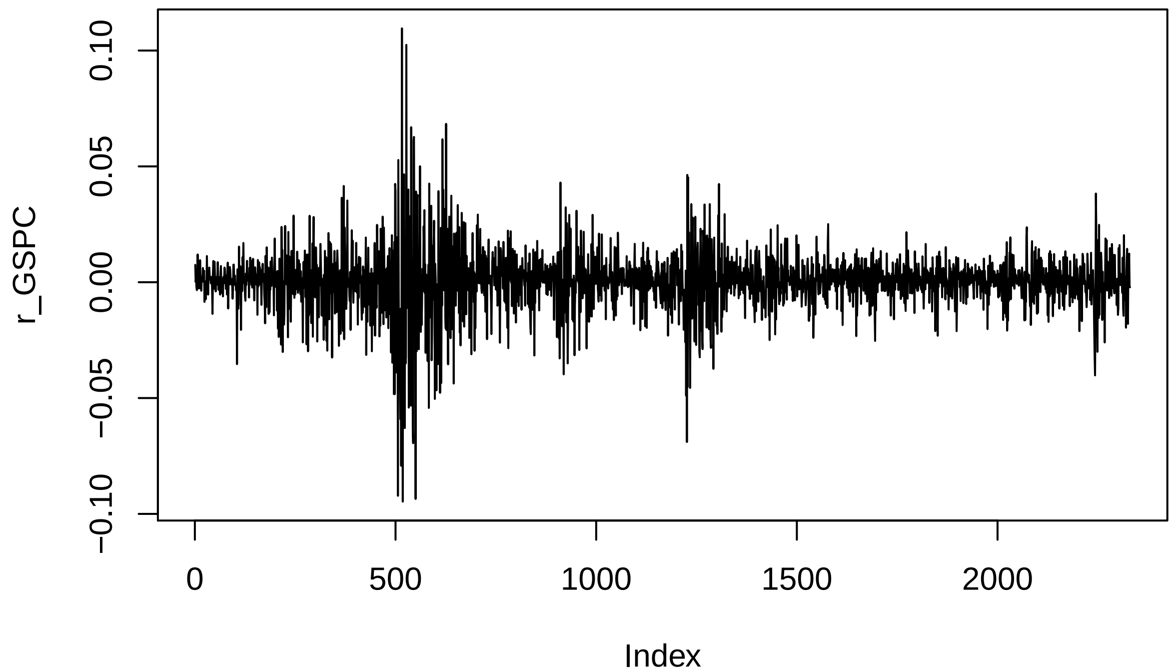
```
DBV = read.csv("DBV.csv")
GSPC = read.csv("GSPc.csv")

DBV_p = DBV$Adj.Close
DBV_p_lag = c(0,DBV_p[1:(length(DBV_p)-1)])
dvd = DBV_p/DBV_p_lag
r_DBV = log(dvd[2:length(dvd)])

GSPC_p = GSPC$Adj.Close
GSPC_p_lag = c(0,GSPC_p[1:(length(GSPC_p)-1)])
dvd2 = GSPC_p/GSPC_p_lag
r_GSPC = log(dvd2[2:length(dvd2)])
```

