LAB REPORT: LAB 4

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Abstract

This lab report describes the basics of what an implicit surface is, how Constructive Solid Geometry (CSG) operators function as well as what quadric surfaces are and how to get a quadric matrix for each type of quadric surface implemented. The CSG operators mentioned in this report are *union*, *intersection* and *difference*, while the type of quadric surfaces shown in the results consists of ellipsoids, double cones, paraboloids and planes. The results show that using the quadric matrices for each type of quadric works well to create clear surfaces and that the use of CSG operators on the resulting objects works rather well. There are however some issues with how new edges created by operations like *difference* become rather jagged and messy, even with differing values for the sampling distance. This could possibly be fixed by implementing a discrete gradient and/or curve operator for implicits as well as implementing super-elliptic blending.

1 Background

This lab focused on implicit surfaces, surfaces which can be represented using a scalar function $f(x, y, z)$. The implementation mainly consisted of two parts, implementation of Constructive Solid Geometry (CSG) operators and implementation of quadric surfaces (implicit surfaces where the scalar function has been set to zero). There are three types of CSG operators used in this lab, mainly *union*, *intersection* and *difference*. Figure 1 shows the two different objects which each operation is being performed on (A in blue and B in red) as well as what would be the result of each operation (represented by a black contour).

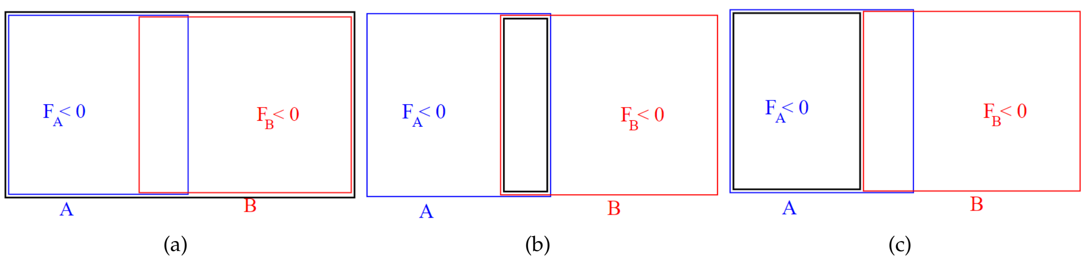


Figure 1: Representation of different CSG operators, the black contour representing the resulting area after using: (a) Union, (b) Intersection, (c) Difference. (Taken from lecture slides.)

By setting different values to the surface of the object, $F_X = 0$, the inside of the object, $F_X < 0$, and the outside of the object, $F_X > 0$, it is possible to use *max* and *min* operators to test whether a part of an object fulfils the conditions to be a union, intersection or difference. For example: If the *min* operation is used on F_A and F_B, a negative result would show that we have met the conditions for a union.

The second part of the lab was, as mentioned above, implementation of quadric surfaces. Quadric surfaces are implicit surfaces which are defined by a quadratic implicit function. Because of this, it is possible to rather easily calculate an approximation of the curvature as well as represent them in matrix form. For this lab six different types of quadric surfaces, mainly: planes, cylinders, ellipsoids, paraboloids and hyperboloids were implemented.

Each type of quadric can be written as a function with three variables (see equation 1), this function can then rather easily be written in matrix form. To get this matrix, the analytical expressions for each type of quadric is used. For example: The analytical expression 2 represents a double cone.

$$f(x, y, z) = Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 2Gy + Hz^2 + 2Iz + J \tag{1}$$

$$f(x, y, z) = x^2 + y^2 - z^2 = 0 \tag{2}$$

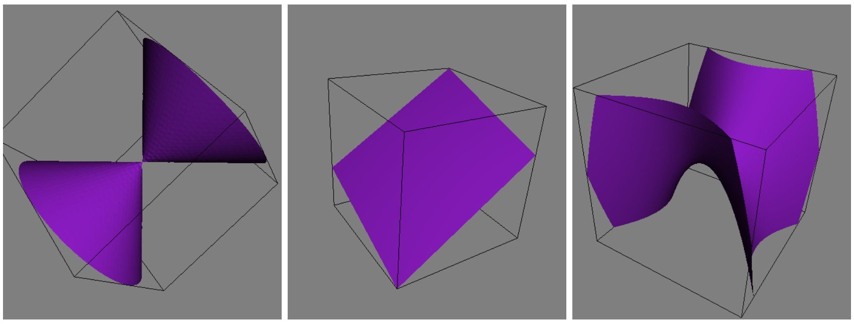
Using equation 2 it is possible to compare with equation 1 and get that $A = 1$, $E = 1$ and $H = -1$, while the rest of the scalars are zero. Then it is as simple as putting the values into the matrix, which results in: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2 Results

Figure 2 shows three of the six different quadric surfaces implemented during the lab. Firstly is 2(a) which is a double cone, then comes 2(b) which is a simple plane and finally 2(c) which is a paraboloid. When creating the cone there was no need to choose any values for any variables, but for the plane however, a was set to 0.5, b to 1 and c to 1.5 while for the paraboloid the sign before y^2 was chosen to be negative.

