Theory and Practice of Bayesian Inference Using JASP

JASP

Johnny van Doorn

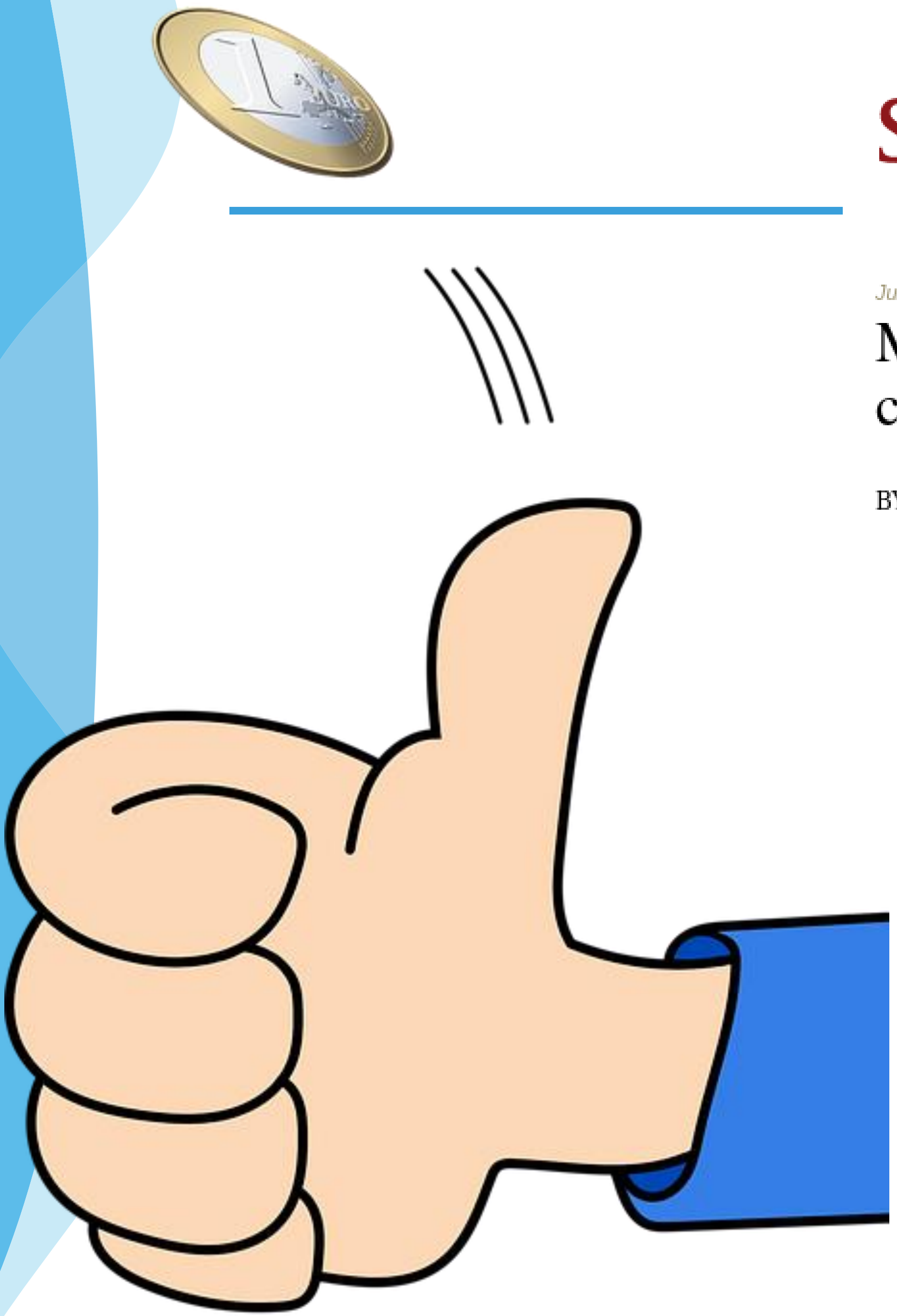
Practical Stuff

- Did everyone manage to install JASP 0.95.4?
- Reading
  - https://johnnydoorn.github.io/IntroductionBayesianInference/
- Introduce myself
- Slides →
  www.edu.nl/4qenh

edu.nl/4qenh

Outline

- Statistical Models
- How do Models Learn from Data?
- Comparing Statistical Models
- Extending to other Tests
- Some Practice



Stanford | News Service

June 4, 2004

Magician-turned-mathematician uncovers bias in coin flipping

BY ESTHER LANDHUIS

Persi Diaconis has spent much of his life turning scams inside out. In 1962, the then 17-year-old sought to stymie a Caribbean casino that was allegedly using shaved dice to boost house odds in games of chance. In the mid-1970s, the upstart statistician exposed some key problems in ESP research and debunked a handful of famed psychics. Now a Stanford professor of mathematics and statistics, Diaconis has turned his attention toward simpler phenomena: determining whether coin flipping is random. Could a simple coin toss -- used routinely to decide which team gets the ball, for instance -- actually be rigged?

Source: https://news.stanford.edu/pr/2004/diaconis-69.html

https://osf.io/6a5hy/

What is Bayesian Inference?

What are the tools we need?

What are Statistical Models?

A statistical model is a combination of a general statistical model (e.g., the binomial model) and a **statement** about a parameter value that describe a certain phenomenon

The general binomial model describes a series of chance-based events with a binary outcome, and is governed by a single parameter θ

$$\frac{n!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}$$

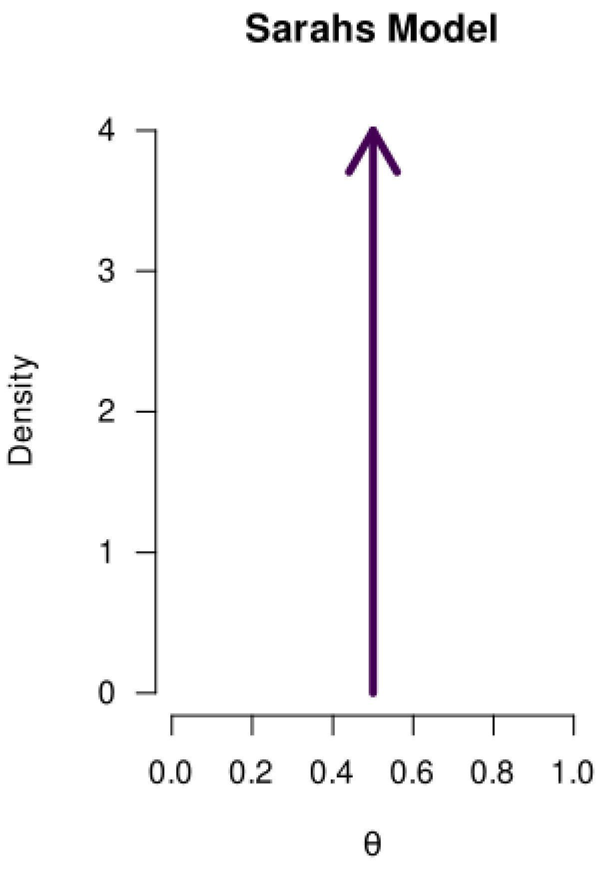
For instance, for flipping a coin:
A binomial model with θ = 0.5

What is the theoretical implication of this model?

What are Statistical Models?

A binomial model with θ = 0.5

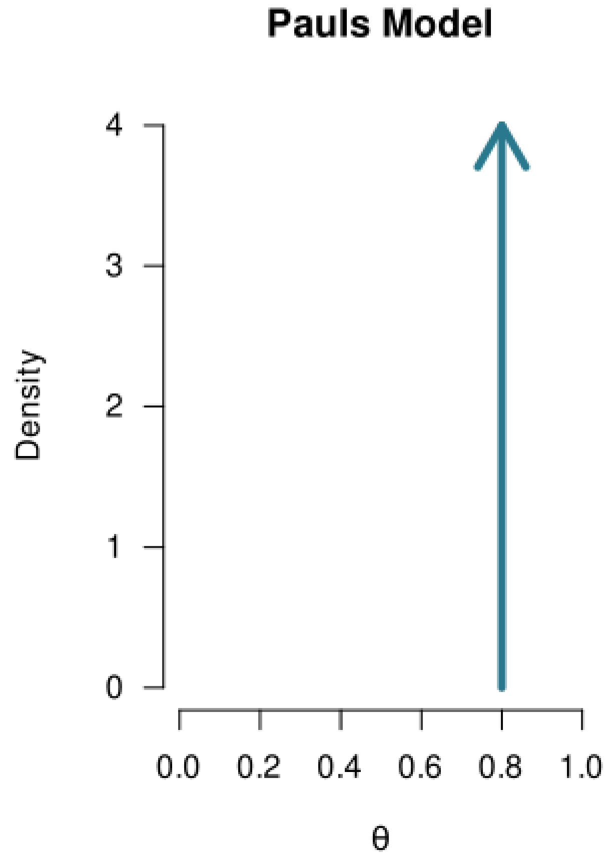
We can reflect a model’s statement by means of a probability distribution



What are Statistical Models?

A binomial model with $\theta = 0.8$

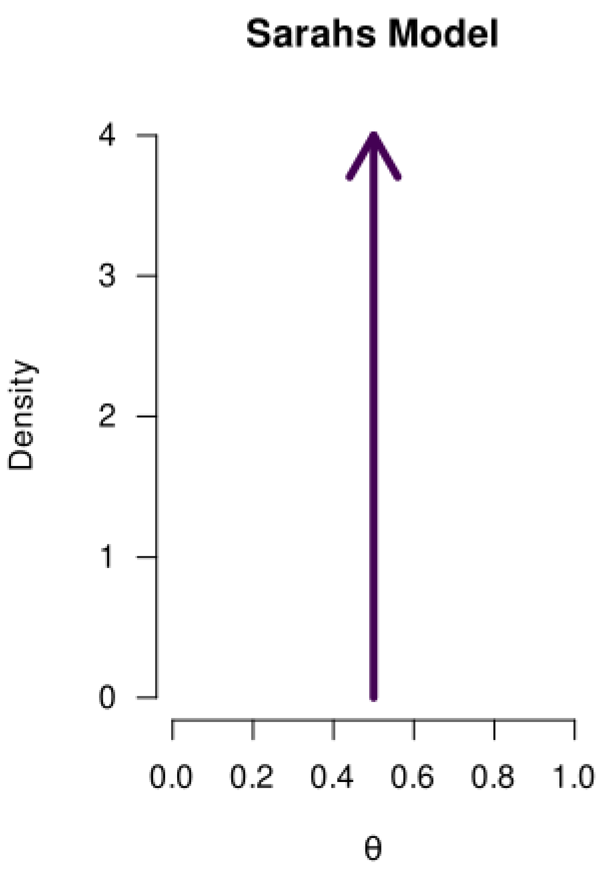
We can reflect a model's statement by means of a probability distribution

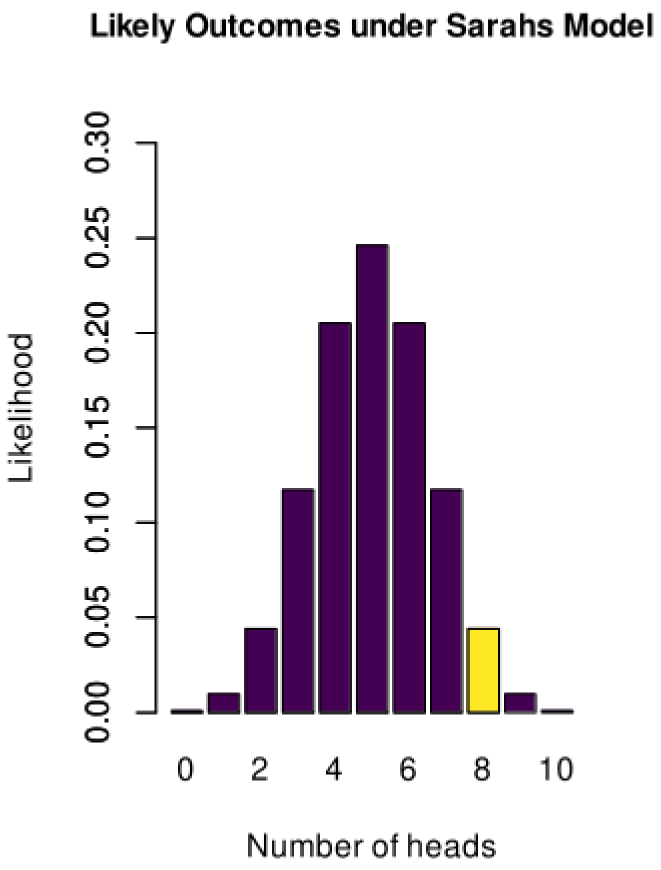


Statistical Models Make Predictions

Based on what these models claim about θ, certain outcomes are more/less likely

The yellow bar indicates how likely an outcome of 8/10 heads is, under Sarah's model





$$\frac{n!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}$$

To create this figure, we take the binomial formula, and fill in θ = 0.5

For instance, for an outcome of k=8 heads, the formula gives 0.0439

Statistical Models Make Predictions

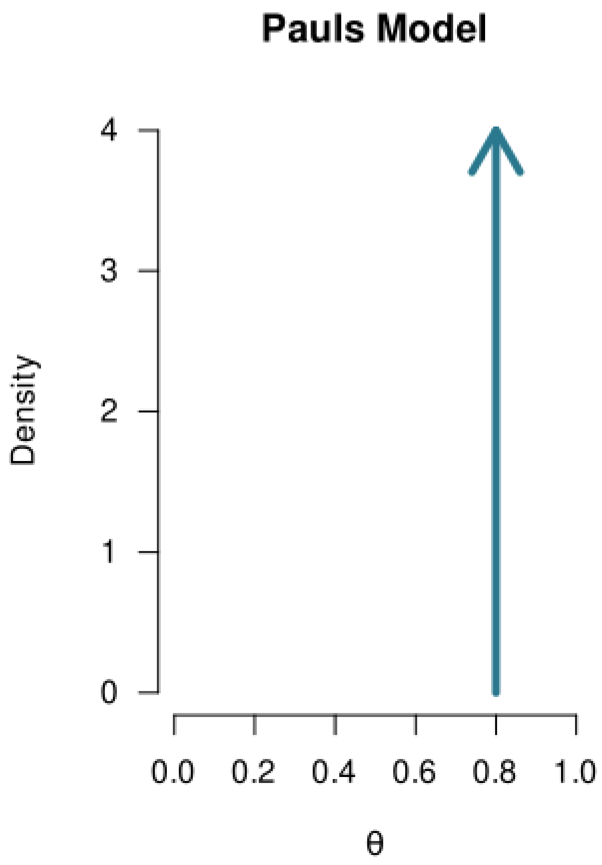
Based on what these models claim about θ, certain outcomes are more/less likely

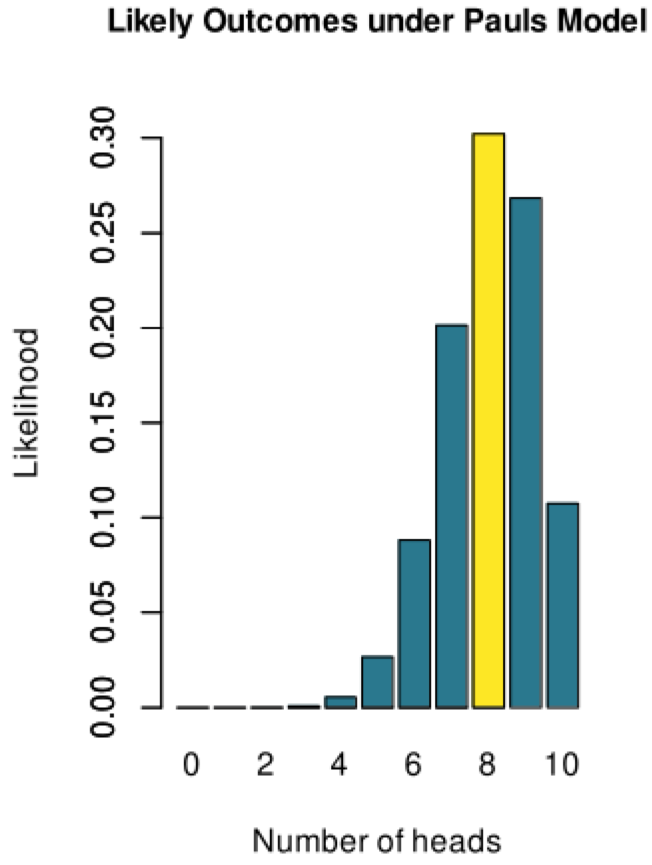
The yellow bar indicates how likely an outcome of 8/10 heads is, under Paul's model

$$\frac{n!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}$$

To create this figure, we take the binomial formula, and fill in θ = 0.8

For instance, for an outcome of k=8 heads, the formula gives 0.302





Statistical Models Make Predictions

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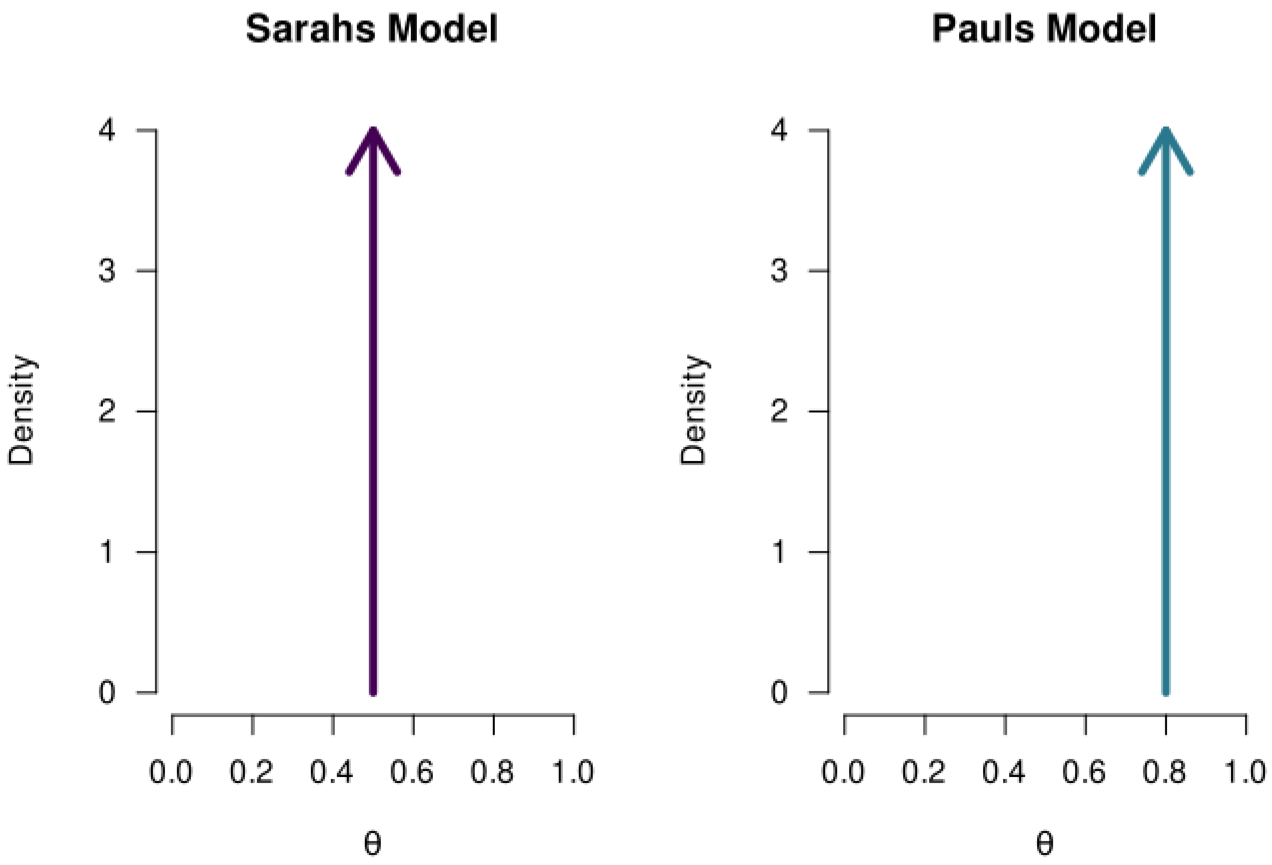
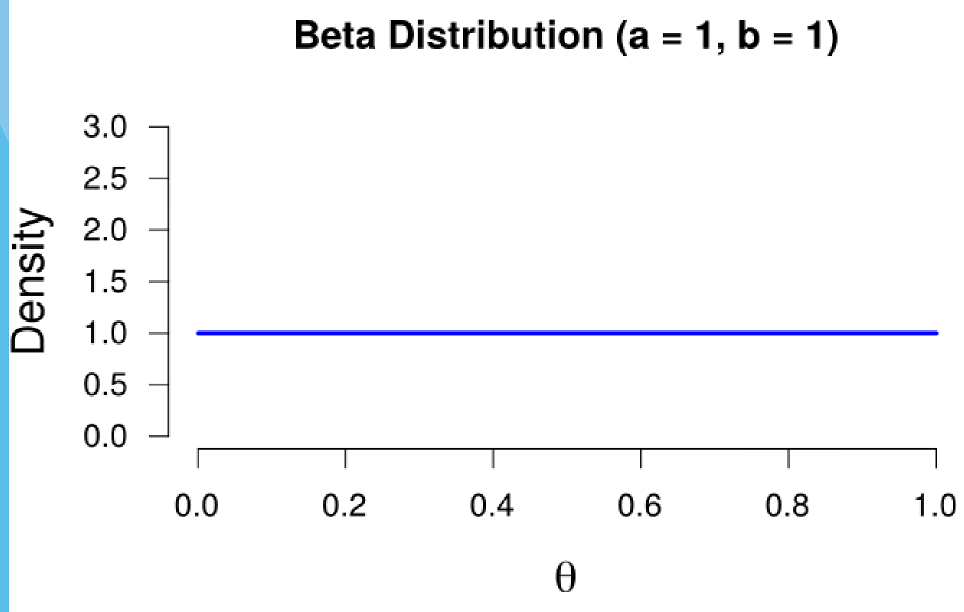


Models Can Also State a Range of Values



Introducing the beta distribution:

- It ranges from 0 to 1
- Its shape is determined by two values: a and b
- If a and b equal 1, it is uniform

We use the beta distribution here because:

- A proportion is also between 0 and 1
- We can create many different shapes, which allows us to reflect many different prior ideas

We can reflect a model's statement by means of a probability distribution

Models Can Also State a Range of Values

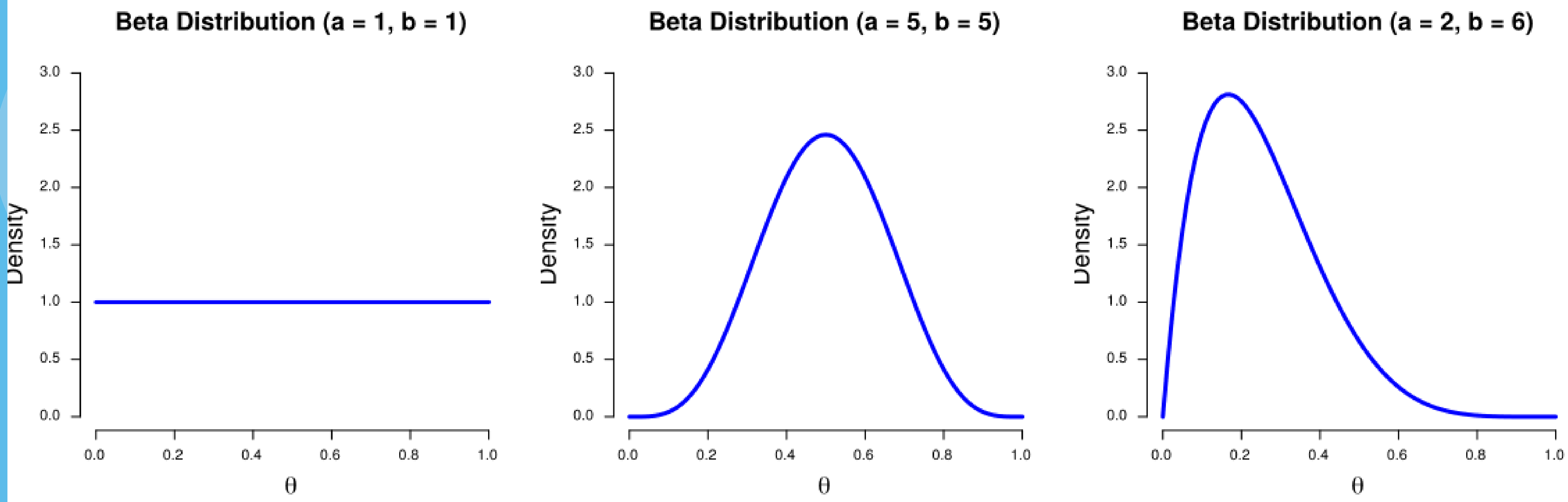
Introducing the beta distribution:

- It ranges from 0 to 1
- Its shape is determined by two values: a and b
- If a and b equal 1, it is uniform

An applet that let you shape a beta distribution

https://maglit.me/bayesian-intro-shiny

Models Can Also State a Range of Values



A model that reflects the idea that all values of the proportion are equally plausible - we call this an *uninformative model*

A model that reflects the idea that values close to 0.5 are more plausible

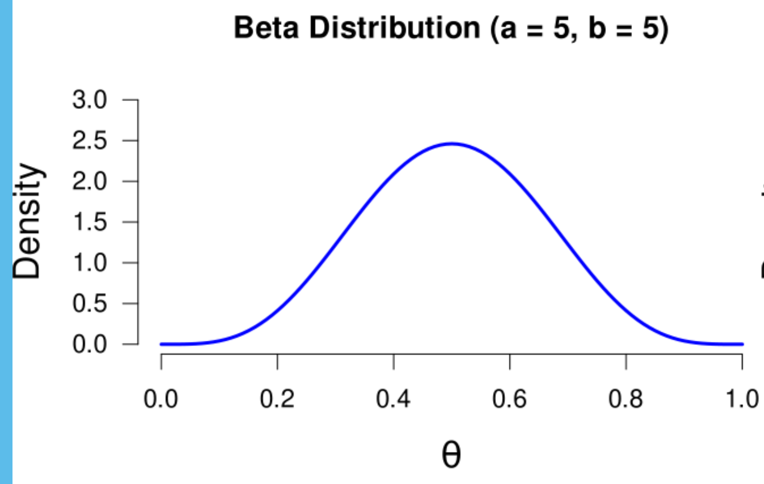
A prior distribution that reflects the idea that values below 0.5 are more plausible (i.e., the coin is biased towards tails)

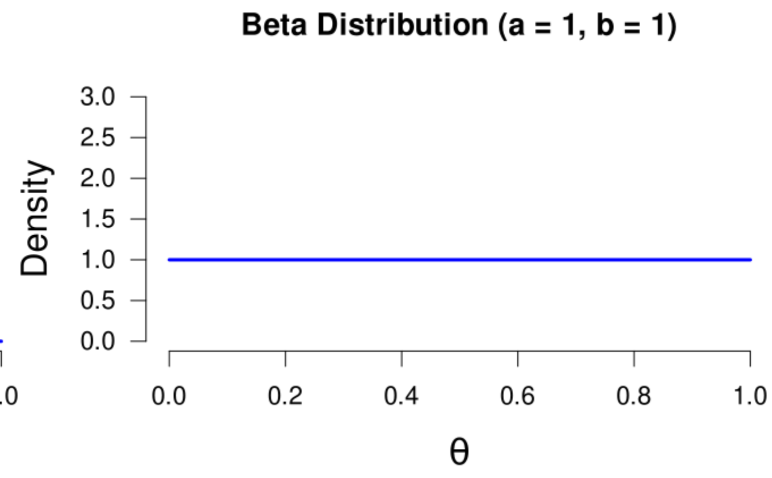
Beta Distribution Interpretation

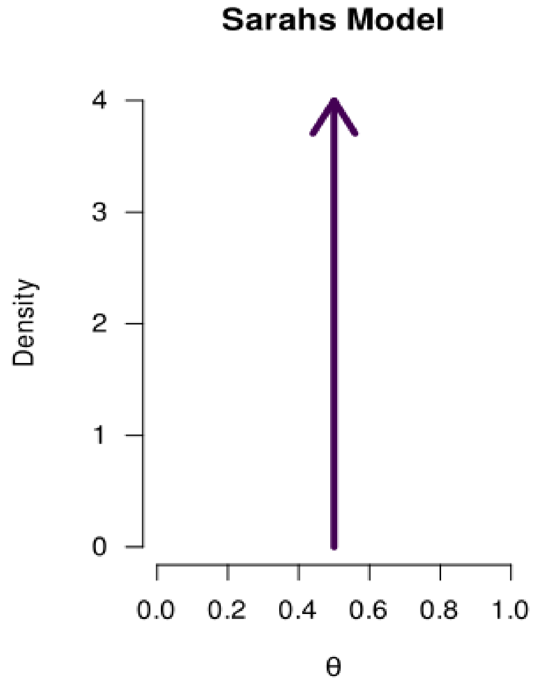
In the context of a prior distribution for a proportion, the a and b can be interpreted as previously observed heads and tails.