(a)

```r
plot(r_DBV, type = "l")
```

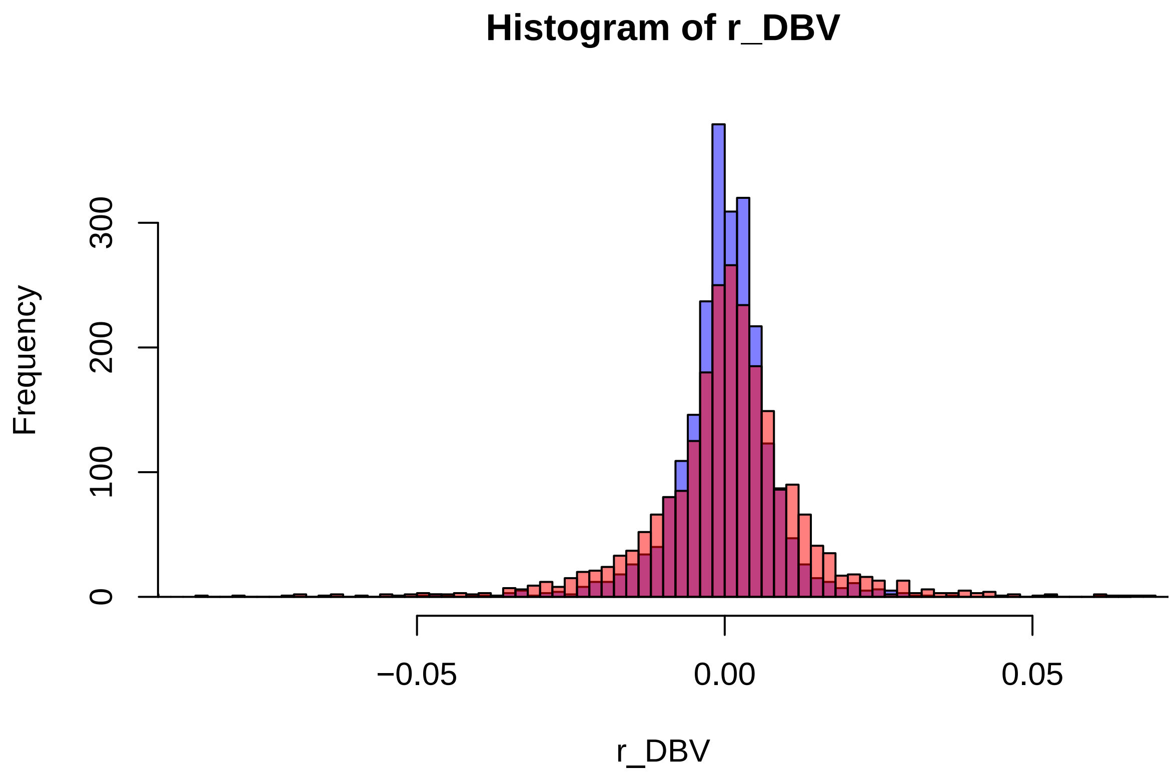



```r
plot(r_GSPC, type = "l")
```



(b)

```r
colB = scales::alpha("blue",0.5)
colR = scales::alpha("red",0.5)
hist(r_DBV, breaks = 100, col = colB)
hist(r_GSPC, breaks = 100, add = T, col = colR)
```



2.

(a)

```
skew_DBV = sum((r_DBV-mean(r_DBV))^3/(sd(r_DBV)^3))/(length(r_DBV)-1)
t_skew_DBV = skew_DBV/sqrt(6/length(r_DBV))
if (abs(t_skew_DBV) > 1.96){
  print(paste("|t| =", round(abs(t_skew_DBV),2), "> 1.96, reject null hypothesis and skewness of DBV is
  }else{
  print("skewness of DBV is not significantly different than 0")}
```

```
## [1] "|t| = 16.69 > 1.96, reject null hypothesis and skewness of DBV is significantly different than 0
```

```
skew_GSPC = sum((r_GSPC-mean(r_GSPC))^3/(sd(r_GSPC)^3))/(length(r_GSPC)-1)
t_skew_GSPC = skew_GSPC/sqrt(6/length(r_GSPC))
if (abs(t_skew_GSPC) > 1.96){
  print(paste("|t| =", round(abs(t_skew_GSPC),2), "> 1.96, reject null hypothesis and skewness of GSPC i
}else{
  print("skewness of GSPC is not significantly different than 0")}
```

```
## [1] "|t| = 6.38 > 1.96, reject null hypothesis and skewness of GSPC is significantly different than 0
```

(b)

```
kurt_DBV = sum((r_DBV-mean(r_DBV))^4/(sd(r_DBV)^4))/(length(r_DBV)-1)
t_kurt_DBV = (kurt_DBV-3)/sqrt(24/length(r_DBV))
if (abs(t_kurt_DBV) > 1.96){
  print(paste("|t| =", round(abs(t_kurt_DBV),2), "> 1.96, reject null hypothesis and kurtosis of DBV is
}else{
```

```
  print("kurtosis of DBV is not significantly different than 0")}
```

```
## [1] "|t| = 133.11 > 1.96, reject null hypothesis and kurtosis of DBV is significantly different than
```

```
kurt_GSPC = sum((r_GSPC-mean(r_GSPC))^4/(sd(r_GSPC)^4))/(length(r_GSPC)-1)
t_kurt_GSPC = (kurt_GSPC-3)/sqrt(24/length(r_GSPC))
if (abs(t_kurt_GSPC) > 1.96){
  print(paste("|t| =", round(abs(t_kurt_GSPC),2), "> 1.96, reject null hypothesis and kurtosis of GSPC i
}else{
  print("kurtosis of GSPC is not significantly different than 0")}
```

```
## [1] "|t| = 96.49 > 1.96, reject null hypothesis and kurtosis of GSPC is significantly different than
```

(c)

```
JB_DBV = t_skew_DBV^2+t_kurt_DBV^2
JB_GSPC = t_skew_GSPC^2+t_kurt_GSPC^2
CHI = qchisq(0.95,2)

if (JB_DBV > CHI){
  print(paste("JB =", round(JB_DBV,2), ">", round(CHI,2), ", reject null hypothesis and DBV is not norma
}else{
  print("DBV is normally distributed")}
```

```
## [1] "JB = 17997.23 > 5.99 , reject null hypothesis and DBV is not normally distributed"
```

```
if (JB_GSPC > CHI){
  print(paste("JB =", round(JB_GSPC,2), ">", round(CHI,2), ", reject null hypothesis and GSPC is not no
}else{
  print("GSPC is normally distributed")}
```

```
## [1] "JB = 9350.26 > 5.99 , reject null hypothesis and GSPC is not normally distributed"
```

