CS515 - Algorithms & Data Structures

Practice Assignment 1

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Problem 1
A fixed point of an array A[1..n] is an index i such that A[i] = i. Given a sorted array of distinct integers A[1..n] as input, give a divide-and-conquer algorithm to determine if A has a fixed point that runs in time $O(logn)$.

$$D_{it} = \begin{cases} 1 & \text{if bank } i \text{ issues ABs at time } t \\ \begin{cases} 1 & \text{if bank } i \text{ issues ABs at time } t \\ & \text{if bank } i \text{ issues CBs at time } t \end{cases} & \text{if bank } i \text{ issues CBs at time } t \end{cases} \tag{1}$$

Problem 2
For a sequence of n numbers $a_1, .., a_n$, a *significant inversion* is a pair (a_i, a_j) such that $i < j$ and $a_i > 2a_j$. Assuming each of the numbers a_i is distinct, give an $O(nlogn)$ time algorithm to count the number of significant inversions in a sequence. (Hint: modify merge sort.)

Problem 3
You are given two sorted arrays of size m and n. Give an $O(logm + logn)$ time algorithm for computing the k-th smallest element in the union of the two arrays.

Problem 4
You are given an $n \times n$ matrix A[1..n, 1..n] where all elements are distinct. We say that an element A[x] is a *local minimum* if it is less than its (at most) four neighbors, i.e. its up, down, left and right neighbors. Give an $O(n)$ time algorithm to find a local minimum of A.