hw3-3

3. Consider the following 4 US Treasury bonds (the par value is \$100):

Bond A is a 5-year bond with a 1% annual coupon. Bond B is a 10-year bond with a 1% annual coupon. Bond C is a 5-year bond with a 4% annual coupon. Bond D is a 10-year bond with a 4% annual coupon.

Assume all of the bonds are currently trading at a 3.5% yield and, as their maturity is over 1 year, they pay semi-annual coupons. Recall if a bond has a 4% annual coupon, it pays 2% of par every 6-months.

(a) Write down the formula for pricing bonds.

$$\text{PV} = \sum_{i=1}^{n} \frac{c}{(1+r_i)^i} + \frac{\text{principle}}{(1+r_n)^n}$$

$$\text{where c = cash flows, n = total number of periods}$$

(b) Compute the current price for each of the bonds.

$$\text{semiannual yield} = 0.035/2 = 0.0175$$

Bond A:

$$\text{CouponA} = 0.01/2 \times 100 = 0.5$$
$$\text{total periods} = 5 \times 2 = 10$$
$$P_A = \sum_{i=1}^{10} \frac{0.5}{(1+0.0175)^i} + \frac{100}{(1+0.0175)^{10}} = 88.62$$

Bond B:

$$\text{CouponB} = 0.01/2 \times 100 = 0.5$$
$$\text{total periods} = 10 \times 2 = 20$$
$$P_B = \sum_{i=1}^{20} \frac{0.5}{(1+0.0175)^i} + \frac{100}{(1+0.0175)^{20}} = 79.06$$

Bond C:

$$\text{CouponA} = 0.04/2 \times 100 = 2$$
$$\text{total periods} = 5 \times 2 = 10$$
$$P_A = \sum_{i=1}^{10} \frac{2}{(1+0.0175)^i} + \frac{100}{(1+0.0175)^{10}} = 102.28$$

Bond D:

$$\text{CouponB} = 0.04/2 \times 100 = 2$$
$$\text{total periods} = 10 \times 2 = 20$$
$$P_B = \sum_{i=1}^{20} \frac{2}{(1+0.0175)^i} + \frac{100}{(1+0.0175)^{20}} = 104.19$$

(c) Suppose the yield increases to 3.8% from 3.5%. Compute the new prices and compute the change in the bond's price.

$$\text{semiannual yield} = 0.038/2 = 0.019$$

Bond A:

$$\begin{aligned}
&\text{CouponA} = 0.01/2 \times 100 = 0.5 \\
&\text{total periods} = 5 \times 2 = 10 \\
&P_A = \sum_{i=1}^{10} \frac{0.5}{(1+0.019)^i} + \frac{100}{(1+0.019)^{10}} = 87.36 \\
&\Delta P_A = \text{new price} - \text{old price} = 87.36 - 88.62 = -1.26
\end{aligned}$$

The price of bond A decreases \$1.26 as the yield increases to 3.8%.

Bond B:

$$\begin{aligned}
&\text{CouponB} = 0.01/2 \times 100 = 0.5 \\
&\text{total periods} = 10 \times 2 = 20 \\
&P_B = \sum_{i=1}^{20} \frac{0.5}{(1+0.019)^i} + \frac{100}{(1+0.019)^{20}} = 76.89 \\
&\Delta P_B = \text{new price} - \text{old price} = 76.89 - 79.06 = -2.17
\end{aligned}$$

The price of bond B decreases \$2.17 as the yield increases to 3.8%.

Bond C:

$$\begin{aligned}
&\text{CouponA} = 0.04/2 \times 100 = 2 \\
&\text{total periods} = 5 \times 2 = 10 \\
&P_C = \sum_{i=1}^{10} \frac{2}{(1+0.019)^i} + \frac{100}{(1+0.019)^{10}} = 100.90 \\
&\Delta P_C = \text{new price} - \text{old price} = 100.90 - 102.28 = -1.38
\end{aligned}$$

The price of bond C decreases \$1.38 as the yield increases to 3.8%.

Bond D:

$$
\begin{aligned}
&\text{CouponB} = 0.04/2 \times 100 = 2 \\
&\text{total periods} = 10 \times 2 = 20 \\
&P_D = \sum_{i=1}^{20} \frac{2}{(1+0.019)^i} + \frac{100}{(1+0.019)^{20}} = 101.65 \\
&\Delta P_D = \text{new price} - \text{old price} = 104.19 - 101.65 = -2.54
\end{aligned}
$$

The price of bond D decreases \$2.54 as the yield increases to 3.8%.

