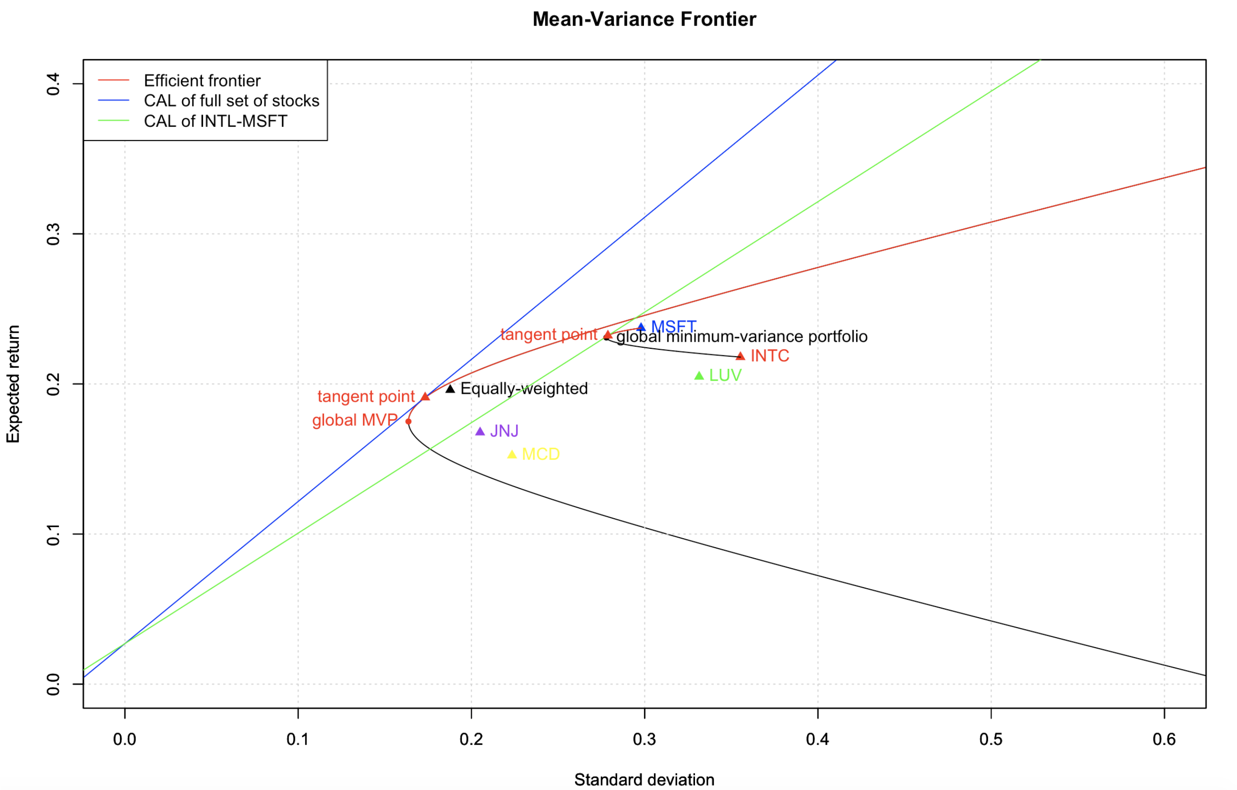
hw7-2-7

Cohort 2 - Group 7 (Huanyu Liu, Hyeuk Jung, Jiaqi Li, Xichen Luo)

The plot of the mean-variance frontier is shown below:



Problem 1. Mean-variance Frontier

The minimum-variance is the vertex of the mean-variance frontier and the effecient frontiers is the red line of the mean-variance frontier. Adding assets shifts the portfolio frontier to the left, which reduces the variance that can be achieved for a given expected return.

Problem 2. Tangent Portfolio Comparison

The Sharp ratio in the Intel-Microsoft case is less than that in the full set of stocks case, because diversification reduces risks for a given expected return. In that case, for a given standard deviation, the expected return in the full set of stocks case is larger than that in the Intel-Microsoft case.

In the full set of stocks case

The return for the tangent point is

$$E(R) = 0.189$$

The standard deviation for the tangent point is

$$\text{sd} = 0.171$$

The weight for each risk asset in the portfolio is:

$$W_{\text{INTC}} = 0.0753$$

$$W_{\text{MSFT}} = 0.2573$$

$$W_{\text{LUV}} = 0.1168$$

$$W_{\text{MCD}} = 0.1768$$

$$W_{\text{JNJ}} = 0.3738$$

In the Intel-Microsoft case

The return for the tangent point is

$$E(R) = 0.232$$

The standard deviation for the tangent point is

$$\text{sd} = 0.279$$

The weight for each risk asset in the portfolio is:

$$W_{\text{INTC}} = 0.259$$

$$W_{\text{MSFT}} = 0.741$$

Problem 3. Optimal Mix of Assets

Knowing the risk aversion is A=5, to maximize utility:

$$U(R, w) = w(E(R) - R_f) + R_f - \frac{A}{2}w^2 V(R_{t+1})$$

$$w^* = argmax_w U(R, w)$$

Take the first derivative to calculate the weight of portfolio that maximize the utility:

$$(E(R) - R_f) - AwV(R_{t+1}) = 0$$

$$w = \frac{1}{A}\frac{(E(R) - R_f)}{V(R_{t+1})}$$

In that case, maximizing the utility is the same as maximizing the Sharp ratio. The optimal mix of assets is also the tangent portfolio, which is shown in the plot.

$$w = 1.092$$

It means that to maximize the utility, short 0.092 risk free asset and long 1.092 risk assets.