We cannot start with an empty distribution (a and b cannot be 0), no matter how clueless you are about something, there will always be starting point (e.g., a = b = 1)

Models that go all in on a single value have a very strong conviction:
Sarah believes as if she has seen infinitely many heads and tails already!





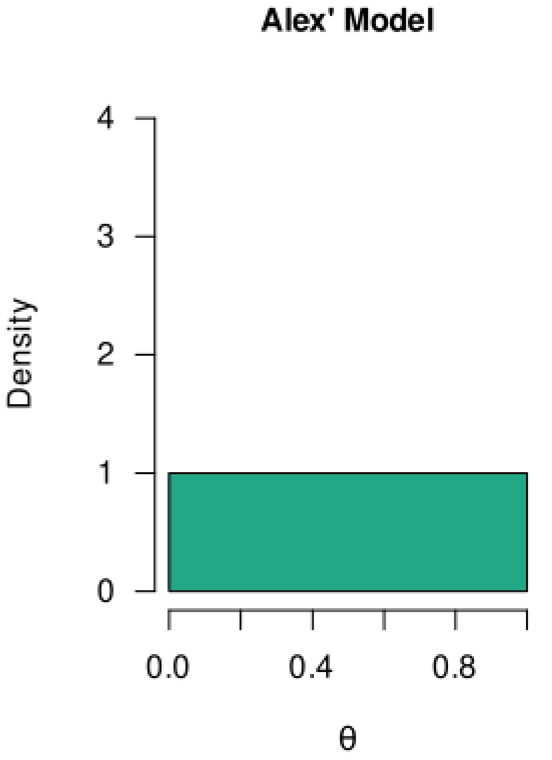


Models Can Also State a Range of Values

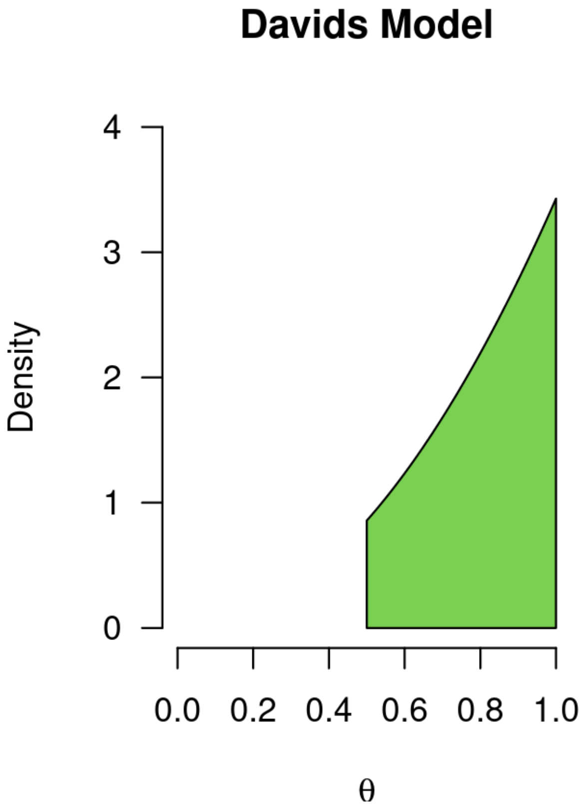
We can reflect a model's statement by means of a probability distribution

A binomial model with θ ~ Beta(1, 1)

A binomial model with θ ~ Beta(3, 1)

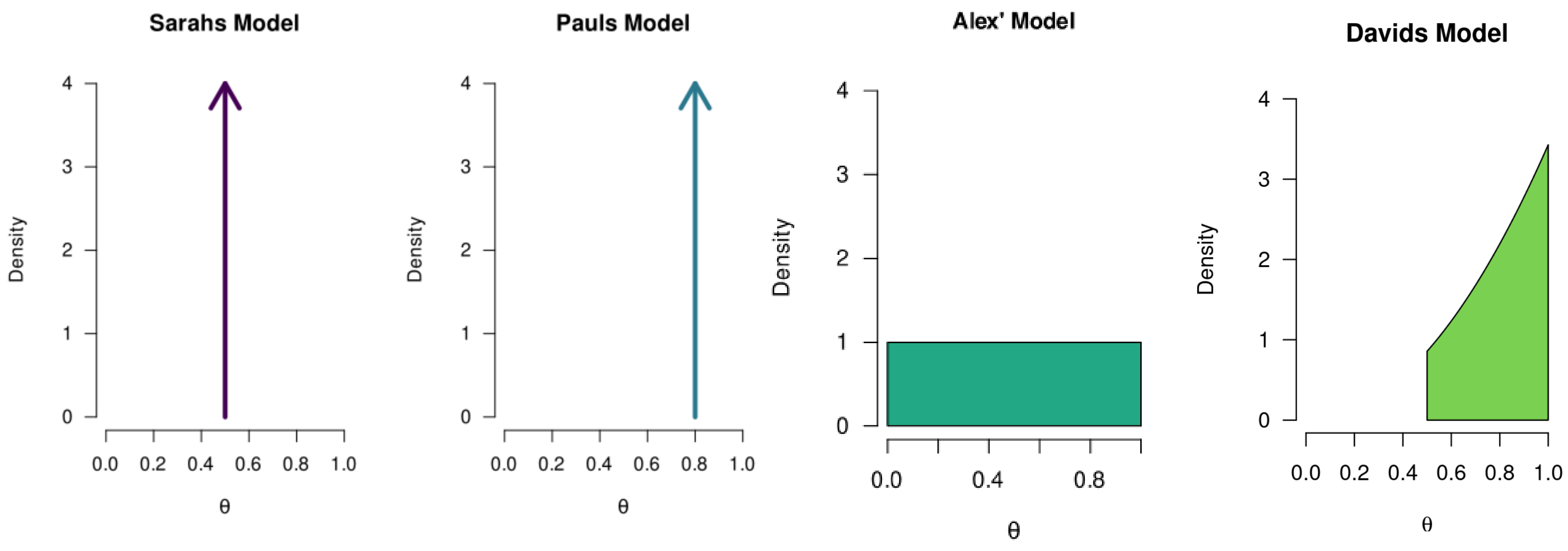


What are the theoretical implications of these models?



It is truncated below 0.5, so Davids model only postulates values > 0.5

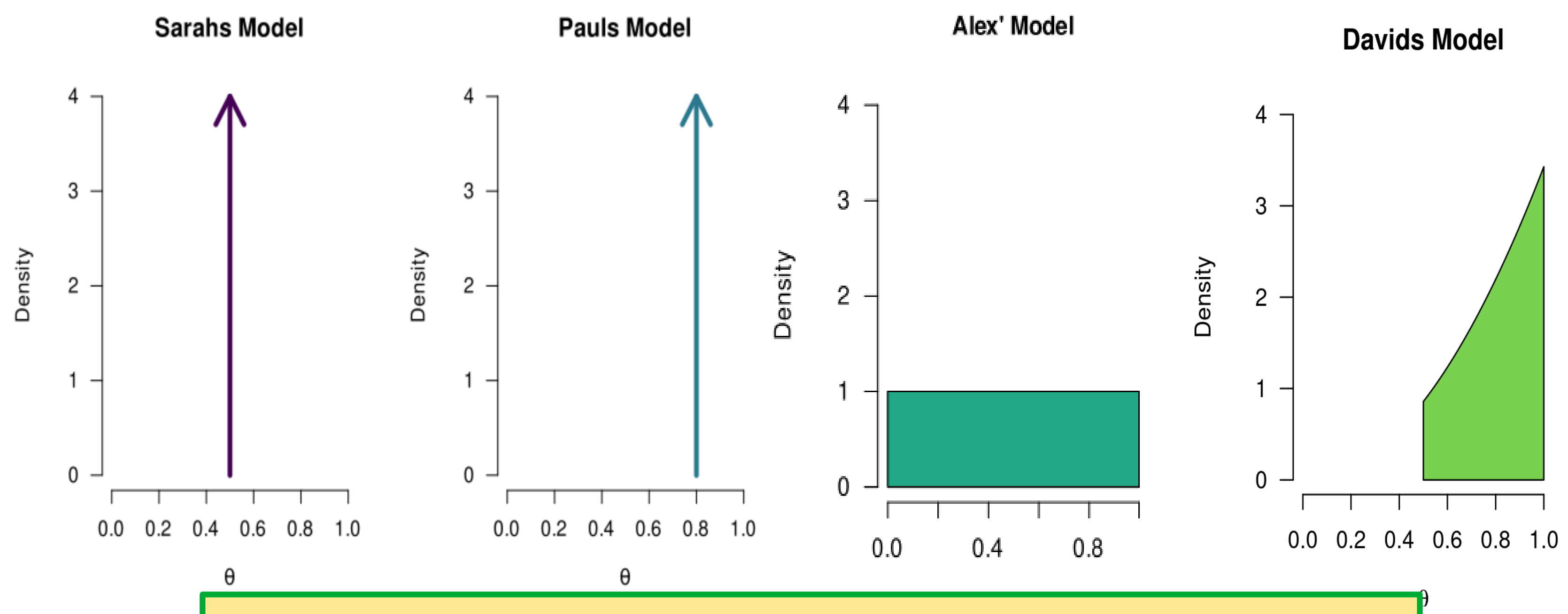
So, We Have All Sorts of Models...



Later we discuss comparing these models to each other

Now we focus on a single model, and how they learn from data

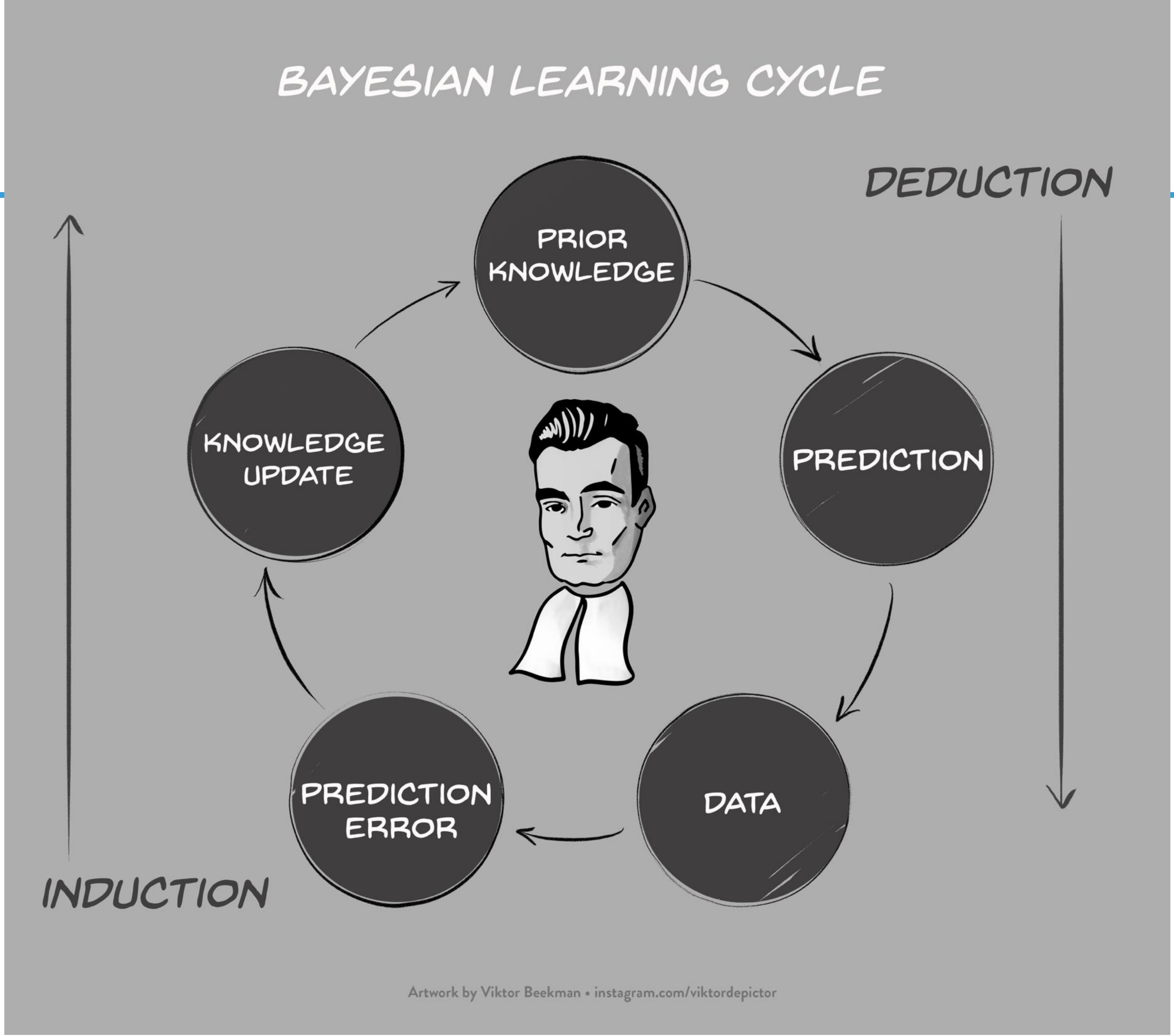
So, We Have All Sorts of Models...



These models reflect prior knowledge/beliefs

We will update this prior knowledge with data

To end up with posterior knowledge



Source:
https://www.bayesianspectacles.org

The Scenario for Today

vs

What is the proportion of Mac users in the population?

Bayes' Theorem

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Applying Bayes' Theorem to Statistical Inference

We can replace "A" and "B", with parameter value "ϑ" and observed "data"

$$P(\theta \mid \text{data}) = \frac{P(\text{data} \mid \theta) P(\theta)}{P(\text{data})}$$

Applying Bayes' Theorem to Statistical Inference

We can rewrite the theorem slightly:

$$P(\theta \mid \text{data}) = P(\theta) \; \frac{P(\text{data} \mid \theta)}{P(\text{data})}$$

Bayesian Knowledge Updating

$$\underbrace{p(\theta \mid \text{data})}_{\substack{\text{Posterior beliefs}\\ \text{about the world}}} = \underbrace{p(\theta)}_{\substack{\text{Prior beliefs}\\ \text{about the world}}} \times \underbrace{\frac{p(\text{data} \mid \theta)}{p(\text{data})}}_{\substack{\text{Predictive}\\ \text{updating factor}}}$$

In the context of statistical analysis, the "world" refers to values of a parameter, such as the population proportion, or population correlation

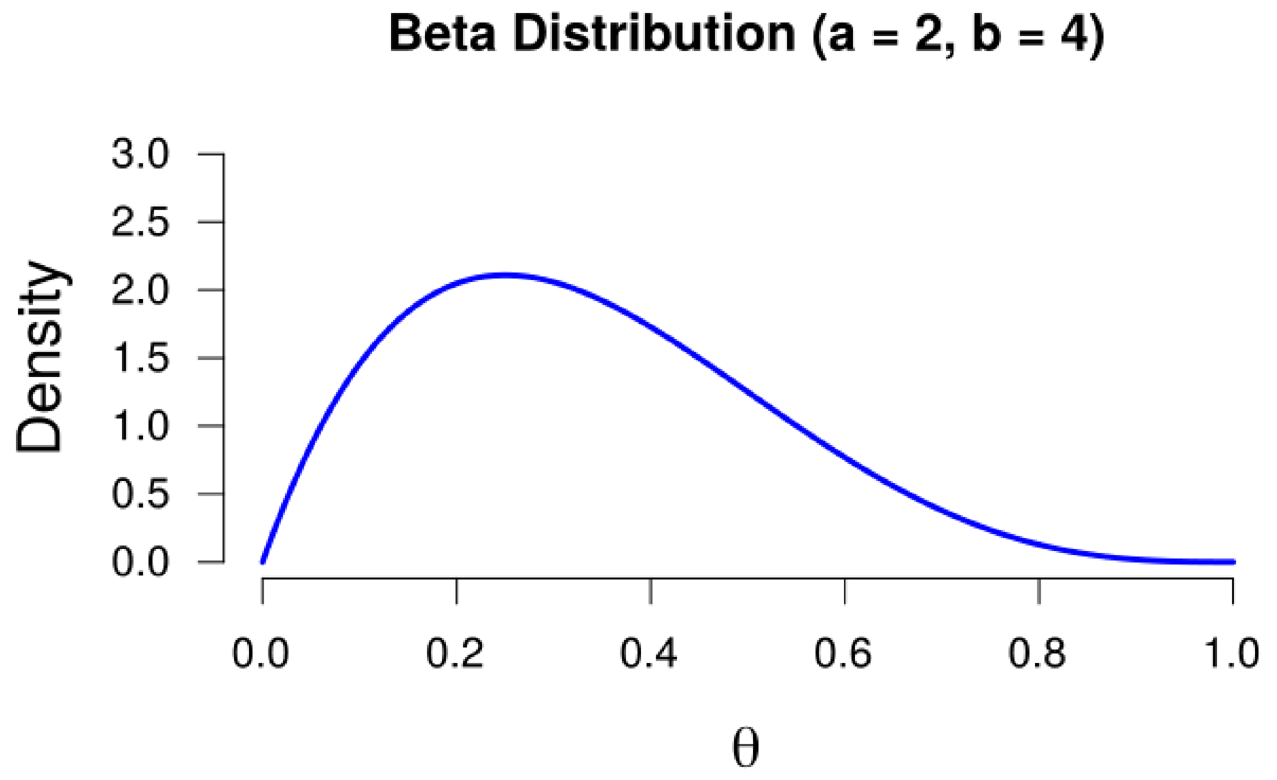
Estimating a Proportion: Prior Distribution

- The prior distribution reflects our beliefs about the parameter, **before** observing the data

- In other words, one of the models we have seen so far!

Estimating a Proportion: Prior Distribution

- Let's use the Beta prior distribution with a = 2, b = 4
    - (would you use a different prior?)



Bayesian Knowledge Updating

$$\underbrace{p(\theta \mid \text{data})}_{\substack{\text{Posterior beliefs} \\ \text{about the world}}} = \underbrace{p(\theta)}_{\substack{\text{Prior beliefs} \\ \text{about the world}}} \times \underbrace{\frac{p(\text{data} \mid \theta)}{p(\text{data})}}_{\substack{\text{Predictive} \\ \text{updating factor}}}$$

We update our prior knowledge by observing data

Estimating a Proportion: Some Frabricated Data

- We observe the following data (n = 10):
  - 3 Mac users
  - 7 Windows users
- Our *statistic* is the observed proportion: 3/10 = 0.3

Estimating a Proportion: Predictive Updating Factor

The **likelihood** of the data, given a certain value of θ in the model

This tells us something about how well a specific value of θ predicted the data (i.e., it is the quality of the prediction for this specific value)

$$\frac{P(\text{data} \mid \theta)}{P(\text{data})}$$

The **marginal likelihood**, across all values of θ in the model

This tells us something how well θ predicted the data, **averaged** over all possible values of θ (i.e., it is the average quality of the prediction by this model)

Taken together, this ratio tells us how well each value of θ in the model predicted the data, **relative** to all other values!

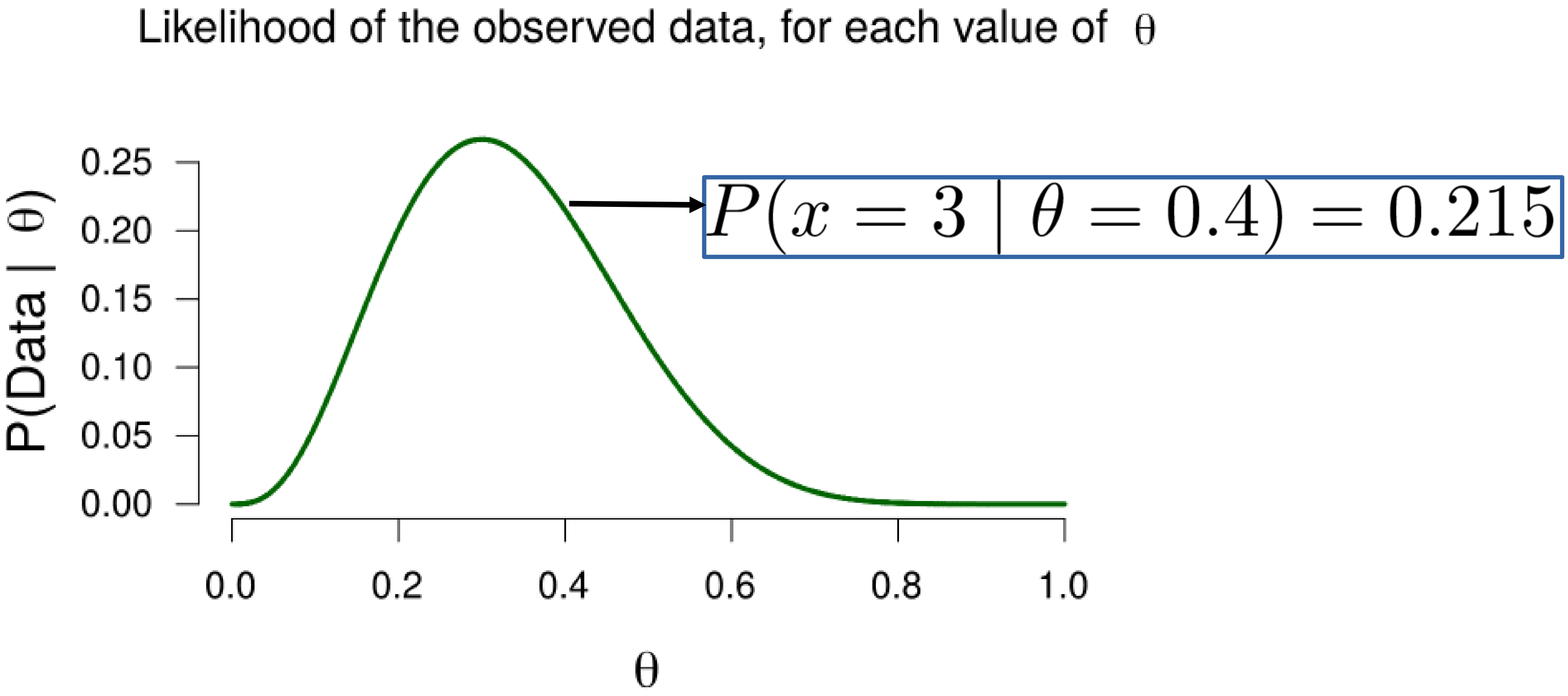
Estimating a Proportion: Predictive Updating Factor

The **likelihood** of the data, given a certain value of θ in the model

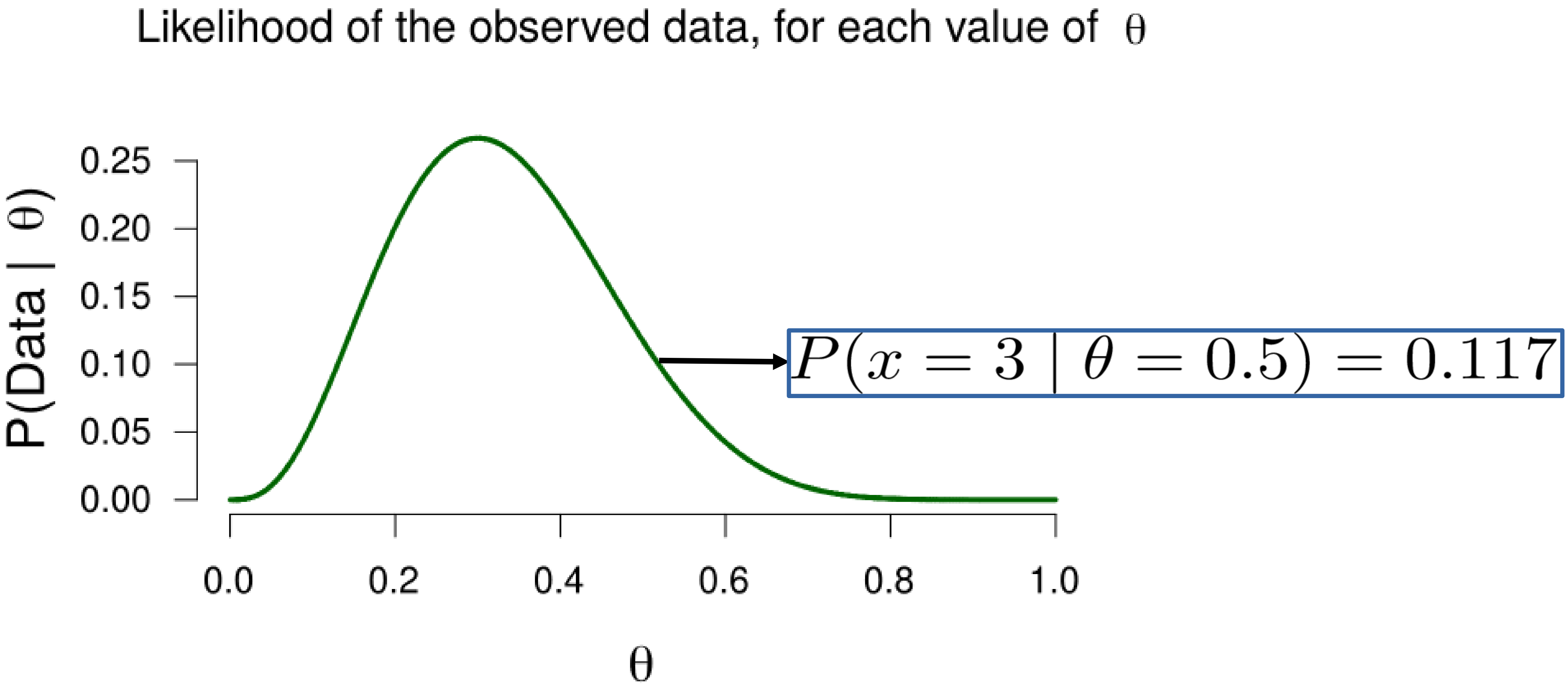
For instance, how likely is our observed data, if θ equals 0.4?