(d) Suppose the yield decreases to 3.2% from 3.5%. Compute the new prices and compute the change in the bond's price.

$$\text{semiannual yield} = 0.032/2 = 0.016$$

Bond A:

$$
\begin{aligned}
&\text{CouponA} = 0.01/2 \times 100 = 0.5 \\
&\text{total periods} = 5 \times 2 = 10 \\
&P_A = \sum_{i=1}^{10} \frac{0.5}{(1+0.016)^i} + \frac{100}{(1+0.016)^{10}} = 89.91 \\
&\Delta P_A = \text{new price} - \text{old price} = 89.91 - 88.62 = 1.29
\end{aligned}
$$

The price of bond A increases \$1.29 as the yield decreases to 3.2%.

Bond B:

$$
\begin{aligned}
&\text{CouponB} = 0.01/2 \times 100 = 0.5 \\
&\text{total periods} = 10 \times 2 = 20 \\
&P_B = \sum_{i=1}^{20} \frac{0.5}{(1+0.016)^i} + \frac{100}{(1+0.016)^{20}} = 81.30 \\
&\Delta P_B = \text{new price} - \text{old price} = 81.30 - 79.06 = 2.24
\end{aligned}
$$

The price of bond B increases \$2.24 as the yield decreases to 3.2%.

Bond C:

$$\text{CouponA} = 0.04/2 \times 100 = 2$$
$$\text{total periods} = 5 \times 2 = 10$$
$$P_C = \sum_{i=1}^{10} \frac{2}{(1+0.016)^i} + \frac{100}{(1+0.016)^{10}} = 103.67$$
$$\Delta P_C = \text{new price} - \text{old price} = 103.67 - 102.28 = 1.39$$

The price of bond C increases $1.39 as the yield decreases to 3.2%.

Bond D:

$$\text{CouponB} = 0.04/2 \times 100 = 2$$
$$\text{total periods} = 10 \times 2 = 20$$
$$P_D = \sum_{i=1}^{20} \frac{2}{(1+0.016)^i} + \frac{100}{(1+0.016)^{20}} = 106.80$$
$$\Delta P_D = \text{new price} - \text{old price} = 106.80 - 104.19 = 2.61$$

The price of bond D increases $2.61 as the yield decreases to 3.2%.

(e) Compute durations of each of these bonds and answer (c) and (d) using duration.

Duration A:

$$D_A = (\sum_{t=1}^{10} w(t) \times t) \times \frac{1}{2} = (\sum_{t=1}^{10} \frac{\text{CouponA} \times t}{P_A \times 1.0175^t} + \frac{100 \times 10}{P_A \times 1.0175^{10}}) \times \frac{1}{2} = 4.88$$
$$D_A^* = \frac{D_A}{1.0175} = 4.80$$

Duration B:

$$D_B = (\sum_{t=1}^{20} w(t) \times t) \times \frac{1}{2} = (\sum_{t=1}^{20} \frac{\text{CouponB} \times t}{P_B \times 1.0175^t} + \frac{100 \times 20}{P_B \times 1.0175^{20}}) \times \frac{1}{2} = 9.466$$
$$D_B^* = \frac{D_B}{1.0175} = 9.30$$

Duration C:

$$D_C = (\sum_{t=1}^{10} w(t) \times t) \times \frac{1}{2} = (\sum_{t=1}^{10} \frac{\text{CouponC} \times t}{P_C \times 1.0175^t} + \frac{100 \times 10}{P_C \times 1.0175^{10}}) \times \frac{1}{2} = 4.59$$
$$D_C^* = \frac{D_C}{1.0175} = 4.51$$

Duration D:

$$D_D = (\sum_{t=1}^{20} w(t) \times t) \times \frac{1}{2} = (\sum_{t=1}^{20} \frac{\text{CouponD} \times t}{P_D \times 1.0175^t} + \frac{100 \times 20}{P_D \times 1.0175^{20}}) \times \frac{1}{2} = 8.38$$
$$D_D^* = \frac{D_D}{1.0175} = 8.24$$

When the yield increases 30 basis points:

Bond A price approximate change $= -0.003 \times D_A^* \times P_A = -1.28$
Bond B price approximate change $= -0.003 \times D_B^* \times P_B = -2.21$
Bond C price approximate change $= -0.003 \times D_C^* \times P_C = -1.38$
Bond D price approximate change $= -0.003 \times D_D^* \times P_D = -2.57$

When the yield decreases 30 basis points:

Bond A price approximate change $= 0.003 \times D_A^* \times P_B = 1.28$
Bond B price approximate change $= 0.003 \times D_B^* \times P_B = 2.21$
Bond C price approximate change $= 0.003 \times D_C^* \times P_C = 1.38$
Bond D price approximate change $= 0.003 \times D_D^* \times P_D = 2.57$

(f) What can you conclude about reaction of bond prices to changes in yields for bonds with differet coupons and maturities?

Coupons:

The higher a bond's coupon, the more income it produces early on and thus the shorter its duration. The lower the coupon, the longer the duration (and volatility).

Maturities:

The longer a bond's maturity, the greater its duration (and volatility).