hw4-2-7

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Yield Curve Spread Trades Report

Executive Summary

In this project, the stretegy at the beginning is randomly chosen. To analyze the viability of the strategy, our group calculate and plot the cumulative return. If the return is positive, which means the strategy would bring profit, then we would conclude that the strategy is effective. If the return is negative, which means the strategy would cause the loss of profit, then we would conclude that the strategy chosen is not effective and should be rejected, and another strategy should be applied.

The strategy we choose is the 10yr - 2yr U.S. Treasury flattener strategy, which is to short 2yr U.S. Treasury bonds and long 10 yr U.S. Treasury bonds. The report is to analyze the effectiveness of the flattener strategy.

Introduction

In the report, the discussion mainly describes the calculation process on the first week. On the last day of the first week, we close the position and immediately reinvest the new 2yr Treasury bonds and 10yr Treasury bonds to maintain the DV01-neutral position and calculate the new capital as the initial capital for the new week. So the calculation after the first week would be initiated again and follow the same process as the first week. If we ever encounter holidays, the data we have on those date will not be available. In that case, we decide to use the data of the next business day of the holiday to close our position and reinvest. Then we will hold our new position until the last day of the week.

Discussion

1. On Dec 30, 1983:

a. The yields of 2yr U.S. Treasury bond and 10yr U.S. Treasury bond are given:

The yield of 2yr U.S. Treasury bond:

$$r_2 = 0.105502$$

The yield of 10yr U.S. Treasury bond:

$$r_{10} = 0.116444$$

b. The starting prices of 2yr U.S. Treasury bond and 10yr U.S. Treasury bond:

The price of 2yr U.S. Treasury bond:

$$P_2 = \frac{100}{e^{2\times r_2}} = 80.98$$

The price of 10yr U.S. Treasury bond:

$$P_{10} = \frac{100}{e^{10\times r_{10}}} = 31.21$$

c. Set up the DV01-neutral yield curve spread trade:

Since both 2yr Treasury bond and 10yr Treasury bond are all zero-coupon bonds, then their Macaulay durations equal to their maturity, which should be:

$$D_2 = T_2 = 2$$

$$D_{10} = T_{10} = 10$$

The calculation of DV01:

(i). For the 2yr U.S. Treasury bond:

$$D_2^* = \frac{D_2}{e^{1\times r_2}}$$

$$DV01_2 = D_2^* \times P_2 \div 10,000$$

(ii). For the 10yr U.S. Treasury bond:

$$D_{10}^* = \frac{D_{10}}{e^{1\times r_{10}}}$$

$$DV01_{10} = D_{10}^* \times P_{10} \div 10,000$$

Build the DV01-neutral yield curve spread trade equition:

$$DV01_2 \times \mathrm{x}_2 = DV01_{10} \times \mathrm{x}_{10}$$

$$|P_2 \times \mathrm{x}_2| + |P_{10} \times \mathrm{x}_{10}| = \frac{\text{Initial Capital}}{\text{Margin Requirement}}$$

Note that the initial capitalis \$1 million dollars:

$$\text{Initial Capital} = 1,000,000$$

The Margin requirement is 10%:

$$\text{Margin Requirement} = 10\%$$

Solve the equitions:

$$\mathrm{x}_2 = -102721.42$$

$$\mathrm{x}_{10} = 53890.85$$

In that case, on Dec 30, 1983, short x_2 shares of 2yr Treasury bonds and long x_{10} shares of 10yr Treasury bonds.

2. One week after Dec 30, 1983, when it is Jan 5, 1984:

a. The calculation of the new prices of the bonds:

The calculation of yields would is based on the Nelson-Siegel-Svensson (NSS) model:

$$r_t = \beta_0 + \beta_1 \times \frac{1-e^{-t_1}}{t_1} + \beta_2 \times [\frac{1-e^{-t_1}}{t_1} - e^{-t_1}] + \beta_3 \times [\frac{1-e^{-t_2}}{t_2} - e^{-t_2}]$$

with $t_j = t/\tau_j$, and where β_k and τ_j are based on the daily estimation posted on the file, the daily U.S. Treasury yield curve.