3.

```
table = matrix(c(skew_DBV, t_skew_DBV, kurt_DBV, t_kurt_DBV, skew_GSPC, t_skew_GSPC, kurt_GSPC, t_kurt_
colnames(table) = c("DBV", "GSPC")
rownames(table) = c("skewness", "t.skewness", "kurtosis", "t.kurtosis")
print(table)
```

```
##                    DBV       GSPC
## skewness    -0.8468559 -0.3239731
## t.skewness -16.6882898 -6.3842704
## kurtosis    16.5096476 12.7924412
## t.kurtosis 133.1117398 96.4857799
```

4.

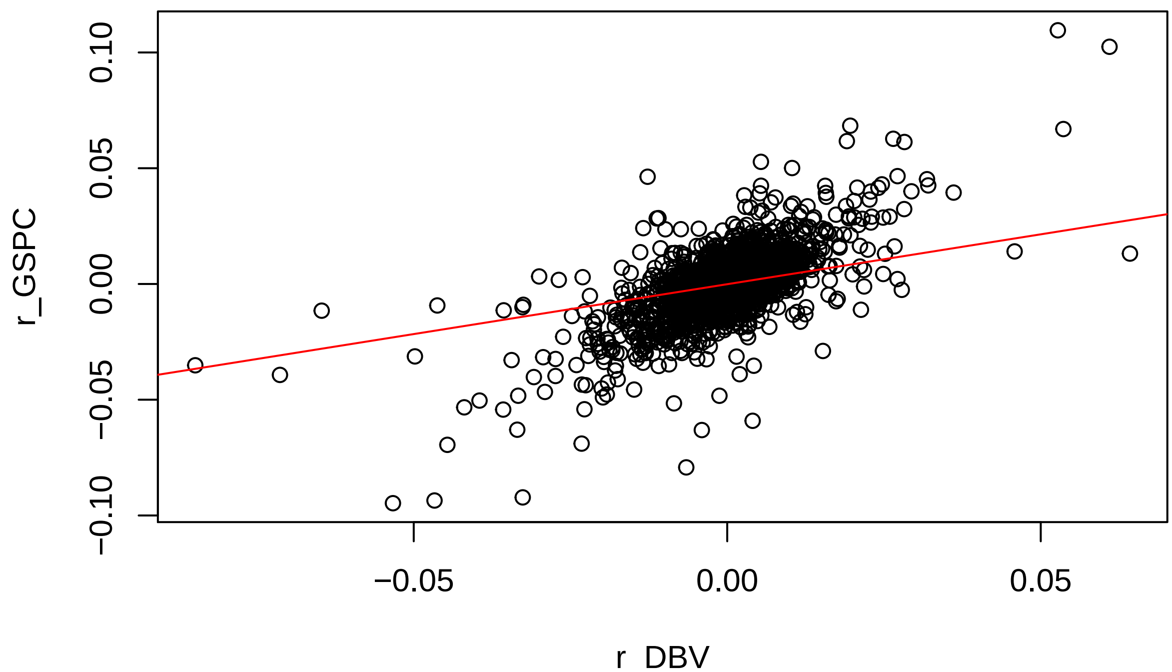
Since the sharpe ratios for equity and currency market are the same and the target return is a constant for both investment, the volatilities of both investment are the same. However, kurtosis and skewness of GSPC are smaller than those of DBV. That means GSPC has less probability of having extreme events. Thus, investing in equity market has less risk.

5.

```
library(sandwich)
```

```
## Warning: package 'sandwich' was built under R version 3.5.2
```

```r
lm.return = lm(r_DBV ~ r_GSPC)
summ_return = summary(lm.return)
std.coeff = data.frame(summ_return$coefficients[1:2,2])
colnames(std.coeff) = "standard error"
plot(r_DBV,r_GSPC)
abline(summ_return$coefficients[1],summ_return$coefficients[2], col= "red")
```




```r
wstd = sqrt(diag(vcovHC(lm.return, type = "HC")))
wstd.coeff = data.frame(wstd)
colnames(wstd.coeff) = "white std"
print(std.coeff)
```

```
##             standard error
## (Intercept)   0.0001361433
## r_GSPC        0.0101101940
```

```r
print(wstd.coeff)
```

```
##                white std
## (Intercept) 0.0001367342
## r_GSPC      0.0181383312
```

For OLS assumption,data is considered as normally distributed with homoskedasticity. However, our data does not satisfy normality and homoskedasticity. The real standard deviation is larger than std with OLS assumptions.