LAB REPORT: LAB 5

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Abstract

This lab report describes the basics of what level sets are, which partial differential equations can be solved to deform said level sets and what rules those equations follow. The concept of reinitialization is briefly looked over and the concepts of erosion, dilation and advection as well as the steps needed to implement them are explained. The results show how cut planes can be used to more easily visualize the result of reinitialization as well as how erosion, dilation and advection can affect different level sets. The conclusion briefly discusses the positives and some negatives with the methods implemented and their results as well as possible improvements, although there are only shortly discussed and not expanded upon.

1 Background

This lab focused on level sets; Implicit surfaces which have the ability to be deformed in several different ways by solving sets of partial differential equations. To do this, different types of differential calculations needed to be implemented. Some examples of the differential equations implemented for this lab are as follows: Φ_x^+ , Φ_x^-, $\Phi_x^\pm$, $\Phi_{xx}^\pm$ and $\Phi_{xy}^\pm$. The differential equations are implemented in the X, Y and Z dimensions, not just for X and/or Y.

When these equations are implemented an implicit surface can be converted into a level set object, making deformations possible. To be able to do this it is generally said that Φ needs to be continuous, but this can to some extent be looked over by instead saying that Φ needs to be *Lipschitz continuous*, which in turn means that it should satisfy the *Eikonal equation* ($|\Phi| = 1$) . The GUI given for the lab therefore gives a "*reinitialize*" button when selecting a level set object, which in turn tries to make Φ satisfy the *Eikonal equation* as well as possible. To visualise this, it is possible to use a cut-plane with a colour map which visualises Iso contours, this can be seen in the results, in Figure 1.

As mentioned several times before, the reason for using level sets is the ability to deform the object with the help of their partial differential equations. In this lab, three different types of deformation were implemented: Erosion, dilation and advection. With the help of erosion and dilation it is possible to perform morphological operations to, for example, remove small protrusions (morphological opening) or to fill in unwanted topological holes or general gaps (morphological closing). Advection is instead used to move the surface of level set objects in a vector field. There are a lot of different types of movements which advection can be used for, the main ones being describing physical events or arbitrary deformations.

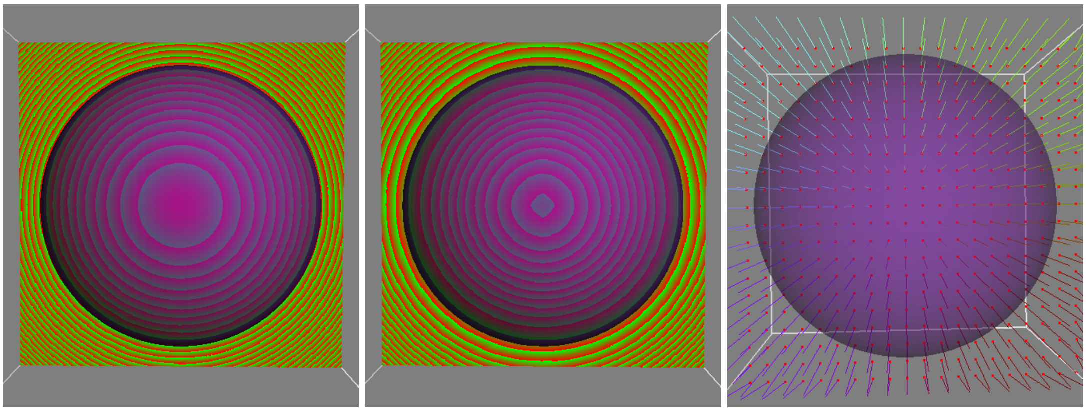
Performing erosion and dilation is done much in the same way, with a simple change in the sign.

By evaluating the speed function (aka the rate of change), then computing a stable time step which in turn is used as the speed at which to propagate the level set. The final step consists of integrating the level set function using Euler integration. The main rule to keep in mind is that the time step needs to always be smaller than $\frac{min\{\Delta X\}}{|F|}$, where F is the speed function mentioned above.

To perform advection the following steps are taken: First the vector field (which is used for different types of physical events), then the gradient is calculated with the help of an upwind scheme, followed by the evaluation of the rate of change. The final step consists of updating the level set function using Euler integration, however other types of integrations can also be used.

2 Results

Figure 1 shows three different images, 1(a) is a sphere object with a scalar cut plane using an Iso contour colour map to showcase the signed distance property, while 1(b) shows the same sphere object and scalar cut plane after a few reinitializations. After using the *reinitialize function*, the cut plane with the Iso contour colour map where the switch between two colours (red and green) represents a change in the level set function with a value of 0.1, it is really easy to see how the "rings" on the cut plane inside of the sphere have gotten closer to one another and how the middle no longer consists of one large circle, but instead several smaller ones. The image 1(c) simply shows the same sphere but instead with the use of a vector cut plane.