$$\frac{P(\text{data} \mid \theta)}{}$$

Estimating a Proportion: Predictive Updating Factor



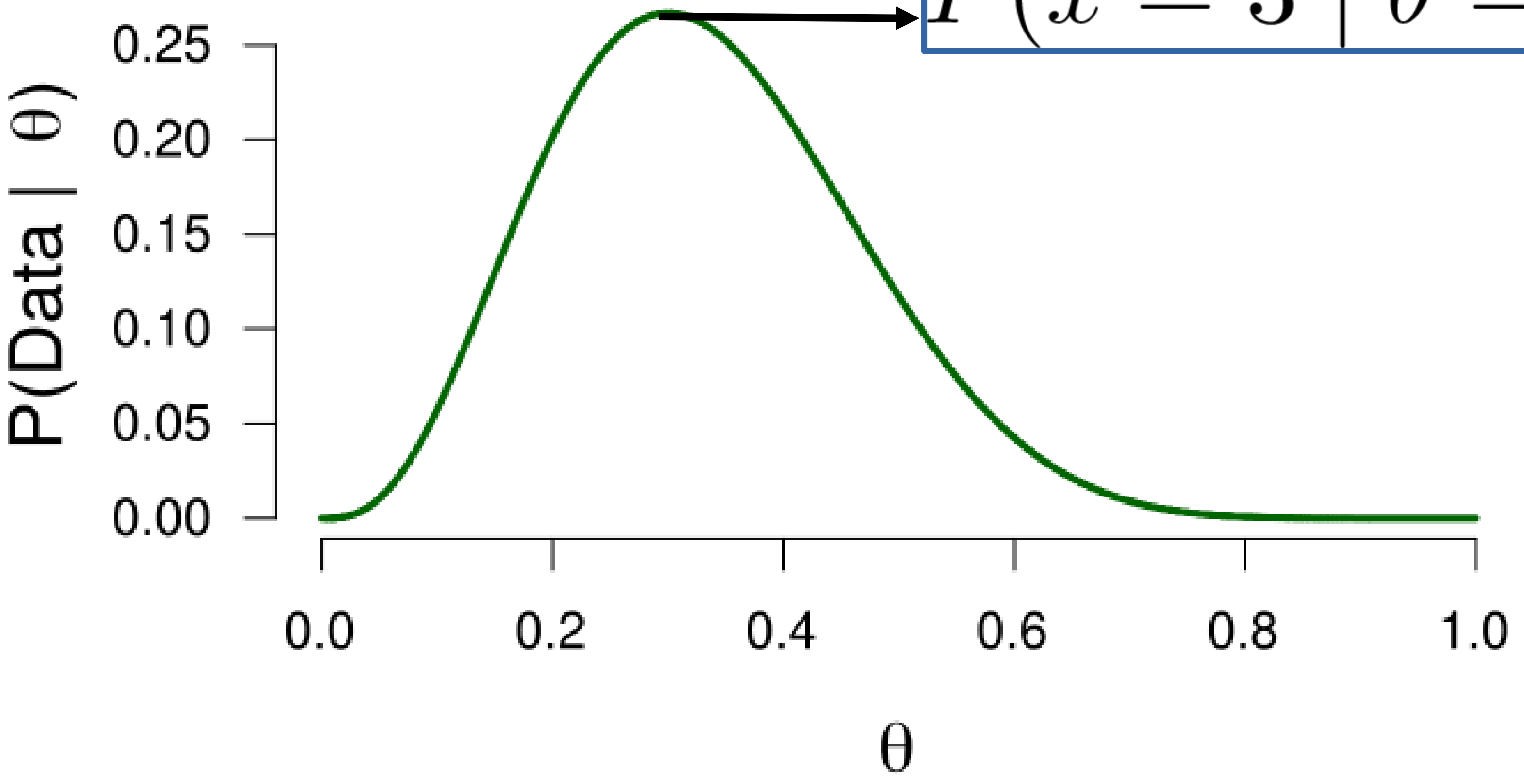
Estimating a Proportion: Predictive Updating Factor



Estimating a Proportion: Predictive Updating Factor

Likelihood of the observed data, for each value of θ

$$\boxed{P(x = 3 \mid \theta = 0.3) = 0.267}$$



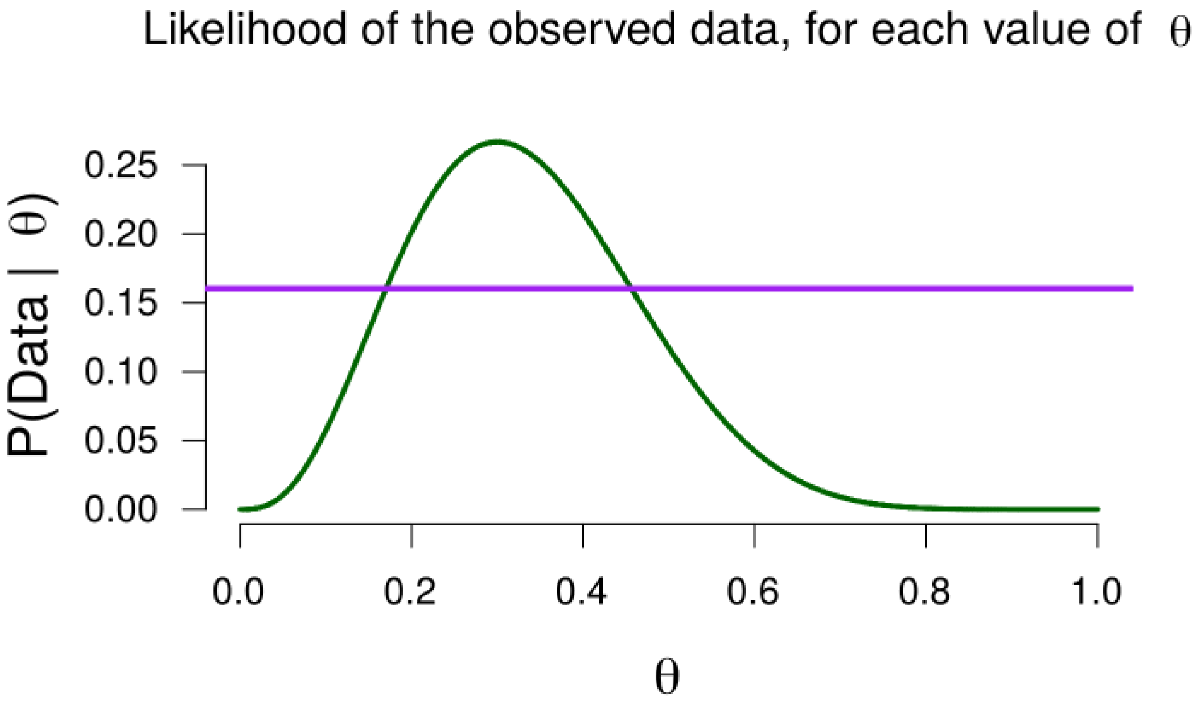
Estimating a Proportion: Predictive Updating Factor

$$P(\text{data})$$

The **marginal likelihood**, across all values of θ

This tells us something how well θ predicted the data, **averaged** over all possible values of θ (i.e., it is the average quality of the prediction by this model)

Usually this is very difficult and we need computers to compute this. In our case, the marginal likelihood equals 0.16



Estimating a Proportion: Predictive Updating Factor

The **likelihood** of the data, given a certain value of θ

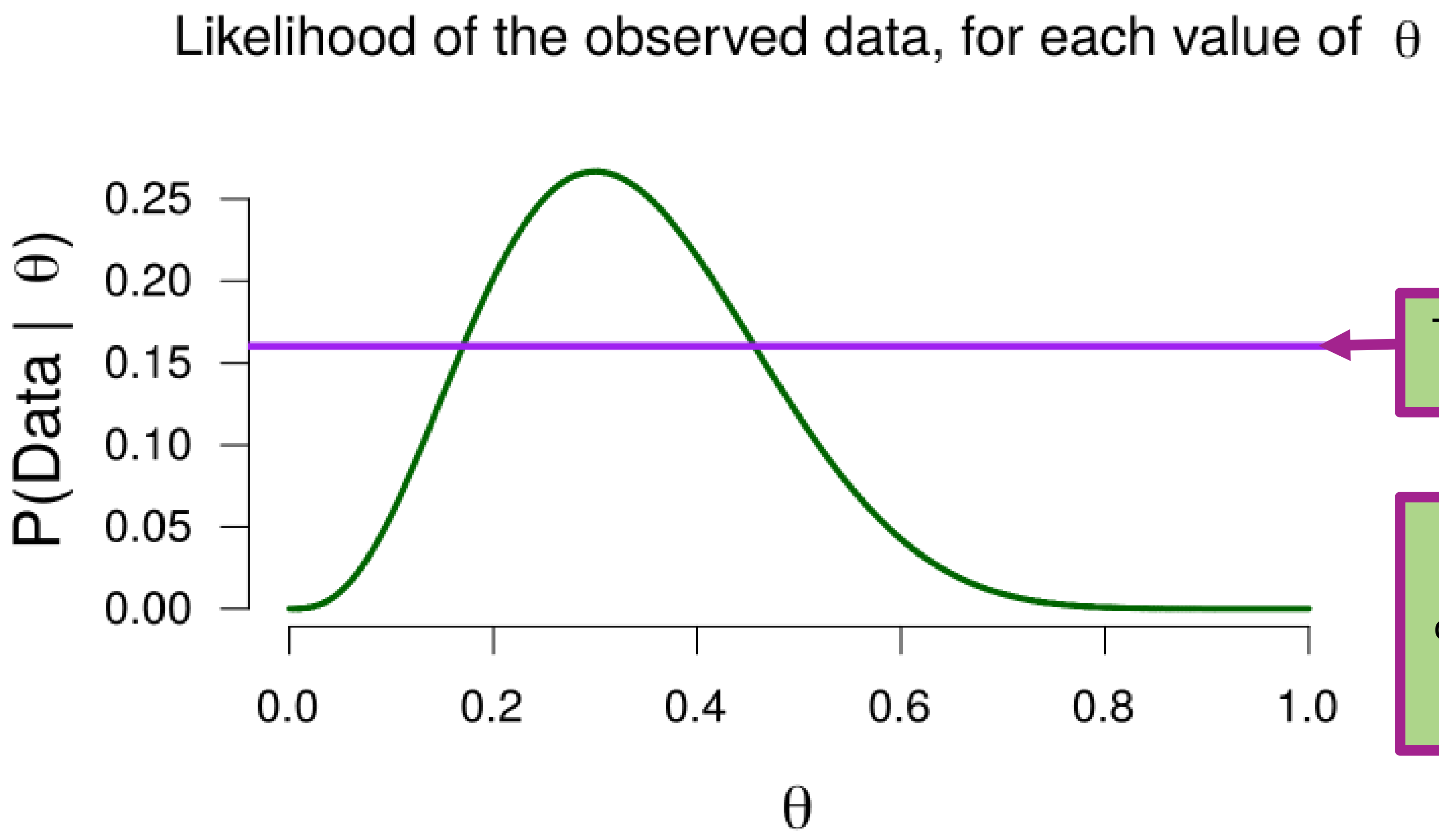
$$\frac{P(\text{data} \mid \theta)}{P(\text{data})}$$

The **marginal likelihood**, across all values of θ

Taken together, this ratio tells us how well each value of θ predicted the data, **relative** to all other values!

Now we know that the marginal (i.e., average) likelihood is 0.16. This means that when the likelihood of the data for a specific value of θ is greater than 0.16, that value of θ has predicted the data above average.
If that is the case, the predictive updating factor is > 1. If a specific value of θ predicted the data worse than average, that ratio is < 1.

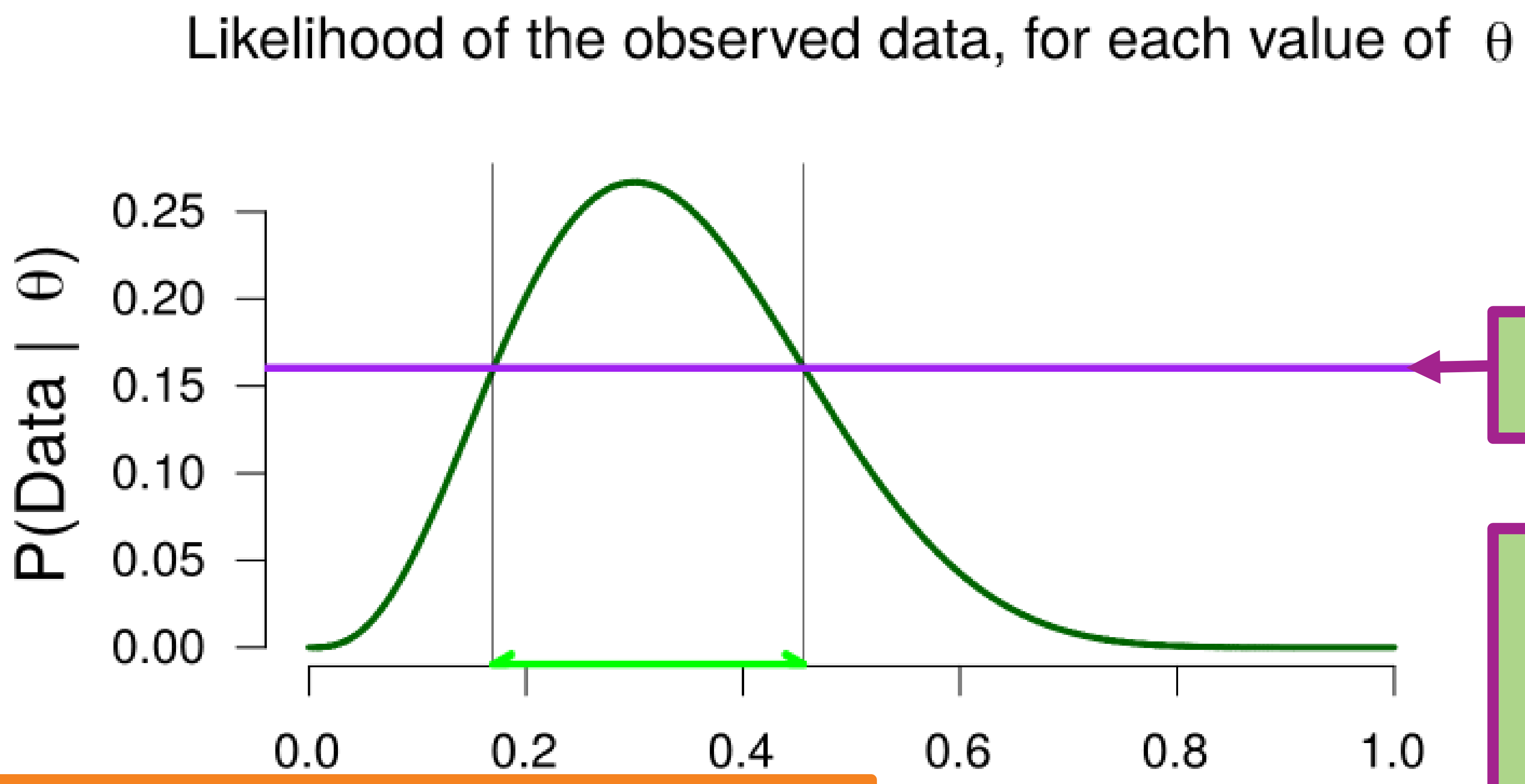
Estimating a Proportion: Predictive Updating Factor



The **marginal likelihood**, across all values of θ (= 0.16)

This tells us something how well θ predicted the data, **averaged** over all possible values of θ (i.e., it is the average quality of the prediction by this model)

Estimating a Proportion: Predictive Updating Factor

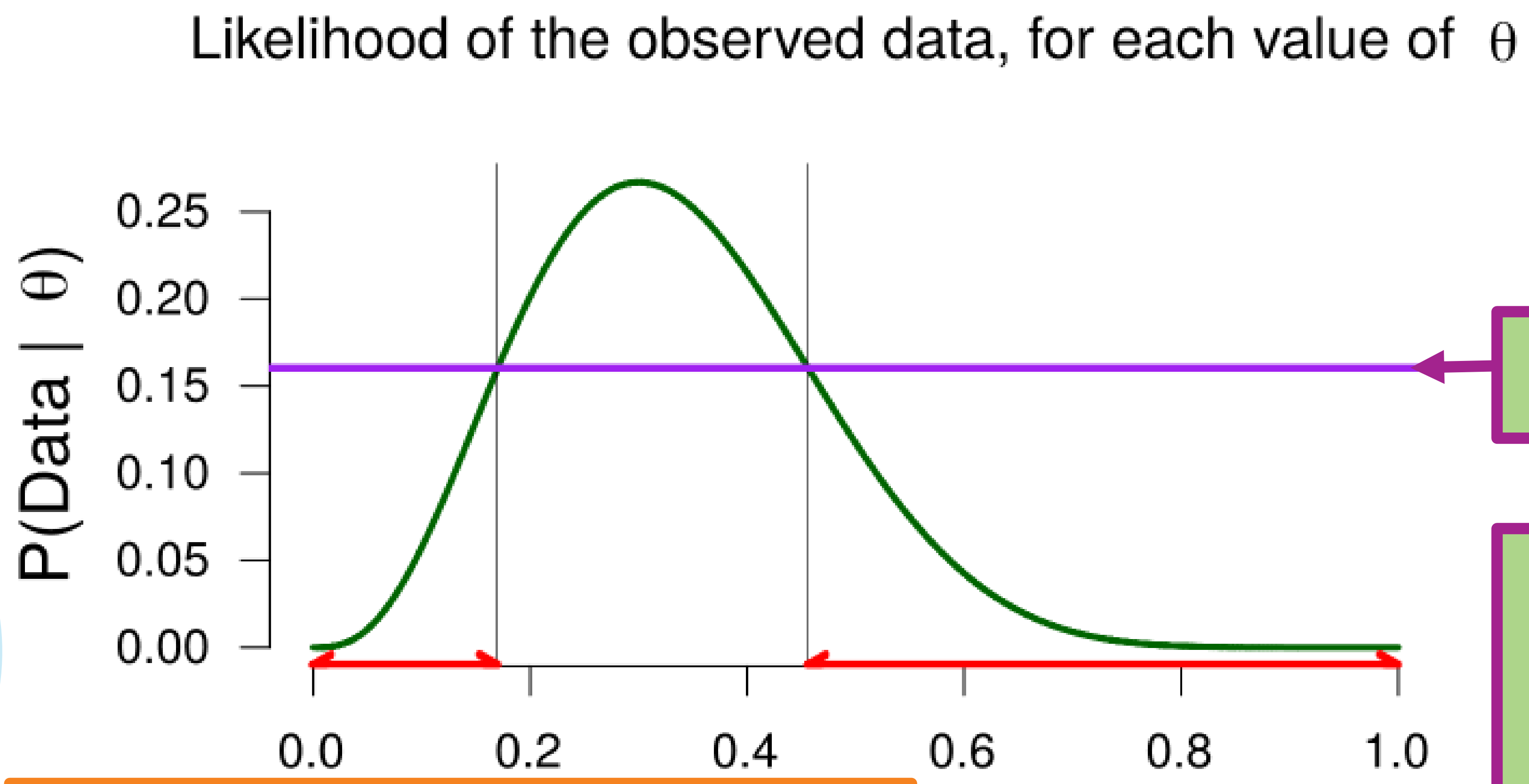


The **marginal likelihood**, across all values of θ (= 0.16)

This tells us something how well θ predicted the data, **averaged** over all possible values of θ (i.e., it is the average quality of the prediction by this model)

Values of θ that predicted the data better than average (i.e., their likelihoods are greater than the marginal likelihood)

Estimating a Proportion: Predictive Updating Factor



The **marginal likelihood**, across all values of θ (= 0.16)

This tells us something how well θ predicted the data, **averaged** over all possible values of θ (i.e., it is the average quality of the prediction by this model)

Values of θ that predicted the data worse than average (i.e., their likelihoods are less than the marginal likelihood)

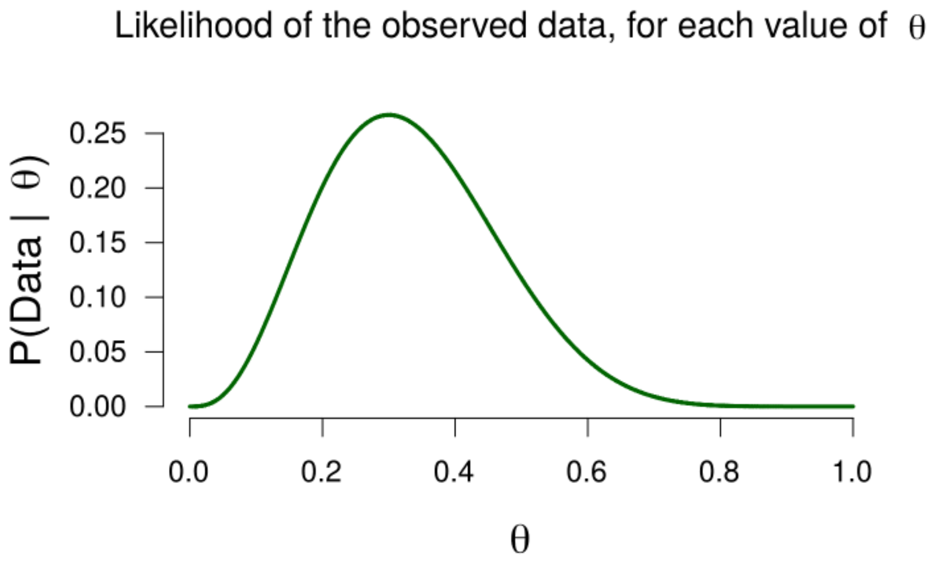
Estimating a Proportion:
Some Notes on the Likelihood

- The likelihood is the same for different prior distributions/models

- However, the marginal likelihood is the average of the likelihoods, weighted by how the model divided its probability mass

- The marginal likelihood is therefore different for different prior distributions

- Since the marginal likelihood reflects the average quality of the predictions made by the model, we will use this later to perform **model comparison**



Bayesian Knowledge Updating

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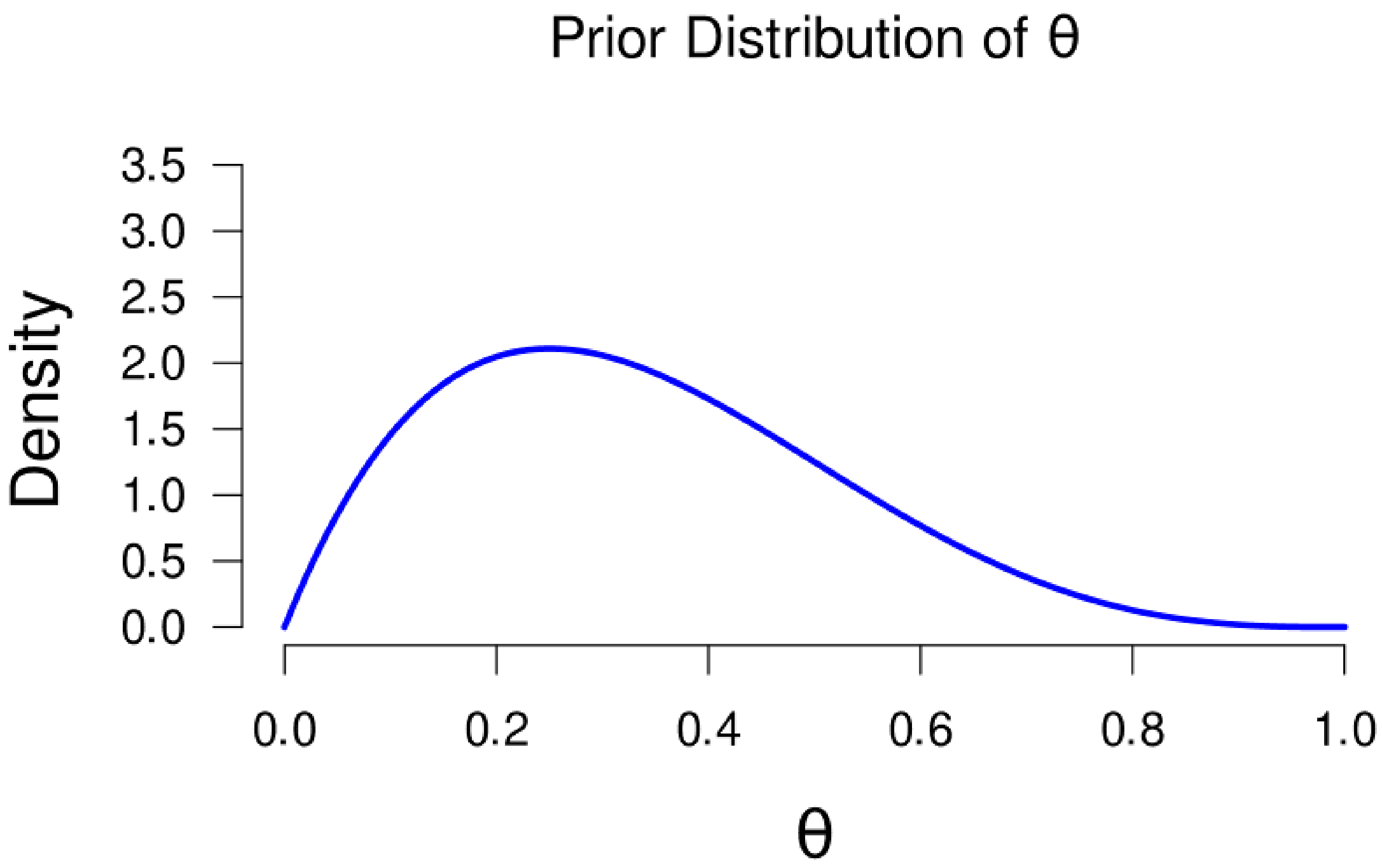
Estimating a Proportion: Posterior Distribution



