hw2-2-7

Cohort 2 - Group 7 (Huanyu Liu, Hyeuk Jung, Jiaqi Li, Xichen Luo)

Question 1

Assume that spot rates are as follows:

Maturity	Spot Rate
1	5%
2	5.5%
3	6%
4	6.3%

Spot rates are with annual compounding, coupon payments are annual, and par values are \$100. Compute the prices of the following bonds:

(a) A zero-coupon bond with 3 years to maturity.

$$\frac{100}{(1+0.06)^3} = 83.96$$

The price of such a zero-coupon bond is 83.96.

$$\frac{100}{(1+\text{ytm})^3} = 83.96$$

Solve and we get ytm = 0.06

(b) A bond with coupon rate 6% and 2 years to maturity.

$$\frac{6}{(1+0.05)} + \frac{106}{(1+0.055)^2} = 100.95$$

The price of such a coupon bond is 100.95.

$$\frac{6}{(1+\text{ytm})} + \frac{106}{(1+\text{ytm})^2} = 100.95$$

Solve and we get ytm = 0.0548.

(c) A bond with coupon rate 8% and 4 years to maturity.

$$\frac{8}{(1+0.05)} + \frac{8}{(1+0.05)^2} + \frac{8}{(1+0.05)^3} + \frac{108}{(1+0.05)^4} = 106.11$$

The price of such coupon bond is 106.11.

$$\frac{8}{(1+\text{ytm})} + \frac{8}{(1+\text{ytm})^2} + \frac{8}{(1+\text{ytm})^3} + \frac{108}{(1+\text{ytm})^4} = 106.11$$

Solve and we get ytm = 0.0623

Question 2

Bond	Coupon (annual)	Price	Maturity
X	100 * 4% = $4	100.98	6 months
Y	100 * 6% = $6	103.58	1 year

Let's denote 6-month spot rate as r.6month and 1-year spot rate as r.1yr.

First, current price of Bond X is calculated as follow:

$$100.98 = \frac{100 + coupon/2}{1 + \frac{r.6month}{2}} = \frac{100 + 3}{1 + \frac{r.6month}{2}}$$

Using the above equation, we can get the 6-month spot rate.

$$1 + \frac{r.6month}{2} = \frac{103}{100.98} = 1.0101$$

Therefore, r.6month = (1.0101 - 1) $\times$ 2 = 0.0202.

Now, using the Bond Y and 6-month spot rate, we can also get the 1-year spot rate.

$$103.59 = \frac{coupon/2}{1 + \frac{r.6month}{2}} + \frac{100 + coupon/2}{(1 + \frac{r.1yr}{2})^2} = \frac{3}{1 + \frac{0.0202}{2}} + \frac{103}{(1 + \frac{r.1yr}{2})^2}$$

$$\frac{103}{(1 + \frac{r.1yr}{2})^2} = 103.59 - \frac{3}{1 + 0.0101} = 103.59 - 2.97 = 100.62$$

$$(1 + \frac{r.1yr}{2})^2 = \frac{103}{100.62} = 1.0237$$

$$1 + \frac{r.1yr}{2} = \sqrt{1.0237} = 1.0118$$

Therefore, r.1yr = (1.0118 - 1) $\times$ 2 = 0.0235.

Question 3

Suppose we buy x Bond A, y Bond B, z Bond C (negative value means short that amount of bond). To eliminate any future payments, we have:

$$100x + 5y + 7z = 0$$
$$105y + 107z = 0$$
$$x = -\frac{2}{105}z$$
$$y = -\frac{107}{105}z$$

To get an arbitrage opportunity at the initial state, there must be positive cash inflow at initial state, so we have:

$$
\begin{aligned}
-95.238x - 98.438y - 103.37z &> 0 \\
-95.238 \times (-\frac{2}{105})z - 98.438 \times (-\frac{107}{105})z - 103.37z &> 0 \\
-1.243z &> 0 \\
z &< 0
\end{aligned}
$$

z is negative, so we have to short bond C.

The arbitrage strategy is:

Suppose we short 1 bond C and we have to long $\frac{2}{105}$ bond A and $\frac{107}{105}$ bond B, so that there will be no future cash flow. The arbitrage opportunity cash inflow is:

$$
\begin{aligned}
&- 95.238 \times \frac{2}{105} - 98.438 \times \frac{107}{105} + 103.37 \times 1 \\
&= 1.2429
\end{aligned}
$$