Stat 461 HW5 Jiaqi Li

1. (a)

$$Y_{it} = \mu + \tau_i + \varepsilon_{it}, \quad i = A, B, C, \quad t = 1, 2.$$

note : C stands for "control".
A stands for "drug A" & B stands for "drug B".

(c) For $\{Y_{it}\}$, $i = A, B, C$, $t = 1, 2$

$$\bar{Y}_{A\cdot} = \frac{-14-4}{2} = -9$$

$$\bar{Y}_{B\cdot} = \frac{5-1}{2} = 2$$

$$\bar{Y}_{C\cdot} = \frac{-2+6}{2} = 2.$$

$$\bar{Y}_{\cdot\cdot} = \frac{-14-4+5-1-2+6}{6} = -\frac{5}{3}$$

$$SST = 2\times(-9+\tfrac{5}{3})^2 + 2\times(2+\tfrac{5}{3})^2 + 2\times(2+\tfrac{5}{3})^2 = \frac{484}{3}$$

$$SSTOT = (-14+\tfrac{5}{3})^2 + (-4+\tfrac{5}{3})^2 + (5+\tfrac{5}{3})^2 + (-1+\tfrac{5}{3})^2 + (-2+\tfrac{5}{3})^2 + (6+\tfrac{5}{3})^2 = \frac{784}{3}$$

$$SSE = SSTOT - SSE = 100$$

(b) $E(\Delta BP \text{ of } B) = \mu + \tau_B = \bar{Y}_{B\cdot} = 2$

ΔBP of B $\sim N(2, \sigma^2/2)$ since $\bar{Y}_{B\cdot} \sim N(2, \sigma^2/2)$

(d)
$$\hat{\sigma}^2 = \frac{SSE}{n-v} \quad \text{where } n = 6, \ v = 3$$
$$= \frac{100}{6-3} = \frac{100}{3}.$$

(e) $E(\Delta BP \text{ of } A - \Delta BP \text{ of } C) = (\mu + \tau_A) - (\mu + \tau_C) = \bar{Y}_{A\cdot} - \bar{Y}_{C\cdot} = -11.$

ΔBP of A $-$ ΔBP of C $\sim N(-11, \sigma^2)$
since $\bar{Y}_{A\cdot} - \bar{Y}_{C\cdot} \sim N(-9-2, \frac{\sigma^2}{2} + \frac{\sigma^2}{2}) = N(-11, \sigma^2)$

(f) When $\tau_A = \tau_C$, $(\mu+\tau_A) - (\mu+\tau_C) = 0$.
Then,
$$\hat{Y}_{A\cdot} - \hat{Y}_{C\cdot} \sim N(0, \sigma^2).$$

(g) Let $k(\sigma) = \sigma$
Then, $\frac{\hat{Y}_{A\cdot} - \hat{Y}_{C\cdot}}{k(\sigma)} \sim N(0/\sigma^2, \sigma^2/\sigma^2) = N(0,1)$.

(h) $$\frac{SSE}{\sigma^2} = \frac{\sum_i\sum_t (Y_{it} - \bar{Y}_{i\cdot})^2}{\sigma^2}$$
$$= \left[\frac{\sum_i\sum_t (Y_{it} - \bar{Y}_{i\cdot})}{\sigma}\right]^2 = \sum_i\sum_t \left(\frac{Y_{it} - \bar{Y}_{i\cdot}}{\sigma}\right)^2$$

Since $\frac{Y_{it} - (\mu+\tau_i)}{\sigma} \sim N(0,1)$ where $Y_{it} \overset{iid}{\sim} N(\mu+\tau_i, \sigma^2)$,

$$\left(\frac{Y_{it} - (\mu+\tau_i)}{\sigma}\right)^2 \sim \chi^2_1$$

Then, $\sum_i\sum_t \left(\frac{Y_{it} - (\mu+\tau_i)}{\sigma}\right)^2 \sim \chi^2_6$
Now, replace $\mu+\tau_i$ with $\bar{Y}_{i\cdot}$, we have

$$\frac{SSE}{\sigma^2} = \sum_i\sum_t \left(\frac{Y_{it} - \bar{Y}_{i\cdot}}{\sigma}\right)^2 \sim \chi^2_{6-3} = \chi^2_3.$$

(i) Since $\frac{SSE}{\sigma^2}$ is chi-square distribution with degree of freedom 3 and $\frac{\hat{Y}_{A\cdot} - \hat{Y}_{C\cdot}}{k(\sigma)}$ is standard normal distribution,
$$\frac{(\hat{Y}_{A\cdot} - \hat{Y}_{C\cdot})/k(\sigma)}{SSE/[(n-v)\sigma^2]} \sim t_3$$

(j) ANOVA (one-way):

Source	Deg. of Freedom	Sum of Square	Mean Square	Ratio
Treatment	$3-1=2$	$484/3$	$(484/3)/2 = 242/3$	$(242/3)/(100/3) = 2.42$
Error	$6-3=3$	100	$100/3$	
Total	$6-1=5$	$784/3$		

H_0: there are no differences in mean blood pressure change between women getting any treatment.

Under null hypothesis, test statistic $T^* = 2.42$
Test statistic is F distribution (i.e. $F_{2,3}$)