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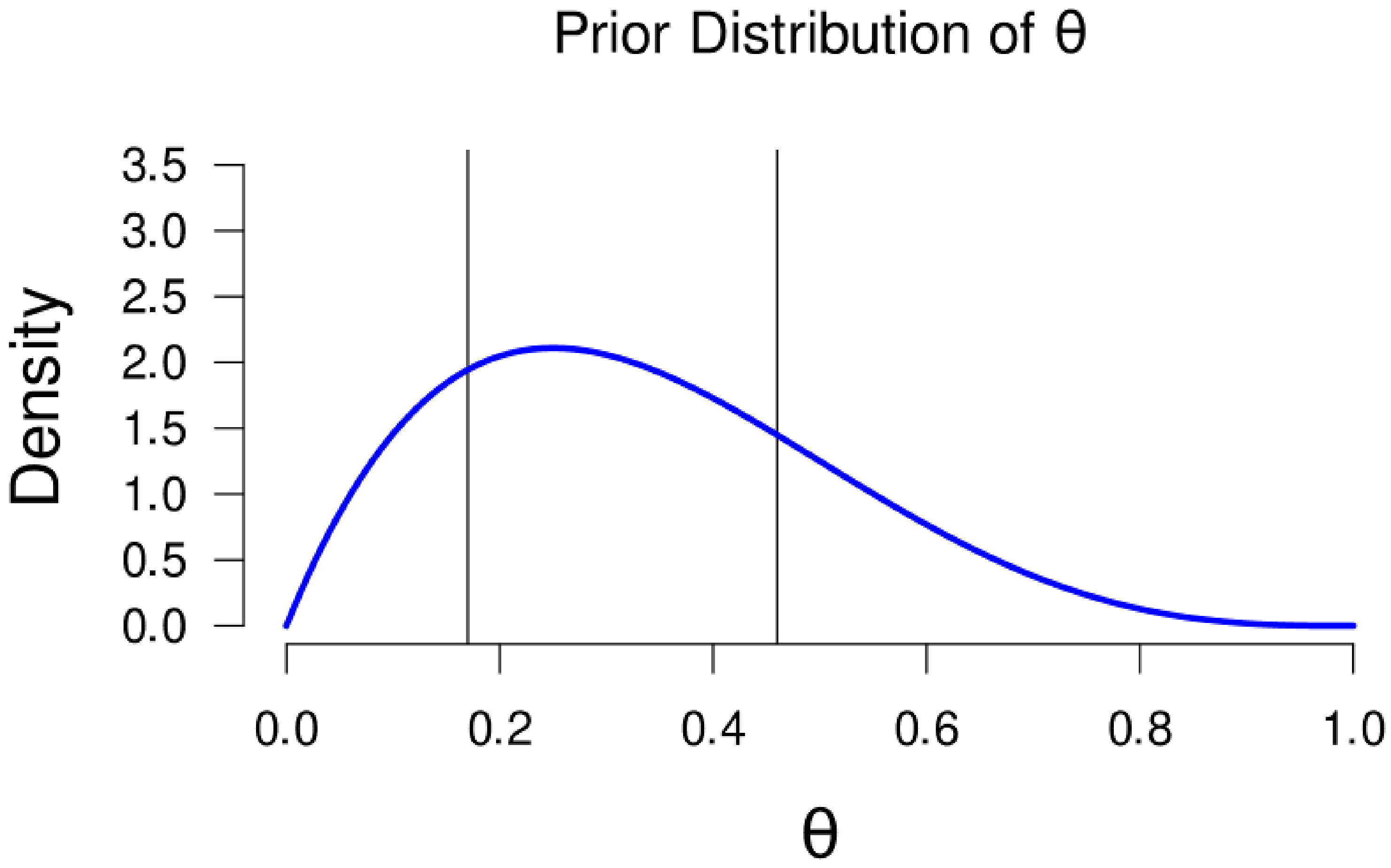
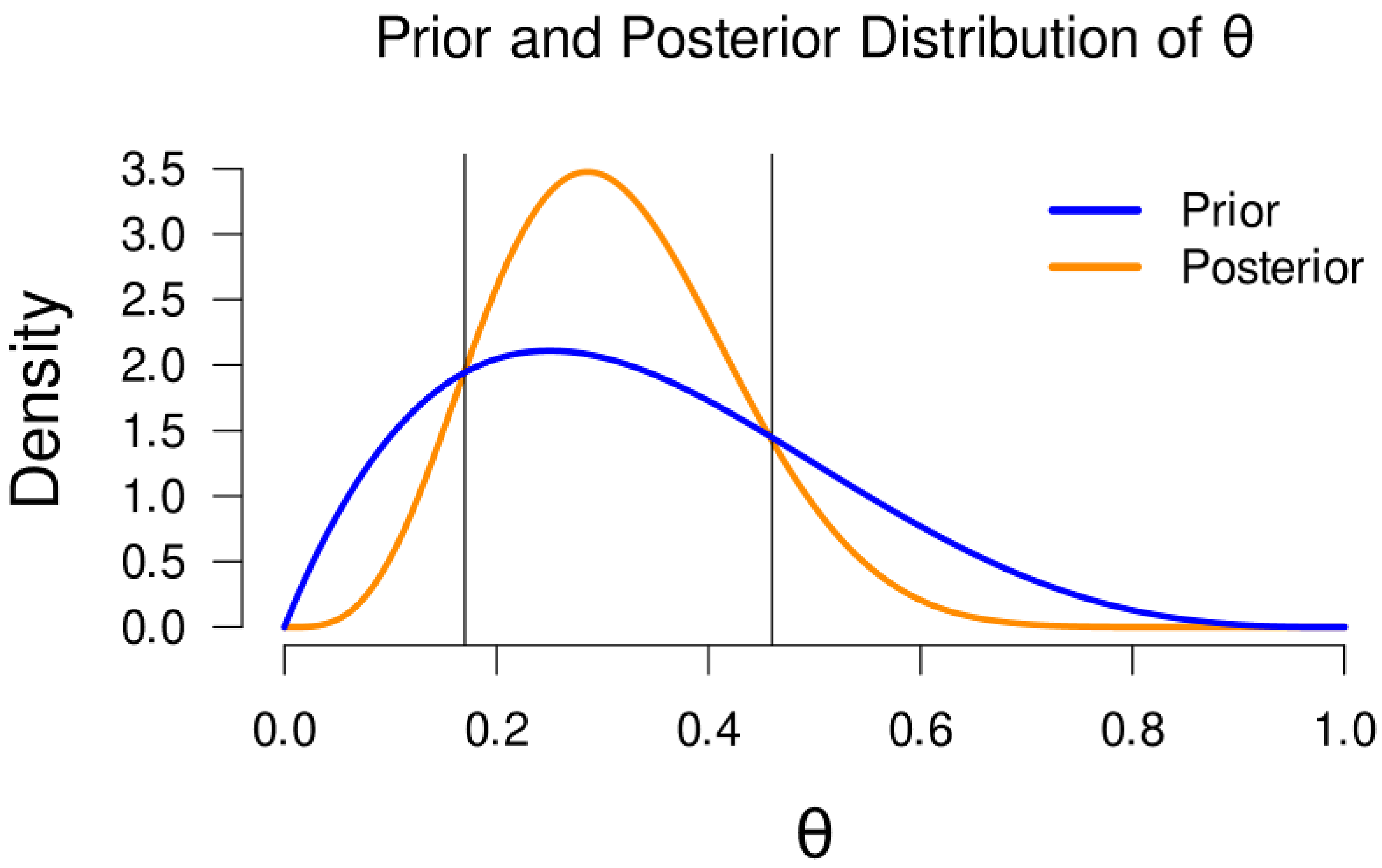


Same lines as on slide 34/35! It shows which values of theta received a decrease/increase in plausibility, because of the data

Estimating a Proportion: Posterior Distribution



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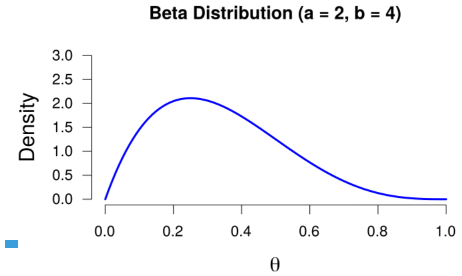
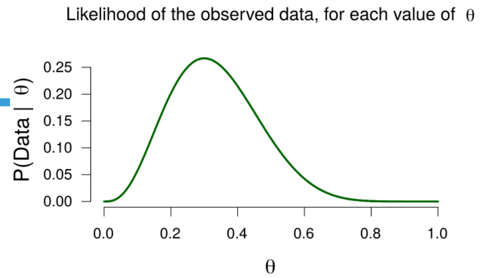


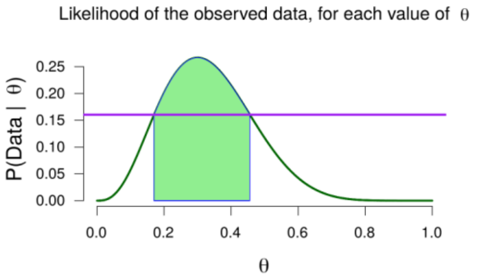
We start with our prior beliefs

We update those with data

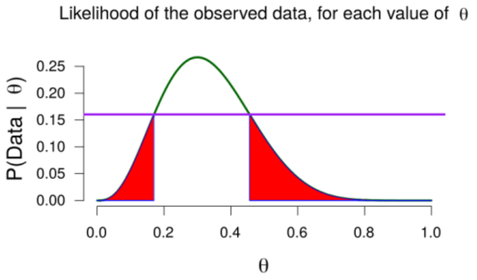


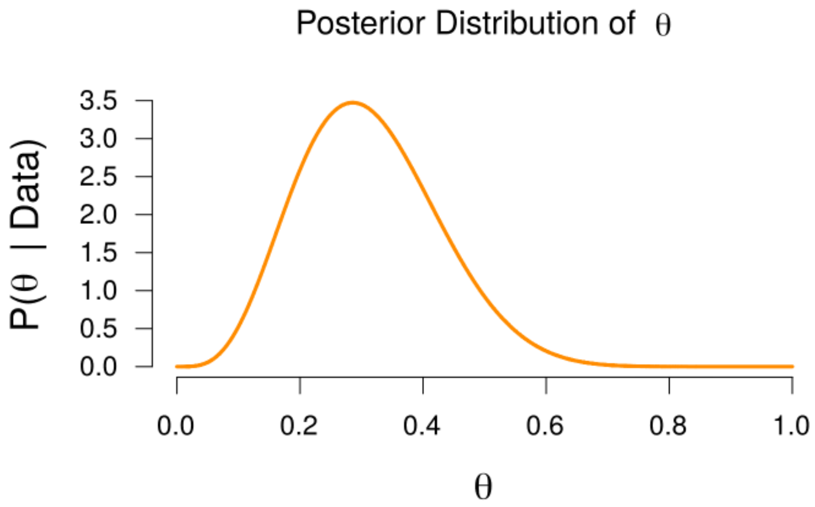
Values of θ that predicted the data better than average receive a boost in plausibility (i.e., their updating ratio >1

Values of θ that predicted the data worse than average receive a penalty in plausibility (i.e., their updating ratio < 1

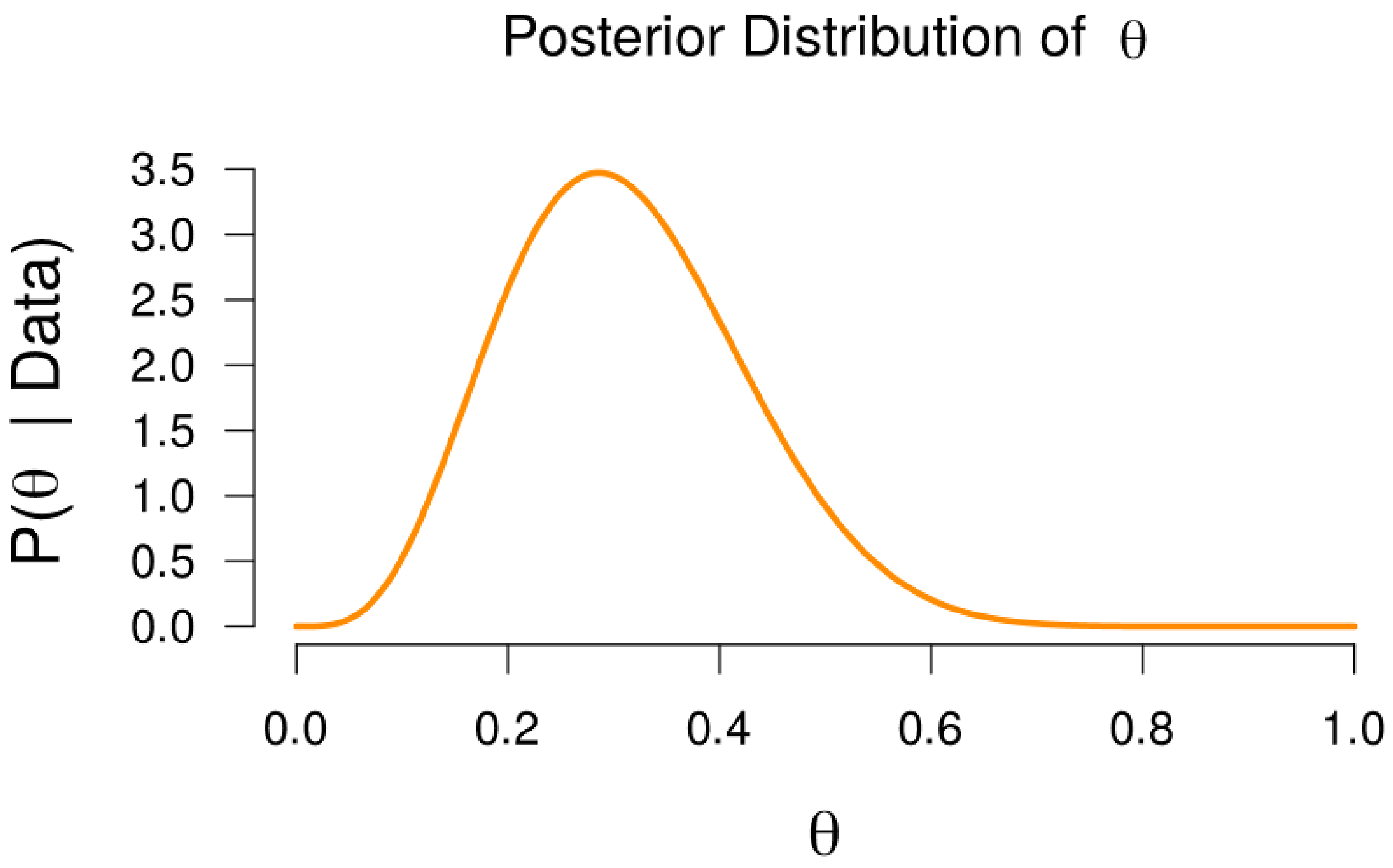


We end with our posterior beliefs



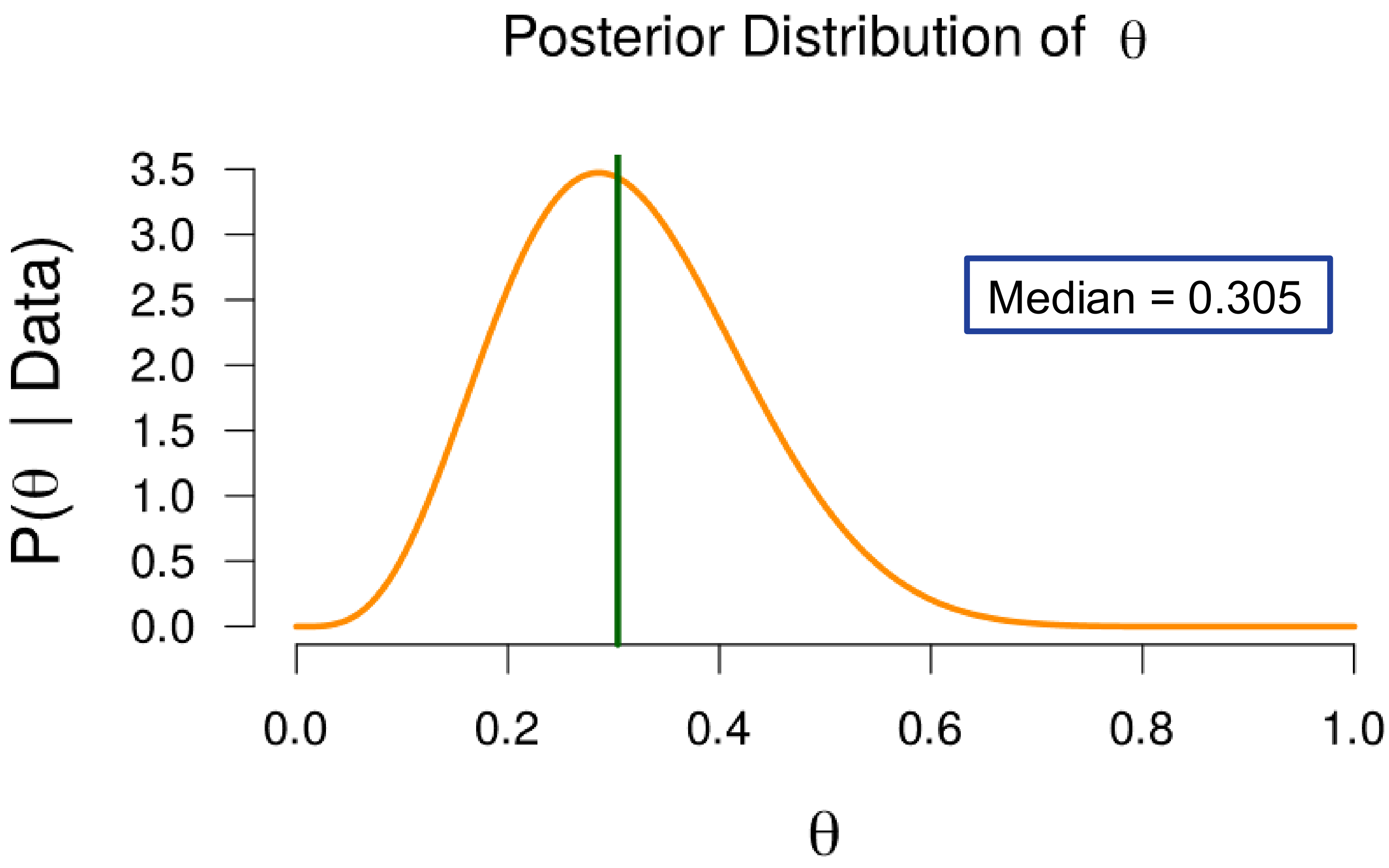


Estimating a Proportion: Posterior Distribution



We can look at the posterior distribution to do paramter **estimation!** This would be the best guess of this model

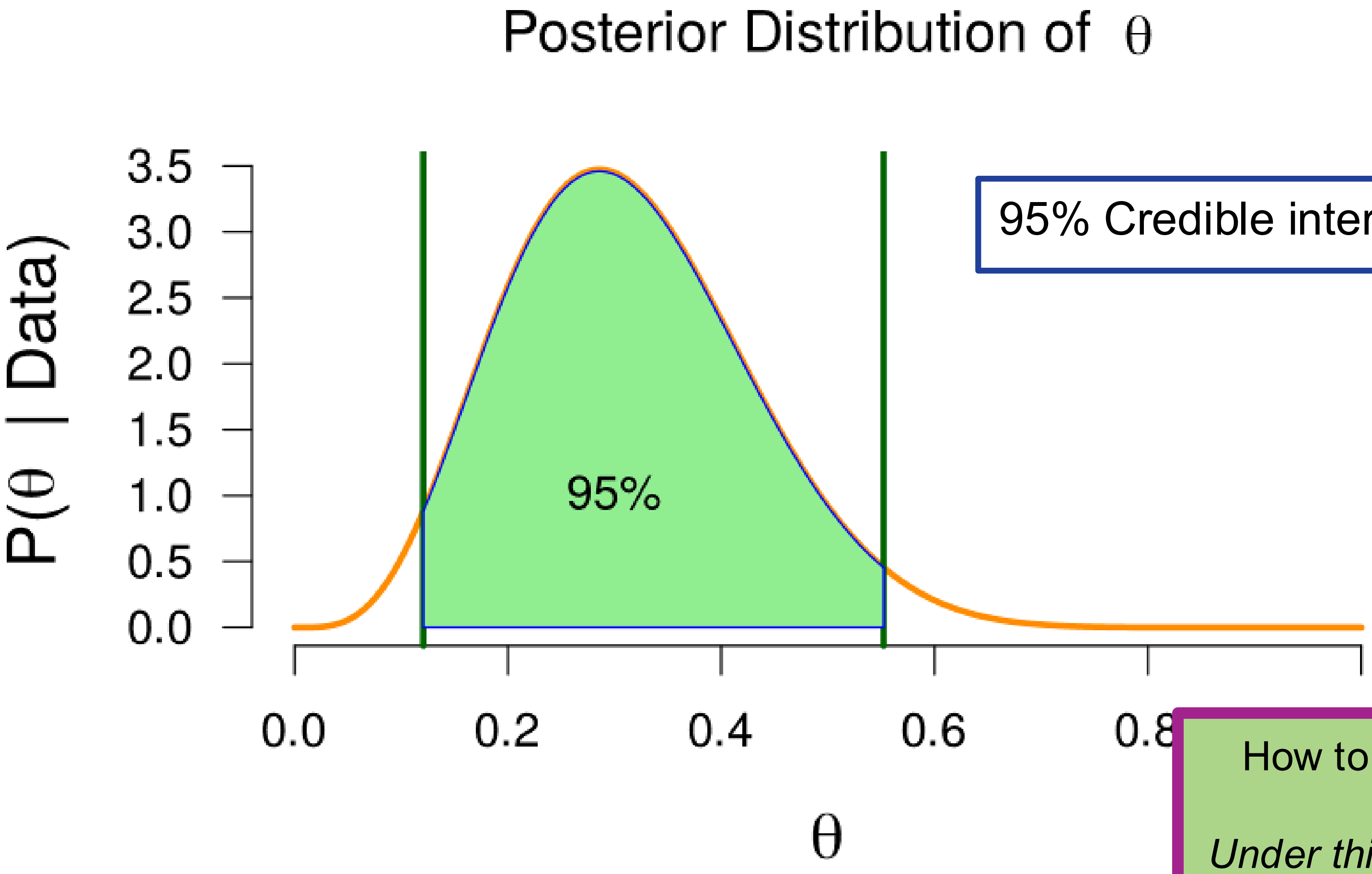
Estimating a Proportion: Posterior Distribution



We can look at the posterior distribution to do paramter **estimation!** This would be the best guess of this model

We can make a point estimate, and take the **posterior median or mean**

Estimating a Proportion: Posterior Distribution



We can make an interval estimate, and take the **central Credible Interval**

95% Credible interval = (0.12, 0.55)

To obtain the x% central credible interval, we take x% of the most central posterior mass, and see which 2 points are the thresholds

How to interpret a x% central credible interval?

Under this model, there is a x% probability that the true proportion is between 0.12 and 0.55

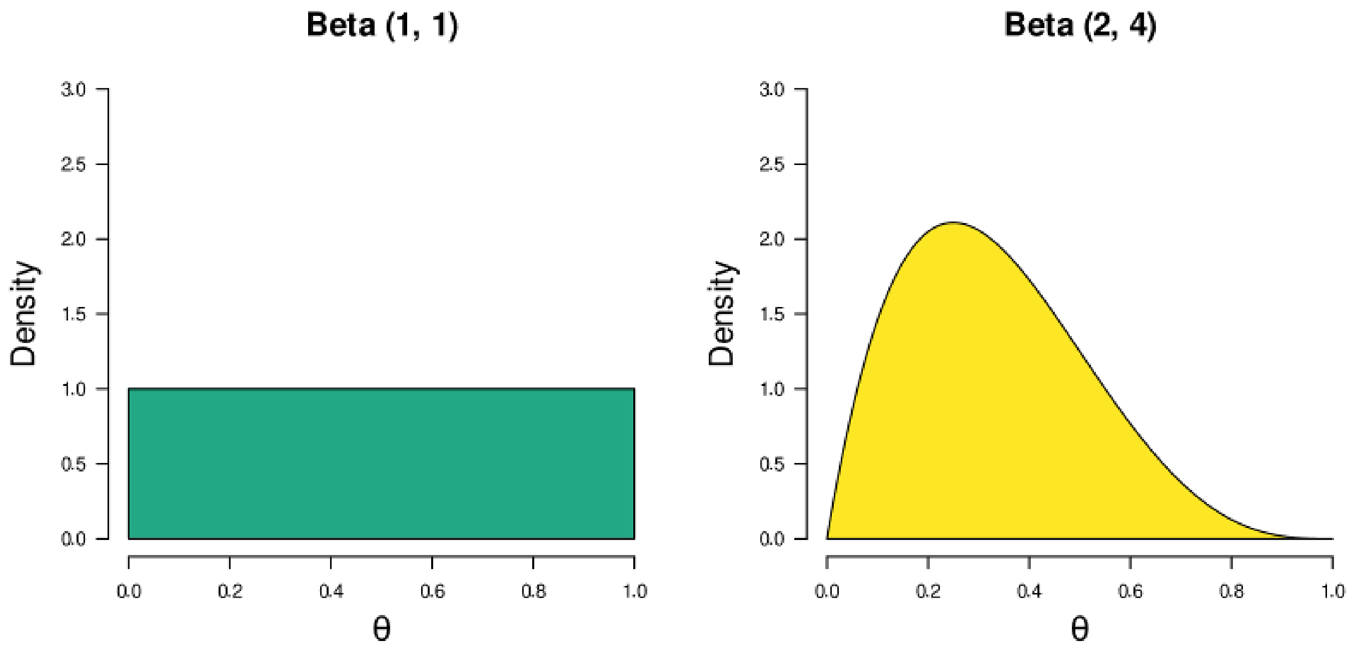
Live Demonstration

VS

- https://maglit.me/bayesian-intro-shiny
- JASP → Learn Bayes →
  Binomial Estimation

Testing a Proportion: Model Comparison

- We have seen we can specify all sorts of models that reflect all sorts of statements about the population proportion



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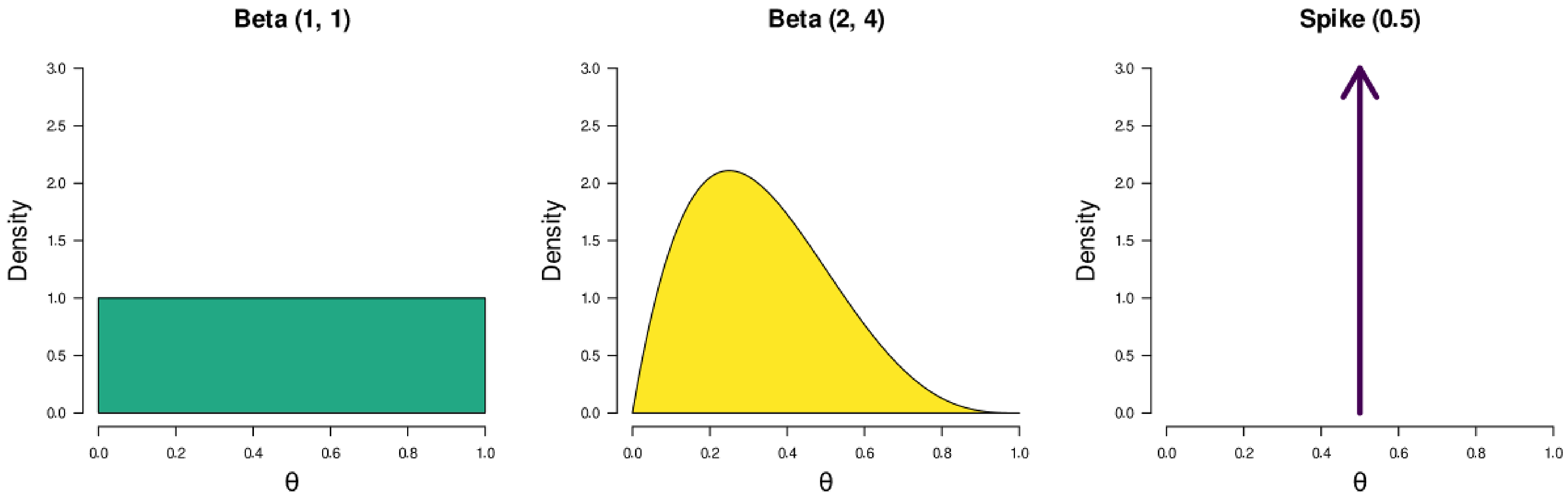
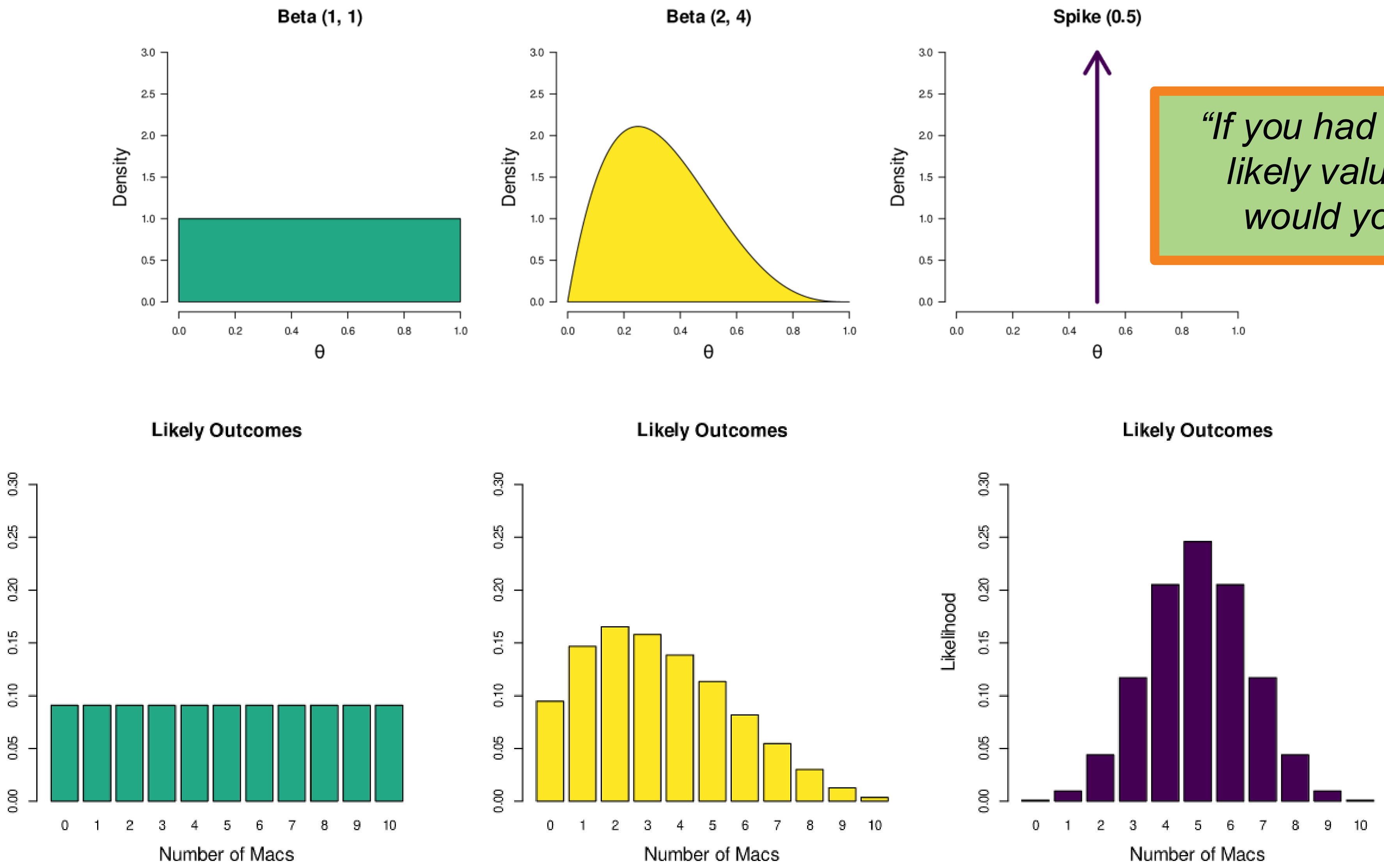


What Do the Models Predict?

