1. VaR for option on 2 underlying.

1. $\text{Gain} = M(S_{1,t+1}, S_{2,t+1}) - M(S_{1,t}, S_{2,t})$.

$$\approx M'(S_{1,t})(S_{1,t+1} - S_{1,t}) + M'(S_{2,t})(S_{2,t+1} - S_{2,t}).$$

$$Var(\text{Gain}) = \Delta_1^2\sigma_1^2S_1^2 + \Delta_2^2\sigma_2^2S_2^2 + 2\Delta_1\Delta_2\rho\sigma_1S_1\sigma_2S_2.$$

$$sd = \sqrt{Var(\text{Gain})}$$

$$Mean(\text{Gain}) = \Delta_1\mu_1S_1 + \Delta_2\mu_2S_2$$

$$VaR = -(\Delta_1\mu_1S_1 + \Delta_2\mu_2S_2) + 2.32 \times \sqrt{\Delta_1^2\sigma_1^2S_1^2 + \Delta_2\sigma_2^2S_2^2 + 2\Delta_1\Delta_2\rho\sigma_1S_1\sigma_2S_2}$$

2. $\text{Gain} = M(S_{1,t+1}, S_{2,t+1}) - M(S_{1,t}, S_{2,t})$.

$$\approx \Delta_1(S_{1,t+1} - S_{1,t}) + \Delta_2(S_{2,t+1} - S_{2,t}) + \frac{1}{2}\Gamma_1(S_{1,t+1} - S_{1,t})^2 + \frac{1}{2}\Gamma_2(S_{2,t+1} - S_{2,t})^2$$
$$+ \frac{1}{2}\Gamma_{1,2}(S_{1,t+1} - S_{1,t})(S_{2,t+1} - S_{2,t}) + \frac{1}{2}\Gamma_{1,2}(S_{2,t+1} - S_{2,t})(S_{1,t+1} - S_{1,t})$$

$$= \Delta_1 dS_1 + \Delta_2 dS_2 + \frac{1}{2}\Gamma_1(dS_1)^2 + \frac{1}{2}\Gamma_2(dS_2)^2 + \Gamma_{1,2}dS_1dS_2$$

$$= \Delta_1 dS_1 + \Delta_2 dS_2 + \frac{1}{2}\Gamma_1\sigma_1^2S_1^2dt + \frac{1}{2}\Gamma_2\sigma_2^2S_2^2dt + \Gamma_{1,2}dS_1dS_2$$

$$= (\Delta_1\mu_1S_1 + \Delta_2\mu_2S_2 + \frac{1}{2}\Gamma_1\sigma_1^2S_1^2 + \frac{1}{2}\Gamma_2\sigma_2^2S_2^2 + \Gamma_{1,2}\sigma_1\sigma_2S_1S_2\rho)dt$$
$$+ \Delta_1\sigma_1S_1dW_1 + \Delta_2\sigma_2S_2dW_2$$

$$Mean(\text{Gain}) = \Delta_1\mu_1S_1 + \Delta_2\mu_2S_2 + \frac{1}{2}\Gamma_1\sigma_1^2S_1^2 + \frac{1}{2}\Gamma_2\sigma_2^2S_2^2 + \Gamma_{1,2}\sigma_1\sigma_2S_1S_2\rho$$

$$Var(\text{Gain}) = \Delta_1^2\sigma_1^2S_1^2 + \Delta_2^2\sigma_2^2S_2^2 + 2\Delta_1\Delta_2\sigma_1\sigma_2S_1S_2\rho$$

$$VaR = -(\Delta_1\mu_1S_1 + \Delta_2\mu_2S_2 + \frac{1}{2}\Gamma_1\sigma_1^2S_1^2 + \frac{1}{2}\Gamma_2\sigma_2^2S_2^2 + \Gamma_{1,2}\sigma_1\sigma_2S_1S_2\rho)$$
$$+ 2.32(\Delta_1^2\sigma_1^2S_1^2 + \Delta_2^2\sigma_2^2S_2^2 + 2\Delta_1\Delta_2\sigma_1\sigma_2S_1S_2\rho).$$

3. $M_T = \max(\min(S_{1,T}, S_{2,T}) - K, 0)$.

$$M(S_1, S_2) = S_1 N_2\left(\gamma_1 + \sigma_1\sqrt{\tau}, \frac{\ln\left(\frac{S_2}{S_1}\right) - \frac{1}{2}\sigma^2\sqrt{\tau}}{\sigma\sqrt{\tau}}, \frac{\rho\sigma_2 - \sigma_1}{\sigma}\right)$$

$$+ S_2 N_2\left(\gamma_2 + \sigma_2\sqrt{\tau}, \frac{\ln\left(\frac{S_1}{S_2}\right) - \frac{1}{2}\sigma^2\sqrt{\tau}}{\sigma\sqrt{\tau}}, \frac{\rho\sigma_1 - \sigma_2}{\sigma}\right)$$

$$- Ke^{-r\tau} N_2(\gamma_1, \gamma_2, \rho)$$

$$\gamma_1 = \frac{\ln\left(\frac{S_1}{K}\right) + (r - \frac{1}{2}\sigma_1^2)\tau}{\sigma_1\sqrt{\tau}}$$

$$\gamma_2 = \frac{\ln\left(\frac{S_2}{K}\right) + (r - \frac{1}{2}\sigma_2^2)\tau}{\sigma_2\sqrt{\tau}}$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2.$$

$N_2(\alpha, \beta, \theta) = P(X_1 \le \alpha, X_2 \le \beta)$ if X_1, X_2 standard Normal with $\mathrm{corr}(X_1, X_2) = \theta$.

$r = 0.005\%$, $\sigma_1 = \sigma_2 = 2\%$, $\rho = 0.4$, $\mu_1 = \mu_2 = 0.03\%$
at date 0, $T = 6$ months, $S_{1,0} = 99$, $S_{2,0} = 101$, $K = 100$.