(i). For the 2yr U.S. Treasury bond:

$t = 2 - \frac{7}{365}$, τ_1, τ_2, β_0, β_1, β_2, and β_3 are from the estimation on Jan 5, 1984. Solve the NSS model, get the yield of 2yr U.S. Treasury bond and denote as:

$$r_2^{'}$$

(ii). For the 10yr U.S. Treasury bond:

$t = 10 - \frac{7}{365}$, τ_1, τ_2, β_1, β_2, and β_3 are from the estimation on Jan 5, 1984. Solve the NSS model, get the yield of 10yr U.S. Treasury bond and denote as:

$$r_{10}^{'}$$

Hence, the new prices of 2yr U.S. Treasury bond and 10yr U.S. Treasury bond:

$$P_2^{'} = \frac{100}{e^{(2-\frac{7}{365})\times r_2^{'}}} = 81.27$$

$$P_{10}^{'} = \frac{100}{e^{(10-\frac{7}{365})\times r_{10}^{'}}} = 31.38$$

b. The calculation of interest earning or paid:

The calculation of interest rate is based on the Nelson-Siegel-Svensson (NSS) model:

Here we assume that the interest rate is the one-week Treasury yield and it is calculated based on the parameters given on Dec 30, 1983.
Then $t = \frac{7}{365}$, τ_1, τ_2, β_0, β_1, β_2, and β_3 are from the estimation on Dec 30, 1983. Solve the NSS model, get the interest rate, and denote as:

$$r_{int}$$

The cash position during the past week is:

$$\text{Cash Position} = -P_2 \times \mathrm{x}_2 - P_{10} \times \mathrm{x}_{10} + \text{Initial Capital}$$

The interest earned or paid during the past week is:

$$\text{Interest} = (e^{\frac{7}{365}\times r_{int}} - 1) \times \text{Cash Position}$$

c. Close out the position:

$$\text{End Capital} = P_2^{'} \times \mathrm{x}_2 + P_{10}^{'} \times \mathrm{x}_{10} + \text{Interest} + \text{Cash Position}$$

The Formula of Returns Calculation

Cumulative Return

The return for each week:

$$\Delta\text{Return} = \text{End Capital} - \text{Initial Capital}$$

$$\text{Cumulative Return} = \sum_{i=1}^{1801} \Delta\text{Return}_i$$

Spread Return

The return due to changes in the yield spread using DV01.

The calculation is based on the same yield curve as the first day of the week, the spread return earned from 2y Treasury bond is:

$$\Delta\text{Spread Return}_2 = -(\mathrm{x}_2 \times P_2) \times D_2^* \times \Delta r_2 = -(\mathrm{x}_2 \times P_2) \times D_2^* \times (r_2^{''} - r_2)$$

The yield of 2y Treasury bond is given:

$$r_2^{''}$$

The spread return earned from 10y Treasury bond is:

$$\Delta\text{Spread Return}_{10} = -(\mathrm{x}_{10} \times P_{10}) \times D_{10}^* \times \Delta r_{10} = -(\mathrm{x}_{10} \times P_{10}) \times D_{10}^* \times (r_{10}^{''} - r_{10})$$

The yield of 10y Treasury bond is given:

$$r_{10}^{''}$$

The spread return for this week is:

$$\Delta\text{Spread Return} = \Delta\text{Spread Return}_2 + \Delta\text{Spread Return}_{10}$$

The cumulative spread return:

$$\text{Cumulative Spread Return} = \sum_{i=1}^{1801} \Delta\text{Spread Return}_i$$

Convexity Return

The return due to changes in the yield from convexity.

The convexity of the bond is:

$$\begin{aligned}\gamma_T &= \frac{1}{P}\frac{\partial^2 P_T}{\partial y^2}\\ &= \frac{e^{yT}}{100}\frac{100}{e^{yT}}T^2\\ &= T^2\end{aligned}$$

The convexity of 2y bond is:

$$\gamma_2 = T^2 = 4$$

The convexity of 10y bond is:

$$\gamma_{10} = T^2 = 100$$

The convexity return from 2y Treasury bond is:

$$\Delta\text{Convexity Return}_2 = \frac{1}{2}(\text{x}_2 P_2) \times \gamma_2 \times (\Delta r_2)^2 = \frac{1}{2}(\text{x}_2 P_2) \times \gamma_2 \times (r_2^{''} - r_2)^2$$

The convexity return from 10y Treasury bond is:

$$\Delta\text{Convexity Return}_{10} = \frac{1}{2}(\text{x}_{10} P_{10}) \times \gamma_{10} \times (\Delta r_{10})^2 = \frac{1}{2}(\text{x}_{10} P_{10}) \times \gamma_{10} \times (r_{10}^{''} - r_{10})^2$$

The convexity return for this week is:

$$\Delta\text{Convexity Return} = \Delta\text{Convexity Return}_2 + \Delta\text{Convexity Return}_{10}$$

The culmulative convexity return:

$$\text{Cumulative Convexity Return} = \sum_{i=1}^{1801} \Delta\text{Convexity Return}_i$$

Time Return

The return due to the passage of time and interest on the cash position.