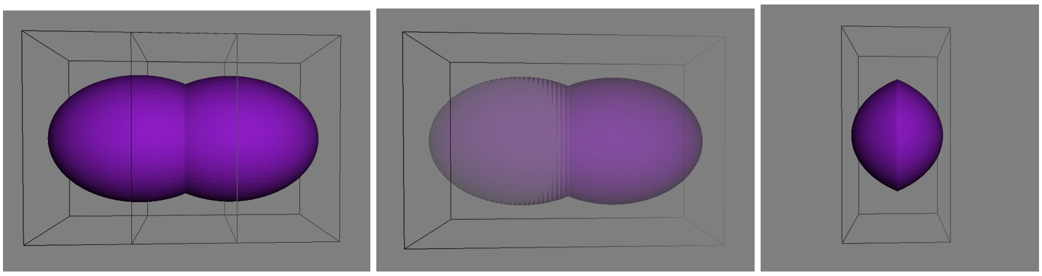
Figure 3 shows how different operations affect an object. The images in the figure represent the following: 3(a) shows the original setup of two overlapping ellipsoids before any operation is performed (other than a translation of one of the ellipsoids to the side), 3(b) shows the result of performing the *union* operation (opacity has been lowered to more clearly show that the ellipsoids have been affected) and 3(c) shows the result of performing the *intersection* operation.

One operation which is not shown in Figure 3 is the *difference* operation, which is shown in Figure 4. This result was put into a separate figure to more easily look at the affect that sampling distance has on the resulting object. Figure 4 consists of two images where 4(a) is the result of using the difference operation between two ellipsoids when using the sampling distance of 0.01 and 4(b) is the result of the same operation done with a sampling distance of 0.05. When looking at the two images side by side it is possible to see how a lower value for sampling distance leads to a much cleaner result at the edges where pieces of the object was removed, other than that, it seems to have no affect on the result of the operation, simply making the object smoother.



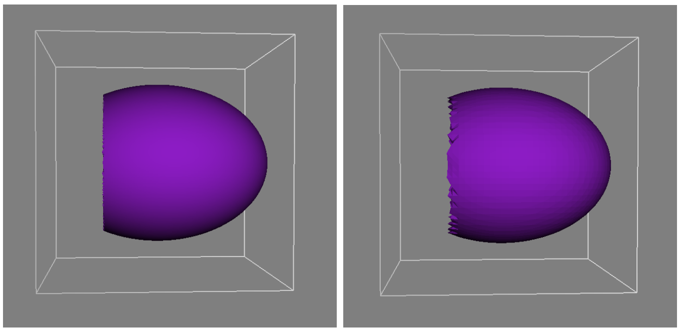
(a) (b) (c)

Figure 2: Some of the implicit quadric surfaces which were implemented: (a) Double cone, (b) Plane, (c) Paraboloid



(a) (b) (c)

Figure 3: (a) No operation performed, (b) Union performed, (c) Intersection performed



(a) (b)

Figure 4: (a) Mesh sampling at 0.01, (b) Mesh sampling at 0.05

3 Conclusion

Using quadric surfaces allows for easier calculations of second partial derivatives, transformations and normals, simply because of how the implicit functions used to define the surfaces is, in a way, simplistic. However, there are still some issues that pop up, for example, the rough edge seen in Figure 4, even when using a low value for the sampling distance.

Another issue is how some surfaces are a bit cut off because of the bounding box (like the paraboloid in Figure 2(c)), something which could be fixed by either setting a larger bounding box or changing the shape/size of the object.

Of course there are ways to make it better, as only what was needed to pass was implemented. For example: Implementing a discrete gradient and/or curve operator for implicits as well as implementing super-elliptic blending.

4 Lab partner and grade

For the records: My lab partner was Felix Lindgren. All assignments for grade 3 were completed, hence, I should get grade 3.