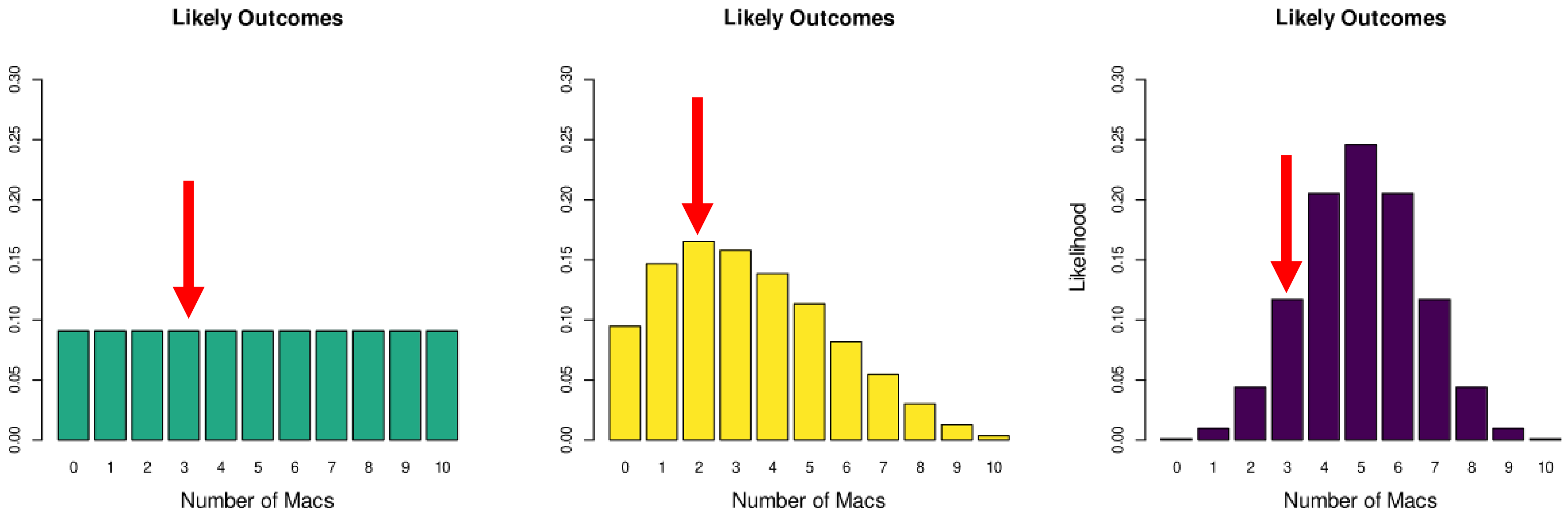


"If you had 100€ to bet on likely values of θ, how would you divide it?"

How Well Did The Models Predict The Data?

- The models placed their bets, we observed 3 Macs
- How much the model bet on the observed outcome equals the quality of its prediction
- The ratio of two such bars is known as the **Bayes factor**

"If you had 100€ to bet on likely values of θ, how would you divide it?"



Bayes Factor

- The workhorse of Bayesian hypothesis testing
- It quantifies which of two models predicted the data better, and how much better
- Usually, we pit two models against each other:
  - The null model (H_0), with a spike at the point of testing
    - (e.g., 0.5 for proportion, 0 for correlation or difference in means)
  - The alternative model (H_1), which predicts multiple values (dictated by its prior distribution)
    - The alternative model can represent a one-sided or two-sided hypothesis

Bayes Factor Interpretation

- **Example 1: BF_{10} = 20**

The data are twenty times more likely under H_1 than under H_0

- **Example 2: BF_{10} = 1/5 → BF_{01} = 5**

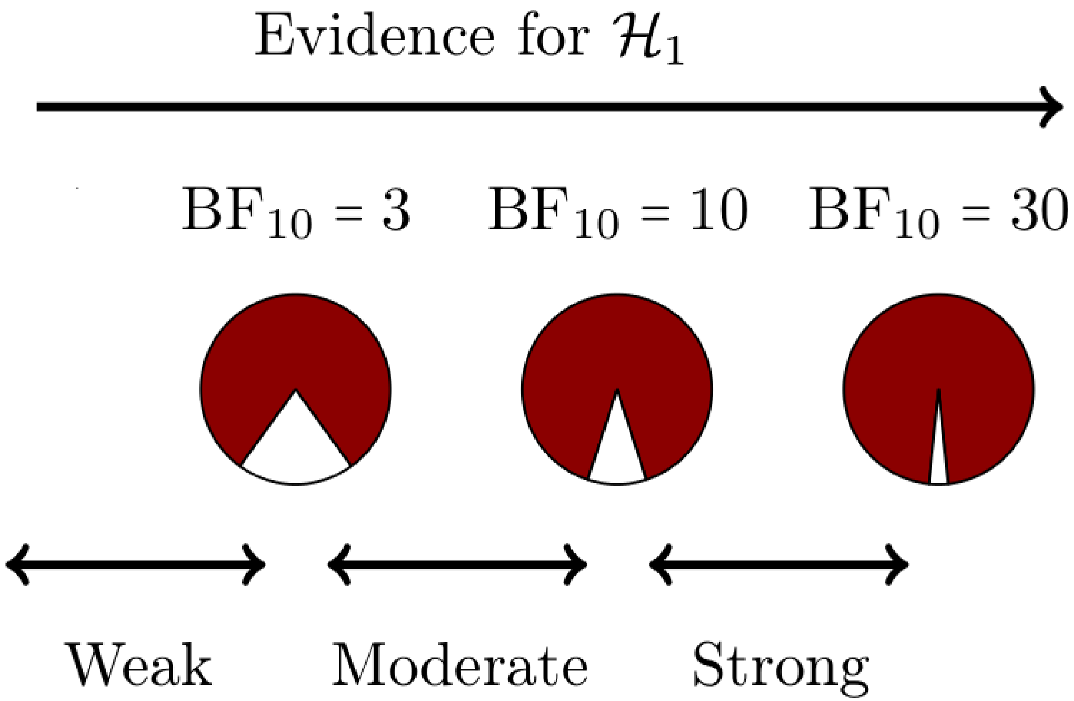
The data are five times more likely under H_0 than under H_1

- **Example 3: BF_{10} = 1**

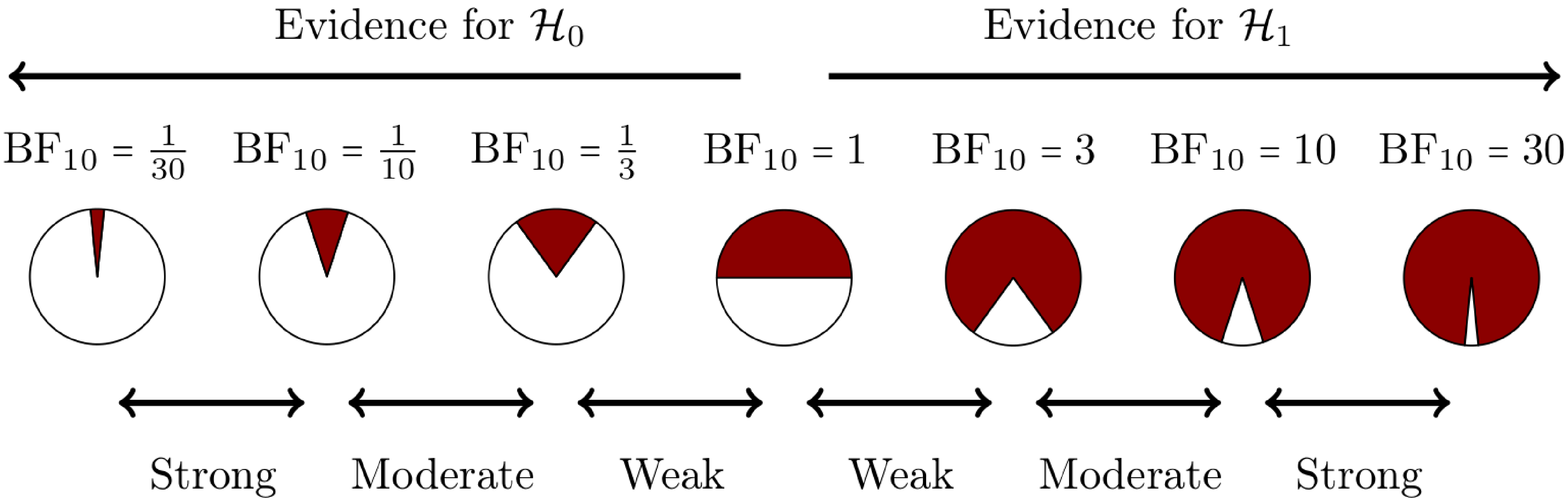
The data are equally likely under H_1 and under H_0

Bayes Factor Interpretation

Bayes Factor Interpretation

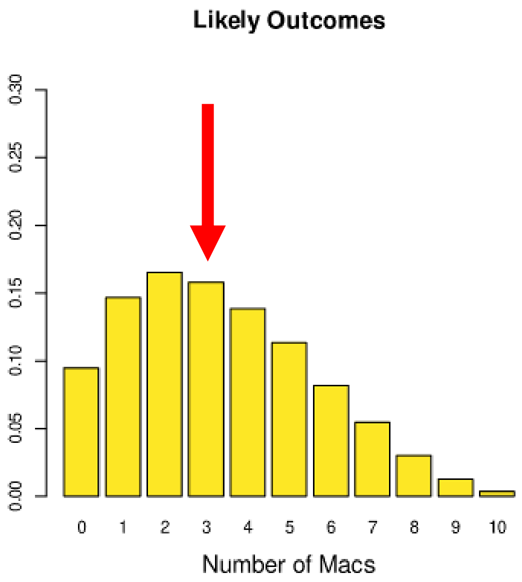


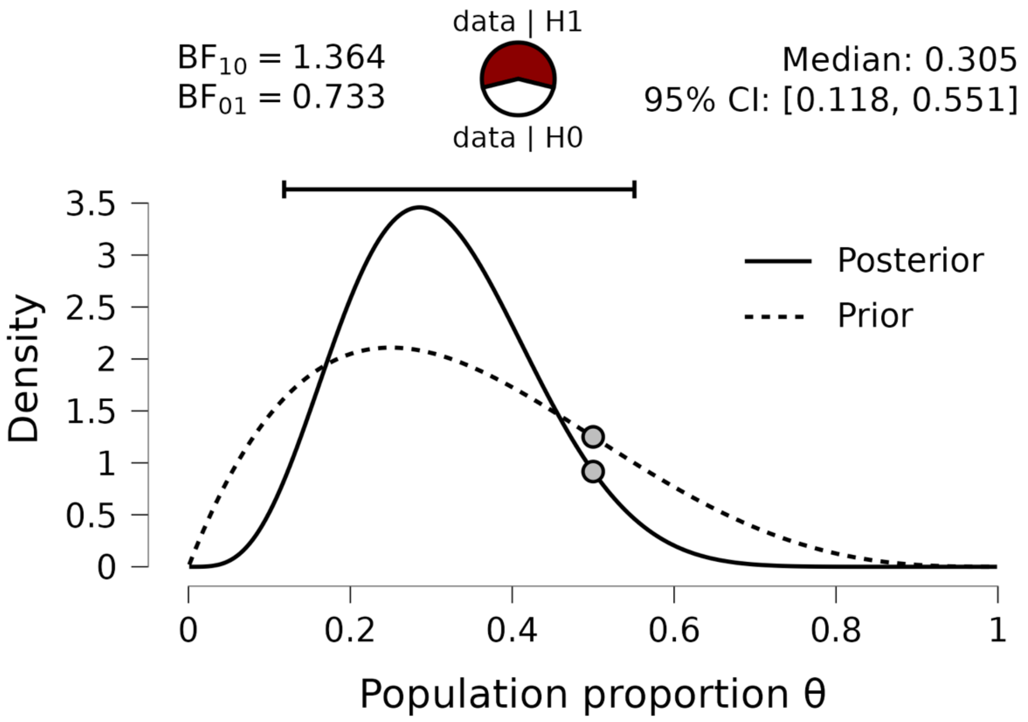
Bayes Factor Interpretation

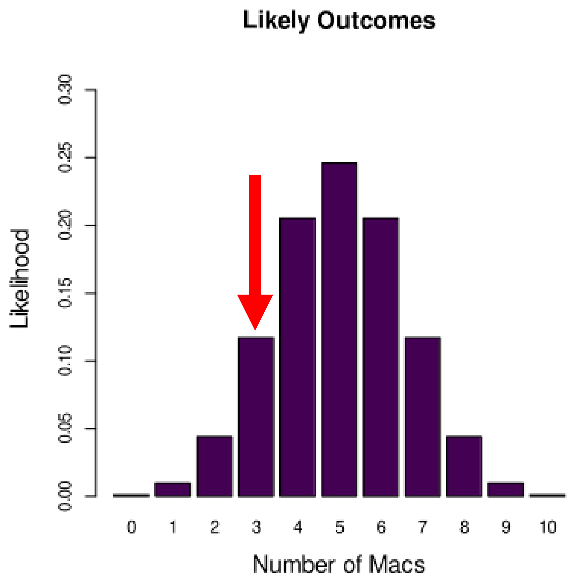


How Well Did The Models Predict The Data?

- For comparing a point model to a range model, we can use the **Savage-Dickey density ratio** as a computational shortcut to obtain the Bayes Factor
  - BF_{10} = prior density at point of testing / posterior density at point of testing





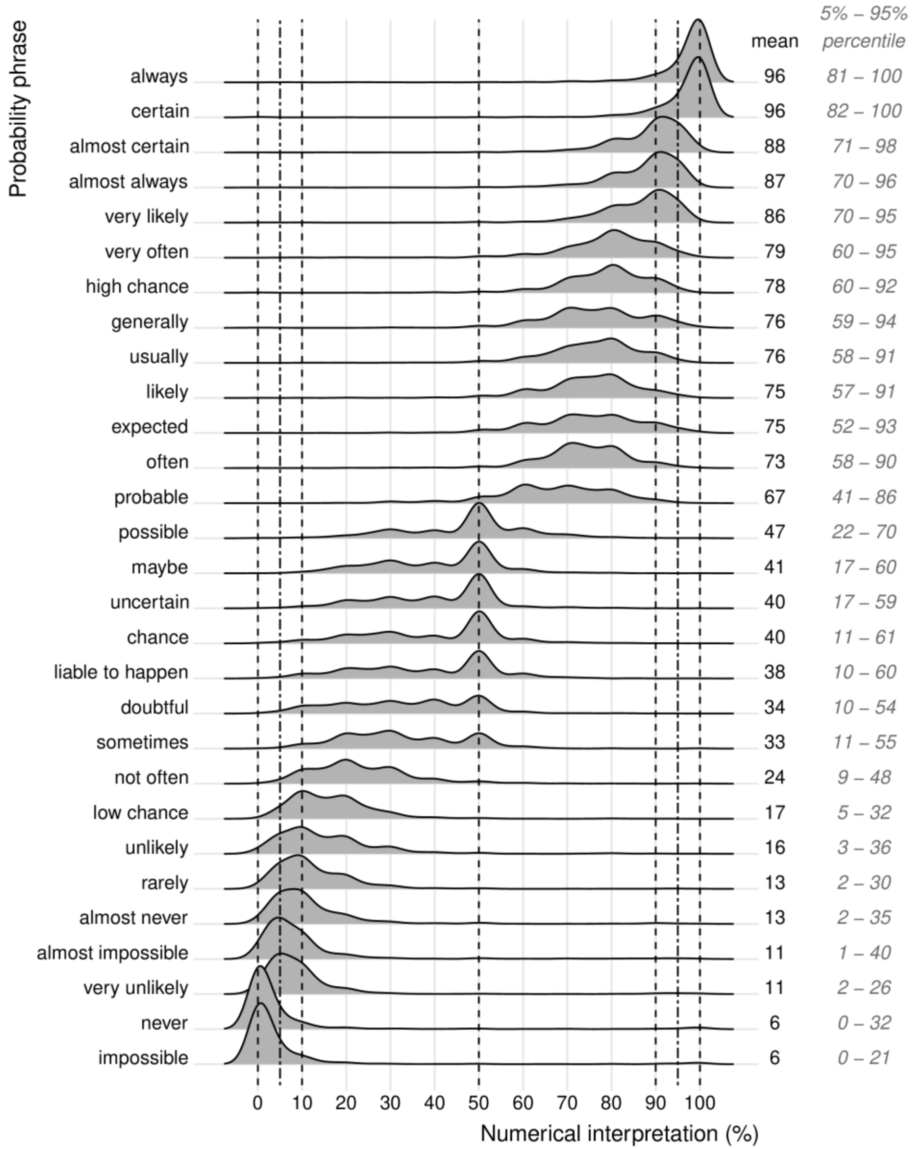


Bayes Factor Interpretation - Caution

But!

- The general challenge of translating between words and numbers:

- People have different interpretations of probabilistic words/phrases



Source:
[Willems, S., Albers, C., & Smeets, I. (2020). Variability in the interpretation of probability phrases used in Dutch news articles — a risk for miscommunication. Journal of Science Communication, 19.]

Bayes Factor Interpretation - Caution

Bayes Factor Interpretation - Caution


ONLY A SITH DEALS IN ABSOLUTES
quickmeme.com

Bayes Factor Features

- Parsimony
  - A built in Occam’s Razor (e.g., one-sided vs two-sided)
- Sensitivity Analysis
  - Investigate the impact of the prior distribution
- Sequential Analysis
  - Monitor evidence as data accumulate

Bayes & Frequentism

Bayes	Frequentism
Compare the predictive quality of two hypotheses → Invites continuous reasoning	Minimize type 1 error (incorrectly reject H_0) → Invites binary reasoning
Can monitor the results as data accumulate	Can only look at the data once: looking twice increases the risk of type 1 error
Check assumptions	Check assumptions
Still incomplete (but steadily expanding)	Has wide range of types of analyses
Sampling plan: can monitor the results and stop when satisfied	Have to specify sampling plan before, based on hypothesized effect size

Live Demonstration

VS

Try It Yourself!

- Try your new-found knowledge on your own data, or try some of the example exercises
  - https://johnnydoorn.github.io/IntroductionBayesianInference/exercises.html
- Bayesian T-test/ANOVA?

Choosing the Prior Distribution

- Before the experiment / looking at the data (so there are no calculations)
- Informed by previous experiments/knowledge or kept uninformative
  (e.g., uniform prior on the population proportion)
- One-sided / two-sided determined by what you want to test
  (e.g., do you want to know if there is a difference, or a positive difference?)
- Assess robustness of result to choice of prior distribution

Choosing the Prior Distribution

- The prior distribution is on the same *domain* as the parameter of interest:
  - For a proportion: [0, 1]
  - For a correlation: [-1, 1]
  - For a difference in means: [-∞, ∞]

Bayesian Correlation

$$P(\rho \mid \text{data}) = P(\rho)\ \frac{P(\text{data} \mid \rho)}{P(\text{data})}$$

Bayesian Correlation: Hypothesis

$$\mathcal{H}_0 : \rho = 0$$

$$\mathcal{H}_1 : \rho \neq 0$$

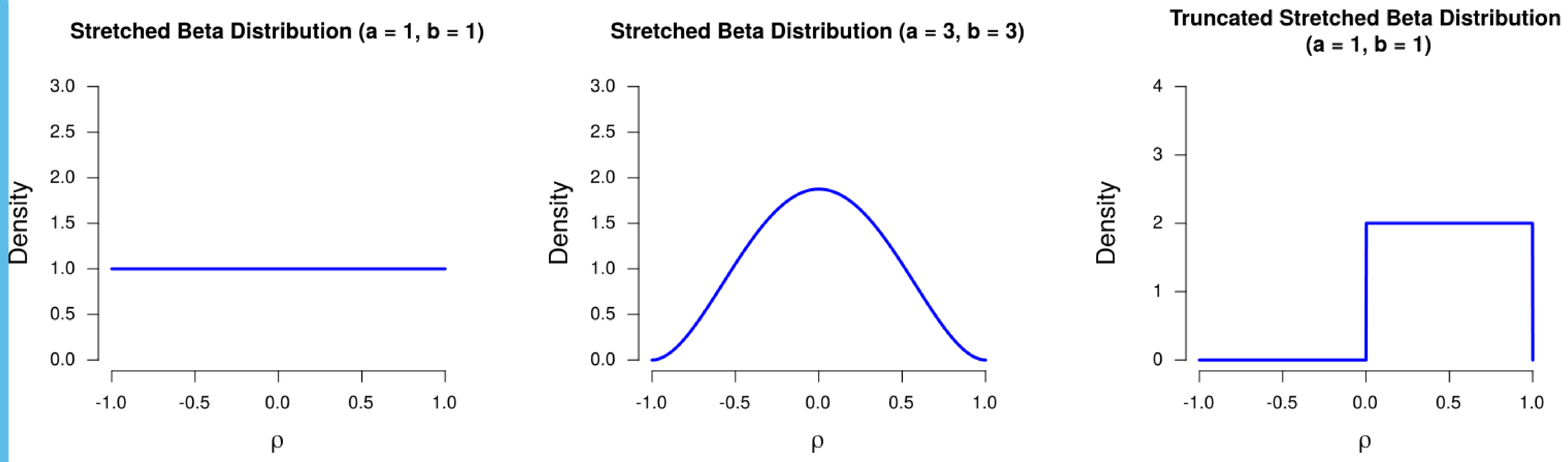
$$\mathcal{H}_+ : \rho > 0$$

$$\mathcal{H}_- : \rho < 0$$

"Alternative Hypothesis"

Bayesian Correlation: Prior Distribution

The prior distribution is on the same domain as the parameter of interest: we can take the **stretched Beta distribution!**
It is the same as Beta, but then stretched to the domain [-1, 1]

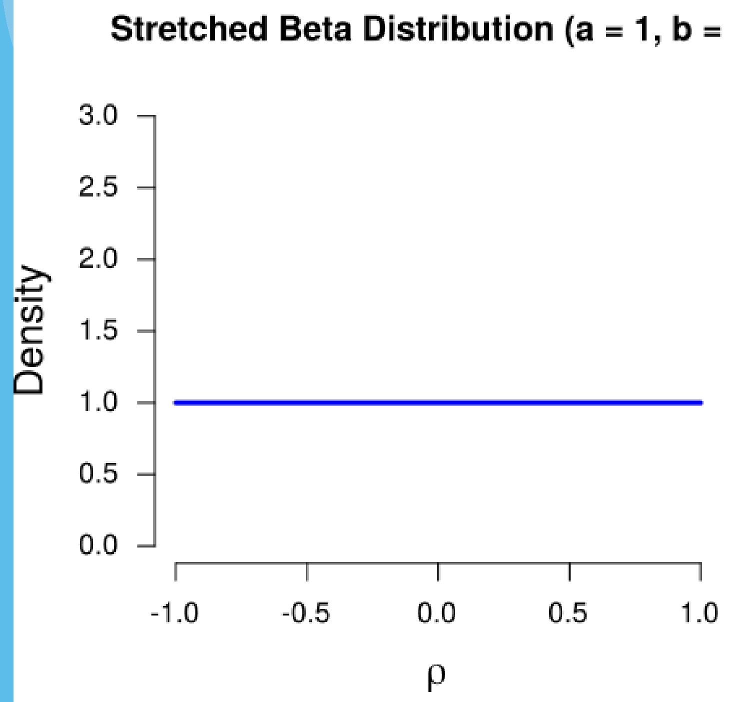


Bayesian Correlation: Prior Distribution

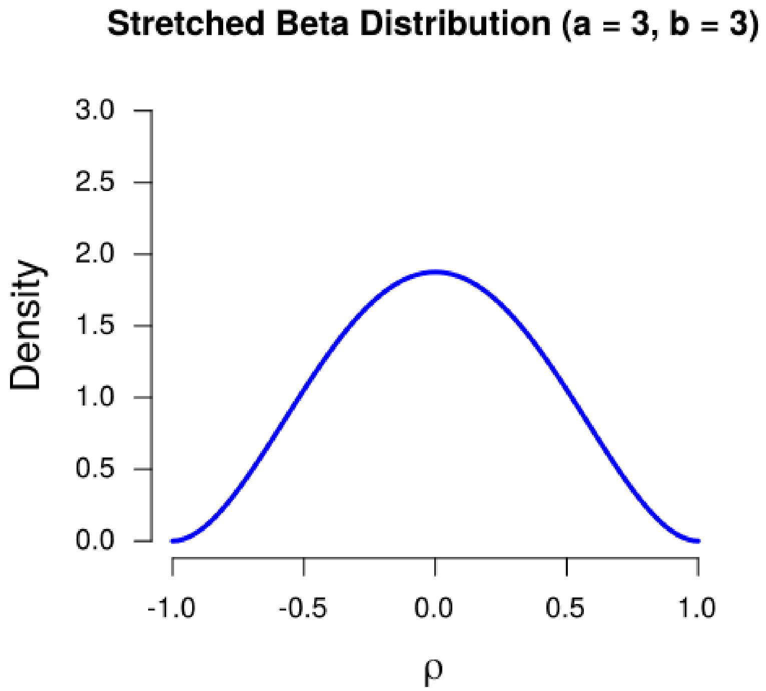
A prior distribution that reflects the belief that all values of ρ are equally plausible, **a priori**, we call this an *uninformative prior*

A prior distribution that reflects the belief that values of ρ close to 0 are more plausible (i.e., the correlation will be around 0 or not be very strong), **a priori**

