

# A Vehicular Communication Routing Algorithm Based on Graph Theory

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**Abstract**—Recently, the vehicular ad hoc network (VANET) has attracted the attention of researchers with the development of the Internet of Things (IoT) and the Intelligent Transport system (ITS). One of the major application scenarios of the fifth generation wireless communication is Massive Machine Type Communication (mMTC). In order to aggregate the data recorded by machines, information packets need to be delivered to bureaus in the network. However, some packets are not very urgent and they don't have to be transferred by the cellular communication due to the fact that the spectrum source is scarce. With the increasing number of vehicles and the increasing computing power of on board units (OBUs), vehicle-to-vehicle (V2V) communications are better to deliver data packets. In order to transfer the packets effectively, it is important to find a reliable vehicular communication route. As the topology and the vehicles velocity change much more rapidly, the existing routing algorithms in other kinds of ad hoc networks are not suitable for the VANET. In this paper, we propose a vehicular routing algorithm based on graph theory. We consider the network situations more comprehensively and the simulation results show that the algorithm proposed is superior to the traditional routing algorithm.

**Index Terms**—mMTC, VANET, vehicular communication, graph theory, routing algorithm

## I. INTRODUCTION

Generally, ad hoc network can be divided into three types, namely wireless ad hoc network (WANET), mobile ad hoc network (MANET) and vehicular ad hoc network (VANET). The nodes can communicate with each other in the network [1]. With the rapid development of the Internet of Things (IoT) and the Intelligent Transport system (ITS), VANET has attracted lots of attention in these years [2]. VANET can provide people with convenient information by utilizing various vehicular applications [3]. For example, passengers and drivers can get the information about real-time road safety, live road conditions, surrounding facilities, maps and so on [4]. Also, vehicles can be connected to the infrastructures and the devices in the surrounding environment for communication in order to support the Massive Machine Type Communication (mMTC), which is one of the three major application scenarios of the fifth generation (5G) wireless communication [5]. The

primary data recorded by machines need to be transferred to the corresponding bureaus.

In order to deliver the packets effectively, it is important to find a reliable route efficiently in real-time between the source and the destination. Due to the increasing number of vehicles and the limited spectrum resource, vehicle-to-vehicle (V2V) communications are becoming a core technology nowadays [7]. Because VANET has the characteristics of large data, rapid topology change, high velocity and restricted driving trajectory [6], which makes VANET is different from WANET and MANET, therefore, the existing routing algorithms in WANET and MANET are not applicable. It is necessary to consider network situations such as the vehicle link connection situations, the network topology and the node situations when designing the routing algorithm.

The graph theory can cope with these problems by introducing the concepts of vertexes and edges in a graph. Evolving graphs such as Markovian evolving graph is a dynamic model where the connections among the nodes in network change every time. Therefore, the evolving graph model is suitable to describe the dynamic-network scenarios [9] where the weights of edges between vertexes can represent the situations of connections between nodes during communication. Also, the evolving graph used graph analysis by considering the characteristics of VANET is more practicable when analyzing the network than the existing methods in static graphs to capture the communication route features in VANET. [11], [12].

Based on the above analysis, we propose a vehicular communication routing algorithm based on graph theory in this paper. The source machine and destination bureau can communicate with each other through the route selected with the help of V2V communications by considering the network situations. And the rest of this paper is organized as follows. Section II introduce the related work. And the system model is introduced in Section III. The corresponding routing algorithm based on the system model is proposed in Section IV. Simulation results are provided in Section V. Finally, we get the conclusion in Section VI.

## II. RELATED WORK

T. Yan et al. [15] designed two algorithms to select a minimum number of intersections to install base stations (BSs) to make a tradeoff between the number of BSs and the percentage of vehicles covered. This paper formulated the topology-aware intersection selection problem (TIS) and solved this NP-complete problem by solving the vertex cover problem in bipartite graphs. Although it can reduce the network cost, too many vehicles connect with one BS at the same time may increase the transmission delay and cause the network congestion because the bandwidth resource is scarce. Y. Huang et al. [16] made full use of the V2V communications. In order to transmit packets without interference among vehicles, this paper got the maximum independent number of concurrent transmission flows in road units by finding the maximum independent number of a graph. However, the interference model of this paper was simple and it ignored the node situations.