Here we assume that the yield during this week is flat, which means the yield on Jan 5, 1984 is the same as the yield on Dec 30, 1983, and the price is changed only because of the change of maturity.

The yield of 2y Treasury bond is calculated based on the NSS model, and we have $t = 2 - \frac{7}{365}$, τ_1, τ_2, β_1, β_2, and β_3 are from the estimation on Dec 30, 1983. Denote as:

$$r_2^{'''}$$

The price of 2y Treasury bond:

$$P_2^{'''} = \frac{100}{e^{(2-\frac{7}{365})\times r_2^{'''}}}$$

The time return from 2y Treasury bond:

$$\Delta\text{Time Return}_2 = \Delta P_2 \times \text{x}_2 = (P_2^{'''} - P_2) \times \text{x}_2$$

The yield of 10y Treasury bond is calculated based on the NSS model, and we have $t = 10 - \frac{7}{365}$, τ_1, τ_2, β_1, β_2, and β_3 are from the estimation on Dec 30, 1983. Denote as:

$$r_{10}^{'''}$$

The price of 10y Treasury bond:

$$P_{10}^{'''} = \frac{100}{e^{(10-\frac{7}{365})\times r_{10}^{'''}}}$$

The time return from 10y Treasury bond:

$$\Delta\text{Time Return}_{10} = \Delta P_{10} \times \text{x}_{10} = (P_{10}^{'''} - P_{10}) \times \text{x}_{10}$$

$$\Delta\text{Time Return} = \Delta\text{Time Return}_2 + \Delta\text{Time Return}_{10} + \text{Interest}$$

The cumulative time return:

$$\text{Cumulative Time Return} = \sum_{i=1}^{1801} \Delta\text{Time Return}_i$$

Residual

The difference between the total return and the sum of the spread return, convexity return, and time return.

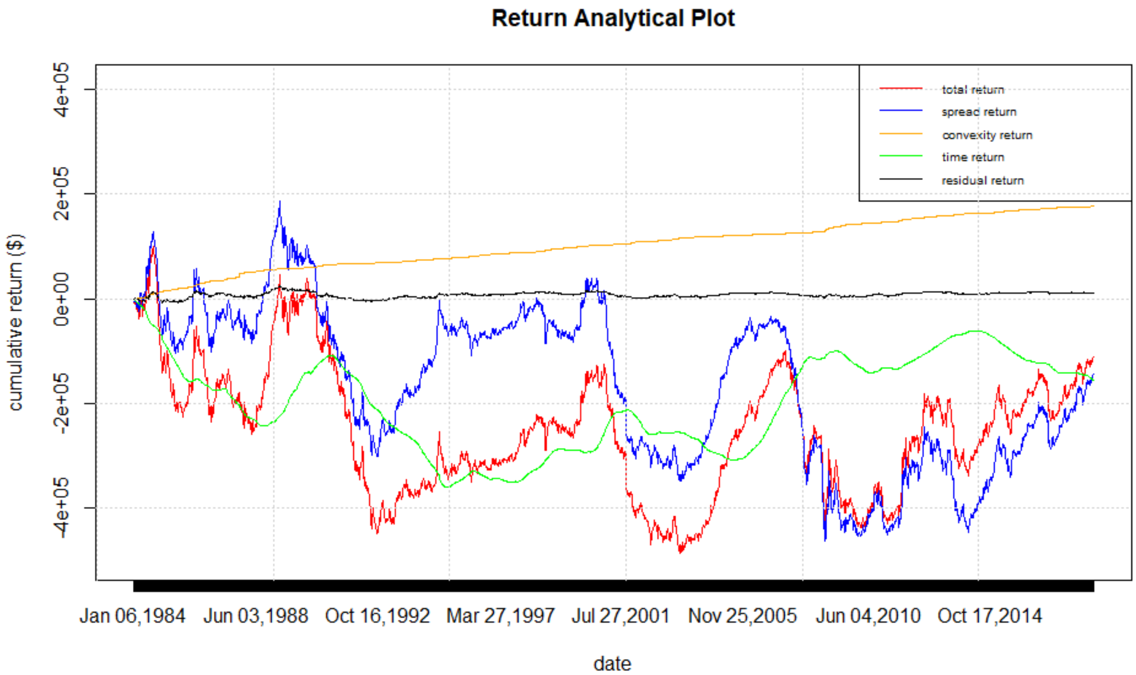
$$\Delta\text{Residual Return} = \Delta\text{Cumulative Return} - (\Delta\text{Spread Return} + \Delta\text{Convexity Return} + \Delta\text{Time Return})$$