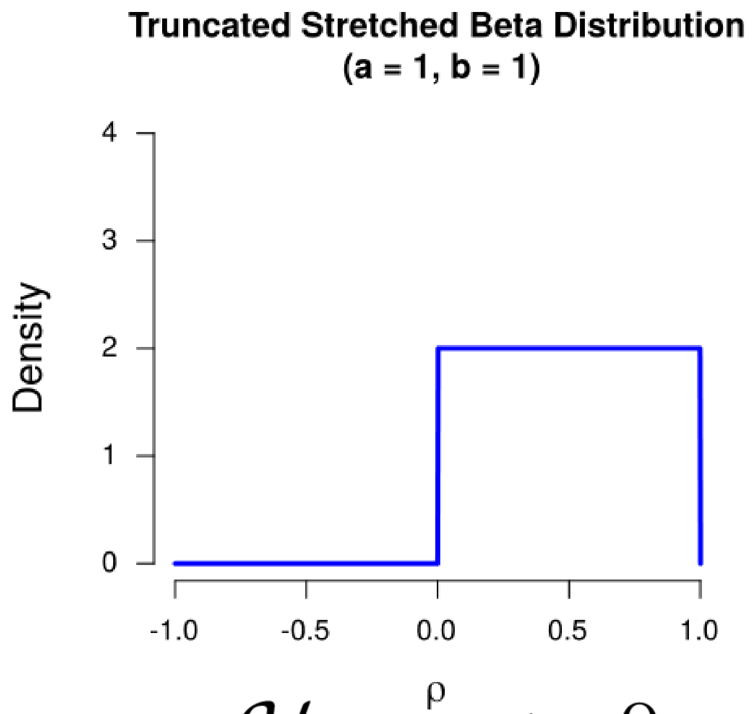
A prior distribution that reflects the belief that only positive values of ρ are possible and that those values of ρ are equally plausible (i.e., there will be a positive correlation), **a priori**



$$\mathcal{H}_1 : \rho \neq 0$$

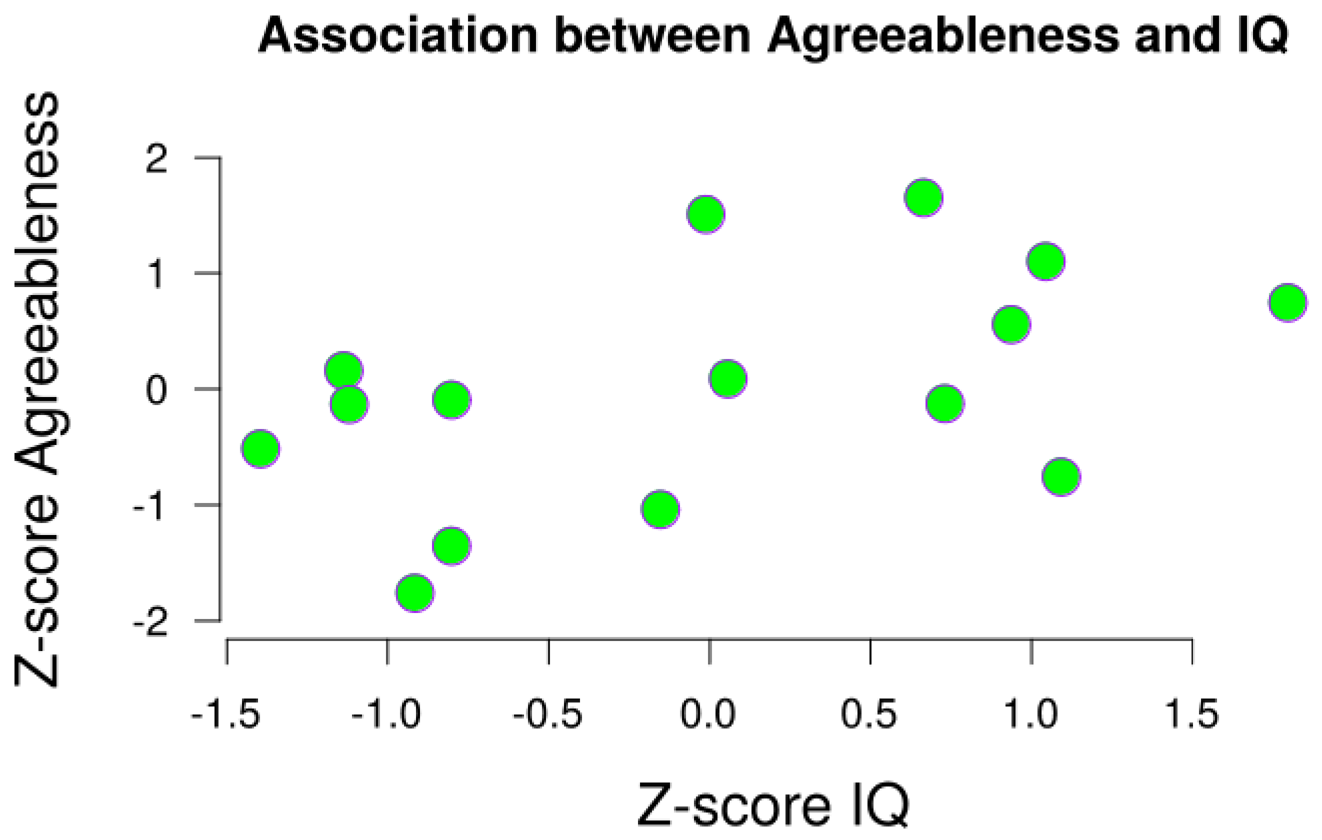


$$\mathcal{H}_1 : \rho \neq 0$$

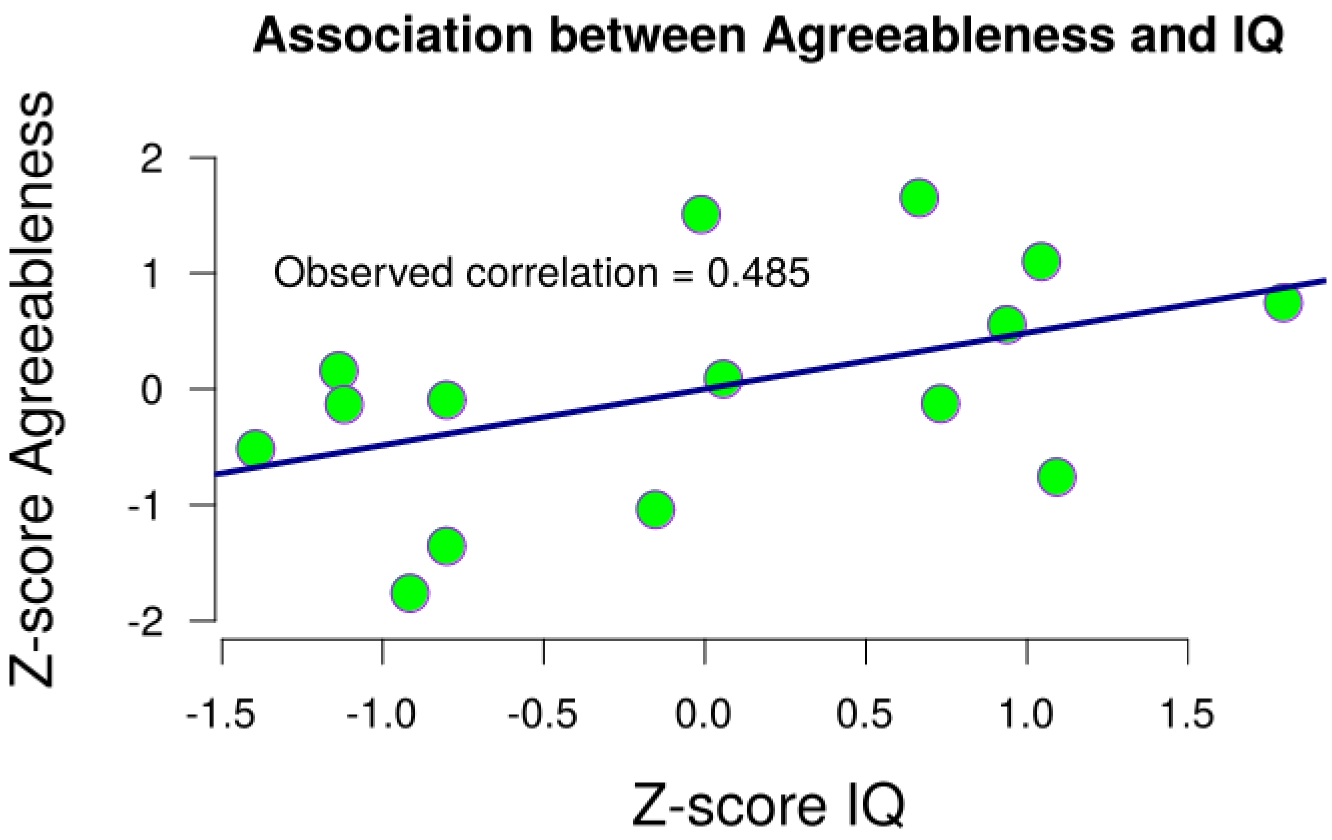


$$\mathcal{H}_+ : \rho > 0$$

Bayesian Correlation: Data



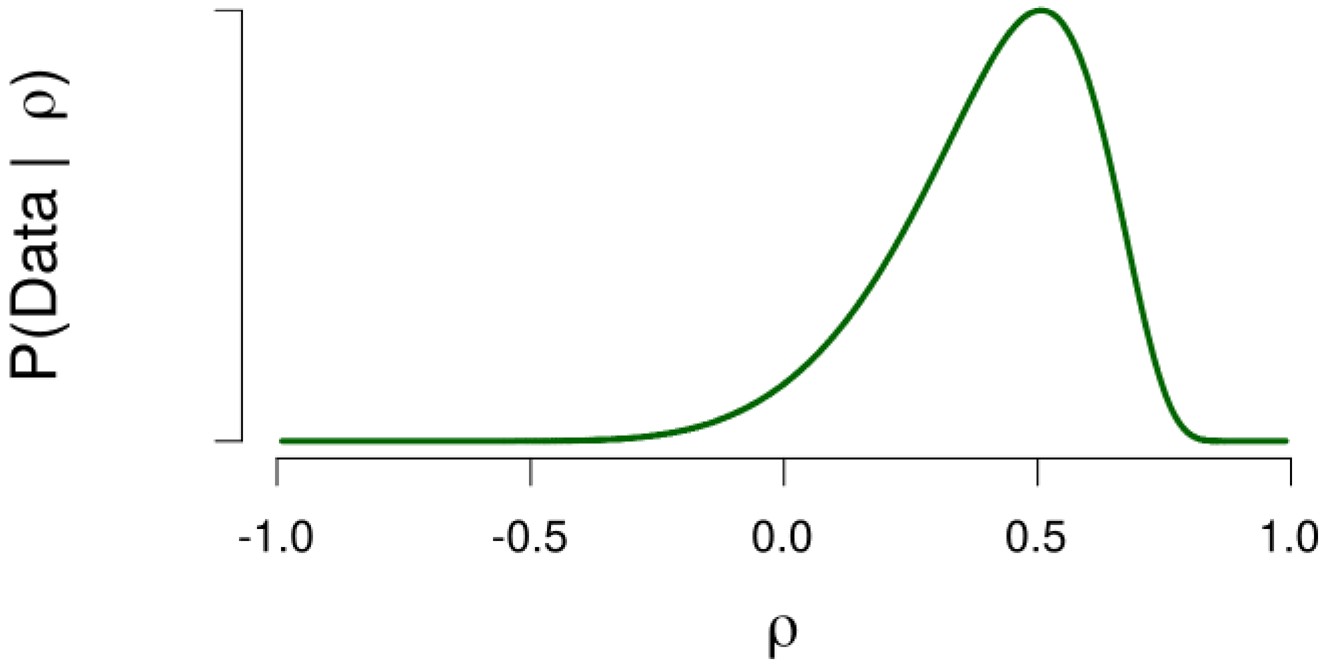
Bayesian Correlation: Data



Bayesian Correlation: Likelihood

$$P(\text{data} \mid \rho)$$

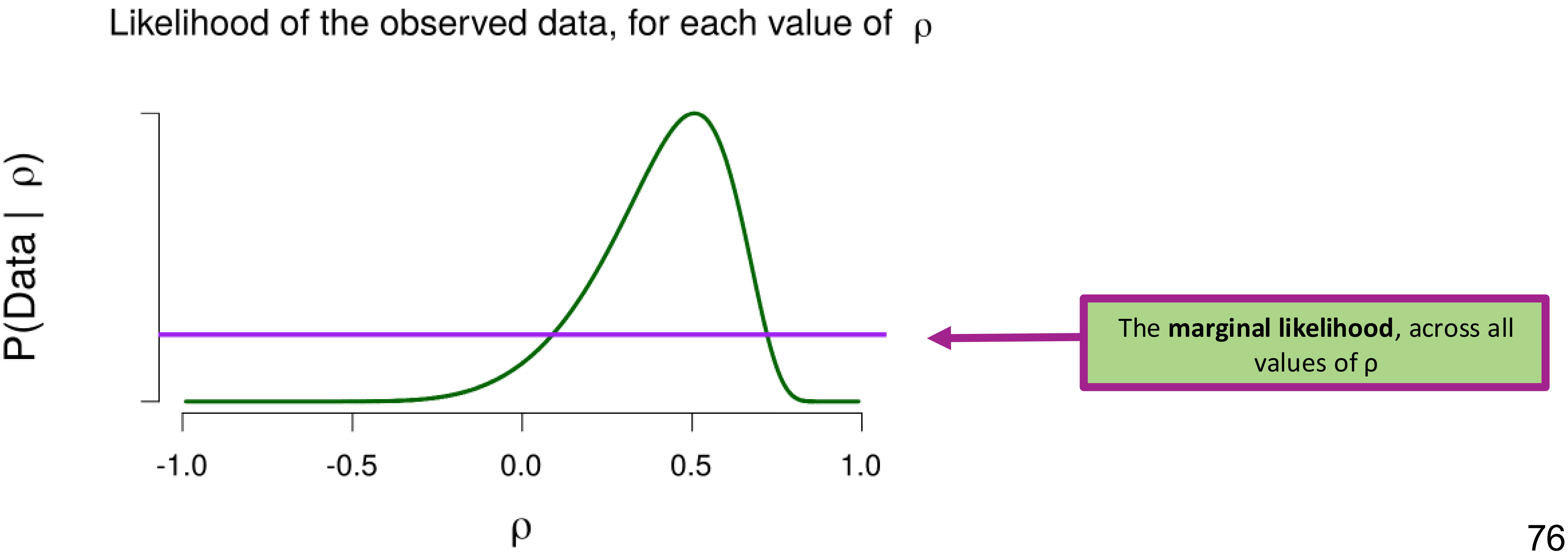
Likelihood of the observed data, for each value of ρ



We see that the data are likely for values of ρ close to 0.5.
This makes sense, because the observed correlation (i.e., the data) is equal to 0.485!

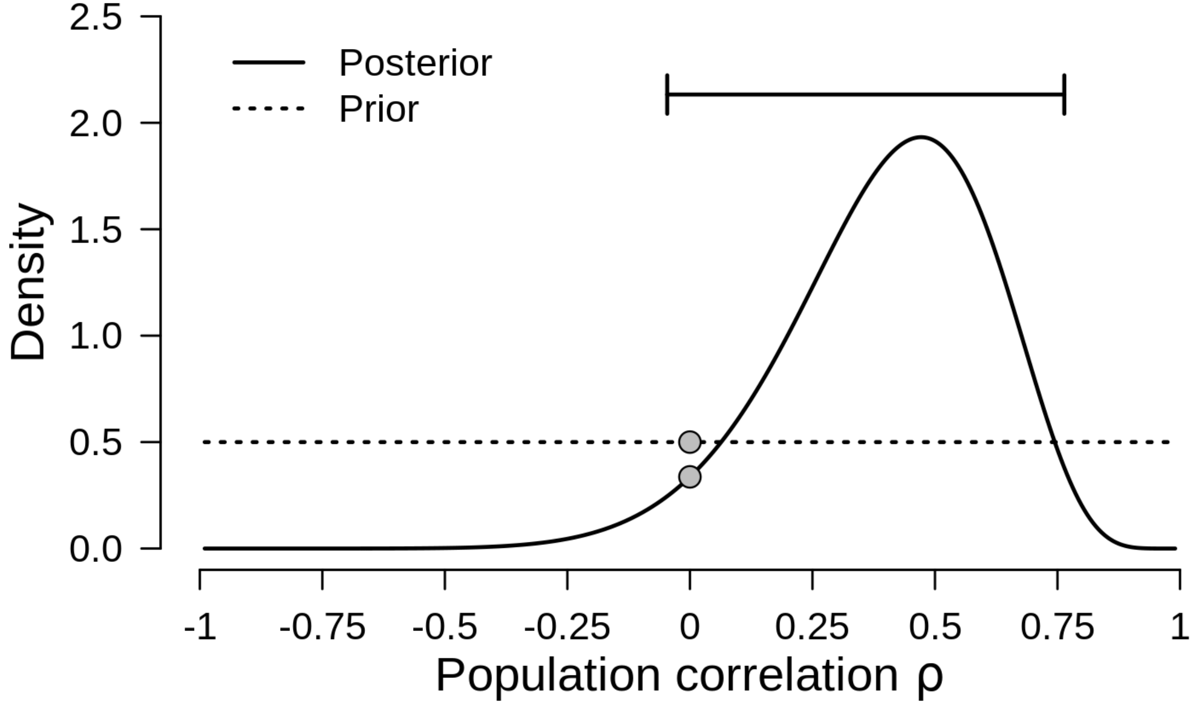
Bayesian Correlation: Likelihood

$$P(\text{data} \mid \rho)$$



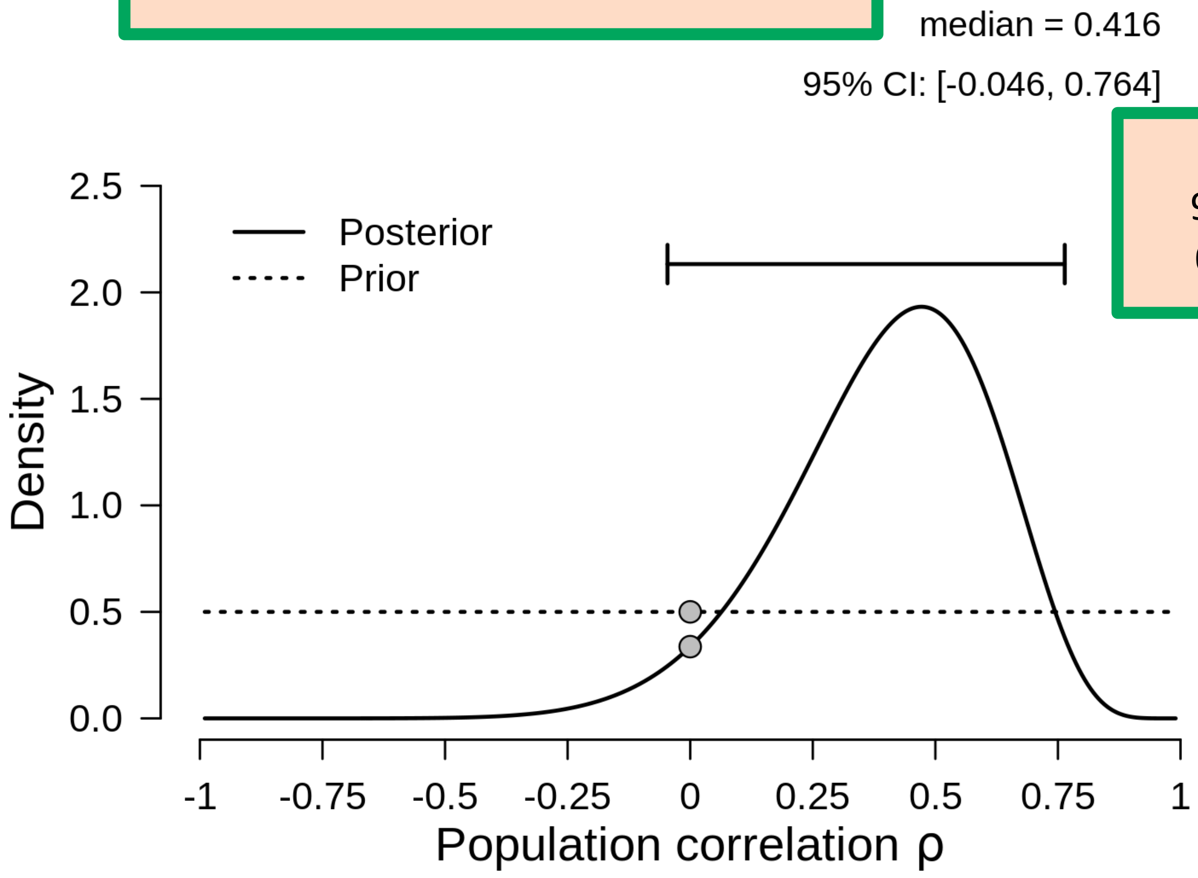
Bayesian Correlation: Posterior Distribution

$$P(\rho \mid \text{data})$$



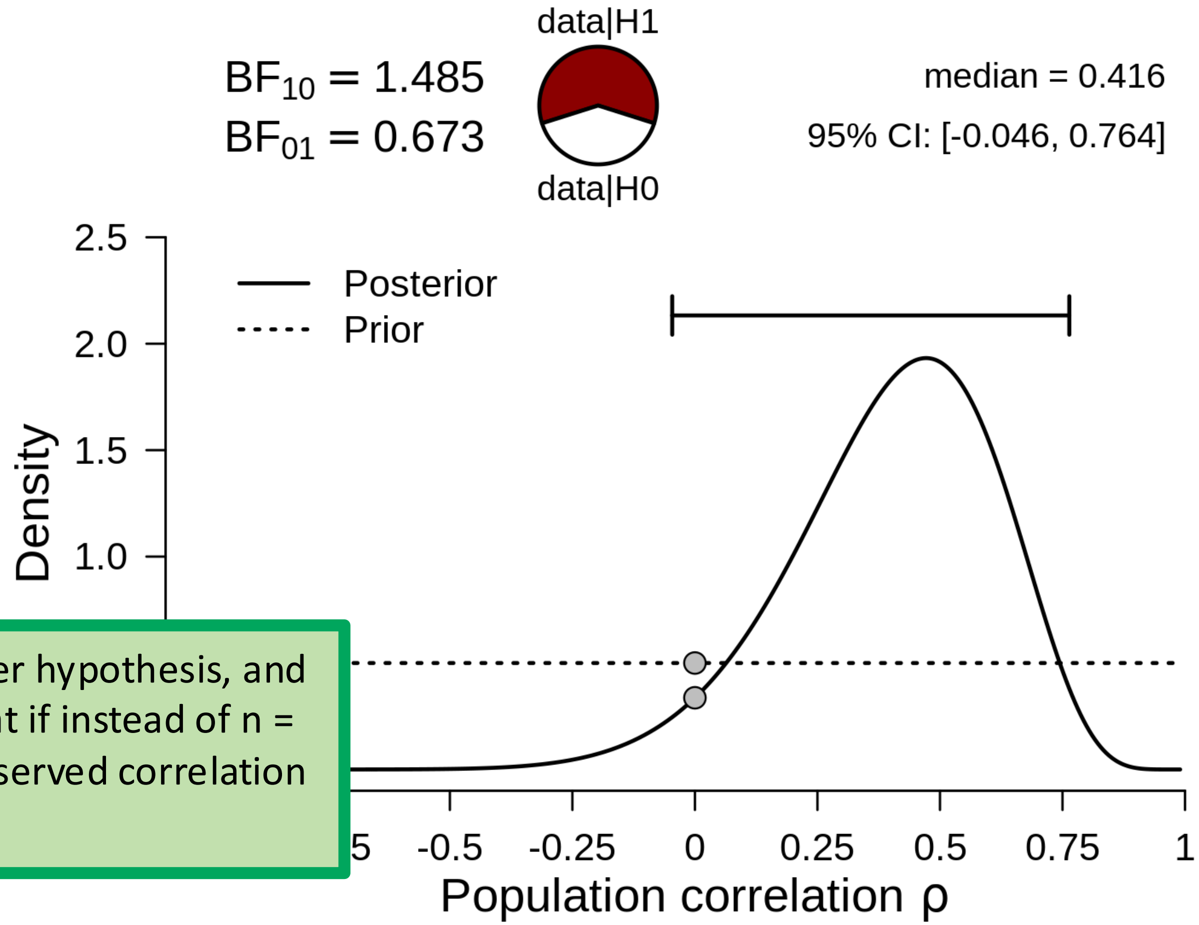
Bayesian Correlation: Posterior Distribution

Median:
50% probability that ρ is equal to or lower than 0.416, under this model



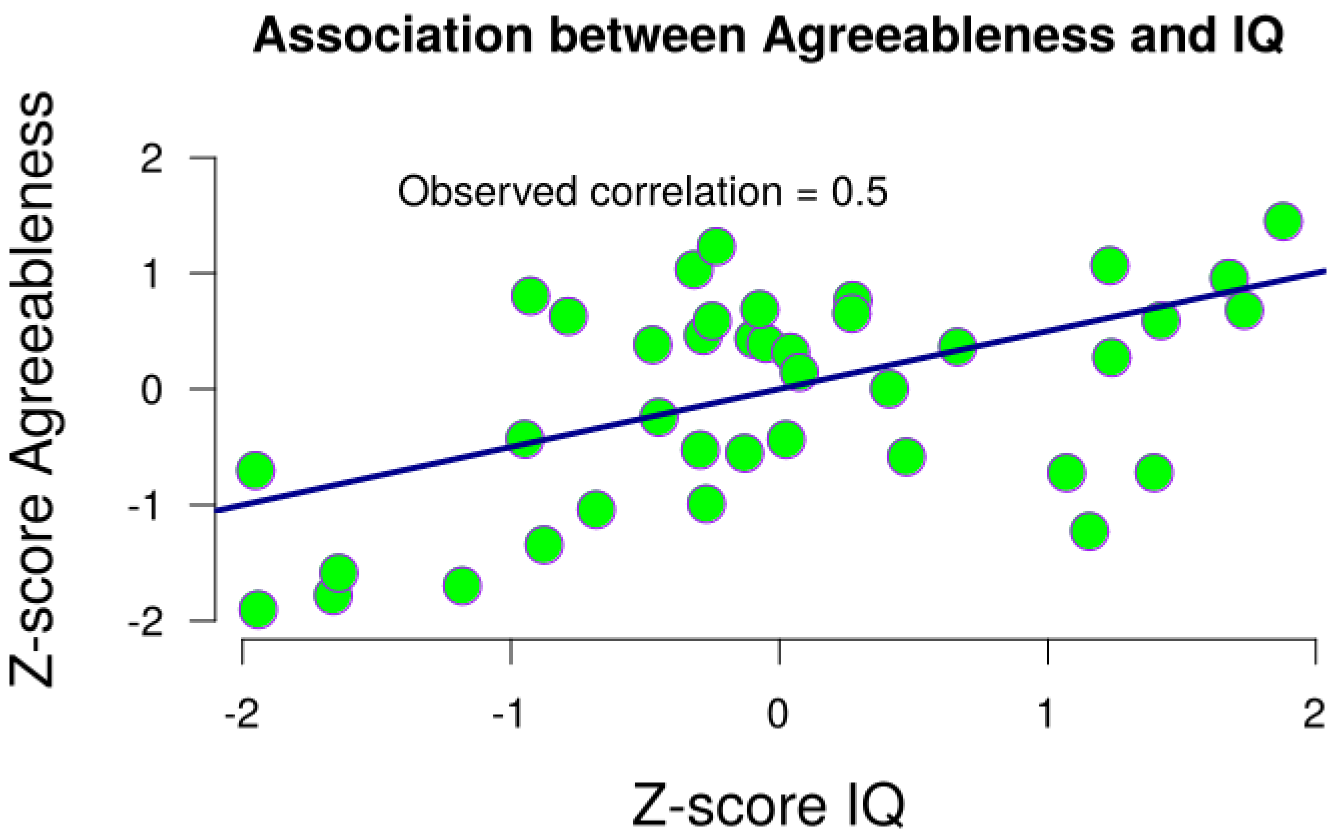
95% Credible interval:
95% probability that ρ is between -0.046 and 0.764, under this model

Bayesian Correlation: Bayes Factor



Very weak evidence in favor of either hypothesis, and a very wide credible interval… What if instead of n = 15, we had n = 40, but the same observed correlation (+- 0.5)?