M. H. Eiza et al. [13] pointed that due to the fact that VANET can not be treated as a fixed scheduled dynamic network, the existing methods in evolving graph theory can not be utilized directly when analyzing the vehicular communication in VANET. Therefore, researchers calculated the links' availability probability to measure the route reliability between nodes in V2V communications [8], [13]. However, they only considered the link connection situations namely the influence of the relative distance and velocity between two nodes. They neglected the network topology and the node situations which can represent the influence of surrounding nodes and the type of vehicles respectively. And the route selected was not very effective.

M. X. Punithan et al. [14] introduced a framework and represented a vehicle's neighborhood topology in network by using the King's graph. It regarded the neighborhood region as the lattices by using the presence or absence status information of neighbors in binary form. Also, L. Zhang et al. [1] proposed a recursive decomposition approach based on percolation theory to calculate the transmission probability between the source and the destination in a two-dimensional VANET lattice. However, the bond probability between each node in this approach is stationary, it isn't suitable for the dynamic VANET with different probabilities of each hop.

In this paper, we select a vehicular communication route efficiently in real-time by considering the network situations.

First of all, we regard the vehicles as the vertexes and the link connections between vehicles as edges in a graph and set up a more comprehensive vehicular communication model to calculate the route reliability among vehicle nodes. We not only consider the link connection situations such as the influence of the relative distance and velocity between two vehicle nodes, we also consider the network topology and the node situations which can be denoted by the degree and the type of vehicles respectively.

What's more, we regard the intersections as the vertexes and the roads between intersections as edges in a graph and

propose a routing algorithm which can dynamically select intersection areas as relays in the route by analyzing the route reliability among intersections in real-time. And the packets will be transferred by V2V communications between two selected adjacent intersections according to the route.

Finally, we execute the routing algorithm proposed and the traditional routing algorithm in scenario which is more similar to the movement of vehicles in the real world compared with the scenario where nodes move randomly. And the simulation results show that the routing algorithm proposed is superior to the traditional routing algorithm.

## III. VEHICULAR COMMUNICATION MODEL BASE ON GRAPH THEORY

### A. Applicable Scenario

With the development of the 5G wireless communication and IoT, there are massive types of machines which are used to record data communicating with the internet. Then the corresponding bureaus can aggregate the primary data. For example, the cameras used to collect the traffic information for the whole day, the electricity meters used to record the energy use need to transfer the data to traffic control department and electric power bureau respectively. We notice that some information don't need to be handled immediately, therefore, they don't have to be delivered by cellular communication because the bandwidth is scarce. Also, it is difficult for workers to record the data directly by hand because they are distributed randomly.

Because there are lots of vehicles with plenty of computing power, we can transfer the information packets with the help of V2V communications. The main purpose of this paper is to find a reliable route efficiently in real-time between the source machine and the destination bureau. The machine will firstly broadcast the packets to the vehicles in the nearest intersection. Then the packets will be transferred by V2V communications between two selected adjacent intersections according to the route until the packets are transmitted to the intersection closest to the destination bureau. Finally, the packets will be offloaded to the bureau from the vehicles in this area. In order to transfer the packets effectively, the route reliability have to be taken into consideration. Also, because vehicles move quickly, we need to find the route efficiently in real-time.

### B. System Model

To solve these problems, we firstly regard the vehicles as the vertexes and the link connections between vehicles as edges in a graph and set up a more comprehensive vehicular communication model to calculate the route reliability among vehicle nodes.

We regard the vehicles in the network at time  $t$  as a weighted and undirected planar graph  $G(V, E)$  as shown in Fig. 1. The vertexes in the graph represent the vehicle nodes and the edges represent the link connections between two nodes at time  $t$ . The weights of each edge represent the route reliability at time  $t$  when vehicles communicate in the network. We use  $(v_i, v_j)$

to represent the edge between the vertex  $v_i$  and the vertex  $v_j$  in the planar graph model  $G(V, E)$ .  $V = \{v_1, v_2, \dots, v_m\}$  is the vertex set of vehicle nodes and  $E = \{e_1, e_2, \dots, e_n\}$  is the edge set of link connections between nodes in the network.