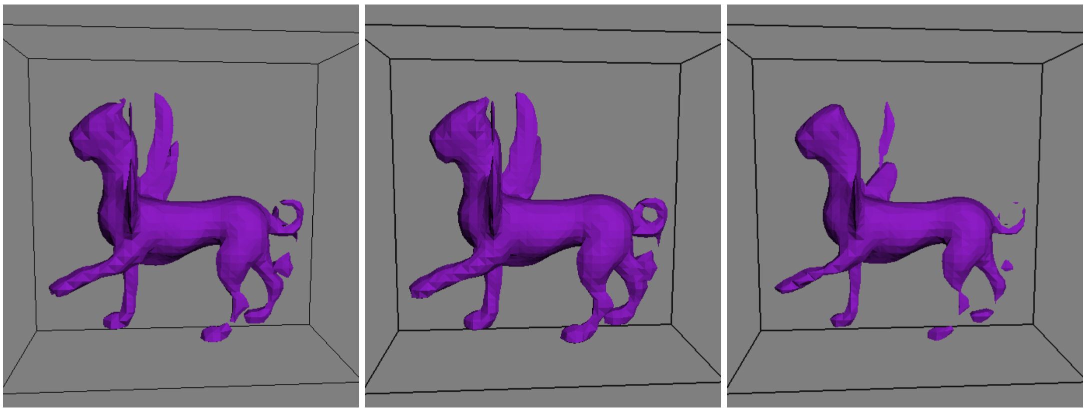


(a) (b) (c)

Figure 1: (a) Original object with scalar cut plane, (b) The same object after several reinitializations, (c) Vector plane representation of the same object.

Figure 2 consists of 2(a) which is the original object without any erosion or dilation performed, 2(b) where several dilations have been performed and 2(c) where several erosions have been performed. A good area to focus on in these images is the hind leg closest to the "camera" and the tail. When performing dilation the gap in the ankle is closed up, something which can be very useful when unwanted topological holes occur, while erosion can remove unwanted extra material, such as the slight spike which sticks out of the bottom of the tail.

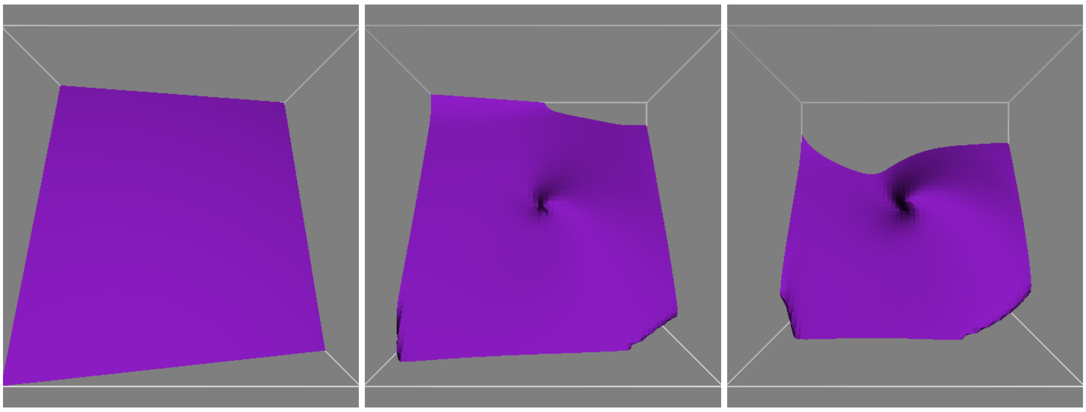
Figure 3 shows a plane, 3(a), which is being moved/manipulated with the help of advection.



(a) (b) (c)

Figure 2: (a) Original object, (b) The same object after several dilations, (c) The same object after several erosions.

After a few advections 3(b) is reached, here it is possible to see how most edges have started to bend and moved closer to the centre, but the the centre is where the interest lies. The vector field used for this advection describes a vortex, making the centre of the plane look as if it is being rotated as the edges are drawn inwards. After several more advections 3(c) is reached, here the affect of the advection is even clearer, as the edges have moved even closer towards the centre of the plane and the vortex like structure in the middle is even more pronounced.



(a) (b) (c)

Figure 3: (a) Original plane object, (b) The plane after several advect operation, (c) The plane after even more advect operatior.

3 Conclusion

Using different types of cut planes together with colour mapping really helps to visualise how reinitialization affects the level sets function, without it it is rather hard to fully wrap your head around it. The only issue with the method is that objects which are more complex than simple spheres can be harder to understand, as the difference after reinitialization can be rather minimal.

Using both the erosion and dilation operators in succession to perform morphological opening or closing could help to remove several of the problems, such as the disconnected ankle or spiky protrusion from the tail, at ones, but there is a risk that they can remove or add details which are not wanted.

Advection can be really useful when wanting to represent different types of physical events such as winds or fluids, as well as the more arbitrary ones like the vortex one used in this lab. It still isn't perfect however, as it depends a lot on the individual object whether the final result will work well. A problem with our result, which is not shown in the figures, is how the vortex has a tendency to, after a while, completely suck up the plane, leaving no material being, which should not be the case, as it isn't meant to model a black hole or something similar.

As the implementation stopped after a passing grade was reached there are clear ways in which the results could, to some extent, be improved, mainly through implementation of mean curvature flow and morphing.

4 Lab partner and grade

For the records: My lab partner was Felix Lindgren. All assignments for grade 3 were completed, hence, I should get grade 3.