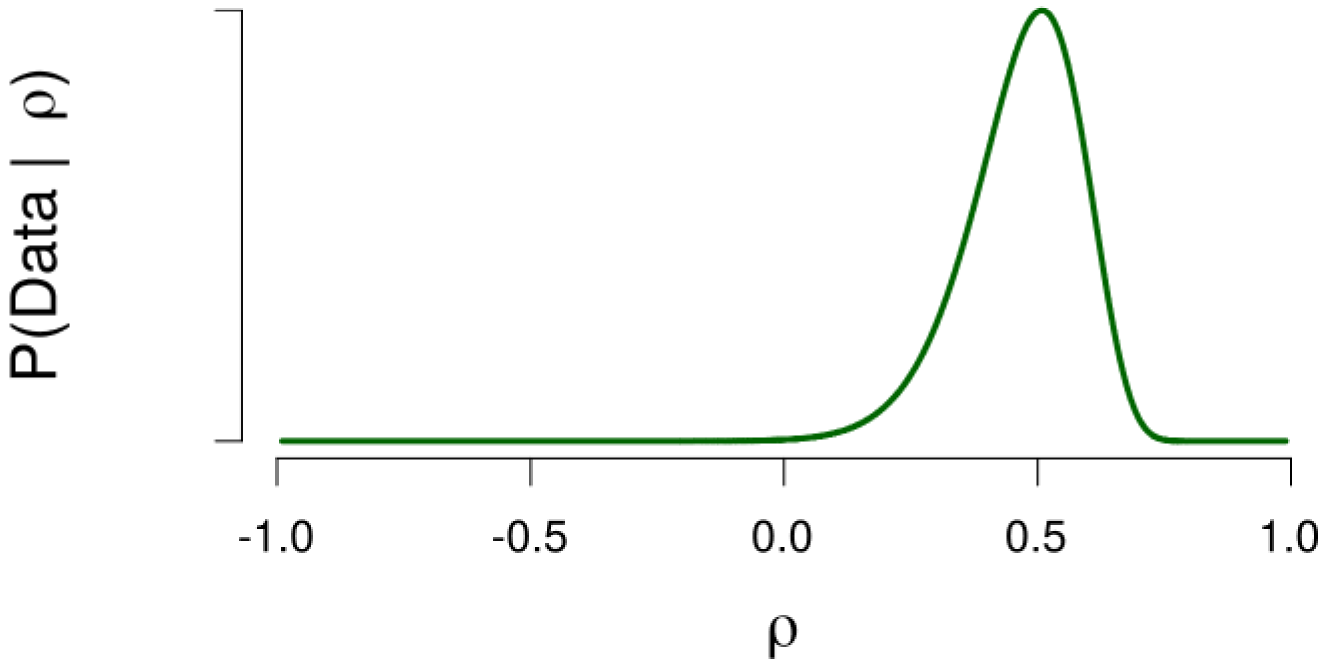
Bayesian Correlation: Data



Bayesian Correlation: Likelihood

$$P(\text{data} \mid \rho)$$

Likelihood of the observed data, for each value of ρ



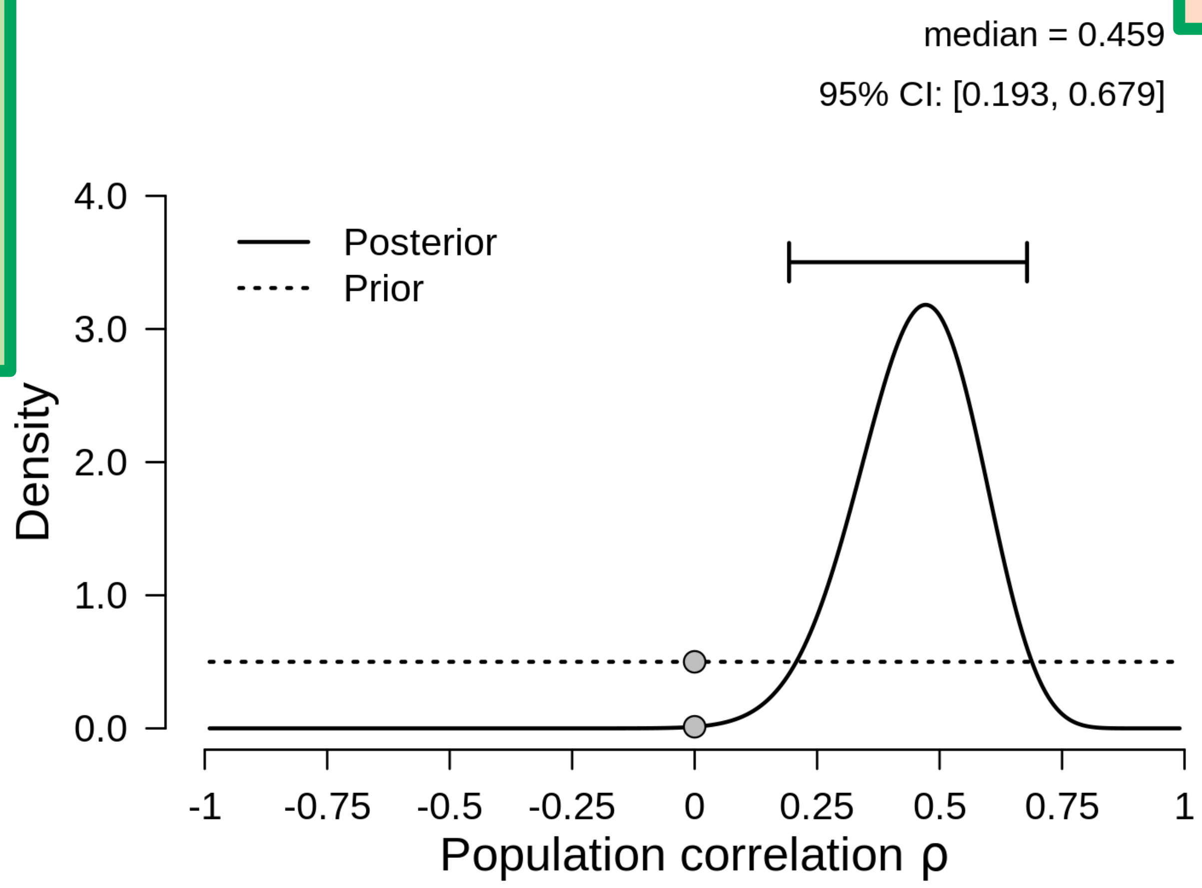
The likelihood grew more narrow, compared to when we had a lower sample size!

Bayesian Correlation: Posterior Distribution

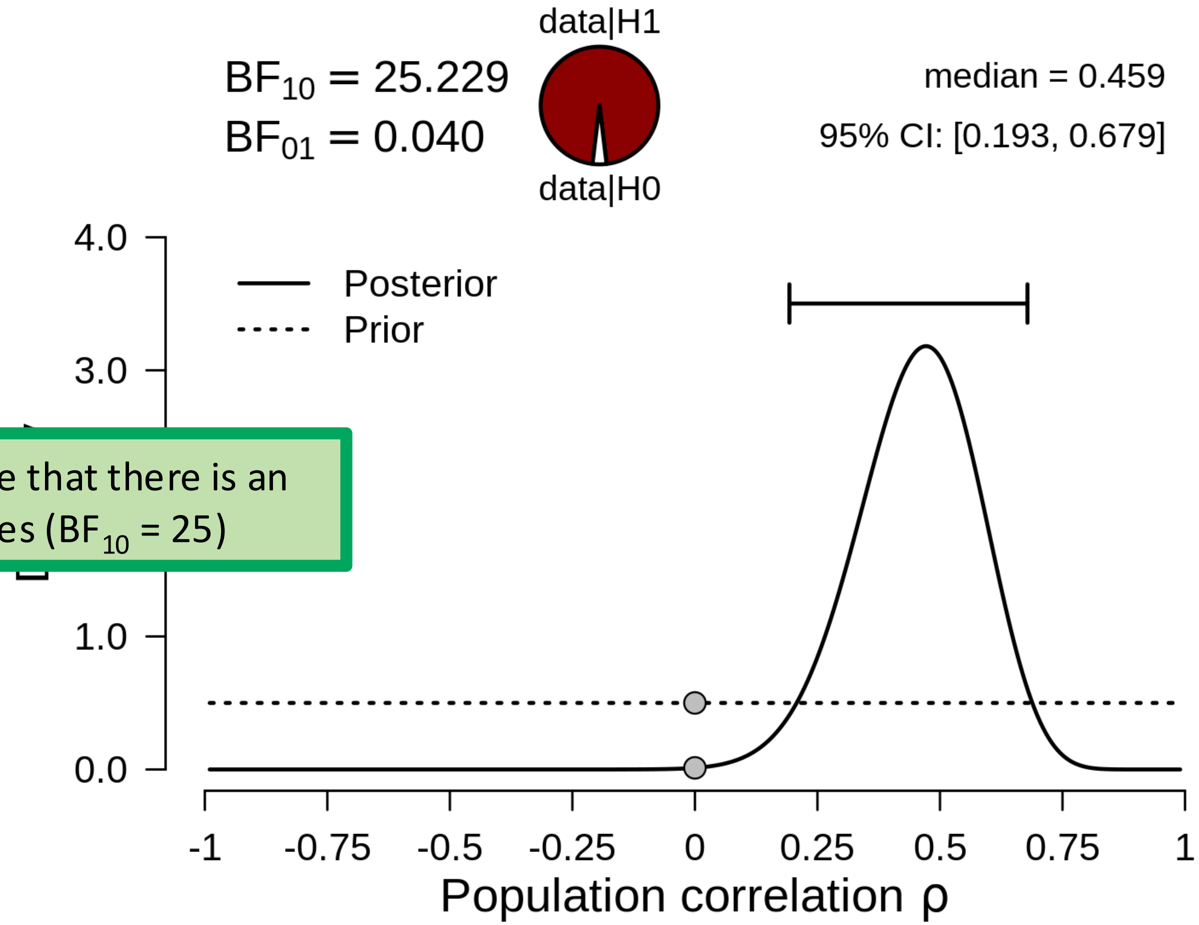
95% Credible interval:
“95% probability that ρ is between 0.193 and 0.679”

The posterior is also more narrow!
This also leads to a more narrow credible interval (still 95%, but the two numbers are closer together (e.g., 0 is no longer in the interval)

We can make a more specific prediction, with the same certainty (95%)

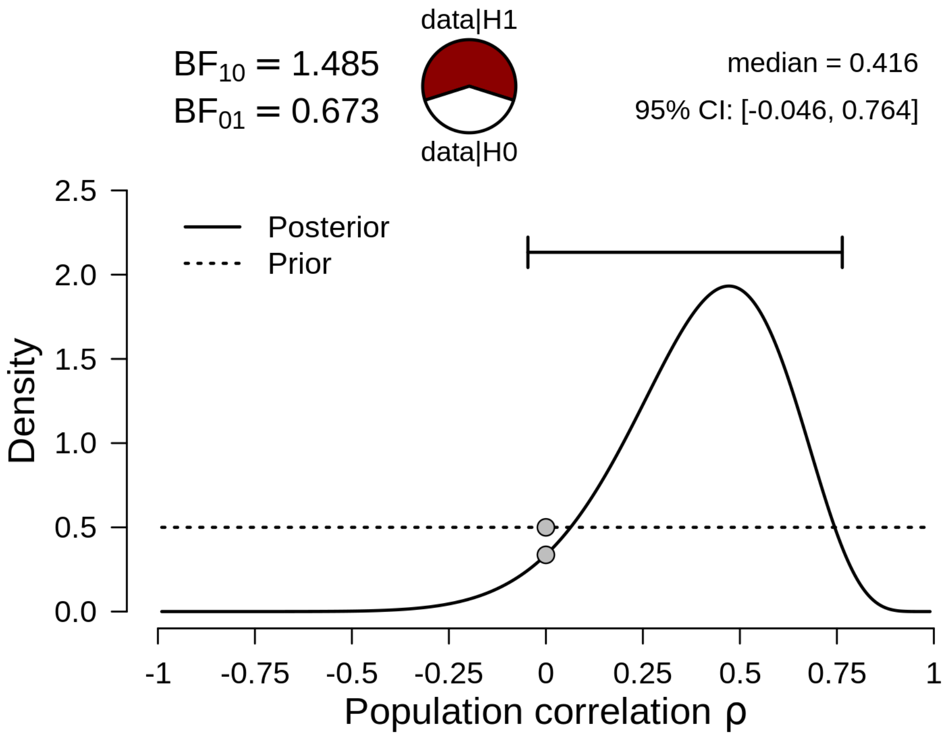


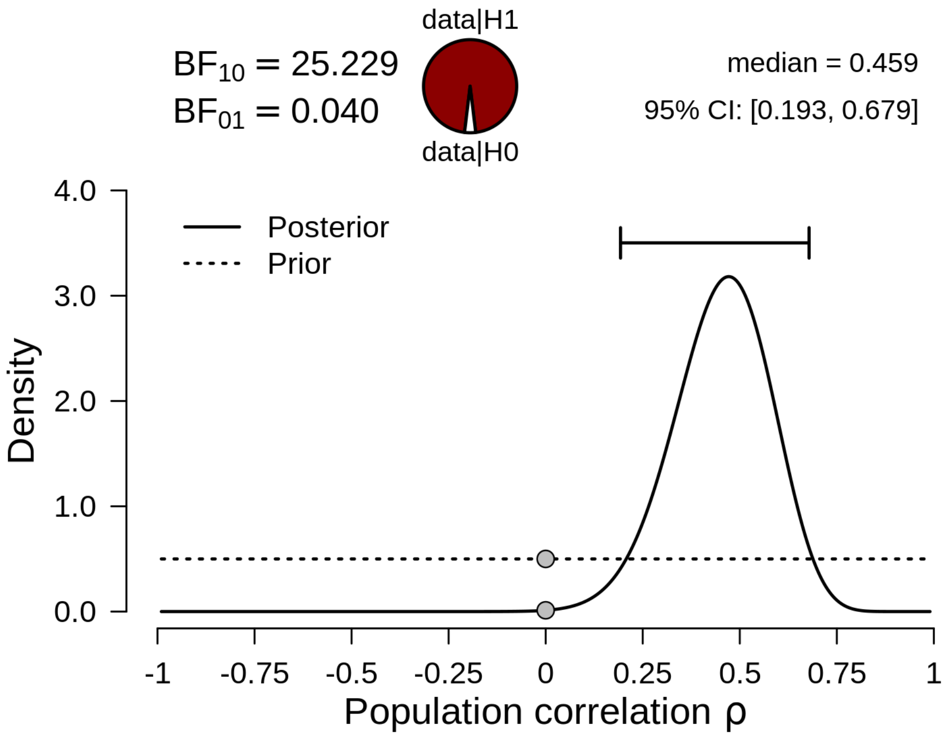
Bayesian Correlation: Bayes Factor



Now we have pretty strong evidence that there is an association between the variables (BF_{10} = 25)

Bayesian Correlation: Bayes Factor

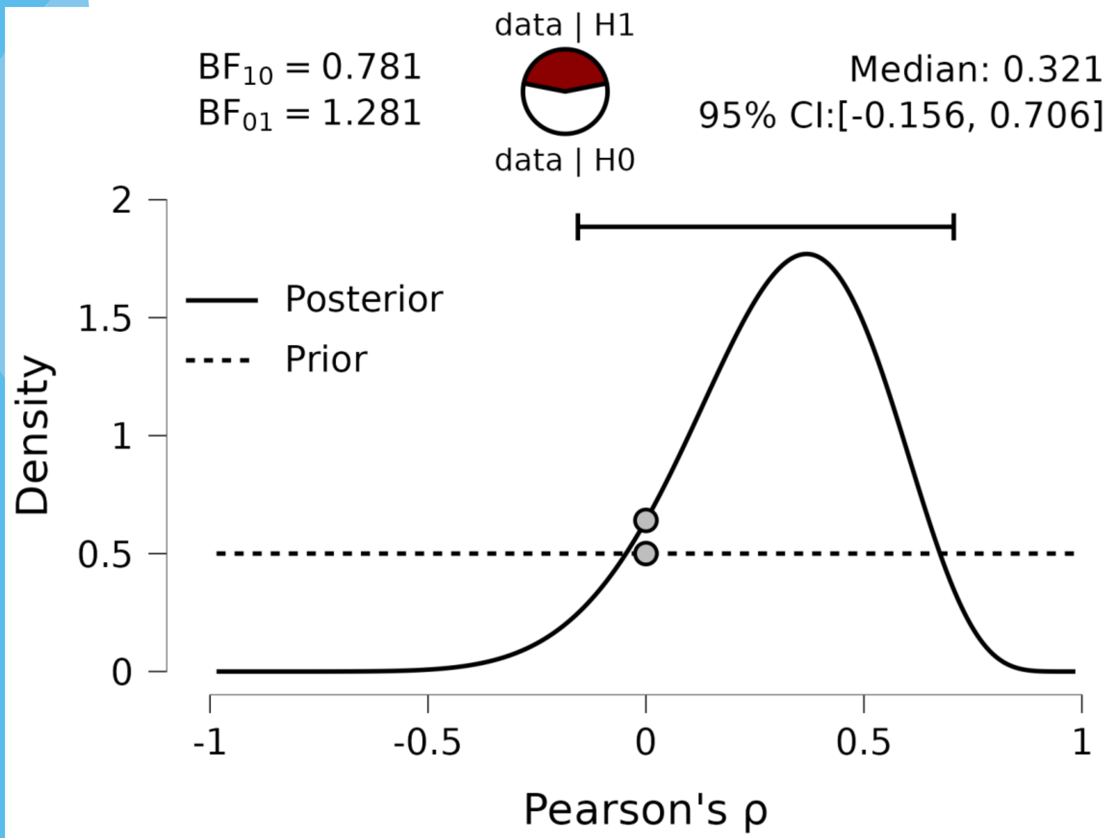


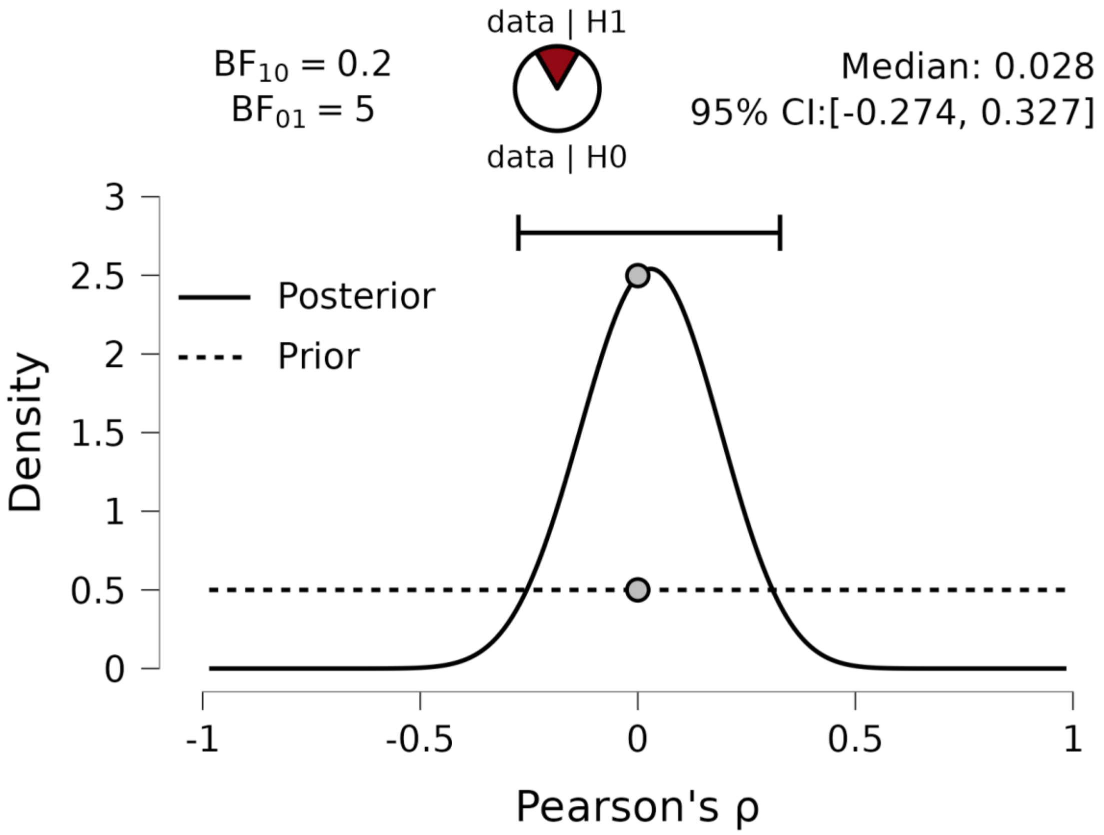


N = 15, observed correlation = 0.485

N = 40, observed correlation = 0.5

“Absence of evidence” vs “evidence of absence”





Both analyses would yield a non-significant p-value – but what does that actually mean?

Bayesian T-Test

- What about differences between/within groups?
- In psychology, differences between groups are characterized by **δ** (“delta”, also known as Cohen’s *d*): a *standardized* difference between groups

Effect size	***d***	**Reference**
Very small	0.01	Sawilowsky, 2009
Small	0.20	Cohen, 1988
Medium	0.50	Cohen, 1988
Large	0.80	Cohen, 1988
Very large	1.20	Sawilowsky, 2009
Huge	2.0	Sawilowsky, 2009

As with the Bayes factor, this is a table to provide some intuition for the magnitude of an effect – there are no hard cut-off values here!

For further reading and formulas
https://en.wikipedia.org/wiki/Effect_size#Difference_family:_Effect_sizes_based_on_differences_between_means

Bayesian T-Test

$$P(\delta \mid \text{data}) = P(\delta) \; \frac{P(\text{data} \mid \delta)}{P(\text{data})}$$

Bayesian T-Test: Hypothesis

$$\mathcal{H}_0 : \delta = 0$$

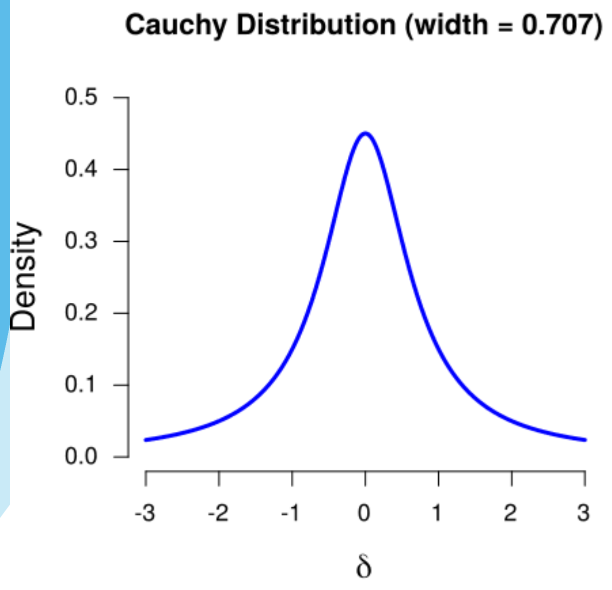
$$\left.\begin{array}{l} \mathcal{H}_1 : \delta \neq 0 \\ \mathcal{H}_+ : \delta > 0 \\ \mathcal{H}_- : \delta < 0 \end{array}\right\} \text{“Alternative Hypothesis”}$$

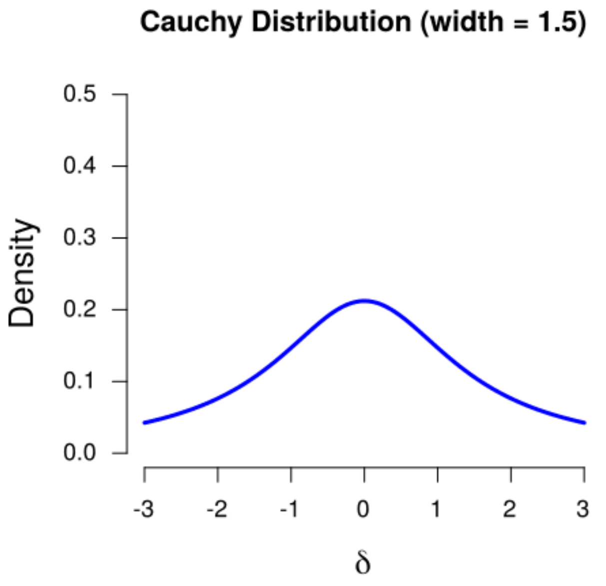
Bayesian T-Test: Prior Distribution

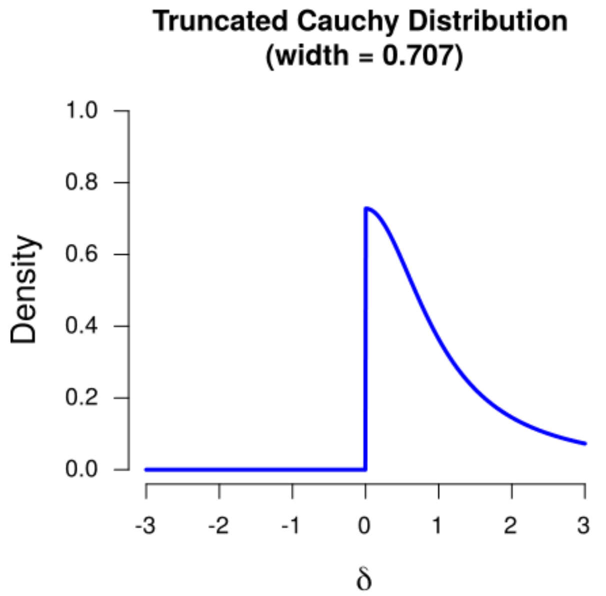
The prior distribution is on the same domain as the parameter of interest: so we need a distribution that is between [-∞, ∞].
A normal distribution could work, but the convention is to use a t-distribution with df = 1. This is known as the **Cauchy** distribution

A uniform distribution would not work here, because delta has an infinite domain

The Cauchy distribution is governed by a single shape parameter that determines how wide it is







Bayesian T-Test: Prior Distribution

A prior distribution that reflects the belief that 50% of the values of δ are located between -0.707 and 0.707, **a priori**

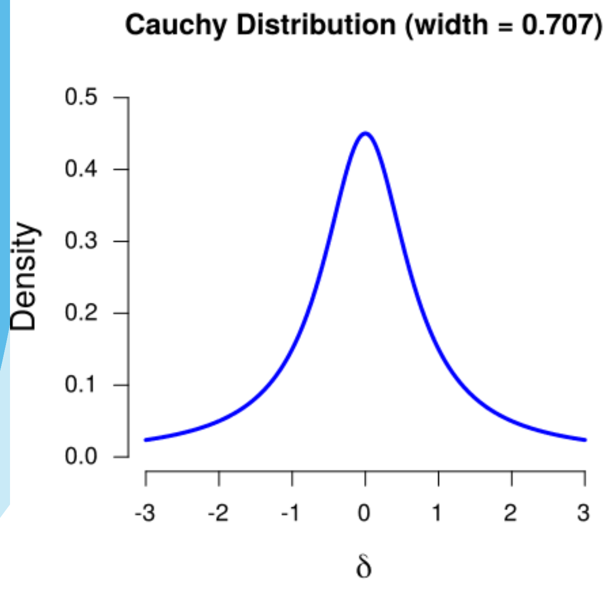
This is often selected as the uninformative option. Why? Unfortunately that is a long and technical story, not suited for this workshop :(

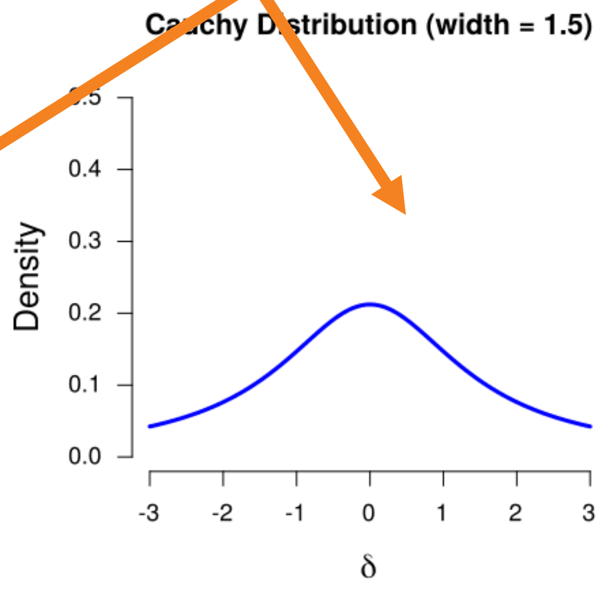
A prior distribution that reflects the belief that 50% of the values of δ are located between -1.5 and 1.5 , **a priori**