We can infer that there are two subgraphs  $G_1$  and  $G_2$  in Fig. 1 which can represent two subnetworks formed from seven vehicles and three vehicles respectively. For simplicity, we introduce the concept of vertexes and edges in subgraph  $G_1$  and introduce the concept of weights in subgraph  $G_2$ . The subgraph  $G_1$  is composed of the vertexes  $\{A, B, C, D, E, F, G\}$  and the edges  $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ . Vehicles can communicate with each other if there is at least one valid edge between two vertexes. For example, Vertex  $A$  can exchange information packets with vertex  $E$  at time  $t$  while it can not communicate with vertex  $H$ . Additionally, there are two edge weights in subgraph  $G_2$ . In this paper, we assume that the reliability of  $e_9$  is better than the reliability of  $e_{10}$  if  $w_1$  is greater than  $w_2$  at time  $t$ .

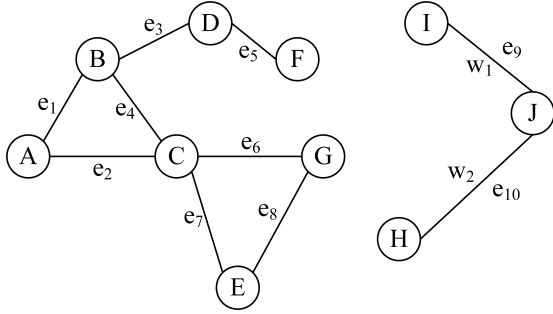


Fig. 1. An example of a graph  $G(V, E)$  at time  $t$ .

### C. Network Situation

In this paper, we use the weights of the edges to denote the route reliability. To calculate the route reliability, we consider the network situations more comprehensively. And the network situations we consider are the link connection situations, network topology and the node situations.

1) *Link Connection Situations*: The first parameter which can affect the link connection situations is the relative distance  $\Delta d_{i,j,t}$  between the node  $i$  and the node  $j$  at time  $t$ . Use the following (1) to calculate the  $\Delta d_{i,j,t}$  where  $x$  and  $y$  are the position coordinate on x axis and y axis of a vehicle node at time  $t$ . We assume that  $\Delta d_{i,j,t} = \Delta d_{j,i,t}$  and  $\Delta d_{i,i,t} = 0$ .

$$\Delta d_{i,j,t} = \sqrt{(x_{i,t} - x_{j,t})^2 + (y_{i,t} - y_{j,t})^2} \quad (1)$$

The second parameter is the relative speed  $\Delta v_{i,j,t}$  between the node  $i$  and the node  $j$  at time  $t$ . We can get the  $\Delta v_{i,j,t}$  by using (2).  $v_{i,x,t}$  and  $v_{j,x,t}$  are the velocity components on the axis.

$$\Delta v_{i,j,t} = \sqrt{(v_{i,x,t} - v_{j,x,t})^2 + (v_{i,y,t} - v_{j,y,t})^2} \quad (2)$$

2) *Network Topology*: Network topology is another parameter that affects network situations. Too many vehicles communicate with the same vehicle may cause the link congestion. Therefore, we use the node degree  $D_{i,t}$  at time  $t$  to denote the influence of neighboring topology of node  $i$  on communication. The detailed steps to calculate  $D_{i,t}$  is below.

We suppose that there are  $m$  vehicles in a certain area. And the relative distance matrix  $\Delta d_t$  which can represent the nodes' relative positions in VANET at time  $t$  is shown in (3).

$$\Delta d_t = \begin{bmatrix} \Delta d_{1,1,t} & \Delta d_{1,2,t} & \cdots & \Delta d_{1,m,t} \\ \Delta d_{2,1,t} & \Delta d_{2,2,t} & \cdots & \Delta d_{2,m,t} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta d_{m,1,t} & \Delta d_{m,2,t} & \cdots & \Delta d_{m,m,t} \end{bmatrix} \quad (3)$$

For example,  $\Delta d_t$  may exist below: (in a unit of m)

$$\Delta d_t = \begin{bmatrix} 0 & 120.13 & \cdots & 610.68 \\ 120.13 & 0 & \cdots & 169.71 \\ \vdots & \vdots & \ddots & \vdots \\ 610.68 & 169.71 & \cdots & 0 \end{bmatrix} \quad (4)$$

We also suppose that the maximum communication range of the on board unit (OBU) on the vehicles is 250 meters. If the relative distance between node  $i$  and node  $j$  is smaller than it, these two nodes will be neighbor nodes at time  $t$ . After that we can get the adjacency matrix in binary form from the relative distance matrix  $\Delta d_t$ . The adjacency matrix  $A_t$  of (4) is shown in (5).