The cumulative residual return:

$$\text{Cumulative Residual Return} = \sum_{i=1}^{1801} \Delta\text{Residual Return}_i$$

Conclusion

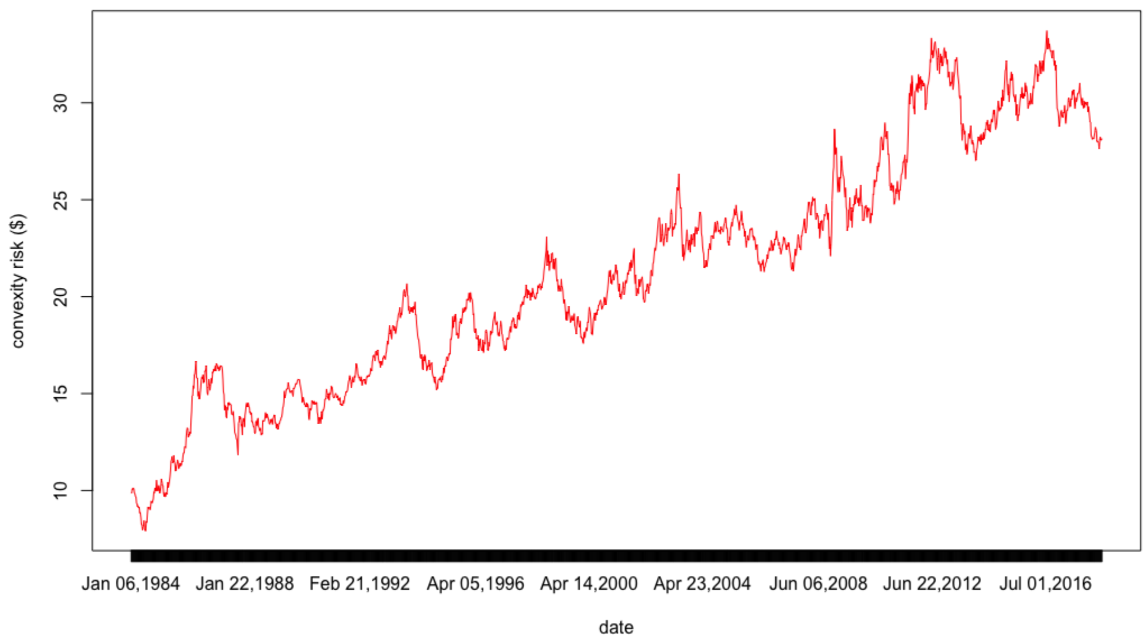
1. The plot of cumulative return, spread return, convexity return, time return and residual.



By observing the graph above, we can tell that the residual curve, marked as the balck line, is close to zero. This means that the sum of the spread return, the time return, and the convexity return explains the total return well enough. Also, based on the shape of the spread return curve and the total return curve, we can conclude that the total return is mainly determined by the spread return.

By plotting the result, we conclude that the total cumulative return is below the 0 level when we chose the flatterner strategy. It means that the application of flatterner strategy cause the loss, which is not effective and should not be chosen The steepener strategy should be used to earn profit.

2. The plot of the convexity risk while holding the constant units of 10yr bonds



Long positon: $1000000 \div 100 = 10000$.
Let short position be x_2.

Since we are using DV01-neutral, we will have:

$$10000 \times P_{10} \times D_{10}^* + x_2 P_2 \times D_2^* = 0$$

Solve the above equation, we will have our short position $= x_2$.

Also, we are given that $\Delta y = 0.0001$.

