# hw3-2-7-Q1

Cohort 2 - Group 7 (Huanyu Liu, Hyeuk Jung, Jiaqi Li, Xichen Luo)

## Question 1

| Maturity | coupon | par value | payment |
|---|---|---|---|
| 3 | 10 | 100 | annual |

| Maturity | Spot Rate |
|---|---|
| 1 | 5% |
| 2 | 5.5% |
| 3 | 6.5% |

(a) Determine the bond's price and YTM

**First, the bond's price.**

$$\begin{aligned} price &= \frac{10}{1.05} + \frac{10}{1.055^2} + \frac{110}{1.065^3} \\ &= 109.5717 \end{aligned}$$

**Using the bond price calculated above, we can also get the YTM.**

$$109.5717 = \frac{10}{1+ytm} + \frac{10}{(1+ytm)^2} + \frac{110}{(1+ytm)^3}$$

**Therefore, ytm = 0.0639**

(b) Determine as many forward rates as you can, based on the spot rates above.

**With given spot rates, we can get *three* forward rates.**

$$\begin{aligned} f_{1,1} &= \frac{1.055^2}{1.05} - 1 = 0.06 \\ f_{1,2} &= \sqrt{\frac{1.065^3}{1.05}} - 1 = 0.0726 \\ f_{2,1} &= \frac{1.065^3}{1.055^2} - 1 = 0.0852 \end{aligned}$$

(c) You would like to get a guaranteed 3-year return on your coupon bond. Explain how this can be achieved using forward rates. Which forward rates should you use? What is your guaranteed 3-year return?

**To lock the 3-year return, we have to roll over the cash flows' investments and exploit the forward rates. We have two methods. First, we can invest the coupons to $f_{1,1}, and f_{2,1}$.**

$$\begin{aligned} futureprice &= 10 \times (1 + f_{1,1}) \times (1 + f_{2,1}) + 10 \times (1 + f_{2,1}) + 110 \\ &= 10 \times 1.06 \times 1.0852 + 10 \times 1.0852 + 110 \\ &= 132.36 \\ return &= \sqrt[3]{\frac{132.36}{109.5717}} - 1 \\ &= 0.065 \end{aligned}$$

**Second, we can invest 1st coupon in $f_{1,2}$ and 2nd coupon in $f_{2,1}$.**

$$\begin{aligned} futureprice &= 10 \times (1 + f_{1,2})^2 + 10 \times (1 + f_{2,1}) + 110 \\ &= 10 \times 1.0726^2 + 10 \times 1.0852 + 110 \\ &= 132.36 \\ return &= \sqrt[3]{\frac{132.36}{109.5717}} - 1 \\ &= 0.065 \end{aligned}$$

**In both methods, we can see that the expected returns are the same, 6.5%.**