$$A_t = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & \cdots & 0 \end{bmatrix} \quad (5)$$

After that we can get the  $D_{i,t}$  by calculating the sum of each row of  $A_t$ . And the equation is shown in (6).

$$D_{i,t} = \sum_{j=1}^n a_{i,j,t} \quad (6)$$

3) *Node Situations*: We use the vehicle type  $T_i$  to measure the node situations. It is an important parameter which can denote the influence of the node itself and the computing power on the communication. We suppose that there are three kinds of vehicles on the road namely the buses, the trucks and the cars. Buses have the most stationary traces and have larger space to install stronger computing power OBUs. Also the government can install the OBUs without having to consider the trace privacy too much due to the public nature of buses. However, although the cars have less space and less stationary traces, the smallest volume with the least shadow fading probability is beneficial for communication. In order to calculate the impact of each type of vehicles on communication, we set the value of  $T_i$ . We suppose that the value of buses is 3 and the value of cars is 2. Also, the value of trucks is 1.

#### D. Route reliability

Based on the analysis of network situations, we can infer that the shorter  $\Delta d_{i,j,t}$  is, the less the transmission delay is when they are communicating with each other. Also, the link connection established will be more stable if the  $\Delta v_{i,j,t}$  is lower. Additionally, the vehicle node with a higher degree may have a higher load which can cause communication congestion. The node situation is an important factor as well. It represents the influence of the node itself and the computing power.

Above all, we propose a more comprehensive equation to calculate the route reliability between node  $i$  and node  $j$  which is shown in (7). It measures the weight of each edge. Furthermore,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_4$  are characteristic parameters.

$$r_{i,j,t} = \begin{cases} \frac{\alpha_1 \cdot T_i T_j}{\alpha_2 \cdot \Delta d_{i,j,t} \times \alpha_3 \cdot D_{i,t} D_{j,t} \times \alpha_4 \cdot \Delta v_{i,j,t}} & (a_{i,j,t} = 1) \\ 0 & (a_{i,j,t} = 0) \end{cases} \quad (7)$$

Also, the route reliability matrix  $r_t$  of a certain area with  $m$  vehicles at time  $t$  can be obtained by (8).

$$r_t = \begin{bmatrix} r_{1,1,t} & r_{1,2,t} & \cdots & r_{1,m,t} \\ r_{2,1,t} & r_{2,2,t} & \cdots & r_{2,m,t} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m,1,t} & r_{m,2,t} & \cdots & r_{m,m,t} \end{bmatrix} \quad (8)$$

#### IV. ROUTING ALGORITHM FOR VEHICULAR COMMUNICATION

To solve the problems in III-A, we regard the intersections as the vertexes and the roads between intersections as edges in a graph and propose a routing algorithm which can dynamically select intersection areas as relays in the route by analyzing the route reliability among intersections in real-time. And the packets will be transferred by V2V communications between two selected adjacent intersections according to the route.

We suppose that there is a certain area where a Manhattan-like road structure is considered. Each road can be represented by an edge and the intersection can be represented by an vertex. The Manhattan-like road is shown in Fig. 2(a). The four corresponding intersections and roads among them in Fig. 2(a) can be modeled in Fig. 2(b). We divide the roads into several independent areas. The length and width of each area are 250 meters and each area has only one intersection. The details are shown in Fig. 2(c).

For simplicity, we assume that there is one machine in a certain realistic scenario which is shown in Fig. 3.

In order to find the route from the machine to the bureau, the routing algorithm will calculate the route reliability  $r_{i,j,t}$  between vehicles in each area at first. After that, it will calculate the reliability  $R_{I,t}$  of each intersection area by using (9) where there are  $m$  vehicles in this area.

$$R_{I,t} = \frac{\sum_{i=1}^{m-1} \sum_{j=i+1}^m r_{i,j,t}}{m(m-1)/2} \quad (9)$$

And the route reliability  $R_{I,J,t}$  between area  $I$  and area  $J$  can be obtained by (10).

