HW6-2-7

Cohort 2 - Group 7 (Huanyu Liu, Hyeuk Jung, Jiaqi Li, Xichen Luo)

Problem 1. The mean and volatility of weekly returns

The calculation of weekly returns is based on the adjusted price, which considers the dividends effect.

$$\mathrm{r} = \frac{\mathrm{p}_1 - \mathrm{p}_0}{\mathrm{p}_0}$$

Note that in the formula above, r is the weekly return, p_1 is the adjusted price of the last day of the week, and p_0 is the adjusted price of the first day of the week.

The mean of weekly returns:

$$\mu = \frac{\sum \mathrm{r}_i}{n}$$

Note that in the formula above, μ is the mean of weekly returns, r_i is the weekly returns, and n is the number of weeks from 12/29/1989 to 9/28/2018.

The mean of Intel weekly returns is:

$$\mathrm{r}_{\mathrm{INTC}} = 0.42\%$$

The mean of Microsoft weekly returns is:

$$\mathrm{r}_{\mathrm{MSFT}} = 0.46\%$$

The standard deviation of weekly returns:

$$\sigma = \sqrt{\frac{\sum \left(\mathrm{r}_i - \mu\right)^2}{n}}$$

Note that in the formula above, σ is the standard deviation of weekly returns.

The standard deviation of Intel weekly returns is:

$$\sigma_{\mathrm{INTC}} = 4.93\%$$

The standard deviation of Microsoft weekly returns is:

$$\sigma_{\mathrm{MSFT}} = 4.13\%$$

The annualized mean of weekly returns:

$$\mu_a = \mu \times 52$$

Note that in the formula above, μ_a is the annualized mean of weekly returns.

The annualized mean of Intel weekly returns is:

$$\mu_{\text{a.INTC}} = 21.79\%$$

The annualized mean of Microsoft weekly returns is:

$$\mu_{\text{a.MSFT}} = 23.73\%$$

The annualized volatility:

$$\sigma_a = \sigma \times \sqrt{52}$$

Note that in the formula above, σ_a is the annualized volatility of weekly returns.

The annualized volatility of Intel weekly returns is:

$$\sigma_{\text{a.INTC}} = 35.53\%$$

The annualized volatility of Microsoft weekly returns is:

$$\sigma_{\text{a.MSFT}} = 29.79\%$$

Problem 2. Portfolio allocation

Calculate the portfolio weight that maximizes utility:

$$w_t^\star = \frac{E_t(R_{r,t+1}) - R_{f,t}}{A \times V_t(R_{r,t+1})}$$

Note that A is 4, $R_{f,t}$ is 1%, $E_t(R_{r,t+1}$ is the annualized mean of weekly returns, and $V_t(R_{r,t+1}$ is the quadratic annualized volatility of weekly returns.

The Intel stock weight in the portfolio that maximizes utility:

$$w_{t.INTC}^\star = 41.18\%$$

In that case, for the portfolio that maximizes the utility, allocate 41.18% capital on Intel stock and 59.82% (= 1 - 41.18%) capital on risk-free asset.

The Microsoft stock weight in the portfolio that maximizes utility:

$$w_{t.MSFT}^\star = 64.02\%$$

In that case, for the portfolio that maximizes the utility, allocate 64.02% capital on Microsoft stock and 35.98% (= 1 - 64.02%) capital on risk-free asset.

Problem 3. Allocation strategy based on utility

The portfolio weight that maximizes utility is calculated above. Then calculate the utility using the portfolio weight calculated.

$$U(R_{t+1}, w_t) = w_t(E_t(R_{r,t+1}) - R_{f,t}) + R_{f,t} - \frac{A}{2} \times w_t^2 V_t(R_{r,t+1})$$

The utility allocating on Intel stock:

$$U_{INTC} = 0.053$$

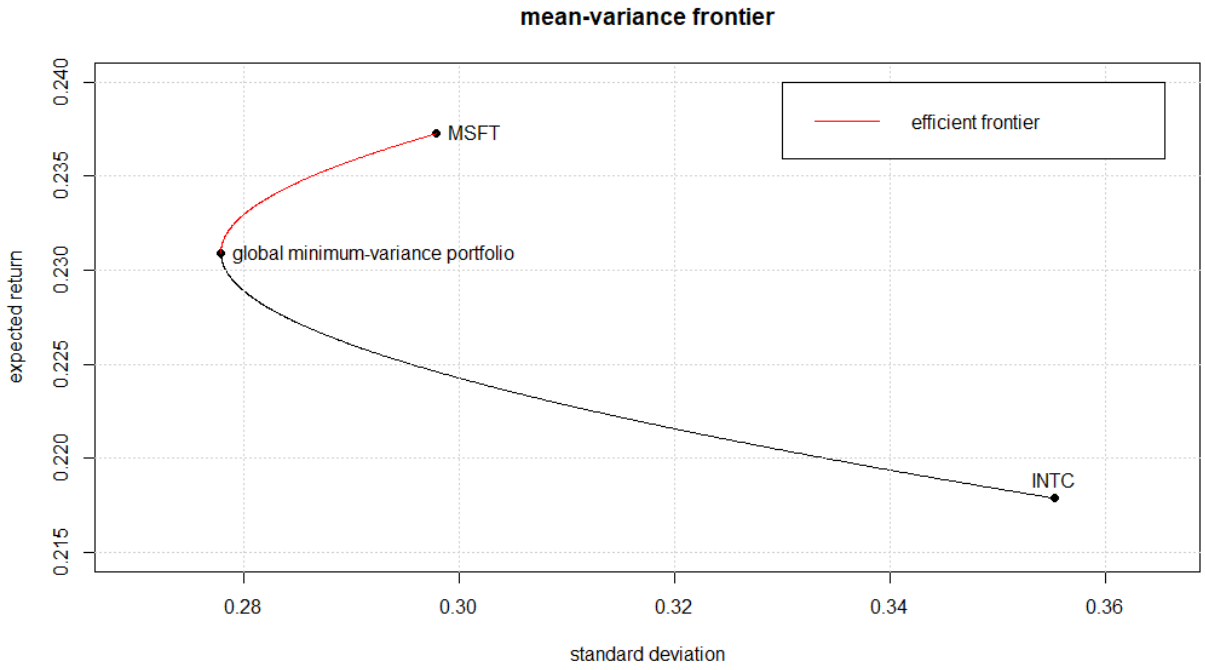
The utility allocating on Microsoft stock:

$$U_{MSFT} = 0.083$$

The utility allocating on Microsoft stock is larger than the utility allocating on Intel stock, in that case, choose to allocate 64.02% capital on Microsoft stock and 35.98% (= 1 - 64.02%) capital on risk-free asset.

Problem 4. Mean-variance frontier for Intel-Microsoft combination

The plot of efficient frontier for Intel-Microsoft combination



The red line in the plot is the efficient frontier and the minimum-variance portfolio is on the dot marked as "global minimum-variance portfolio". Because this dot is the lowest standard deviation (also the lowest variance) that the portfolio could reach.