Then, convexity risk from 2y bond is:

$$\Delta \text{Convexity Risk}_2 = \frac{1}{2}(x_2 P_2) \times \gamma_2 \times (\Delta r_2)^2$$

Convexity risk from 10y bond is:

$$\Delta \text{Convexity Risk}_{10} = \frac{1}{2}(10000 \times P_{10}) \times \gamma_{10} \times (\Delta r_{10})^2$$

Then, the total convexity risk is:

$$\Delta \text{Convexity Risk} = \Delta \text{Convexity Risk}_2 + \Delta \text{Convexity Risk}_{10}$$

The cumulative convexity risk is:

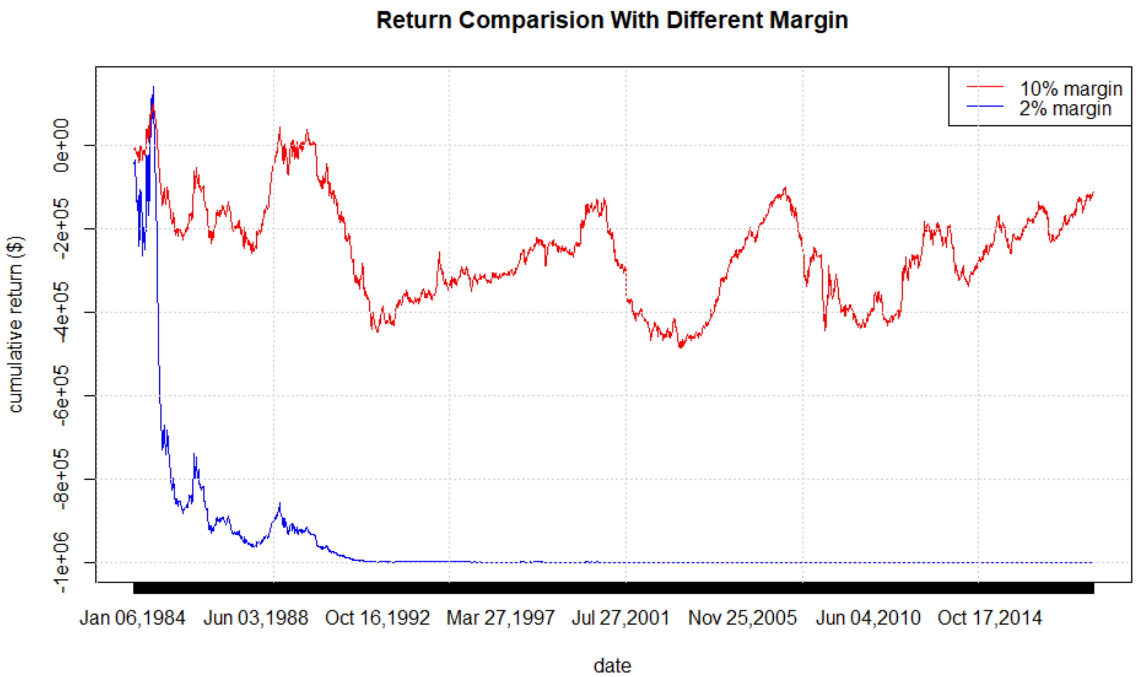
$$\text{Cumulative Convexity Risk} = \sum_{i=1}^{1801} \Delta \text{Convexity Risk}_i$$

For the convexity, the longer the maturity, the higher the convexity of a bond. As a result, holding long position for 10yr Treasury bonds means that the portfolio is exposed to the higher interest rate risk.

During the investment horizon, the yield for both 2yr and 10yr Treasury bonds fall down gradually and the portfolio should experience increasing convexity return. As the 10yr Treasury bonds have higher convexity exposure, the portfolio also experienced positive convexity return. **Not sure to mention in this way**

From the result, we can conclude that the flattener strategy benefits from the convexity.

3. The plot of the cumulative total return of the 2% margin requirement and the 10% margin requirement



When the margin requirement is 2%, the capital run out in 1993. The reason is that the less the margin requirement is, the larger the leverage is. For the 10% margin requirement, the leverage is 10 times. For the 2% margin requirement, the leverage is 50 times. When the leverage is too large, the volatility will also become heady. In that case, even though the portfolio only drifts down a little, the loss could cause the loss of the most part of the capital, even the loss of the whole capital.