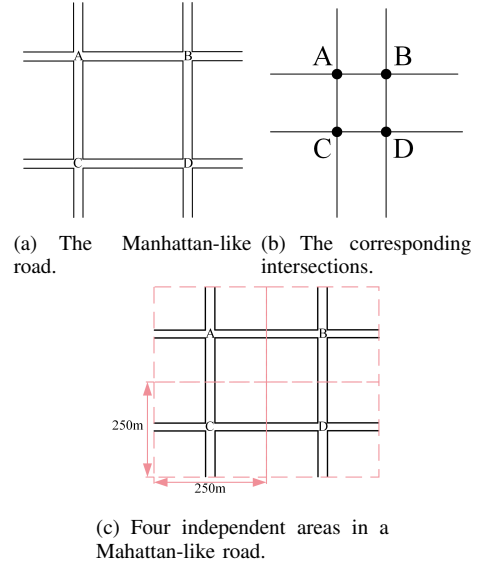


Fig. 2. A Realistic Scenario in VANET.

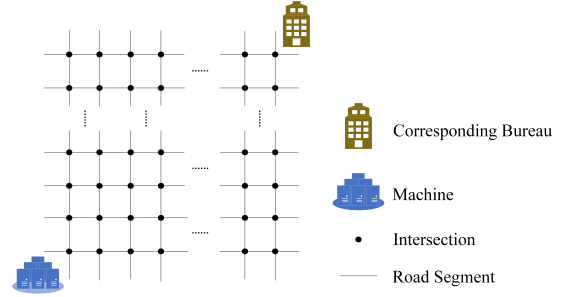


Fig. 3. A machine and bureau distributed in a certain area.

$$R_{I,J,t} = \begin{cases} R_{I,t} \times R_{J,t} & (I \text{ and } J \text{ are adjacent areas}) \\ 0 & (I \text{ and } J \text{ are not adjacent areas}) \end{cases} \quad (10)$$

Similarly, the route reliability matrix  $R_t$  can be calculated by (11)

$$R_t = \begin{bmatrix} R_{1,1,t} & R_{1,2,t} & \cdots & R_{1,n,t} \\ R_{2,1,t} & R_{2,2,t} & \cdots & R_{2,n,t} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n,1,t} & R_{n,2,t} & \cdots & R_{n,n,t} \end{bmatrix} \quad (11)$$

Then, the routing algorithm will find the route from the machine to the bureau by applying the Dijkstra algorithm. The normal Dijkstra algorithm is a classical method to find a shortest path on a graph by using the distance matrix [17]. However, the normal Dijkstra algorithm is not suitable for the dynamic graphs in VANET. Therefore, we propose a routing algorithm based on the normal Dijkstra algorithm to find a reliable route efficiently in real-time between two intersections by considering the route reliability between two areas. And the details are shown in Algorithm 1.

**Algorithm 1** Routing algorithm**Input:** the initial intersection route reliability matrix  $R_t$ ;**Output:** reliable route;

- 1: get the initial source area  $s$  and destination area  $d$ ;
- 2: create a vertex set  $V$ ;
- 3: calculate the most reliable route by using Dijkstra algorithm and the initial  $R_t$
- 4: **while** the next hop  $n$  of  $s$  is not  $d$  **do**
- 5:     put  $n$  into set  $V$
- 6:      $s = n$
- 7:     update the intersection route reliability matrix  $R_t$
- 8:     calculate the most reliable route by using Dijkstra algorithm and  $R_t$
- 9: **end while**
- 10: **return**  $V$

The routing algorithm proposed can dynamically select every intersection-relay in the route. And the vertexes in set  $V$  are the real-time intersection-relays between the area closest to the machine and the area closest to the bureau. After selecting the route, the packets can be transferred by V2V communications between the selected adjacent intersections according to the route.

## V. SIMULATION RESULTS

In this section, we execute the routing algorithm proposed and the traditional routing algorithm AODV in scenario which is more similar to the movement of vehicles in the real world compared with the scenario where nodes move randomly. The simulation results show that the routing algorithm proposed is superior to the traditional routing algorithm AODV.

The road constructure we set is like Fig. 2(c). We set three horizontal roads and four vertical roads with twelve intersections in the scenario. We suppose there are two lanes in each road and set different numbers of vehicles on each road. We also assume that the speed of vehicles follows the normal distribution. And the other simulation parameters are shown in Table I.