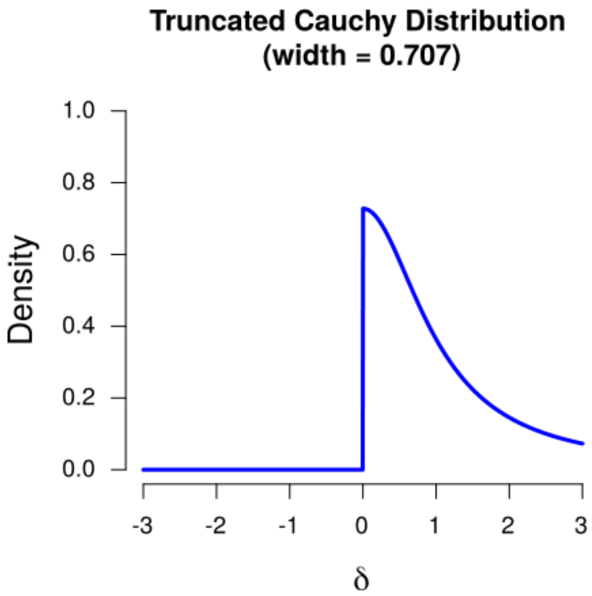
A prior distribution that reflects the belief that only positive values of δ are possible and that 50% of the values of δ are between 0 and 0.707, **a priori**

$$\mathcal{H}_1 : \delta \neq 0$$

$$\mathcal{H}_+ : \delta > 0$$



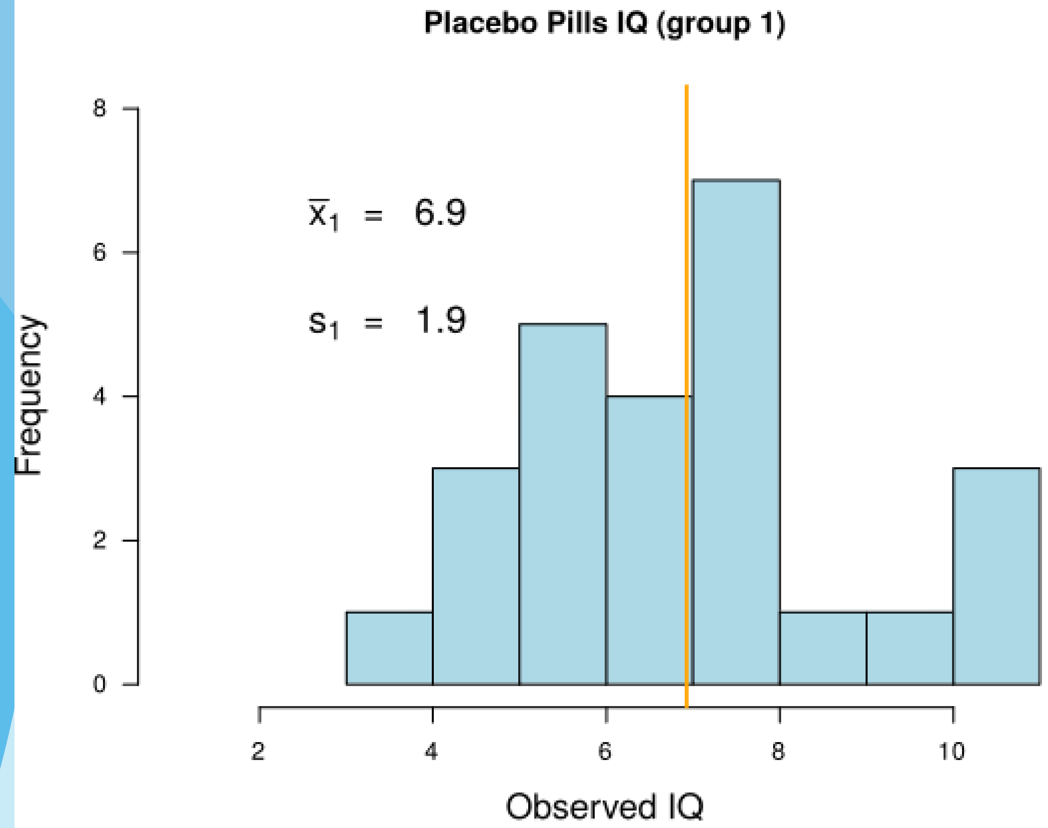


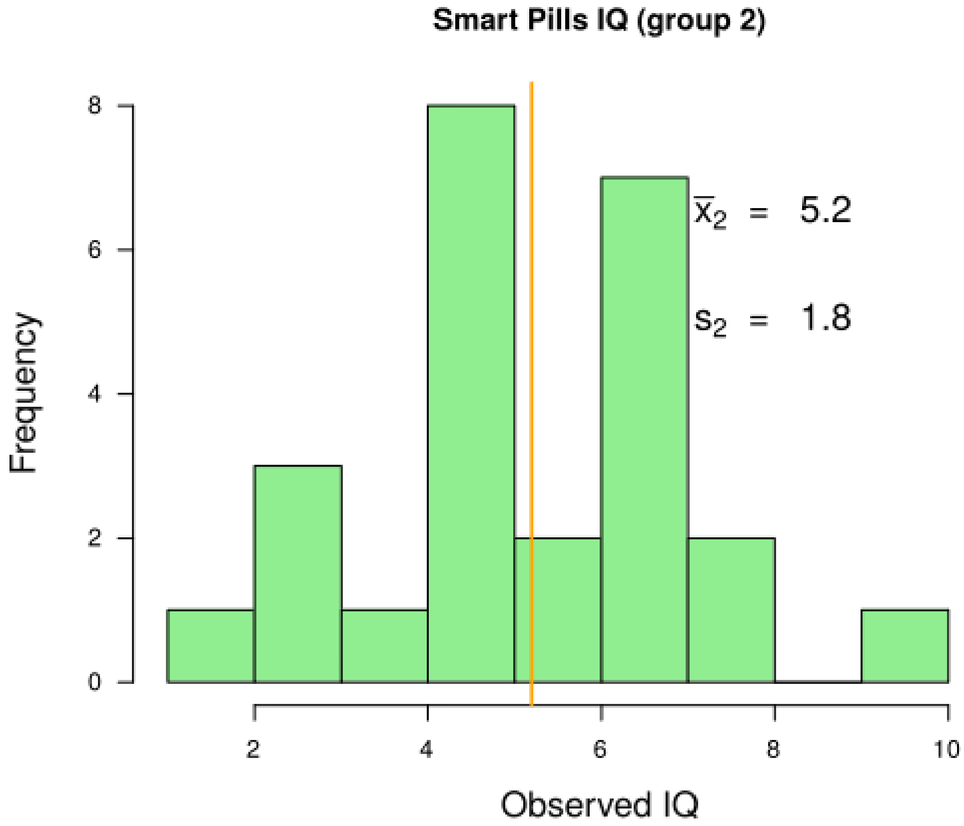


Bayesian T-Test: Data

Observed δ = 0.95
(t = 3.27)

n = 25

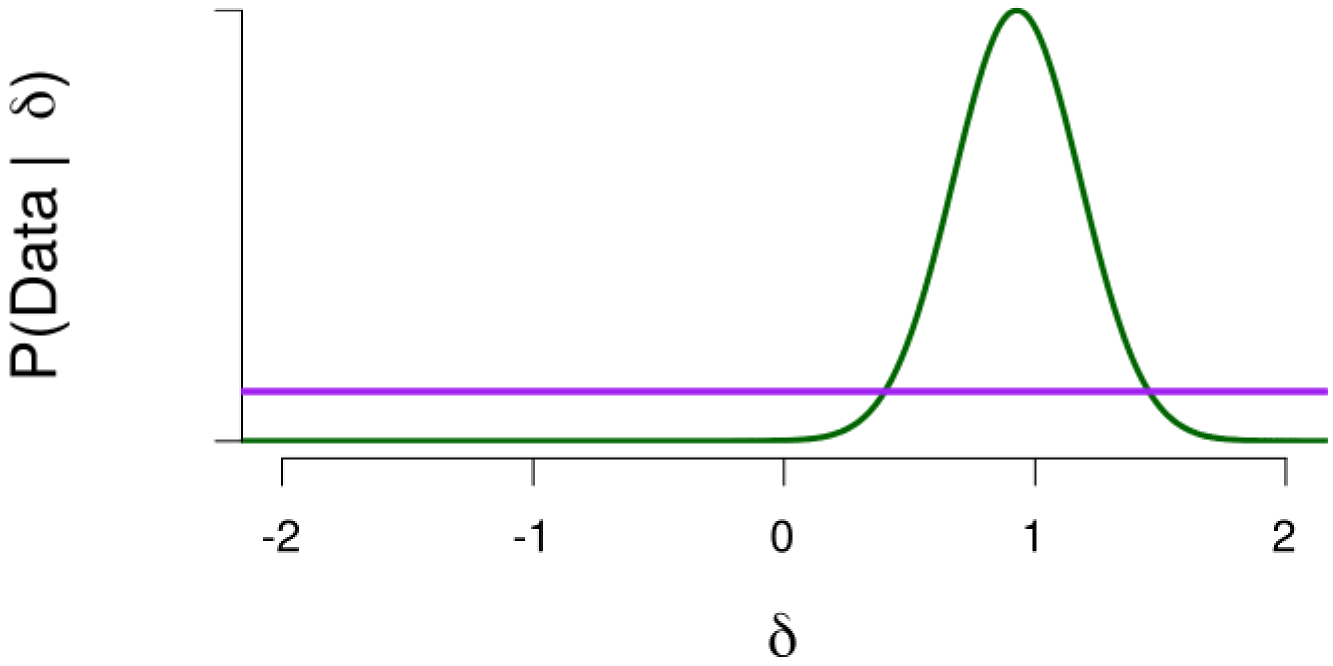




Bayesian T-Test: Likelihood

$$P(\text{data} \mid \delta)$$

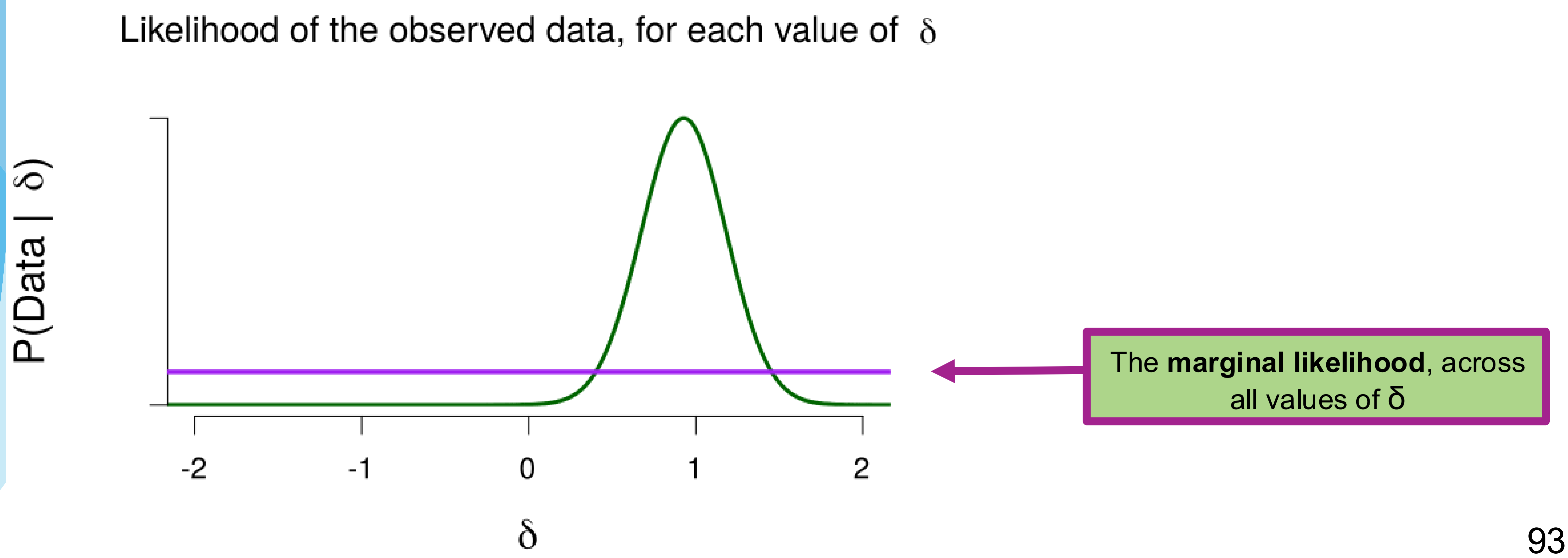
Likelihood of the observed data, for each value of δ



We see that the data are likely for values of δ close to 1.
This makes sense, because the observed delta (i.e., the data) is equal to 0.95!

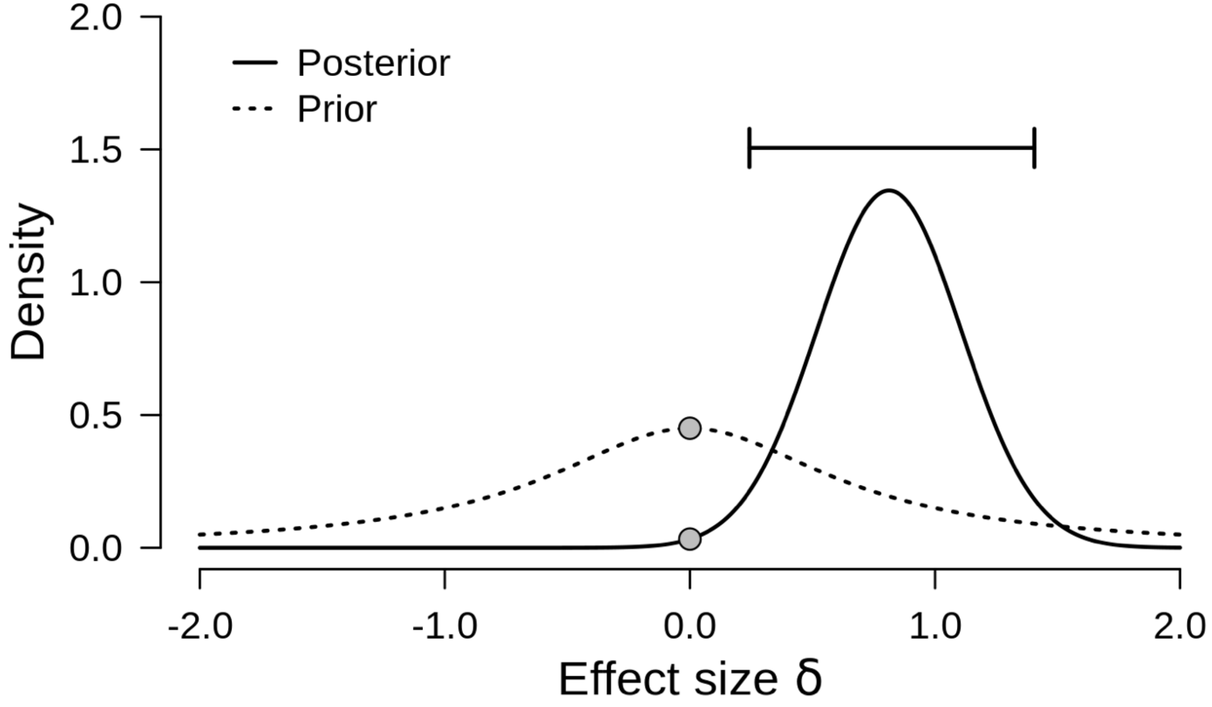
Bayesian T-Test: Likelihood

$$P(\text{data} \mid \delta)$$

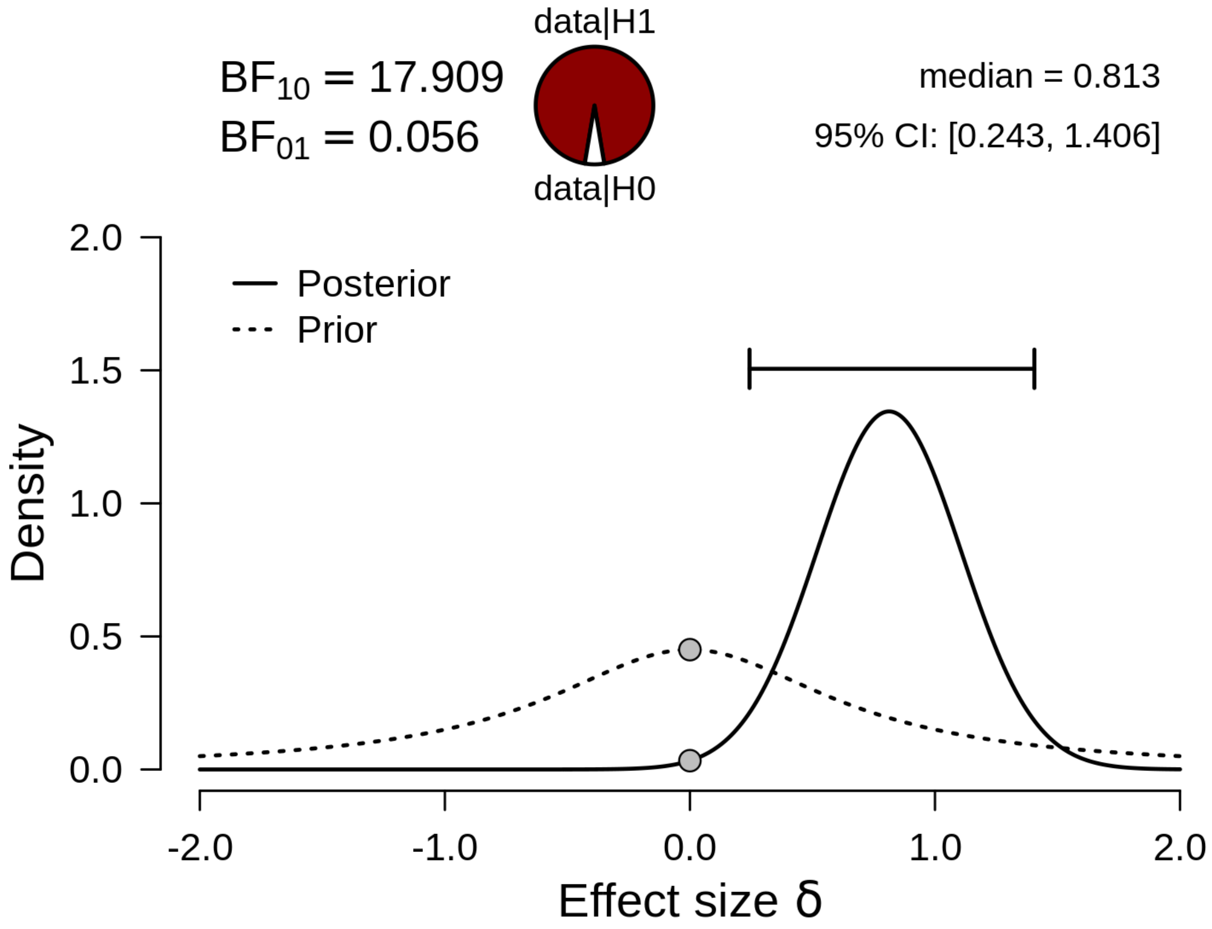


Bayesian T-Test: Posterior Distribution

$$P(\delta \mid \text{data})$$



Bayesian T-Test: Bayes Factor

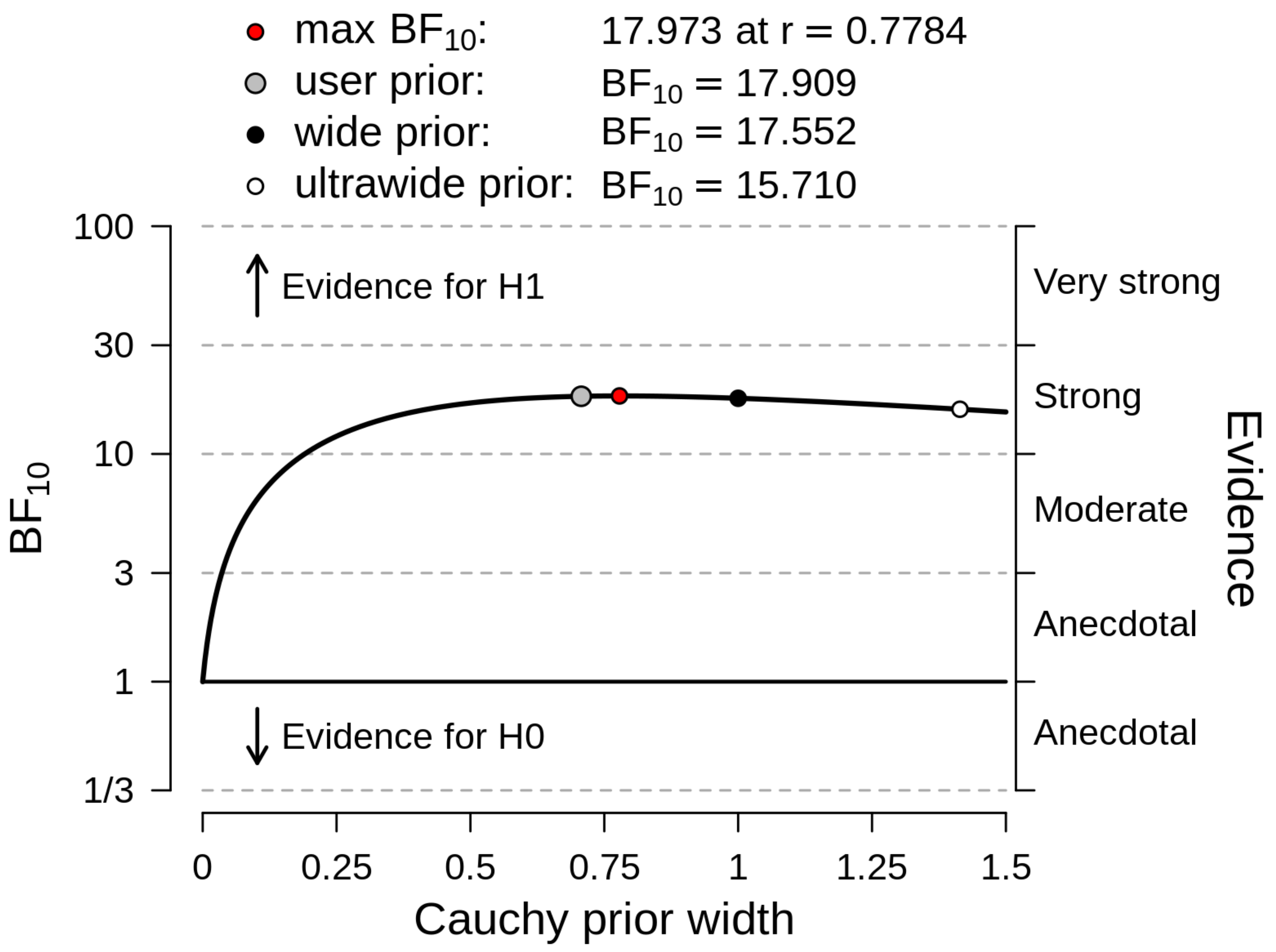


Bayesian T-Test: Robustness Check

Because choosing the prior setting is fairly subjective, we can explore what would have happened if we had chosen a different value for the prior width. This is known as a **Bayes factor robustness check**, **or sensitivity analysis**

We see here that the Bayes factor is pretty stable (i.e., flat line) across many values for the prior width. Except for very strong prior settings (0.15 and lower) does the evidence for H_1 decrease

"Here, the Bayes factor does not qualitatively differ for a wide range of prior specifications (0.25 – 1.5)



Singer Heights

This data set, "Singers", provided "Heights in inches of the singers in the New York Choral Society in 1979. The data are grouped according to voice part.

Variables:

- Height - Height (in inches) of the singers.
- Gender - Gender of the singers (female, male).
- Pitch - Voice pitch of the singers (very low, low, high, very high)

Research question: to what extent does height differ for different vocal pitches and gender?

Source:

Chambers, J. M., Cleveland, W. S., Kleiner, B., & Tukey, P. A. (1983). Graphical Methods for Data Analysis. New York: Chapman & Hall.

Singer Height

MODEL

$$S_i = \alpha + \beta_1 W_i + \beta_2 P_i + \varepsilon_i$$

Height = base level + pitch + gender + error

$$S_i = \alpha + \beta_1 W_i + \varepsilon_i$$

Height = base level + pitch + error

$$S_i = \alpha + \beta_2 P_i + \varepsilon_i$$

Height = base level + gender + error

Singer Height

Model Comparison ▼

Models	P(M)	P(M|data)	BF_M	BF_{10}	error %
Gender + Pitch	0.2000	0.9000	36.0148	1.0000	
Gender + Pitch + Gender ✻ Pitch	0.2000	0.0958	0.4238	0.1064	3.6429
Gender	0.2000	0.0042	0.0167	0.0046	2.0183
Pitch	0.2000	8.8662e−39	3.5465e−38	9.8509e−39	2.0184
Null model	0.2000	1.0914e−40	4.3655e−40	1.2126e−40	2.0183

Singer Height

Model Comparison ▼

Models	P(M)	P(M|data)	BF_M	BF_{10}	error %
Gender + Pitch	0.2000	0.9000	36.0148	1.0000	
Gender + Pitch + Gender ✻ Pitch	0.2000	0.0958	0.4238	0.1064	3.6429
Gender	0.2000	0.0042	0.0167	0.0046	2.0183
Pitch	0.2000	8.8662e−39	3.5465e−38	9.8509e−39	2.0184
Null model	0.2000	1.0914e−40	4.3655e−40	1.2126e−40	2.0183

Analysis of Effects - Height

Effects	P(incl)	P(excl)	P(incl|data)	P(excl|data)	BF_{incl}
Gender	0.6000	0.4000	1.0000	0.0000	∞
Pitch	0.6000	0.4000	0.9958	0.0042	159.7825
Gender ✻ Pitch	0.2000	0.8000	0.0958	0.9042	0.4238

Many Models? Again?

- In ANOVA (and regression) we compare models
- Differs from classical analysis, where we assess effects
- Each model instantiates a theoretically interesting assumption
- All tests of interest are pairwise
- Remember: "Best" model is:
  - Not necessarily good (check assumptions)
  - Not necessarily much better than second best

Handy Literature

- van Doorn, J., van den Bergh, D., … & Wagenmakers, E.-J. (2021). The JASP guidelines for conducting and reporting a Bayesian analysis. Psychonomic Bulletin & Review, 28, 813-826.
- Etz, A., & Vandekerckhove, J. (2018). Introduction to Bayesian inference for psychology. Psychonomic bulletin & review, 25(1), 5-34.
- Wagenmakers, E.-J.,… & Morey, R. D. (2018). Bayesian inference for psychology. Part II: Example applications with JASP. Psychonomic Bulletin & Review, 25, 58-76.
- van den Bergh, D., Van Doorn, J., Marsman, M., Draws, T., Van Kesteren, E. J., Derks, K., … & Wagenmakers, E. J. (2020). A tutorial on conducting and interpreting a Bayesian ANOVA in JASP. LAnnee psychologique, 120, 73-96.
- Field, A., van Doorn, J., & Wagenmakers, E.-J. (in press). Discovering statistics using JASP. London: Sage.

Handy Resources

- The JASP Video Library
- The JASP Data Library
- Discovering Statistics Using JASP
- How to Use JASP – Inventory of blogs/videos/gifs for frequentist and Bayesian analyses
- JASP YouTube page
- Step By Step Guide: 1. Bayesian One-Way ANOVA and the full playlist
- JASP on Bluesky - https://bsky.app/profile/jaspstats.bsky.social
- JASP forum - https://forum.cogsci.nl/index.php?p=/categories/jasp-bayesfactor
- Found a bug? Please report on Github: https://github.com/jasp-stats/jasp-issues/issues
- JASP Verification Project
- More JASP workshops: https://jasp-stats.org/workshop/

The Future of JASP

- Feature roadmap
  - Full syntax mode
  - More data manipulation
  - Selective filtering
  - Module library
- [JASP Community]