TABLE I  
THE PARAMETERS OF THE SCENARIO

Parameter	Value
The number of vehicles on each road	10,20,30,40,50,60,70,80,90,100
The width of each lane (m)	3.75
Time (s)	100
The means of speed (km/h)	45
The standard deviation of speed (km/h)	10

Different numbers of vehicles move in the scenario in 100 seconds during each simulation. The routing algorithm proposed will firstly calculate the reliability of each intersection area by calculating the route reliability among vehicles in each area. Then it will find a route from the area closest to the machine to the area closest to the bureau. The AODV routing algorithm will calculate the distance among the vehicles in the whole scenario and find a shortest route from the vehicle

closest to the machine to the vehicle closest to the bureau. The simulation results as follows.

Fig. 4 shows the average time to find a route in 100 seconds when a certain number of vehicles move. We can find that the average time to find a route increases when the number of vehicles increases. Although the average time to find a route of AODV routing algorithm is slightly less than that of the routing algorithm proposed, it increases rapidly when the number of vehicles is greater than 50. And the average time to find a route of AODV routing algorithm is much greater than that of the routing algorithm proposed. We also notice that the average time to find a route of the routing algorithm proposed only has a small steady rise all the time. Therefore, the routing algorithm proposed can find the route much more efficiently in real-time.

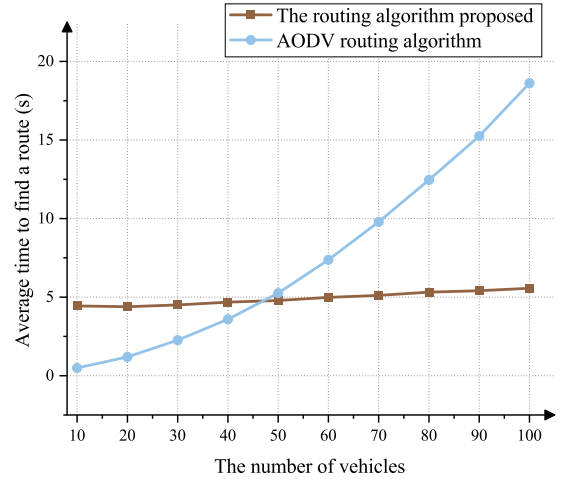


Fig. 4. The average time to find a route for the different numbers of vehicles.

Fig. 5 shows the route changes in 100 seconds when a certain number of vehicles move. During the simulation, the route selected by the routing algorithm proposed changes very little compared with the AODV routing algorithm, therefore, the packets can be delivered much more stably. Correspondingly, the rate of packet loss and the routing overhead may be less. As a result, we can find that the routing algorithm can select a much more reliable route.

To sum up, the routing algorithm proposed is superior to the traditional routing algorithm AODV. It can find a much more reliable route efficiently in real-time between the source and the destination.

## VI. CONCLUSIONS

In this paper, we propose a vehicular communication routing algorithm based on graph theory to transfer the packets from the machines to the bureaus. In order to select a reliable route efficiently in real-time, we firstly consider the network situations comprehensively. The factors we consider are the link connection situations, the network topology and the node situations. Then we analyze the influence of these factors to the route and calculate the route reliability between vehicles. We

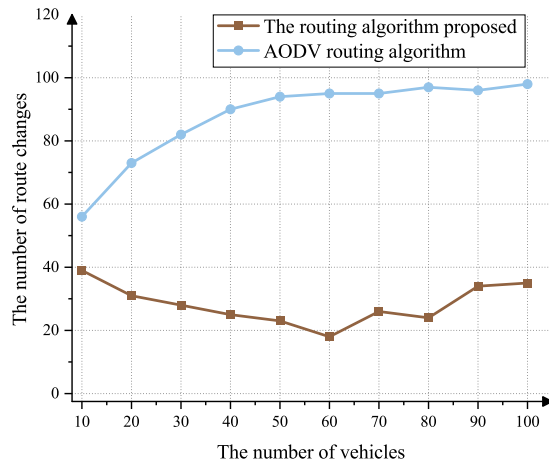


Fig. 5. The number of route changes for the different numbers of vehicles.

also calculate the route reliability between intersections. After that we select the route from the machines to the bureaus by using the routing algorithm proposed. The route regards the intersections as the relays. From the simulation results, we can get the conclusion that the routing algorithm proposed is superior to the traditional routing algorithm.

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